VALIDITY OF THE PAULI PRINCIPLE IN THE EXTERIOR BRANCH OF HADRONIC MECHANICS

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1989

Abstract

A Lie-isotopic generalization of the usual anticommutation algebra of the creation and annihilation operators is defined. The eigenvalues of the T-isotopically lifted number operator, $N = a^+ * a = a^+ Ta$, can be calculated exactly, the Pauli principle being not violated. The possibility of violation of the Pauli principle in the open interior problem of hadrons within the Lie-admissible framework is briefly discussed.

Hadronic J. **13** (1990) 183-190.

1 Introduction

A number of mathematical studies, and applications to the hadronic systems, of the isotopic generalization of the conventional operator theory on Hilbert space have been presented by Myung and Santilli [1] and Santilli [2]. Various implications of the generalization have been rewieved in Refs. [3, 4]. Let us briefly recall the main ideas of the approach.

1a. Let $\mathcal{A}$ be a complex associative algebra equipped with the product AB, $A, B \in \mathcal{A}$. Let T be a fixed invertible element of $\mathcal{A}$. The T-isotope of $\mathcal{A}$, denoted by $\mathcal{A}^{(T)}$, is defined as the algebra with multiplication

$$A * B = ATB = (AT)B = A(TB), \qquad A, B, T \in \mathcal{A}, \tag{1}$$

on the same underlying linear space as $\mathcal{A}$. Algebra $\mathcal{A}^{(T)}$ has the generalized identity element $I^* = T^{-1}$. To define an action of the T-isotope of operators on physical states, one needs to develop also a notion of T-isotope of $\mathcal{A}$-module.

Let V be an unital left $\mathcal{A}$-module under the composition Ax, $A \in \mathcal{A}$, $x \in V$. Define mapping

$$\begin{gathered} \mathcal{A}^{(T)} \times V \to V, \\ (A, x) \to A * x, \qquad A \in \mathcal{A}^{(T)}, \quad x \in V. \end{gathered} \tag{2}$$

Under this definition, V becomes an unital left $\mathcal{A}^{(T)}$-module V^T, since $I^* * x = x$ for all $x \in V$.

1b. Another possibility of generalization is to define the multiplication $A \circ B = ARB - BSA$ which arised from a generalization of the Heisenberg equation by Santilli (so-called (R, S)-mutation of $\mathcal{A}$). The structure of $\mathcal{A}(R, S)$ has been studied in considerable detail in relation with $\mathcal{A}^{(T)}$ (see refs. in [1]). Under the commutators $[A, B]^\circ = A \circ B - B \circ A$ and $[A, B]^* = ATB - BTA$, the algebras $\mathcal{A}(R, S)$ and $\mathcal{A}^{(T)}$ respectively proved to be algebras reffered to as Lie-admissible algebras [1, 2, 3, 5].

2 T-isotopes of the anticommutation algebra of the creation and annihilation operator

2a. In the present paper, we define T-isotopic generalization of the anticommutation algebra of the fermionic creation and annihilation operators,

a^H and a (H denotes usual Hermitian conjugation). The eigenvalues of the T-isotopically lifted number operator N, acting on elements of T-isotopic Hilbert space, can be calculated exactly, the Pauli principle is not violated.

Let $\mathcal{A}^{(T)}$ be as in 1a. Following Remark 2.2 of Ref. [1] define the anticommutator

$$\{A, B\}^* = A * B + B * A. \quad (3)$$

Let us now consider the following anticommutation relations for creation and annihilation operators defined in $\mathcal{A}^{(T)}$:

$$\{a, a^+\}^* = I^*, \{a, a\}^* = 0, \{a^+, a^+\}^* = 0. \quad (4)$$

(in the limit $I^* = I$ the usual anticommutation relations are recovered). Here, a^+ is T-Hermitian conjugated quantity. The T-Hermitian conjugation [1] shares all properties with that of the usual Hermitian one, and can be expressed in the conventional terms as $a^+ = T^H a^H T^{-1}$.

It is well known that in the usual case the number operator $N = a^H a$ has the eigenvalues $n = 0, 1$. This means that the spin-1/2 particles quantized by Fermi obey the Pauli principle. Is in the case of T-isotopic generalization (4) Pauli principle is garanteed?

The question is reasonable in view of the renewed interest of late in Pauli principle violation mechanisms [6, 7] originated earlier in Ref. [5]. Some aspects of the proposed experimental test for Pauli principle were considered in Refs. [8, 9, 10]. Also, the so-called Q-algebra by Kuryshkin [11] with nonlinear commutation relations has been intended to provide useful "bosonization" methods [2] the concept of bozonization [13] being applied to several problems of noncanonical physics, with the difference that transformations are not longer bilinear in general. In the recent paper by Ignatiev and Kuzmin [4] the idea of non-bilinear realization of the creation and annihilation operator algebra is suggested to be important for the description of a small violation of the Pauli principle. Also, Lie-admissible generalization of the Fock space has been studied by Jannussis et al. [15]

2b. Let us define the generalized number operator

$$N = a^+ * a \equiv a^+ T a, \quad (5)$$

which is evidently T-Hermitian, $N^+ = N$, and consider the eigenvalue problem

$$N * |\phi> = n_T * |\phi> . \quad (6)$$

Here, the true scalar $n_T = nI^*, n \in C$. It is easy to verify then that the following analogues of the usual commutation relations hold:

$$[N,a]^* = -a, \qquad [N,a^+]^* = -a^+, \tag{7}$$

$$N^{2T} \equiv N * N = N, \tag{8}$$

where the anticommutation rules (4) have been used. The relation (8) means that

$$N^{2T} * |\phi >= N * |\phi > \tag{9}$$

and hence

$$n_T^{2T} = n_T, \tag{10}$$

i.e. $nT^{-1} * nT^{-1} \equiv n^2 T^{-1} = nT^{-1}$, so that $n = 0$ or 1 exactly as in the usual Fermi quantization case.

Note that in the usual terms Eq.(6) reads

$$NT * |\phi >= n|\phi >, \tag{11}$$

that is n is the eigenvalue of the operator NT with the eigenvector $|\phi >$, $|\phi > \in V$, while in the T-isotopic terms, n_T is the T-eigenvalue of the operator N with T-eigenvector $*|\phi >$, $*|\phi > \in V^T$.

3 Discussion

We conclude with a brief discussion of possible Lie-admissible generalizations of the underlying algebra, when the Pauli principle may be violated.

To begin with, we note that the proposed Lie-isotopic lifting of the anticommutation rules means that one deals with the extension of the conventional description when the non-Hamiltonian interactions, provided by the isotopic operator T, is incorporated.

3a. In this respect, we note that the violation can happen when T is defined as a non-Hermitian operator $T^H \neq T$. So, the possibility to obtain the violation scheme arises from the Lie-admissible generalization [16] of the Lie-isotopic framework, i.e. when genotopy is used instead of the isotopy.

3b. The isotensorial product of isoHilbert spaces developed by Myung [17] recovers the T-isotopic lifting of the $\mathcal{A}$-module of section 2, and allows us to use two types of generalized Hermitian conjugation, "dual" to each other.

The product gives a real envelope of the usual spin half-integer algebras so that, in particular, a mutation of the 1/2 spin may take place. From this point of wiev, possible violations of Pauli principle, formulated for particles with exact 1/2 spin, can be naturally interpreted as washing out of the strict spin 1/2 due to the enveloping algebra under the non-Hamiltonian interactions.

However, there is a natural way to construct the anticommutation algebra for creation and annihilation operators, and the number operator N when Pauli principle appears to be garanteed in the isoHilbert space. The way is similar to that of the T-isotopic lifting of section 2, and is shown in the Appendix 4.

3c. Returning to the T-isotopic lifting of the anticommutation algebra of section 2, we note that within the hadronic mechanical approach, the validity of Pauli principle for a hadron as a whole does not garantee, in general, the validity of the same principle for its spin-1/2 constituents, in the same sense as it had been emphasized [18] for the uncertainty principle.

Acknowledgment

I am much indebted to Prof. R.M.Santilli for his encouragement and invaluable suggestions and comments during the preparation of this work.

4 Appendix

Let us define the lifting of the usual anticommutation relations and the number operator N to the isoHilbert space, and calculate the eigenvalues of N. The lifting is four-fold due to the fact that one can use the G-isoproduct or T-isoproduct, and $\tilde{G}$-isoHermitian or $\hat{T}$-isoHermitian conjugations (hereafter, we use the notation and definitions of Ref. [17]). Accordingly, one can write down the following four sets of anticommutation rules:

$$\text{(a) } \{a^{\tilde{}}, a\}^* = I^*, \{a, a\}^* = 0, \{a^{\tilde{}}, a^{\tilde{}}\}^* = 0, \tag{12}$$

$$\text{(b) } \{a^{\hat{}}, a\}^* = I^*, \{a, a\}^* = 0, \{a^{\hat{}}, a^{\hat{}}\}^* = 0, \tag{13}$$

$$\text{(c) } \{a^{\tilde{}}, a\}^\circ = I^\circ, \{a, a\}^\circ = 0, \{a^{\tilde{}}, a^{\tilde{}}\}^\circ = 0, \tag{14}$$

$$\text{(d) } \{a^{\hat{}}, a\}^\circ = I^\circ, \{a, a\}^\circ = 0, \{a^{\hat{}}, a^{\hat{}}\}^\circ = 0, \tag{15}$$

where $I^* = G^{-1}$, $I^\circ = T^{-1}$, $a^{\tilde{}} = T^{-1}G^+a^+TG^-$, $a^{\hat{}} = G^{-1}Ta^+G^+T^{-1}$, $\{A,B\}^* = A{*}B - B{*}A$, $\{A,B\}^\circ = A{\circ}B + B{\circ}A$, $A{*}B = AGB$, $A{\circ}B = ATB$, A^+ denotes the usual Hermitian conjugation, and T is assumed Hermitian, $T^+ = T$. The corresponding liftings of the usual fermionic number operator $N = a^+a$ will be

$$\text{(a) } N_a = a^{\tilde{}} * a, \qquad \text{(b) } N_b = a^{\hat{}} * a, \tag{16}$$

$$\text{(c) } N_c = a^{\tilde{}} \circ a, \qquad \text{(d) } N_d = a^{\hat{}} \circ a, \tag{17}$$

According to Ref. [17], an operator A is $\tilde{G}$-isoHermitian or $\hat{T}$-isoHermitian if and only if $(TAG)^+ = TAG$ or $(GAT)^+ = GAT$, respectively. One can easily verify the latter conditions for the number operators $N_{a,b,c,d}$ and find that (a) N_a is $\tilde{G}$-isoHermitian, $N_a^{\tilde{}} = N_a$, (b) N_b is $\hat{T}$-isoHermitian, $N_b^{\hat{}} = N_b$, (c) N_c is $\tilde{G}$-isoHermitian, $N_c^{\tilde{}} = N_c$, if G is Hermitian, (d) N_d is $\hat{T}$-isoHermitian, $N_d^{\hat{}} = N_d$.

It is now a matter of straightforward calculations to verify that

$$N_a * N_a = N_a, \quad N_b * N_b = N_b, \quad N_c * N_c = N_c, \quad N_d * N_d = N_d, \tag{18}$$

and

$$[N_a, a]^* = -a, \quad [N_a, a^{\tilde{}}]^* = a^{\tilde{}}, \quad [N_b, a]^* = -a, \quad [N_b, a^{\hat{}}]^* = a^{\hat{}}, \tag{19}$$

$$[N_c, a]^\circ = -a, \quad [N_c, a^{\tilde{}}]^\circ = a^{\tilde{}}, \quad [N_d, a]^\circ = -a, \quad [N_d, a^{\hat{}}]^\circ = a^{\hat{}}, \tag{20}$$

where the anticommutation relations (12)-(15) have been used.

The (right) actions of the number operators (16) and (17) on physical states are defined due to the associated $\mathcal{E}^G$- and $\mathcal{E}^T$-modules with G-isoket $*| >$ and T-isoket $\circ| >$ vectors, respectively [3, 17]. Considering the eigenvalues problems

$$N_{a,b} * | >= n^G_{a,b} * | >, \quad N_{c,d} \circ | >= n^T_{c,d} \circ | >, \tag{21}$$

where $n^G = nI^*$ and $n^T = nI^\circ$, and using the relations (18), we find that $n = 0, 1$ for all the cases (a)-(d). Thus, we arrive at the conclusion that under the appropriate definitions of the anticommutation algebras and the number operators, presented above, the Pauli principle remains valid in the isoHilbert space for the exterior, Lie-isotopic branch of hadronic mechanics.

However, when the Lie-isotopic brackets are replaced with the more general Lie-admissible brackets representing the open conditions of the interior branch of hadronic mechanics [1] then all results (18) are violated, as reader can easily verify.

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