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Mathematical, physical and chemical studies on Einstein's prediction that "quantum mechanics is not a complete theory," II: Apparent confirmation of Einstein's prediction

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Abstract. In 1935, A. Einstein expressed his historical view, jointly with B. Podolsky and N. Rosen, that quantum mechanics could be "completed" into a form recovering classical determinism at least under limit conditions (*EPR argument*). In the preceding Paper I, we have assumed the exact validity of quantum mechanics for point particles in vacuum under linear, local and potential interactions (*exterior dynamical systems*) and outlined the basic mathematical and physical methods underlying the "completion" of quantum mechanics into hadronic mechanics for the representation of extended particles within physical media (such as such as the constituents of hadrons, nuclei and stars) under additional non-linear, non-local and non-potential interactions (*interior dynamical systems*). In this Paper II, we study the *isosymmetries* for extended particles in time-reversal invariant interior conditions; we study the apparent proof by the author that interior dynamical systems admit a classical counterpart; we review the apparent additional proof by the author that quantum uncertainties tend to zero with the increase of the density of the medium; and we provide illustrative examples of the progressive recovering of Einstein's classical determinism for extended particles in interior conditions and it's full recovering for gravitational collapse.

1. INTRODUCTION

1.1. The EPR argument.

As it is well known, Albert Einstein did not accept quantum mechanical uncertainties as being final, for which reason he made his famous quote "God does not play dice with the universe."

More particularly, Einstein believed that "quantum mechanics is not a complete theory," in the sense that it could be broadened into such a form to recover classical determinism at least under limit conditions.

Einstein communicated his views to B. Podolsky and N. Rosen and they jointly published in 1935 the historical paper [1] that became known as the *EPR argument.*

In view of the rather widespread belief that quantum mechanics is a final theory valid for all conceivable conditions existing in the universe, objections against the EPR argument has been voiced by numerous scholars, including by N. Bohr [2], J. S. Bell [3] [4],

J. von Neumann [5] and others (see Ref. [6] for a review and comprehensive literature). The field became known as *local realism* and included the dismissal of the EPR argument based on claims that quantum axioms do not admit *hidden variables* λ [7] [8].

1.2. Outline of Paper I.

This paper, and the preceding Ref. [9] (hereinafter referred to as Paper I), are dedicated to the review and upgrade of decades of studies by mathematicians, physicists, and chemists (see Refs. [10] to [70] and papers quoted therein) on the apparent proof of the EPR argument via the "completion," also called *isotopic lifting,* of quantum mechanics into the axiom-preserving *hadronic mechanics* (see the 1995 monographs [28] [29] [30] and literature quoted therein).

More specifically, in Section I-1.1, we have outlined the EPR argument [1] jointly with representative objections [2] to [6].

In Section I-1.2, we have outlined the apparent first proof by R. M. Santilli [10] that interior dynamical systems represented with hadronic mechanics admit classical counterparts.

In the same Section I-1.2, we have outlined the apparent second proof by Santilli [11] that classical determinism is progressively approached in the interior of hadrons, nuclei, stars and gravitational collapse as predicted by Einstein.

In support of the plausibility of the EPR argument, in the subsequent Sections I-1.3 to I-1.7, we have outlined insufficiencies of quantum mechanics for time-irreversible processes, particle physics, nuclear physics, chemistry, and other fields. We have also provided various references indicating the apparent resolution of said insufficiencies by hadronic mechanics.

In Section I-2, we have outlined the *Lie-admissible covering of Lie's theory* [12] [13], with ensuing time-irreversible *Lie-admissible brach of hadronic mechanics,* also known as *genomechanics,* [12] [14] allowing studies on the compatibility of mechanics with thermodynamics, said compatibility being notoriously impossible for quantum mechanics.

Quantum mechanics and the objections against the EPR argument are formulated for time-reversal invariant systems of exterior dynamical systems. Therefore, in preparation for the proof of the EPR argument studied in Section 3, we have outlined and upgraded in Section I-3 the time-reversal invariant *Lie-isotopic subclass of Lie-admissible mathematics,* also known as *isomathematics,* [15] [17] which is used for the representation of time-reversible invariant interior dynamical systems.

In the same Section I-3, we have devoted particular attention to the "completion" of conventional Hilbert spaces [18], numeric fields [19] and Newton-Leibnitz differential calculus [20] into forms defined on *volumes,* rather than points.

In the same Section I-1.3, we have provided particular attention to the main methods for the proofs of the EPR argument, namely, the *axiom-preserving, isotopic lifting of Lie's theory* [25], today known as the *Lie-Santilli isotheory* [37].

Finally, in Section I-4, we have outlined and upgraded the time-reversal invariant *isotopic branch of hadronic mechanics,* also known as *isomechanics* [29] which provides the dynamical foundations of the proofs of the EPR argument [10] [11].

1.3. Basic assumptions.

The most dominant aspects underlying the studies here considered are:

1) The validity of quantum mechanics for point-like particles in vacuum with ensuing

Figure 1: *In this figure, we present a conceptual rendering of the tacit assumption underlying the objections against the EPR argument [2] - [6], namely, the representation of particles as being point-like because it is solely possible under the differential calculus underlying quantum mechanics, namely, the representation of particles as isolated points in empty space. A first consequence is that, being dimensionless, particles can only be at a distance, with ensuing Einstein's argument on the need for superluminal interactions to explain quantum entanglement [1]. A second consequence is that, being at a distance, the sole possible interactions are of linear, local and potential type, under which assumptions the objections against the EPR argument are indeed valid.*

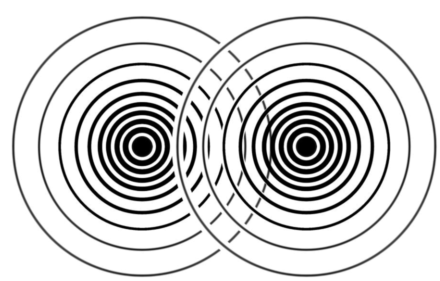

Figure 2: *A conceptual rendering of the main assumption of the apparent proofs [10] [11] of the EPR argument [1], zs in the representation of particles as extended, deformable and hyperdense in conditions of mutual overlapping/entanglement with ensuing continuous contact at a distance eliminating the need for superluminal interactions to explain quantum entanglement. A first implication is the need, for consistency, of generalizing Newton-Leibnitz differential calculus from its historical form solely definable on isolated points, to a covering form definable on volumes [*__?__*]. Another implication is the emergence of contact, non-linear, non-local and non-potential interactions that, being not representable by Hamiltonians, require a structural lifting of the Lie-algebra structure of quantum mechanics under which the objections against the EPR argument are inapplicable (Section 3). Intriguingly, the "completions" here considered turned out to be of isotopic/axiom-preserving type, thus being fully admitted by quantum mechanical axioms, merely subjected to a realization broader than that of the Copenhagen school. The apparent proofs of the EPR argument [10] [11] becomes an unavoidable consequence of the indicated "completions" (Section 3).*

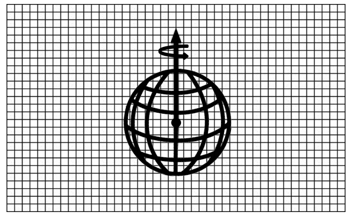

Figure 3: *A conceptual rendering of the central notion used for the study of the EPR argument, namely, a mathematically consistent representation invariant over time of an extended, deformable and hyperdense particle, such as a proton, in the interior of a physical media, such as hadrons, nuclei or stars, under the most general known (non-singular) interactions of linear and non-linear, local and non- local and potential as well as non-potential type.*

linear, local and action-at-a-distance/potential interactions (*exterior dynamical problems*) occurring in atomic structures, particles in accelerators, crystals and numerous other systems in nature (Figure 1);

2) The "completion" of quantum mechanics into hadronic mechanics for the representation of extended, therefore deformable and hyperdense particles within physical media with ensuing, additional, non-linear, non-local and contact/non-potential interactions (*interior dynamical problems*), occurring in the structure of hadrons, nuclei and stars, with limit conditions occurring in the interior of gravitational collapse where the inapplicability (rather than the violation) of quantum mechanics is already accepted by the majority of serious scholars (Figure 2, 3).

The central assumption of these studies is the axiom-preserving lifting of the conventional associative product $ab = a \times b$ between *all* possible quantum mechanical quantities (numbers, functions, matrices, etc.) into the *isoproduct* [14] [25] (Section 3)

$$a \star b = a \, \hat{T} \, b, \tag{1}$$

where $\hat{T}$, called the *isotopic element,* is restricted to be positive-definite, $\hat{T} > 0$, but possesses otherwise an unrestricted functional dependence on all needed local variables.

Refs. [14] [25] constructed an axiom-preserving isotopy of the various branches of Lie's theory, resulting in a theory today known as the *Lie-Santilli isotheory* [37] (Section I-3.7) with isotopic lifting of lie algebras of the type [10]

$$[X_i \hat{,} X_j] = X_i \star X_j - X_j \star X_i = C_{ij}^k X_k. \;\; i, j = 1, 2, ..., N. \tag{2}$$

Following laborious efforts for the achievement of mathematical maturity, Ref. [10] applied the Lie-Santilli isotheory to the isotopy $\hat{SU}(2)$ of the $SU(2)$ spin with three-dimensional isoalgebras of type (2) and introduced the realization of hidden variables [7] [8] of the type

$$\hat{T} = Diag.(1/\lambda, \lambda), \;\; Det\hat{T} = 1. \tag{3}$$

Ref. [10], therefore establishing that, contrary to objections [2] to [6], *the abstract axioms of quantum mechanics do indeed admit explicit and concrete realizations of hidden variables.*

The proof in Ref. [10] that interior systems admit identical classical counterparts was consequential (Section 3).

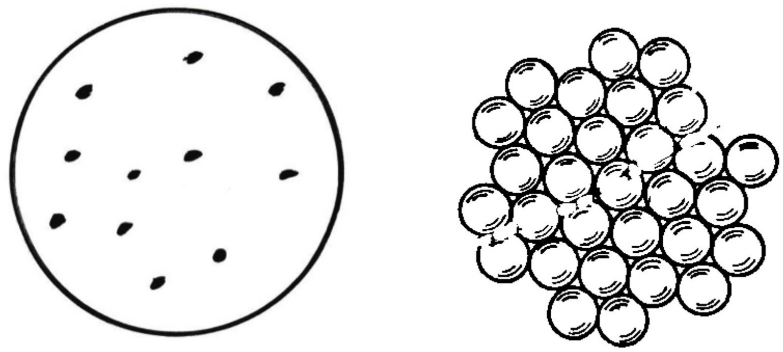

Figure 4: *In the l.h.s. of this picture, we present a conceptual rendering of the structure of nuclei as a sphere with isolated point-like particles in its interior, which is an inevitable consequence of the elaboration of quantum mechanics via the conventional differential calculus, resulting in rather serious insufficiencies in nuclear physics outlined in Section I-1.5. In the r.h.s. of this picture, we present a conceptual rendering of the representation of nuclei as occurring in the physical reality, namely, as a collection of extended charge distributions in condition of partial mutual penetration according to Eq. (4) of isomathematics and related isomechanics, whose resolution of at least some of the insufficiencies of quantum mechanics have been indicated in Section I-1.5.*

Isoproduct (1) also allows a direct and immediate representation of extended particles in conditions of mutual penetration with realizations of the type (Figure 3) [32]

$$\hat{T} = \Pi_{k=1,...,N} Diag.(\tfrac{1}{n_{1k}^2}, \tfrac{1}{n_{2k}^2}, \tfrac{1}{n_{3k}^2}, \tfrac{1}{n_{4k}^2}) e^{-\Gamma},$$
$$k = 1, 2, ..., N, \quad \mu = 1, 2, 3, 4, \tag{4}$$

where n_1^2, n_2^2, n_3^2, (called *characteristic quantities*) represent the deformable semi-axes of the particle normalized to the values $n_k^2 = 1, `k = 1, 2, 3$ for the sphere; n_4^2 represents the *density* of the particle considered normalized to the value $n_4 = 1$ for the vacuum; and Γ represents non-linear, non-local and non-Hamiltonian interactions caused by mutual penetrations/entanglement of particles.

The smaller than 1 absolute value of the isotopic element $\hat{T}$ occurring in all known applications [25]-[35]

$$|\,\hat{T}\,| \; \leq \; 1, \tag{5}$$

permitted Ref. [?] to show that *the standard deviations Δr and Δp appear to progressively tend to zero with the increase of the density of the medium, and appear to achieve full classical determinism in the interior of gravitational collapse, as originally conceived by Einstein.*

The initial construction of the isotopies of 20th century applied mathematics with isoproduct (1) defined over conventional numeric fields $F(n, \times, 1)$ [25] turned out to be inconsistent because the underlying time evolution is *non-unitary,* thus causing the lack of invariance over time of the traditional basic unit 1, with ensuing inapplicability over time of the entire field $F(n, \times, 1)$.

The above occurrence mandated the construction of *isofields* $\hat{F}(\hat{n}, \star, \hat{I})$ [19] [40](Section I-3.3) with basic *isounit*

$$\hat{I} = 1/\hat{T} > 0, \tag{6}$$

and *isonumbers* $\hat{n} = n\hat{I}$ equipped with isoproduct (1).

Ref. [19] essentially established that *the abstract axioms of a numeric field do not require that the multiplicative unit of the field be the trivial number* 1*, since said unit can be an arbitrary quantity with an unrestricted functional dependence on local variables, provided that said multiplicative unit is positive definite and the field is lifted into a compatible form.*

Despite all the above efforts, the ensuing isomathematics was still inapplicable to the proof of the EPR argument because it lacked the crucial *invariance over time,* namely, the prediction of the same interior dynamical systems under the same conditions but at different times.

The above occurrence forced the construction of the covering of the Newton-Leibnitz differential calculus into the covering *isodifferential isocalculus* [20] [43] (Section I-3.6) with basic *isodifferential* (Figure 2) [?]

$$\hat{d}\hat{r} = \hat{T}d[r\hat{I}(r, ...)] = dr + r\hat{T}d\hat{I}(r, ...), \tag{7}$$

and corresponding *isoderivative*

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I}\frac{\partial\hat{f}(\hat{r})}{\partial\hat{r}}. \tag{8}$$

In essence, Ref. [20] established the inapplicability of the conventional differential calculus whenever the axioms of numeric fields admit multiplicative units with a dependence on the differentiation variable, with ensuing inapplicability of quantum mechanics, as well as of the objections against the EPR argument, for interior dynamical systems.

The "completion" of the differential calculus into an isotopic form compatible with basic isoproduct (1) finally allowed the achievement of invariance over time (Section I-3.9), thus signaling the achievement of maturity for the apparent proof of the EPR argument reviewed.

In Section 2 of this paper, we complete the methodological needs by outlining and upgrading the time-reversal invariant coverings of conventional spacetime symmetries, known as *isosymmetries,* which is needed for systems of extended particles in interior conditions; in Section 3, we review and upgrade the Lie-isotopic $SU(2)$-spin symmetry and related proofs [10] [11] of the EPR argument; and in Section 4, we present illustrative examples. This paper ends with comments and the identification of open problems.

A few comments on terminologies appear to be recommendable.

The word "completion" is used in these studies to honor the memory of Albert Einstein and should not be intended to indicate "final" theories. In fact, isomathematics and isomechanics admit coverings of Lie-admissible character [12] (Section I-2) that, in turn, admits coverings of hyperstructural character [42], with additional coverings remaining possible in due time.

The terms "non-Hamiltonian interactions" are intended to indicate interactions that are not representable with a Hamiltonian, and are technically identified as interactions violating the integrability conditions for the existence of a Hamiltonian, namely, the *conditions of variational self-adjointness* [24].

When dealing with stable and isolated interior dynamical systems, the terms "nonconservative forces" are strictly referred to *internal* non-Hamiltonian exchanges verifying conditions (1-55) for the verification of the ten conventional total conservation laws for the total energy, momentum, angular momentum and the uniform motion of the center of mass.

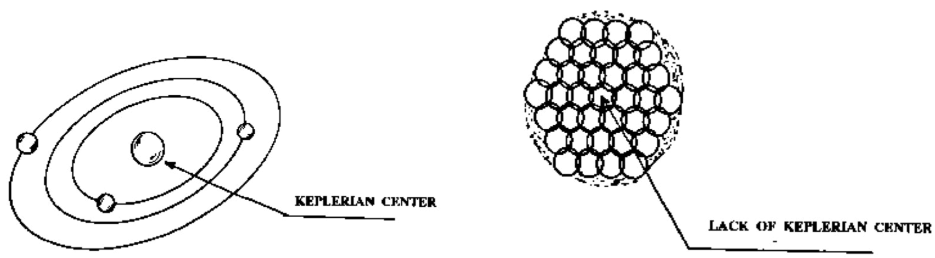

Figure 5: *The l.h.s. of this figure illustrates the Keplerian systems for which space-time symmetries have been constructed, namely, exterior dynamical systems of point-like particles orbiting in vacuum around a heavier point-like particle known as t he Keplerian center. The r.h.s. of this figure illustrates the interior systems for which isosymmetries have been built, namely, systems of extended particles in conditions of mutual penetration without any Keplerian center.*

The terms "physical media" refer to media composed by matter in its various states, and are often referred to as *hadronic media,* in the sense that the media are *not* composed by empty space, thus requiring the use of hadronic mathematics and mechanics for their quantitative treatment.

The terms "extended particles" refer to: the wavepacket of elementary particles such as the electron assumed to be of about $1\ fm = 10^{-15}\ cm$; extended charge distributions for protons and neutrons when members of a nuclear structure, also assumed to have a diameter of about $1\ fm$; and stable nuclei when considering the structure of stars. Due to its crucial significance for the structure of interior systems, a technical definition of the notion of "extended particles" will be given in Section 3 via the notion of *isoparticle* as isorepresentations of space-time isosymmetries.

2. ISOSYMMETRIES

2.1. Foreword.

In this section, we study the axiom-preserving "completion" (or isotopic lifting) of conventional space-time symmetries, known as *Lie-isotopic symmetries,* or *isosymmetries* for short, which provide the invariance of stable and isolated (thus time reversible) interior dynamical systems of extended particles at mutual distances smaller than their size which occur, e.g., in nuclear structures (Figure 3).

Lie-isotopic symmetries were first introduced by Santilli in the 1978 Harvard University paper [13] as a particular case of the broader *Lie-admissible symmetries* for irreversible, non-conservative systems [14]. Isosymmetries were then studied in various subsequent works quoted in this section.

The understanding of this section requires a knowledge of the *Lie-Santilli isotheory* (Section I-3.7), which was first formulated in monographs [24] [25] over the field of real numbers. Isosymmetries were then formulated in monographs [28] [29] with the full use of isomathematics, including the use isofields [19] [40] and the isodifferential calculus [20] [43] (see Refs. [37] [45] [46] [?] for works on the Lie-Santilli isotheory, and Ref. [44] for a general review with applications and experimental verifications).

The assumption at the foundation of isosymmetries is *the preservation of the abstract axioms of 20th century space-time symmetries, and the mere construction of their broadest* *possible realization permitted by isomathematics.*

Consequently., criticisms of isosymmetries and their novel implications are *de facto* criticisms on 20th century space-time symmetries and their implications

2.2. Inapplicability of space-time Lie symmetries.

A first aspect needed for a direct study of the EPR argument is a knowledge of the "inapplicability" (rather than the "violation") for interior dynamical systems of conventional space-time symmetries that have been proved to be so effective for exterior dynamical systems.

Said inapplicability can be first seen from the fact that the Galileo and the Lorentz-Poincaré symmetries can only provide a non-relativistic and relativistic characterization, respectively, of *Keplerian systems,* namely, systems of point-like masses orbiting in vacuum around a heavier mass called the *Keplerian nucleus* [25].

However, interior dynamical systems do not admit a Keplerian structure because *nuclei hav no nuclei* [32] and the same happens for hadrons, stars and gravitational collapse (Figure 8).

It is then possible to prove, e.g., via the imprimitivity theorem, that the lack of existence of a Keplerian structure implies the lack of exact validity of conventional space-time symmetries [25] [29].

On more technical grounds, Lie's theory is known to be solely applicable to exterior systems of point-like particles in vacuum with ensuing sole possible, linear, local and Hamiltonian interactions.

Experimental evidence on interior dynamical systems, e.g., on nuclear volumes compared to the volumes of individual nucleons, establishes that nuclei are composed of extended charge distributions in conditions of partial mutual penetration/entanglement with the ensuing existence of additional, non-linear, non-local and non-Hamiltonian interactions under which Lie's theory is inapplicable.

Hence, the transition of particles from exterior to interior conditions implies the inapplicability of the $SU(2)$-spin symmetry with consequential inapplicability of Bell's inequality [3] and other objections against the EPR argument [6] in favor of suitable covering vistas [10] [11].

In any case, the $SU(2)$ symmetry, while unquestionable effective for exterior dynamical systems, has been unable to provide a consistent representation of the spin of particles and nuclei, thus warranting the search for a suitable "completion."

2.3. The fundamental theorem on isosymmetries.

The construction of isosymmetries requires the full use of isomathematics with particular reference to the Lie-Santilli isotheory formulated on isospaces over isofield and elaborated via the isodifferential calculus (Section I-2.7).

Said construction can be done with the following theorem (for brevity, see the proof in Section 1.2 , Vol. I of Refs. [35]):

THEOREM 2.3.1: Let G be an N-dimensional Lie symmetry of the line element of a k-dimensional metric or pseudo-metric space $S(x, m, I)$ over a numeric field F with coordinates *x, metric m over a numeric field F with conventional unit I,*

$$G: \quad x' = \Lambda(w)x, \quad y' = \Lambda(w)y, \quad x, y \in S,$$

$$(x' - y')^\dagger \Lambda^\dagger m \Lambda(x' - y') \equiv (x - y)^\dagger m(x - y), \tag{9}$$

$$\Lambda^\dagger(w) m \Lambda(w) \equiv m. \ w \in F.$$

Then, all infinitely possible (non-singular) Lie-Santilli isotopies $\hat{G}$ of G on isospace $\hat{S}(\hat{x}, \hat{M}, \hat{I})$ with isocoordinates

$$\hat{x} = xI, \tag{10}$$

isometric

$$\hat{M} = \hat{m}\hat{I} = (\hat{T}_i^k m_{kj})\hat{I}, \tag{11}$$

and isounit

$$\hat{I} = 1/\hat{T} > 0, \tag{12}$$

over an isofield $\hat{F}$ with isounit $v\hat{I}$ leave invariant the isoline element of the isospace $\hat{S}(\hat{x}, \hat{M}, \hat{I})$:

$$\hat{G}: \quad \hat{x}' = \hat{\Lambda}(\hat{w}) \star \hat{x}, \quad \hat{y}' = \hat{\Lambda}(\hat{w}) \star \hat{y}, \quad \hat{x}, \hat{y} \in \hat{S},$$

$$(\hat{x}' - \hat{y}')^\dagger \star \hat{\Lambda}^\dagger \star \hat{M} \star \hat{\Lambda} \star (\hat{x}' - \hat{y}') \equiv (x - y)^\dagger \hat{m}(x - y), \tag{13}$$

$$\hat{\Lambda}^\dagger(\hat{w}) \star \hat{M} \star \hat{\Lambda}(\hat{w}) \equiv \hat{M}.$$

All infinitely possible so constructed isosymmetries $\hat{G}$ are locally isomorphic to the original symmetry G.

The reader should note that, while a given Lie symmetry G is unique as well known, there can be an infinite number of covering isosymmetries $\hat{G}$ with generally different explicit forms o the isotransformations due to the infinite number of possible isotopic elements representing the infinitely different internal interactions of extended particles within physical media.

Note also that all possible isotopic images of a given Lie symmetry can be explicitly and uniquely constructed via the sole knowledge of the original Lie symmetry and of the isotopic element $\hat{T} > 0$, or of the isounit $\hat{I} = 1/\hat{T}$, which property shall be hereon tacitly assumed.

2.4. Isospaces and isogeometries.

As it is well known, the fundamental representation space of relativistic space-time symmetries is the conventional *Minkowski space* $M(x, \eta, I)$ formulated on the field of real numbers $\mathcal{R}$ with coordinates $x = (x^1, x^2, x^3, x^4 = ct)$, metric $\eta = Diag.(1, 1, 1, -1)$, unit $I = Diag(1, 1, 1, 1)$ and invariant

$$x^2 = (x^\mu \eta_{\mu\nu} x^\nu) I = \\ = (x_1^2 + x_2^2 + x_3^3 - c^2 t^2) I, \tag{14}$$

where the trivial multiplication by the conventional unit $I = Diag.(1, 1, 1, 1`)$ is done for compatibility with isomathematics.

The fundamental isospaces of space-time isosymmetries are given by the infinite family of *iso-Minkowski isospaces,* also called *Minkowski-Santilli isospaces,* $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$ formulated on the isofield of isoreal isonumbers $\hat{\mathcal{R}}$. (Section I-3.9), which isospaces were first introduced by R. M. Santilli in Ref. [22] of 1983 and then treated in details in works ([28] [29].

Iso-Minkowskian isospaces are characterized by *space-time isocoordinates* $\hat{x} = x\hat{I}$; *isounit* $\hat{I} = 1/\hat{T}$, *isometric*

$$\hat{\Gamma} = (\hat{T}_{\mu}^{\rho}\eta_{\rho\nu})\hat{I}, \tag{15}$$

(where one should note the necessary structure of an isomatrix [28]), positive-definite *isotopic element* (4) representing a system of extended particles in interior dynamical conditions with an restricted functional dependence on local quantities such as coordinates x, momenta p, energy E, frequency ν, density α, temperature τ, pressure pi, wavefunction ψ, etc., under the conditions

$$n_{\mu} = n_{\mu}(x, p, E, \nu, \alpha, \tau, \pi, \psi, \partial\psi, ...) > 0, \;\; \mu = 1, 2, 3, 4, \tag{16}$$

$$\Gamma(x, p, E, \nu, \alpha, \tau, \pi, \psi, \partial\psi, ...) \geq 0, \tag{17}$$

$$\hat{T} = e^{-\Gamma} \;\ll\; 1, \tag{18}$$

Iso-Minkowskian isospaces are characterized by the infinite family of isoinvariants (I-28) with isotopic element (4) that, for the case of one single extended particle can be written

$$\begin{aligned} \hat{x}^{\hat{2}} = \hat{x}^{\mu} \star \hat{\Omega}_{\mu\nu} \star \hat{x}^{\nu} = (x^{\mu}\hat{\eta}_{\mu\nu}x^{\nu})\hat{I} = \\ = (\tfrac{x_1^2}{n_1^2} + \tfrac{x_2^2}{n_2^2} + \tfrac{x_3^2}{n_3^2} - t^2\tfrac{c^2}{n_4^2})\hat{I}, \end{aligned} \tag{19}$$

where the exponential $exp{-}\Gamma$ has been absorbed in the characteristic quantities n_{μ}, and the final multiplication by the isounit is necessary for the isoinvariant to be an isoscalar, namely, an element of the isoreal isofield [19] (Section I-3-5).

The following aspects treated in Paper I are important for the understanding of the apparent proof of the EPR argument:

1. The characteristic quantities n_1^2, n_2^2, n_3^2, admit the first interpretation as representing the deformable semi-axes of elementary or composite particles normalized to the values for the sphere $n_1^2 = n_2^2, = n_3^2 = 1,$;

2. The characteristic quantity n_4^2 admit the first interpretation as representing the *density* of the hadronic medium normalized to the value $n_4 = 1$ for the vacuum;

3. The function $\Gamma \geq 0$ provides an invariant representation (Section I-3-9) of all non-linear, non-local and non-Hamiltonian interactions;

4. Property (18) is verified for all applications of isosymmetries to date [10] to [67].

5. The correct elaboration of iso-Minkowskian isospaces requires the use of the *iso-spherical and isohyperbolic isocoordinates* (see Refs. [28] [29]).

6. Isoinvariant (19) provides a unified representation of both exterior and interior gravitational problems. In fact, K. Schwartzchild wrote in 1916 *two* important papers, the

first paper [48] on the *exterior gravitational problem* which became world famous for its initiation of gravitational singularities, and the second paper [49] in the *interior gravitational problem* which has been vastly ignored, except rare studies (such as that in Section 23.2, page 609, Ref. [51]). Such an oblivion is essentially due to the fact that Schwartzchild's second paper is not aligned with the widespread tendency of reducing masses to point-like constituents, in which case all differences between exterior and interior gravitational problems disappear to the detriment of the depth of the gravitational analysis. Readers should keep in mind the full parallelism between exterior and interior dynamical problems for *particles and gravitation.*

7. The *exterior gravitational interpretation* of isoinvariant (19) is given by the following identical representation of Schwartzchild's exterior metric [49]

$$\hat{T}_{kk} = \frac{1}{1 - \frac{2M}{r}}, \quad \hat{T}_{44} = 1 - \frac{2M}{r}. \tag{20}$$

The corresponding *interior gravitational representation* is given by the following isotopy of Schwartzchild exterior metric

$$\hat{T}_{kk} = \frac{1}{(1 - \frac{2M}{r})n_k^2}, \quad \hat{T}_{44} = (1 - \frac{2M}{r})/n_4^2. \tag{21}$$

In view of the arbitrariness of the functional dependence of the characteristic quantities n_μ, it is easy to prove that Schwartzchild's interior metric [49] is a particular case of the much broader class of interior gravitational models (21).

8. The geometry of the iso-Minkowskian isospaces, first presented by Santilli in Ref. [23] under the name of *iso-Minkowskian isogeometry,* contains the machinery of the Riemannian geometry (due to the dependence of the isometric $\hat{\eta}$ on the local coordinates x), although such a machinery is formulated for consistency over isofields [19] and elaborated via the isodifferential isocalculus [20] (Section I-3.5). Hence, *the isominkowskian isogeometry can unify exterior and interior problems for both particles and gravitation.*

9. Recall that iso-Minkowskian isospaces are locally isomorphic to the conventional Minkowski space (Refs. [22] [23] and Theorem 2.3.1). Therefore, *the iso-Minkowskian isogeometry has a null curvature.* This is due to the fact that, under isotopic lifting, the conventional Minkowski metric $\eta = Diag.(1, 1, 1, -1)$ is lifted into a coordinate-dependent isometric $\hat{T}(x)\eta = \hat{\eta}(x)$ which is *identical* to any given Riemannian metric

$$\eta \ \rightarrow \ \hat{\eta}(x) = \hat{T}(x)\eta = g(x). \tag{22}$$

Jointly, the original unit of the Minkowski space $\hat{I} = Diag.(1, 1, 1, 1)$ is lifted by the *inverse* amount

$$I > 0 \ \rightarrow \ \hat{I}(x) = 1/\hat{T}(x) > 0, \tag{23}$$

resulting in no actual curvature. The above features have suggested the introduction of the new notion of *isoflat isospace,* referred to an isospace that has null curvature when formulated on isofields, while recovering conventional curvature when formulated on conventional fields. Readers should be aware that the achievement of the universal symmetry of (non–singular) Riemannian line elements studied in the next sections is due precisely to the isoflatness of the iso-Minkowski isospace since no such symmetry is possible for a convemtional Riemannian space, as well known.

Recall that the fundamental representation space of symmetries in 3-space dimensions is the conventional Euclidean space $E(r, \times, I$ with coordinates $r = (x^1, x^2, x^3)$, metric $\delta = Diag.(1, 1, 1)$ and unit $I = Diag.(1, 1, 1)$ on the conventional field of real numbers.

Similarly, the fundamental representation space of isosymmetries in 3-dimensions is the *iso-Euclidean isospace* $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$, also called *Euclid-Santilli isospace* (Refs. [14] [25] [28] and Section I-3.5) which is the space component of the iso-Minkowskian isospace. As such, the iso-Euclidean isospace is hereon tacitly assumed to be known.

2.5. Lorentz-Poincaré-Santilli isosymmetries.

2.5.1. Main references. Following, and only following the construction of the isotopies of Lie's theory, Santilli conducted systematic studies on the isotopies of the various aspects of the Lorentz-Poincaré symmetry for the achievement of the universal invariance of spacetime isoinvariant (19), including:

1) The classical isotopies $\hat{SO}(3.1)$ of the Lorentz symmetry $SO(3.1)$ [52];

2) The operator isotopies $\hat{SO}(3.1)$ of the Lorentz symmetry $SO(3.1)$ [53];

3)The isotopies $\hat{SO}(3)$ of the rotational symmetry $SO(3)$ [54] [55] [56];

.4) The isotopies $\hat{SU}(2)$ of the $SU(2)$ spin symmetry [10] [57];

5) The isotopies $\hat{P}(3.1)$ of the Poincaré symmetry $P(3.1)$ [58] [59], which included the universal symmetry of (non-singular) Riemannian line elements;

6) The isotopies $\hat{\mathcal{P}}(3.1)$ of the spinorial covering $\mathcal{P}(3.1)$ of the Poincaré symmetry [60] [61];

7) The isotopies $\hat{M}(3.1)$ of the Minkowskian geometry $M(3.1)$ [23]

A general presentation is available in the 1995 monographs [28] [29] with the full use of isomathematics, including isofields and isodifferential calculus, with up grades in the 2008 monographs [35].

The resulting infinite family of isosymmetries $\hat{SO}(3.1)$ are known as the *Lorentz-Santilli (LS) isosymmetries* while the broader isosymmetries $\hat{P}(3.1)$ and $\hat{\mathcal{P}}(3.1)$ are known as *Lorentz-Poincaré-Santilli isosymmetries* (see Refs. [36] [?] [44] and papers quoted therein).

Experimental verifications of LPS isosymmetries for interior dynamical systems are available in monographs [30] and in Section 3 of the more recent review [62].

In inspecting the subsequent sections, the reader should be aware of the "direct universality" of the LPS isosymmetries for the considered infinite family of interior dynamical systems [63]), including the treatment of exterior and interior, particle and gravitational problems (Section 4).

2.5.2. Basic definitions. As it is well known, the conventional Lorentz-Poincaré (LP) symmetry is the symmetry of line element (14) which we rewrite in the form

$$(x - y)^2 = (x^\mu - y^\mu)\eta_{\mu\nu}(x^\nu - y^\nu)I =$$

$$= [(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2 - (t_1 - t_2)^2 c^2)]\, I, \tag{24}$$

$$\eta = Diag.(1, 1, 1, -c^2), , \quad I = Diag.(1, 1, 1, 1)$$

where the exponential component $exp{-}\Gamma$ is again embedded for simplicity in the characteristic quantities $n\mu^2$.

The LPS isosymmetry is the universal symmetry of the isoline element (19) in the iso-Minkowski isospace $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$ over the isoreal isonumbers $\hat{\mathcal{R}}$ rewritten in the form

$$(\hat{x} - \hat{y})^{\hat{2}} = \left[(\hat{x}^\mu - \hat{y}^\mu) \star \hat{\Omega}_{\mu\nu} \star (\hat{x}^\nu - \hat{y}^\nu)\right] =$$

$$= [x^\mu - y^\mu)\hat{\eta}_{\mu\nu}(x^\nu - y^\nu)]\, \hat{I} =$$

$$= \left[\tfrac{(x_1-y_1)^2}{n_1^2} + \tfrac{(x_2-y_2)^2}{n_2^2} + \tfrac{(x_3-y_3)^2}{n_3^2} - (t_1 - t_2)^2 \tfrac{c^2}{n_4^2}\right] \hat{I}, \tag{25}$$

$$\hat{\eta} = \hat{T}\eta, \ \ \hat{T} = Diag((\tfrac{1}{n_1^2}, \tfrac{1}{n_2^2}, \tfrac{1}{n_3^2}, \tfrac{1}{n_4^2}),$$

$$n_\mu = n_\mu(x, v, a, E, d, \omega, \tau, \psi, \partial\psi, ...) > O, \ \ \hat{I} = 1/\hat{T} > 0.$$

2.5.3. Isotransformations. By following Theorem 2.3.1, the isotransformations of the LPS isosymmetries can be written

$$\hat{x}' = \hat{\Lambda}(\hat{w}) \star \hat{x}, \tag{26}$$

where $\hat{\Lambda}(\hat{w}) = \Lambda(\hat{w})\hat{I}$, resulting in generally *non-linear* isotransformations, including isotranslations of the type

$$\hat{x}' = \hat{x} + \hat{A}(\hat{x}, ...), \tag{27}$$

verifying the following property

$$\hat{\Lambda}^\dagger \star \hat{\eta} \star \hat{\Lambda} = \Lambda\hat{\eta}\Lambda^\dagger. \tag{28}$$

Under the *condition of isomodularity*

$$\hat{D}et\ (\hat{\Lambda}) = +\hat{I}, \tag{29}$$

we have the *isoconnected LS isosymmetries* $\hat{SO}^0(3.1)$ and the *isoconnected LPS isosymmetries* $\hat{P}^0(3.1)$.

Consider the conventional generators of the Poincaré symmetry

$$(J_k) = (J_{\mu\nu}), \ P_\mu, k = 1, 2, 3, 4, 5, 6, \ \mu, \nu = 1, 2, 3, 4. \tag{30}$$

By keeping in mind isoexponentiation (I-16), the isotransformations of $\hat{SO}^0(3.1)$ can be written [59]

$$\begin{aligned} \hat{x}' &= (\hat{e}^{iJ_k w_k}) \star \hat{x} \star (\hat{e}^{-iJ_k w_k}) = \\ &= \left[(e^{iJ_k\hat{T}w_k})x(e^{-iw_k\hat{T}J_k})\right]\hat{I}, \end{aligned} \tag{31}$$

and the isotranslations $\hat{A}(3.1)$ can be written

$$\begin{aligned} \hat{x}' &= (\hat{e}^{iP_\mu a_\mu}) \star \hat{x} \star (\hat{e}^{-iP_\mu a_\mu}) = \\ &= \left[(e^{iP_\mu\hat{T}a_\mu})x(e^{-ia_\mu\hat{T}P_\mu})\right]\hat{I}, \end{aligned} \tag{32}$$

It is evident that the above isotransformations do constitute Lie-Santilli isogroups according to Theorem I-2.7.3.

2.5.4. Isocommutation rules. As recalled earlier, the total quantities of an isolated, stable, interior system must be conserved for consistency.

In order to represent this evidence, the Lie-Santilli isotheory was constructed [25] in such a way to preserve conventional generators, because they represent total conservation laws, and isotopically lift their product.

By expanding the preceding finite isotransforms in terms of the isounit, the *LPS isoalgebra* $\hat{so}^0(3.1)$ is characterized by the conventional generators of the LP algebra and the isocommutation rules [29] [59] (here written in their projection on conventional spaces over conventional fields)

$$[J_{\mu\nu}\hat{,}J_{\alpha\beta}] =$$

$$= \imath(\hat{\eta}_{\nu\alpha}J_{\beta\mu} - \hat{\eta}_{\mu\alpha}J_{\beta\nu} - \hat{\eta}_{\nu\beta}J_{\alpha\mu} + \hat{\eta}_{\mu\beta}J_{\alpha\nu}), \tag{33}$$

$$[J_{\mu\nu}\hat{,}P_\alpha] = i(\hat{\eta}_{\mu\alpha}P_\nu - \hat{\eta}_{\nu\alpha}P_\mu)$$

$$[P_\mu\hat{,}P_\nu] = 0. \tag{34}$$

$$\hat{\eta}_{\mu\nu} = \hat{T}\eta = (\hat{T}^\rho_\mu\eta_{\rho\nu}) \tag{35}$$

where one should note the appearance of the *structure functions* $\hat{\eta}(x, p, E, \nu, \alpha, \tau, \psi, ...)$, rather than the traditional structure constants (Theorem I-2.7.2).

The presence of structure functions $\hat{\eta}$ in isocommutation rules (33)-(35), Theorem I-3.7.2 and the analysis of Section I-3.8 imply the following important property (Section I-3.8):

LEMMA 2.5.1: LPS isosymmetries cannot be derived via non-unitary transformations of the conventional LP symmetry.

Despite the above non-equivalence, the property $\hat{T} > 0$, the topological structure $(+1, +1, +1, -1)$ of the isometric $\hat{\eta} = \hat{T}\eta$ and Theorem 2.3.1 imply that:

LEMMA 2.5.2. *All LPS isosymmetries are locally isomorphic to the conventional LP symmetry.*

Recall from Section I-1 that an important limitation of quantum mechanics for the study of the EPR argument is the inability to achieve a consistent and effective treatment of non-linear interactions that are expected in the structure of hadrons, nuclei and stars. In Section I-4.12 , we have shown that the isotopic"completion" of quantum mechanics into hadronic mechanics does indeed allow a consistent and effective treatment of non-linear interactions via their embedding in the isotopic element $\hat{T}$.

Due to the unrestricted functional dependence of the isotopic element $\hat{T}$ and, therefore, of the isometric $\hat{\eta} = \hat{T}\eta$, it is easy to see that the LPS isosymmetries are indeed non-linear as a necessary condition to provide the invariance of non-linear dynamical equations.

Note that *isolinear isomonenta $\hat{P}_\mu$ isocommute on isospaces over isofields, but they do not commute on conventional spaces over conventional fields,* Eqs. (35), thus confirming that *the LPS isosymmetry is isolinear, that is, linear on isospaces over isofields but generally non-linear in their projection on conventional spaces over conventional fields.*

This important property can be illustrated by recalling the isolinear isomomentum (I-79) on a Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ with isostates $\hat{\psi} >$ over the isocomplex isonumbers $\hat{\mathcal{C}}$

$$\hat{P}_\mu \star |\hat{\psi} >= -i\hat{I}\partial_\mu|\hat{\psi} > . \tag{36}$$

Isocommutators (35) on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$ can then be explicitly written

$$\begin{aligned}[][\hat{P}_\mu,\hat{P}_\nu] \star |\hat{\psi} >= (\hat{P}_\mu \star \hat{P}_\nu - \hat{P}_\nu \star \hat{P}_\mu) \star |\hat{\psi} >= \\ = (-i\hat{I}\partial_\mu)\hat{T}(-i\hat{I}\partial_\nu) - (-i\hat{I}\partial_\nu)\hat{T}(-i\hat{I}\partial_\mu)\hat{T}|\hat{\psi} >= \\ = (i\hat{I}\partial_\mu\partial_\nu - i\hat{I}\partial_\nu\partial_\mu)|\hat{\psi} >= 0.\end{aligned} \tag{37}$$

By contrast, the projection of the same isocommutators (35) on a conventional Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$ no longer commutes,

$$\begin{aligned}[][\hat{P}_\mu, \hat{P}_\nu]|\hat{\psi} >= (\hat{P}_\mu\hat{P}_\nu - \hat{P}_\nu\hat{P}_\mu)|\hat{\psi} >= \\ = (-i\hat{I}\partial_\mu)(-i\hat{I}\partial_\nu) - (-i\hat{I}\partial_\nu)(-i\hat{I}\partial_\mu)|\hat{\psi} >\neq 0.\end{aligned} \tag{38}$$

because, in general, $\partial_\mu\hat{I} \neq \partial_\nu\hat{I}$, and this proves the isolinear character of the isomomentum.

Besides a direct relevance for the structure of hadrons, nuclei and stars, the above isolinearity has important implications , such as a new consistent operator form of gravitation , a new grand unification and other advances [34].

The presence of the structure functions in the isocommutation rules, the capability to provide the invariance under non-linear interactions and other features and applications outlined in Section 4 illustrate the non-triviality of the Lie-Santilli isotheory.

2.5.5. Iso-Casimir Isoinvariants. The simple direct use of isocommutation rules (33)-(35) establishes that the *iso-Casimir-isoinvariants* of $\hat{p}^0 0(3.1)$ are given by [59]

$$\begin{aligned}\hat{C}_1 = \hat{I}((t, r, p, E, \mu, \tau, \psi, \partial\psi, ...) > 0, \\ \hat{C}_2 = \hat{P}^{\hat{2}} = \hat{P}_\mu \star \hat{P}^\mu = (\hat{\eta}_{\mu\nu}P^\mu P^\nu)\hat{I} = \\ = (\textstyle\sum_{k=1,2,3} \frac{1}{n_k^2}P_k^2 - \frac{c^2}{n_4^2}p_4^2)\hat{I}, \\ \hat{C}_3 = \hat{W}^{\hat{2}} = \hat{W}_\mu \star \hat{W}^\mu, \quad \hat{W} = W\hat{I}, \\ \hat{W}_\mu = \hat{\epsilon}_{\mu\alpha\beta\rho} \star J^{\alpha\beta} \star P^\rho,\end{aligned} \tag{39}$$

and they are at the foundation of classical and operator *relativistic isomechanics* (Section I-4) with deep implications for structure models of interior dynamical systems [30].

2.4.6. Isorotations. By using isotransforms (32), the explicit form of the isorotations $\hat{SO}(3)$, first derived in Refs. [54] [55], can be written in the isoplane $(\hat{x},^1, \hat{x}^2)$ of iso-Euclidean isospaces $\hat{E}(\hat{x}, \hat{\Delta}, \hat{I})$ over the isoreals $\hat{\mathcal{R}}$, here formulated for simplicity in their

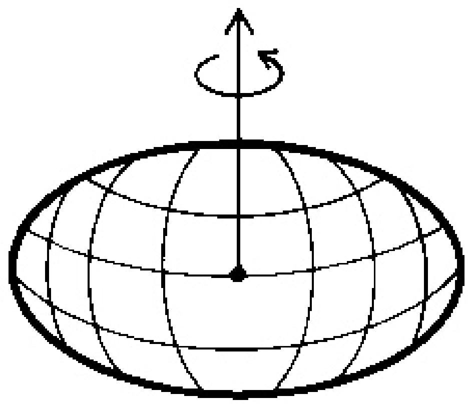

Figure 6: *It was generally believed in the 20th century physics that the rotational symmetry is broken for ellipsoids. Santilli isorotational isosymmetry has restored the exact character of the rotational symmetry for all possible (topology preserving) deformations of the sphere [29].*

projection on the conventional Euclidean space (see Ref. [29] for the general case)

$$x^{1'} = x^1 \cos[\theta(n_1 n_2)^{-1}] - x^2 \tfrac{n_1^2}{n_2^2} \sin[\theta(n_1 n_2)^{-1}],$$
$$x^{2'} = x^1 \tfrac{n_2^2}{n_1^2} \sin[\theta(n_1 n_2)^{-1}] + x^2 \cos[\theta(n_1 n_2)^{-1}]. \tag{40}$$

It was generally believed in the 20th century that the $SO(3)$ symmetry is broken for ellipsoid deformations of the sphere. By contrast, as shown by isotransforms (40) the $\hat{SO}(3)$ isosymmetry achieves the invariance of ellipsoids (Figure 6). But $SO(3)$ and $\hat{SO}(3)$ are locally isomorphic (Theorem 2.3.1). We therefore have the following property [54] [55]:

LEMMA 2.5.3: The Lie-Santilli $\hat{SO}(3)$ isosymmetry restores the exact character of the rotational symmetry for all ellipsoid deformations of the sphere.

.

This property is due to the fact that the mutation of the semiaxes of the sphere occur jointly with the *inverse* ,utation of the related units, thus maintaining the perfect spherical shape in isospaces over isofields

$$Radius \quad 1_k \rightarrow 1/n_k^2, \;\; Unit \quad 1_k \rightarrow n_k^2. \tag{41}$$

Note the crucial role of isonumbers for the reconstruction of the exact rotational symmetry because said reconstruction occurs thanks to the isoinvariant by the isounit.

2.5.7. Lorentz-Santilli isotransforms. The infinite family of isoconnected Lorentz-Santilli (LS) isotransforms $\hat{SO}^0(3.1)$ on iso-Minkowskian isospaces $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$ over the isoreals $\hat{\mathcal{R}}$, first derived by in Ref. [52] of 1983, can be written in the $(\hat{x}^3, \hat{x}^4)$-isoplane in their projection in the conventional l Minkowski space $M(x, \eta, I)$, as follows (see Ref. [29] for the general case):

$$x^{1'} = x^1, \quad x^{2'} = x^2,$$

$$x^{3'} = \hat{\gamma}(x^3 - \hat{\beta}\tfrac{n_3}{n_4}x^4), \tag{42}$$

$$x^{4'} = \hat{\gamma}(x^4 - \hat{\beta}\tfrac{n_4}{n_3}x^3),$$

where

$$\hat{\beta} = \frac{v_3/n_3}{c/n_4}, \quad \hat{\gamma} = \frac{1}{\sqrt{1-\hat{\beta}^2}}. \tag{43}$$

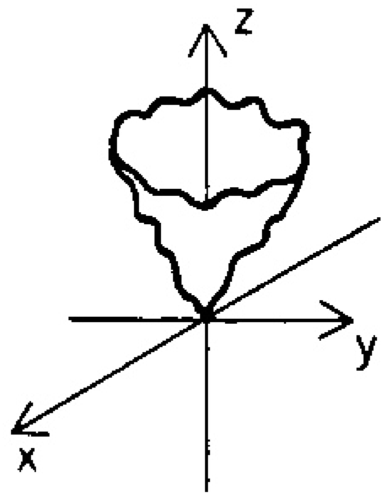

Figure 7: *It was generally believed in the 20th century physics that the Lorentz symmetry is broken for locally varying speeds of light within physical media (here represented with a wiggly light cone). The Lorentz-Santilli isosymmetry has restored the exact validity of the Lorentz symmetry for interior dynamical problems [52] [29]*

A significant aspect of Ref. [52] is the solution of the *historical Lorentz problem,* namely, the invariance of locally varying speeds of light within physical media

$$C = \frac{c}{n_4} \tag{44}$$

In fact, Lorentz first attempted the invariance of the speed of light $C = c/n_4$, but had to restrict his study to the invariance of the constant speed of light in vacuum c, due to insurmontable technical difficulties. Santilli has shown that Lorentz's difficulties were due to the use of Lie's theory, because, under the use of the covering Lie-Santilli isotheory, the invariance of $C = c/n)4$ was achieved in two pages of the 1983 letter [52].

A second significant aspect of Ref. [52] is the achievement of the first invariant formulation of extended, thus deformable and hyperdense particles, as stated beginning with the title of the quoted paper.

It was generally believed in the 20th century that the Lorentz symmetry $SO^0(3.1)$ is broken for locally varying speed of light within physical media represented with the wiggly circle of Figure 7. Ref. [52] proved that the isosymmetry $\hat{SO}^0(3.1)$ achieves the invariance of $C = c/n_4$. But $SO^0(3.1)$ and $\hat{SO}^0(3.1)$ are locally isomorphic, thus restoring the exact character of the abstract axioms of the Lorentz for all possible values $C = c/n_4$. We therefore have the following important property [29]:

LEMMA 2.3.5: The Lie-Santilli $\hat{SO}^0(3.1)$ *isosymmetry restores the exact validity of Lorentz's axioms for locally varying speeds of light.*

This property is due to the reconstruction of the exact light cone on the iso-Minkowskian isospace over isofields with maximal causal value c, called the *light isocone,*

$$\hat{x}^{\hat{2}} = \hat{x}_3^2 + \hat{x}_4^2 = 0 \tag{45}$$

while its projection on the conventional Minkowski space over conventional fields represents a locally varying speed

$$\hat{x}^2 = (\frac{x_3^2}{n_3^2} - t^2\frac{c^2}{n^4})\hat{I} = 0. \tag{46}$$

This property is due to the fact that the mutation of the $\hat{x}^3$ and $\hat{x}^4$ isocoordinates occurs jointly with the *inverse* mutation of the corresponding isounits, by therefore preserving the original perfect light cone with c as the maximal causal speed (see the 1966 monograph [29] for details)

$$\begin{gathered} x^3 \ \rightarrow \ \tfrac{x^3}{n_3}, \qquad I_3 = 1 \ \rightarrow \ \hat{I}_3 = n_3 \\ x^4 = tc \ \rightarrow \tfrac{x^4}{n_4} = t\tfrac{c}{n_4}, \quad I_4 = 1 \ \rightarrow \ \hat{I}_4 = n_4. \end{gathered} \tag{47}$$

Another significant aspect of Ref. [52] is the achievement of the first known invariance of non-linear, non-local and non-Hamiltonian interactions thanks to their embedding in the characteristic n-quantities of the isoinvariant (25).

2.5.8. Isotranslations. In view of their non-linearity, isotranslations in four parameters a_μ can be written in their projection in the conventional Minkowski space [29]

$$x'^\mu = x^\mu + A^\mu(a, x, \ldots), \tag{48}$$

and can be written via a power series expansion of the general expression

$$A^\mu = a^\mu(n_\mu^{-2} + a^\alpha[n_\mu^{-2} \hat{,} P_\alpha]/1! + \ldots), \tag{49}$$

The understanding of the isotopic completion of 20th century space-time symmetries requires the knowledge that, when properly written on iso-Minkowskian isospace over isofields, isotranslations recover their conventional form . [29].

2.5.9. isodilatations. Santilli introduced in Ref. [59] a novel one-dimensional isoinvariance denoted $\hat{D}$ which is given by the dilatation of the isometric caused by its multiplication by as parameter w, while the isounit is jointly subjected to the *inverse* dilatation

$$\begin{gathered} \hat{\Omega} = \hat{\eta}\hat{I} \ \rightarrow \ \hat{w} \star \hat{\Omega} = w\hat{\eta}\hat{I}' \\ \hat{I} \ \rightarrow \ \hat{I}' = \tfrac{1}{w}\hat{I}. \end{gathered} \tag{50}$$

under which isoinvariant (25) remain manifestly unchanged.

In essence, the new symmetry originates from the fact that, for mathematical consistency, isoinvariants must be elements of t isofields, thus having structure (25), namely, isoinvariants must be given by a conventional invariant multiplied by the isounit.

Ref. [59] showed that, by writing conventional invariants with the multiplication, in this case, by the trivial unit 1, the new dilatation symmetry persists for conventional space-time symmetries,

$$\eta \rightarrow \eta' = w\eta, \quad 1 \ \rightarrow \ 1' = \frac{1}{w}1 \tag{51}$$

The above properties imply the following

LEMMA 2.5.5: The conventional Lorentz-Poincaré symmetry is eleven-dimensional with structure

$$P^0(3.1) = so^0(3.1) \times A(3.1) \times D, \tag{52}$$

and, consequently, the Lorentz-Poincaré-Santilli isosymmetry is also eleven-dimensional with the structure

$$\hat{P}^0(3.1) = \hat{so}^0(3.1) \star \hat{A}(3.1) \star \hat{D}, \tag{53}$$

The above seemingly trivial property has permitted Santilli the study of a new grand unification of electrioweak and gravitational interactions based on the embedding of gravitation in the isotopic degree of freedom of the theory [34].

2.5.10. Isoinversions. The isotopic "completion" of conventional inversions has been studied in details in Refs. [29] and consists of the *isotime isoinversions*

$$\hat{\tau}\hat{t} = (\tau\hat{t})\hat{I} \tag{54}$$

plus the *isospace isoinversions*

$$\hat{\pi}\hat{r} = (\pi\hat{r})\hat{I} \tag{55}$$

where τ and π are conventional time and space inversions, respectively.

Despite their simplicity, Santilli has shown in Ref. [29] that *not only continuous, but also discrete space-time symmetries can be reconstructed as being exact on isospaces over isofields when assumed to be broken on conventional spaces over conventional fields.*.

2.5.11. Isospinorial LPS isosymmetry. Recall that the spinorial covering $\mathcal{P}^0(3.1)$ of the connected component of the LP symmetry $P^0(3.1)$ is constructed via the use of the Dirac gamma matrices. In fact, the conventional generators are realized via suitable combination of Dirac gamma matrices.

By following the same historical pattern, Santilli proposed in the 1995 communication [60] of the *Joint Institute for Nuclear Research,* Dubna, Russia (see also the subsequent paper [61]) the following eleven-dimensional isotopic "completion" $\hat{\mathcal{P}}^0(3.1)$ of $\mathcal{P}^0(3.1)$

$$\hat{\mathcal{P}}(3.1) = \hat{SL}(2.\hat{C}) \star \hat{\mathcal{A}}(3.1) \star \hat{\mathcal{D}}, \tag{56}$$

with realization of the generators in terms of the Dirac-Santilli isogamma isomatrices $\hat{\Gamma}_\mu = \hat{\gamma}_\mu \hat{I}$, Eqs. (I-89)

$$\hat{SL}(2.\hat{C}) : \quad \hat{R}_k = \tfrac{1}{2}\epsilon_{kij}\hat{\Gamma}_i \star \hat{\Gamma}_j, \quad \hat{S}_k = \tfrac{1}{2}\hat{\Gamma}_k \star \hat{\Gamma}_4,$$

$$\hat{\mathcal{A}}(3.1) : \quad \hat{P}_\mu, \tag{57}$$

$$k = 1, 2, 3, 4, 5, 6, \quad \mu = 1, 2, 3, 4.$$

The verification by the above isogenerators of isocommutation rules (33)-(35) is an instructive exercise for the interested reader. The proof that the Dirac-Santilli isoequations (I-88) transform isocovariantly under $\hat{\mathcal{P}}^0(3.1)$ is equally instructive.

2.5.12. Galilean isosymmetries As it is well known, the Galileo symmetry $G(3.1)$ characterizes the non-relativistic motion of point particles in vacuum, with consequential absence of resistive or non-potential forces (see the vertical line of Figure 8).

The isotopies of the Galileo symmetry are intended to characterize the non-relativistic motion of extended particles within physical media, by therefore experiencing resistive non-potential forces (see the wiggly line of Figure 8).

The resulting infinite family of isosymmteries $\hat{G}(3.1)$ are here called *Galilean isosymmetries* to stress the preservation of the basic axioms of the Galileo symmetry and the mere construction of the broadest possible realizations permitted by isomathematics.

The Lie-isotopic lifting of the Galileo symmetry were introduced by Santilli in the 1978 paper [12] as a particular case of the covering *Lie-Admissible symmetries,* also called *genosymmetries,* which are intended fo characterize the *time rate of variation of physical quantities.*

The first direct study of Galilean isosymmetries was done in Section 5.3, pages 225 on, of the 1981 monograph [25] formulated over conventional fields. These isotopies were then systematically studies and upgraded in the two 1991 volumes [26] [27]. The formulation of Galilean isosymmetries with thee full use of isomathemaics was done in the 1995 monographs [28] [29] with a final study presented in Ref.[31].

The above studies attracted the attention of Abdus Salam, founder and president of the *International Center for Theoretical Physics* (ICTP), Trieste, Italy, who invited Santilli in 1991 to deliver at his Center a series of lectures in the isotopies of the Galileo symmetry and relativity , said invitation being apparently the last by Salam prior to his death.

During his visit at the ICTP, Santilli wrote papers [64] through [70]. The notes from Santilli's lectures were collected by A. K. Aringazin, A. Jannussis, F. Lopez, M. Nishioka and B. Vel-janosky and published in volume [36] of 1992.

This work is primarily intended for relativistic isosymmetries. Additionally, all primary applications require relativistic treatments. Therefore, we regret to be unable to review Galilean isosymmetries to prevent a prohibitive length.

Nevertheless, the reader should be aware that an introductory knowledge of the Galilean isosymmetries is suggested, e.g., from the readIng of the ICTP papers [64] to [70].

3. APPARENT PROOF OF THE EPR ARGUMENT

3.1. Foreword.

As it is well known, the conventional Pauli matrices $\sigma_k,\ k = 1, 2, 3$, are the fundamental (also called adjoint), irreducible unitary representation of the $SU(2)$-spin symmetry and play a crucial role for the objections against the EPR argument [2] - [6] .

In this section, we review the isotopic "completion' of Pauli's matrices into isomatrces

$$\hat{\Sigma}_k = \hat{\sigma}_k \hat{I} \tag{58}$$

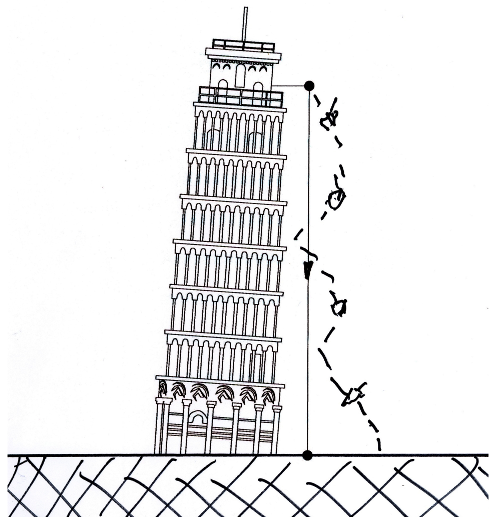

Figure 8: *This figure presents a conceptual rendering of the free fall of point-masses in vacuum studied by Galileo (represented with a straight line), and the free fall of extended masses experiencing resistive forces from our atmosphere studied by Santilli (represented with a wiggling line) [25] [29] [36]. It is symptomatic to note that the achievement of the symmetry for exteded masses required the construction of a covering of the mathematics used for the point masses with particular reference to the generalization of Newton-Leibnitz differential calculus, from its hiostorical formulation for isolated point, to a covering formulation for volumes [20].*

which constitute the isofundamental, isoirreducible, isounitary isorepresentation of the Lie-Santilli $\hat{SU}(2)$ isosymmetry and play a crucial role in the apparent proof of the EPR argument for extended particles within physical media studied later on in this section.

By recalling that the $SU(2)$ symmetry characterizes the spin of point-particles in vacuum, the "completed $\hat{SU}(2)$ isosymmetry is intended to characterize the spin of extended particles within hyperdense media called *hadronic spin,* such as the spin of an electron in the core of a star.

The isotopic "completion" of Pauli's matrices was introduced by Santilli in 1993 while visiting the *Joint Institute for Nuclear Research,* Dubna, Russia [57]. Said "completion" was presented systematically in Refs. [28] [29], used for the apparent proofs of the EPR argument [10] [11], and they are nowadays known as the *Pauli-Santilli isomatrices* [44].

In particular, the preceding studies have shown that, unlike the case for the $SU(2)$ symmetry, the isotopic $\hat{SU}(2)$ isosymmetry admits an explicit and concrete realization of hidden variables λ [3] [4] via realizations of the isotopic element of type $\hat{T} = Diag.(1.\lambda, \lambda)$ Eq.)(3).

In this section, we shall review the construction of $\hat{SU}(2)$ isosymmetry and of Pauli-Santilli isomatrices of regular and irregular type with hidden variables. We shall then use the methods acquired in this and in the preceding paper [9], for the proof that *interior dynamical systems represented via isomathematics and isomechanics appear to admit identical classical counterparts* [10] (Section 3.7), and to *progressively approach the classical EPR determinism [1] in the stricture of hadrons, nuclei and stars., while achieving the EPR determinism in the interior of gravitational collapse* [11] (Section 3.8).

A first understanding of this section requires a knowledge of the Lie-Santilli isotheory (Section I-2.7) [25] [29] [37] [45] [46] [48] A technical understanding of this section requires a technical knowledge of hadronic mechanics [28]- [30].

3.2. Pauli matrices.

As it is well known (see, e.g., Ref. [71]), the carrier space of the two-dimensional group of special unitary transformations $SU(2)$ is the two-dimensional complex Euclidean space $E(z, \delta, I)$ with coordinates $z = (z_1, fz_2)$, metric $\delta = Diag.(1,1)$ and unit $I = Diag/(1,1)$.

The two-dimensional , fundamental (also called adjoint), irreducible, unitary representation of the special unitary Lie algebra $su(2)$ of the $SU(2)$-spin symmetry is given by the celebrated $Pauli\ matrices$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i1 \\ i1 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}., \tag{59}$$

with commutations rules

$$[\sigma_i, \sigma_j] = \sigma_i\sigma_j - \sigma_j\sigma_i = i2\epsilon_{ijk}\sigma_k \tag{60}$$

and eigenvalues on a Hilbert space $calH$ over the field of complex numbers $\mathcal{C}$ with basis $|b>$

$$\begin{gathered} \sigma^2|b>= (\sigma_1\sigma_1 + \sigma_2\sigma_2 + \sigma_3\sigma_3)|b>= 3|b>, \\ \sigma_3|b>= \pm 1|b> \end{gathered} \tag{61}$$

Among the various properties of Pauli's matrices, we should recall their uniqueness in the sense that their expression is invariant under the class of equivalence admitted by quantum mechanics, that under unitary transformation.

We should also recall that Pauli's matrices are also fundamental for the structure of Dirac's equation, Eq. (I-9) since they appear in the very definition of Dirac's gamma matrices, Eqs. (I-89).

3.3. Regular Pauli-Santilli isomatrices

By following Ref. [57], the carrier isospace of the two-dimensional Lie-Santilli isogroup of isospecial isounitary isotransformations $\hat{SU}(2)$ is the isocomplex iso-Euclidean isospace $\hat{E}(\hat{z}, \hat{\Delta}, \hat{I})$ with isocoordinates

$$\hat{z} = z\hat{I} = (z_1, z_2) = (z_1, z_2)\hat{I}; \tag{62}$$

isounit and isotopic element

$$\hat{I} = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix} = 1/\hat{T} > o, \tag{63}$$

$$\hat{T} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2}; \end{pmatrix} \tag{64}$$

isometric

$$\hat{\Delta} = \hat{\delta}\hat{I} = (\hat{T}_i^k\delta_{ki})\hat{I} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix}\hat{I}; \tag{65}$$

positive-definite characteristic quantities n_k with unrestricted functional dependence on the variables for interior dynamical problems

$$n_k = n_k(z, \bar{z}, E, \mu, \alpha, \tau, \psi, \partial\psi, ...) > 0, \quad k = 1, 2; \tag{66}$$

and basic isoinvariant

$$\hat{z}_i \star \hat{\Delta}_{ij} \star \hat{\bar{z}}_j = (z_i \hat{\delta}_{ij} z_j)\hat{I} = \\ = (\frac{z_1 \bar{z}_1}{n_1^2} + \frac{z_2 \bar{z}_2}{n_2^2})\hat{I}. \tag{67}$$

By also following Refs. [28] [57], the isogroup of regular, isospecial, isounitary, iso-transformations $\hat{SU}(2)$ leaving invariant isoline element (67), is characterized by the iso-transforms

$$\hat{z}' = \hat{U}(\hat{\theta}) \star z = \hat{U}(\hat{\theta})\hat{T}\hat{z}, \tag{68}$$

verifying the following conditions [29]:

1. Isounitarity

$$\hat{U}(\hat{\theta}) \star \hat{U}^\dagger(\hat{\theta}) = \hat{U}^\dagger(\hat{\theta}) \star \hat{U}(\hat{\theta}) = \hat{I}; \tag{69}$$

2. Isogroup isoaxioms

$$\hat{U}(\hat{\theta}_1) \star \hat{U}(\hat{\theta}_2) = \hat{U}(\hat{\theta}_1 + \hat{\theta}_2), \\ \hat{U}(\hat{\theta}) \star \hat{U}(-\hat{\theta}) = \hat{U}(0) = \hat{I}, \quad k = 1, 2, 3; \tag{70}$$

and

3. Isospecial isounitarity

$$IsoDet\hat{U}(\hat{\theta}) = \hat{I}, \; Det(U\hat{T}) = 1. \tag{71}$$

The latter condition essentially restricts the isogroup $\hat{SU}(2)$ to its isoconnected component $\hat{SU}^0(2)$, which is hereon tacitly assumed.

The above conditions imply the local isomorphism

$$\hat{SU}(2) \approx SU(2) \tag{72}$$

and the following explicit realization in terms of isoexponential (I-22)

$$\hat{U}(\hat{\theta}) = \Pi_k U_k(\theta_k)\hat{I} = \Pi_k \hat{e}^{\hat{i}\star\hat{J}_k\star\hat{\theta}_k} = \Pi_k(e^{iJ_k\hat{T}\theta_k})\hat{I}, \\ U_k(\theta_k) = e^{iJ_k\hat{T}\theta_k} \quad , k = 1, 2, 3, \tag{73}$$

where $\hat{J}_k$ represents the isogenerators of the Lie-Santilli isoalgebra $\hat{su}(2)$ verifying the conditions

$$IsoTr\hat{J}_k = 0, \; Tr(\hat{J}_k\hat{T}) = 0, \tag{74}$$

and the isocommutation rules

$$[\hat{J}_i \hat{,} \hat{J}_j] = \hat{J}_i \star \hat{J}_j - \hat{J}_j \star \hat{J}_i = \\ = \hat{J}_i\hat{T}\hat{J}_j - \hat{J}_j\hat{T}\hat{J}_i = \epsilon_{ijk}\hat{J}_k \tag{75}$$

Note that, in accordance with Theorem I-2.7.2, the isorepresentations here considered are called *regular* because theycan be constructed via non-unitary transformations of the conventional $su(2)$ algebra, resulting in the preservation of the conventional structure *constants* ϵ_{ijk}.

However, as we shall see in the next section, the isotopies of the $su(2)$ algebra imply realizations called *irregular* that cannot be constructed via non-unitary representations of $su(2)$ [57], in which case the structure constants ϵ_{ijk} are replaced by *structure functions* with an arbitrary (non-singular) functional dependence on local variables,

$$\hat{C}_{ijk} = C_{ijk}(z, \bar{z}, E, \nu, \alpha, \tau, \psi, \partial\psi, ...)\hat{I}. \tag{76}$$

As one can verify, $\hat{su}(2)$ admits the following iso-Casimir isoinvariant

$$\hat{J}^{\hat{2}} = \Sigma_k \hat{J}_k \star \hat{J}_k = \\ = \hat{J}_1 \hat{T} J_1 + \hat{J}_2 \hat{T} J_2 + \hat{J}_3 \hat{T} J_3 \tag{77}$$

The maximal set of isocommuting isooperators is then given by $\hat{J}_3$ and $\hat{J}^{\hat{2}}$.

By again following Ref. [57], in order to compute the explicit form of the isorepresentations of $\hat{su}(2)$, we introduce the Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$[18] over the isofield of isocomplex isonumbers $\hat{\mathcal{C}}$ [19] with d-dimensional isobasis $|\hat{b}^d_k>$ verifying isonormalization (I-75),

$$<\hat{b}^d_k| \star |\hat{b}^d_k> = <\hat{b}^d_k|\hat{T}|\hat{b}^d_k> = \hat{I}, \\ d = 1, 2, 3, ...N, , \quad k = 1, 2, , 3. \tag{78}$$

From the local isomorphism $\hat{su}(2) \approx su(2)$ we know that the isoeigenvalue equations have the structure

$$\hat{J}_k \star |\hat{b}^d_k> = b^d_k|\hat{b}^d_k>, \\ \hat{J}^{\hat{2}} \star |\hat{b}^d_k> = \Sigma_k b^d_k(b^d_k + W)|\hat{b}^d_k> \ , \\ W = Det\hat{T} = 1/n_1^2 n_2^2 \tag{79}$$

where $W = 1$ for regular isorepresentation , otherwise W is an arbitrary function of local quantities to be identified via subsidiary constraints from the medium in which extended particles are immersed.

The explicit form of the isorepresentations of $\hat{su}(2)$ is then given by the simple isotopy of the conventional case [71]

$$\hat{J}_\pm = \hat{J}_1 \pm \hat{J}_2, \\ (\hat{J}_1)_{ij} = i\tfrac{1}{2} <\hat{b}^d_k| \star (\hat{J}_- - \hat{J}_+) \star |\hat{b}^d_k>, \\ (\hat{J}_2)_{ij} = i\tfrac{1}{2} <\hat{b}^d_k| \star (\hat{J}_- + \hat{J}_+) \star |\hat{b}^d_k>, \\ (J_3)_{ij} = <\hat{b}^d_k| \star \hat{J}_3 \star |\hat{b}^d_k> . \tag{80}$$

By continuing to follow Ref. [57], we now restrict our attention to the two-dimensional isofundamental (isoadjoint) isorepresentation of $\hat{su}(2)$ occurring for $d = 2$, in which case we assume

$$\hat{J}_k = \frac{1}{2}\hat{\sigma}_k, \quad k = 1, 2, 3, \tag{81}$$

and select the basic isounitary isotransform according to Sections I-2.8 and I-2.9

$$UU^\dagger = f(W) > 0, \;\; W = Det.\hat{I} = n_1^2 n_2^2 \tag{82}$$

where $f(W)$ is a smooth function such that $f(1) = 1$.

By using the above procedure, we have the following *regular Pauli-Santilli isomatrices* first introduced by Santilli in Ref. [57], Eqs. (3.2) (where the isometric elements are denoted $g_{kk} = n_k^{-2}, \; k - 1, 2$)

$$\hat{\Sigma}_k = \hat{\sigma}_k \hat{I},$$

$$\hat{\sigma}_1 = (n_1 n_2) \begin{pmatrix} 0 & n_1^{-2} \\ n_2^{-2} & 0 \end{pmatrix}, \;\; \hat{\sigma}_2 = (n_1 n_2) \begin{pmatrix} 0 & -\dot{\imath} n_1^{-2} \\ \dot{\imath} n_2^{-2} & 0 \end{pmatrix}, \tag{83}$$

$$\hat{\sigma}_3 = (n_1 n_2) \begin{pmatrix} n_2^{-2} & 0 \\ 0 & -n_1^{-2} \end{pmatrix}.$$

with isocommutation rules

$$[\hat{\sigma}_i \hat{,} \hat{\sigma}_j] = i2\epsilon_{ijk}\hat{\sigma}_k, \tag{84}$$

in which the 'regular' character of the isomatrices is established by the presence of the conventional (constant) structure constants.

We then have the isoeigenvalues isoequations

$$\hat{\sigma}_3 \star |\hat{b}_m^2 >= \hat{\sigma}_3 \hat{T} |\hat{b}_m^2 >= \pm \tfrac{1}{n_1 n_2} |\hat{b}_m^2 >$$

$$\hat{\sigma}^{\hat{2}} = (\sigma_1 \hat{T} \hat{\sigma}_1 + \sigma_2 \hat{T} \hat{\sigma}_2 \overset{+}{} \sigma_3 \hat{T} \hat{\sigma}_3) \hat{T} |\hat{b}_m^2 >= \tag{85}$$

$$= 3 \tfrac{1}{n_1^2 n_2^2} |\hat{b}_m^2 >$$

showing that the regular Pauli-Santilli isomatrices preserve the conventional structure constants ϵ_{ijk} of Pauli matrices, but exhibit structure (84) with generalized isoeigenvalues containing two characteristic quantities n_1^2, n_2^2,.

It is evident that, under isounimldularity condition (71),

$$Det\hat{T} = 1, \;\; n_1 = 1/n_2, \tag{86}$$

isomatrices (83) reduce to

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1^{-2} \\ n_2^{-2} & 0 \end{pmatrix}, \;\; \hat{\sigma}_2 = \begin{pmatrix} 0 & -\dot{\imath} n_1^{-2} \\ \dot{\imath} n_2^{-2} & 0 \end{pmatrix}, \tag{87}$$

$$\hat{\sigma}_3 = \begin{pmatrix} n_2^{-2} & 0 \\ 0 & -n_1^{-2} \end{pmatrix}.$$

by verifying conventional commutation rules (84) and conventional eigenvalues

$$\hat{\sigma}_3 \star |\hat{b}_m^2 >= \pm |\hat{b}_m^2 >$$

$$\hat{\sigma}^{\hat{2}} \star |\hat{b}_m^2 >= 3 |\hat{b}_m^2 > \tag{88}$$

In order to search for additional realizations of regular Pauli-Santilli isomatrices, we now assume the following non-unitary transform

$$U = \begin{pmatrix} n_1 & 0 \\ 0 & n_2 \end{pmatrix} = U^\dagger, \tag{89}$$

under which we have the following second realization of regular Pauli-Santilli isomatrices

$$\hat{\sigma}_k = U \sigma_k U^\dagger,$$

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1 n_2 \\ n_1 n_2 & 0 \end{pmatrix}, \ \hat{\sigma}_2 = \begin{pmatrix} 0 & -\dot{\imath} n_1 n_2 \\ \dot{\imath} n_1 n_2 & 0 \end{pmatrix}, \tag{90}$$

$$\hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & -n_2^2 \end{pmatrix}.$$

It is an instructive exercise for the interested reader to verify that the above isomatrices verify the isocommutation rules (84) and conventional isoeigenvalue(99), namely, *the second realization of the Pauli-Santilli isomatrices, Eqs. (873), also admit conventional structure constants and eigenvalues despite the degrees of freedom permitted by the two characteristic quantities* $n_1^2, \ n_2^2$.

We now assume the following non-diagonal realization of the non-unitary transform

$$U = \begin{pmatrix} 0 & n_1 \\ n_2 & 0 \end{pmatrix}, U^\dagger = \begin{pmatrix} 0 & n_2 \\ n_1 & 0 \end{pmatrix}, \tag{91}$$

$$U U^\dagger = \hat{I} = 1/\hat{T} > 0$$

which characterizes the following third realization of the regular Pauli-Santilli isomatrices

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1 n_2 \\ n_1 n_2 & 0 \end{pmatrix}, \ \hat{\sigma}_2 = \begin{pmatrix} 0 & -\dot{\imath} n_1 n_2 \\ \dot{\imath} n_1 n_2 & 0 \end{pmatrix}, \tag{92}$$

$$\hat{\sigma}_3 = \begin{pmatrix} -n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix},$$

It is easy to see that the above third realization of the regular Pauli-Santilli isomatrices also verify conventional commutation rules (84) and eigenvalues (88).

Note that, while Pauli's matrices are invariant under unitary transforms, there exist a number of Pauli-Santilli isomatrices each of which is invariant under isounitary isotransforms (Section I-3.9).

3.4. Irregular Pauli-Santilli isomatrices.

3.4.1. Historical notes. Recall that stars initiate their life as an aggregate of hydrogen. H. Rutherford [72] conjectured in 1910 that, when the temperature and pressure at the center of the star reaches certain values, the electron e^- is "compressed" inside the proton p^+ resulting in a neutral particle n which is called the *neutron* according to the reaction (see Section 4)

$$e^- + p^+ \ \rightarrow \ n. \tag{93}$$

Rutherford hypothesis was experimentally confirmed by J. Chadwick in 1932 [?], and the neutron became part of scientific history.

Following the experimental verification that the neutron has the same spin $1/2$ of the electron and of the proton, in an attempt at maintaining the conservation of the angular momentum predicted by conventional space-time symmetries, E. Fermi [73] suggested that the synthesis of the neutron occurs with the emission of a hypothetical massless and chargless particle ν with spin $1/2$ according to the reaction widely accepted for about one century

$$e^- + p^+ \ \rightarrow \ n. \tag{94}$$

After joining Harvard University in September 1977, R. M. Santilli noted that, despite the salvaging of space-time symmetries and related conservation laws, reaction (95) was not compatible with quantum mechanical laws [15] [16] because the rest energy of the neutron E_n is $0.782\ MeV$ *bigger* than the sum of the rest energies of the proton E_p and of the electron E_e,

$$E_p = 938.272\ MeV, \ \ E_e = 0.511\ MeV, \ \ E_n = 939.565\ MeV, \tag{95}$$

$$E_n - (E_p + E_e) = 0.782\ MeV > 0. \tag{96}$$

By recalling that quantum mechanics has an excellent consistency for bound states with *negative potentials* causing a *mass defect* (as it is typically the case for nuclear fusions), a representation of experimental data (96) via quantum mechanics is therefore impossible because it would require a *positive potential* capable of producing a *mass excess,* which features imply the loss of physical consistency of Schrödinger equation and quantum mechanical laws (e.g., because the indicial equation of Schrödinger equation admits no consistent solutions for positive potential energies, see Table 5 of Ref. [16]).

Various conjectures, aimed at maintaining for the neutron structure the theory so effective for the hydrogen atom, were proved not to be consistent. For instance, the hypothesis that the missing energy of $0.782\ MeV$ was carried as a relative energy between the electron and the proton had to be dismissed because the cross section $e - p$ is so small at $0.782\ MeV$ to prevent any fusion between the electron and the proton.

In the absence of viable hypotheses salvaging quantum mechanics, Santilli interpreted the above insufficiency as evidence of Einstein's view that *quantum mechanics is not a complete theory.* Therefore, Refs. [15] [16] initiated the search for a "completion" of conventional Lie symmetries and quantum mechanics with particular reference to the central need: identify a suitable "completion"of Lie's theory at large and, in particular, of the $SU(2)$-spin symmetry for the electron when "compressed" inside the hyperdense proton.

The above 1978 remarks lead to conditions of *variational selfadjointness* and to the *axion-preserving isotopies of Lie's theory for variationally non-selfadjoint systems* presented at a post Ph. D. Seminar Course delivered in 1978 at the Lyman Laboratory of Physics of Harvard University, which Seminar Course in monographs [24] [25] whose content has been reviewed in Paper I and in the preceding sections of this paper.

The 1995 papers [54] [55] and the ensuing 1963 paper [57] on the regular isotopies $\hat{SU}(2)$ of the spin symmetry (reviewed in the preceding section) lead to the apparent 1998 proof of the EPS argument [10].

To understand the complexity of the problem of the neutron synthesis and its value for basic advances, one should note that *there exist no consistent quantum mechanical basis for the emission of the neutrino in synthesis (95) due to the lack of* $0.782\ MeV$ *for the* *synthesis of the neutron itself, without any possible energy available for the creation of the neutrino.*

In view of the above occurrence, Santilli noted more recently in the 2007 paper [74] that *as a necessary condition to carry the missing* $0.782\ MeV$, *a third particle should be added in the l.h.s. of the reaction for the neutron synthesis, rather in its r.h.s.*

This remark lead to the conjecture of the *etherino* represented with the symbol a conceived as a longitudinal impulse (rather than a particle as conventionally understood) delivering the missing $0.782\ MeV$ according to the reaction [74]

$$e^- + a + p^+ \ \rightarrow \ n, \tag{97}$$

which, unfortunately, cannot be studied in this paper for brevity.

The motivation of the studies reviewed in this section stems from the fact that, despite their significance for the EPS argument, *regular isotopies of the* $SU(2)$ *spin symmetry are insufficient for the characterization of the spin of the neutron in its synthesis from the hydrogen due to their conventional eigenvalues, while interior dynamical systems at large, and the neutron synthesis in particular, require alterations (called mutations) of conventional eigenvalues that can be solely represented via irregular isorepresentations of* $\hat{SU}(2)$.

The irregular isorepresentations of the $SU(2)$-spin symmetry were identified, apparently for the first time, by Santilli in the 1990 paper [75] and used to achieve a non-relativistic representation of *all* characteristics of the neutron in its synthesis from the hydrogen.

In the 1995 paper [61], Santilli presented a relativistic study of $\hat{SU}(2)$ as an isosubalgebra of the irregular isospinorial covering of the Lorentz-Poincaré symmetry (Section 2.5.11) and used the results to achieve a relativistic representation of the neutron synthesis.

The above irregular isotopies are particularly significant for the identification of interior dynamical systems providing concrete illustrations of the validity of the EPR argument, and can be summarized as follows:

3.4.2. Non-relativistic formulation. The first irregular isotopies of Pauli's matrices, today known as *irregular Pauli-Santilli isomatrices* [44], have been introduced in Eqs. (2.32) of Ref. [75] via the use of the isorepresentations of $\hat{SU}(2)$ worked our in the preceding papers [54] [55], and are given by

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & -n_1 \\ n_2 & 0 \end{pmatrix}, \ \ \hat{\sigma}_2 = \begin{pmatrix} 0 & -in_1 \\ in_2 & 0 \end{pmatrix},$$
$$\hat{\sigma}_3 = \tfrac{1}{n)1n_2} \begin{pmatrix} n_1^2 & 0 \\ 0 & -n_2^2 \end{pmatrix}, \tag{98}$$

with irregular isocommutation rules

$$[\hat{\sigma}_i \hat{,} \hat{\sigma}_j] = 2i \frac{1}{n_1 n_2} \epsilon_{ijk} \hat{\sigma}_k, \ \ i, j, k, = 1, 2, 3 \tag{99}$$

and isoeigenvalues

$$\hat{\sigma}_3 \star |\hat{u}>= \pm \tfrac{1}{n_1 n_2} |\hat{u}>,$$
$$\hat{\sigma}^{\hat{2}} \star |\hat{u}>= \tfrac{1}{n_1 n_2} (\tfrac{1}{n_1 n_2} + 2) |\hat{u}>, \tag{100}$$

It is easy to see that, when the hyperdense medium surrounding the immersed particle is homogeneous and isotropic, the characteristics quantities can be normalized to the values $n_1 = n_2 = n_3 = 1$, in which case isoeigenvalues (100) are conventional. We therefore have the following

LEMMA 3.1: Irregular isorepresentations of the Lie-Santilli isosymmetry $\hat{su}(2)$ represent the inhomogeneity and anisotropy of media in which extended particles are immersed.

Among a number of additional irregular Pauli-Santilli isomatrices with isotopic element $\hat{T}$ in Eq. (64) we quote Eqs. (3.2) of Ref. [10]

$$\hat{\sigma}_1 = n_1 n_2 \begin{pmatrix} 0 & n_1^{-2} \\ n_2^{-2} & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = n_1 n_2 \begin{pmatrix} 0 & -in_1^{-2} \\ in_2^{-2} & 0 \end{pmatrix},$$
$$\hat{\sigma}_3 = n_1 n_2 \begin{pmatrix} n_2^{-2} & 0 \\ 0 & -n_1^{-2} \end{pmatrix}. \tag{101}$$

with irregular isocommutation rules

$$[\hat{\sigma}_i \hat{,} \hat{\sigma}_j] = 2i \frac{1}{n_1 n_2} \hat{\sigma}_k, \tag{102}$$

and isoeigenvalues

$$\hat{\sigma}_3 \star |\hat{u}> = \pm \tfrac{1}{n_1 n_2} |\hat{u}>,$$
$$\hat{\sigma}^{\hat{2}} \star |\hat{u}> = 3 \tfrac{1}{n_1^2 n_2^2} |\hat{u}>, \tag{103}$$

The above isorepresentation appears to be significant when the medium causes a proportional alteration/mutation of both the third component as well as the total value of the spin of a particle having the value $1/2$ in vacuum..

Another example of irregular Pauli-Santilli isomatrices is given by of Eqs. (3.7) of Ref. [10]

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_2 \\ n_1 & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -in_2 \\ in_1 & 0 \end{pmatrix},$$
$$\hat{\sigma}_3 = \begin{pmatrix} n_1^2 & \\ 0 & -n_2^2 \end{pmatrix}. \tag{104}$$

with irregular isocommutation rules

$$[\hat{\sigma}_1 \hat{,} \hat{\sigma}_2] = 2i \tfrac{1}{n_1^2 n_2^2} \hat{\sigma}_3, \quad [\hat{\sigma}_2 \hat{,} \hat{\sigma}_3] = 2i \hat{\sigma}_1,$$
$$[\hat{\sigma}_3 \hat{,} \hat{\sigma}_1] = 2i \hat{\sigma}_2, \tag{105}$$

and mutated isoeigenvalues

$$\hat{\sigma}_3 \star |\hat{u}> = \pm |\hat{u}>,$$
$$\hat{\sigma}^{\hat{2}} \star |\hat{u}> = \tfrac{2}{n_1^2 n_2^2} |\hat{u}>, \tag{106}$$

The above isorepresentation may be useful when the anisotropy and inhomogeneity of the medium maintain the spin value $1/2$ along the third axis, yet they are such to deform the remaining components.

Additional example of irregular Pauli-Santilli isomatrices are available from Refs. [10] and [57], and can be readily constructed by interested readers.

3.4.3. Relativistic formulation. Consider the iso-Minkowskian isospace $\hat{M}(\hat{x}, \bar{\Omega}, \hat{I})$ with isometric

$$\hat{\Omega} = \hat{\eta}\hat{I}, \quad \bar{\eta} = \hat{T}\eta,$$

$$\hat{T} = Diag.(\tfrac{1}{m_1^2}, \tfrac{1}{m_2^2}, \tfrac{1}{m_3^2}, \tfrac{1}{m_4^2}). \tag{107}$$

$$m_\mu = m_\mu(r, p, E, \nu, \alpha, \tau, \pi, \psi, ...) > 0. \;\; \mu = 1, 2, 3, 4.$$

where the new characteristic quantities m_μ have been introduced to avoid confusion with the previously used symbols n_μ.

The relativistic formulation of the irregular isorepresentation of $\hat{SU}(2)$ were derived, apparently for the first time, in Eqs. (6.4c)-(6.4d) of Ref. [61], and can be written

$$J_k = \frac{1}{2}\epsilon_{kij}\hat{\gamma}_i \star \gamma_j \tag{108}$$

where $\hat{\gamma}$ are the regular Dirac-Santilli isomatrices (I-89), i.e.,

$$\hat{\gamma}_k = \tfrac{1}{m_k}\begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix},$$

$$\hat{\gamma}_4 = \tfrac{i}{m_4}\begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{109}$$

and $\hat{\sigma}_k$ are the *regular Pauli-Santilli isomatrices.*

The irregular character of isorepresentation (108) is established by the presence of structure functions in the isocommutation rules, Eqs, (6.4c) of Ref. [61],

$$[J_i \hat{,} J_j] = \epsilon_{ijk}\frac{1}{m_k^2}J_k, \tag{110}$$

and in the irregular isoeigenvalues

$$J_3 \star |\hat{\psi}> = \pm\tfrac{1}{2}\tfrac{1}{m_1 m_2}|\hat{\psi}>,$$

$$J^{\hat{2}} \star |\hat{\psi}> = \tfrac{1}{4}(\tfrac{1}{m_1 m_2} + \tfrac{1}{m_2 m_3} + \tfrac{1}{m_3 m_1})|\hat{\psi}> \tag{111}$$

that, as shown in Ref.[61], permit ra relativistic representation of the spin of the neutron in its synthesis from the hydrogen.

Again one should note that, when the medium is homogeneous and isotropic, isoeigenvalues (101) are conventional.

Note that the assumption of mutated spin for an extended particle within a hyperdense medium implies the *inapplicability (rather than the violation) of the Fermi-Dirac statistics, Pauli's exclusion principle and other quantum mechanical laws* with the understanding that said mutations are internal, thus solely testable under *external* strong interactions, as indicated beginning with the *title* of Harvard's 1978 paper [15].

3.5. Isotopies of hadronic spin and angular momentum.

3.5.1. Historical notes Am electron orbiting in vacuum around the proton in the hydrogen atom experiences no resistive forces, thus verifying known symmetries and conservation laws.

When the same electron has been "compressed" inside the proton according to Rutherford [?], Santilli [75] argued that the sole possible angular moment is that permitted by ıconstraints exercised on the electron by the internal medium

Since the electron is about $2,000$ times lighter than the proton, *the most stable configuration is that for which the electron is "constrained" to orbit with a value of the angular momentum equal to the proton spin,* since any different configuration would imply big resistive forces (Figure 9).

Needless to say, fractional angular momenta are anathema for the quantum mechanical description of point-particles in vacuum [71].

However, the angular momentum of extended particles immersed within hyperdense hadronic media can acquire values other than integers, depending on the local physical conditions of the medium surrounding the particle, such as pressure, density, anisotropy, inhomogeneity, etc.

The first known quantitative study of *constrained angular momenta* of extended particles within hyperdense hadronic media was done at the non-relativistic level by Santilli in Ref. [75] of 1990 following the preceding isotopies of the rotational symmetry, Refs. [54] [55]. The study was then extended to the relativistic level in Ref.[61] of 1990.

These studies are crucial for quantitative representations of the synthesis of hadrons providing apparent verifications of the EPR argument, and can be summarized as follows.

3.4.2. Non-relativistic representation. Recall the central assumption of isosymmetries according to which conventional generators are preserved (because representing conventional total conservation laws), and only their product is lifted into the isotopic form (1) (to represent the extended character of the particles and their non-Hamiltonian interactions.

Hence, the definition of the *isoangular isomomentum,* also called *hadronic angular momentum,* on an iso-Euclidean isospace is the same as that of quantum mechanics

$$L_k = \epsilon_{ijk} \hat{r}_i \star \hat{p}_j = \epsilon_{ijk} r_i p_j, \tag{112}$$

although it is defined on a Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ with isostates $|\hat{\psi}>$ on an isocomplex isofield $\hat{\mathcal{C}}$, with *isolinear isomomentum* Eqs, (I-79), and isocommutation rules are then given by Eqs. (I-81).

It is then easy to verify the following isocommutation rules for the hadronic angular isomomentum, Eqs (2.22b) [75]

$$[L_i \hat{,} L_j] = i\hat{I} \epsilon_{ijk} L_k, \tag{113}$$

where, as one can see, the characteristics of the medium, represented by the isounit $\hat{I}$, enter directly in the isocommutation rules.

The use of the *isosperical isoharmonic isofunctions* (see Page 240 of Ref. [29] for details)

$$\begin{gathered}\hat{Y}_{\ell m}(\hat{\theta}, \hat{\phi}) = UY(\theta, \phi)U^\dagger = \hat{T}^{-1} Y_{\ell m}(\theta, \phi), \\ UU^\dagger = \hat{I} = 1/\hat{T} \neq I,\end{gathered} \tag{114}$$

where $Y_{\ell m}(\theta, \phi$ are the conventional spherical harmonic functions, yields the following isoeigenvalues, Eqs. (2.25), Ref. [75],

$$L_3 \star \hat{Y}_{\ell m}(\hat{\theta}, \hat{\phi}) = \hat{I} m \hat{Y}_{\ell m}(\hat{\theta}, \hat{\phi}),$$

$$L^{\hat{2}} \star \hat{Y}_{\ell m}(\hat{\theta}, \hat{\phi}) = \hat{I}\ell(\hat{I}\ell + 1)\hat{Y}_{\ell m}(\hat{\theta}, \hat{\phi}), \tag{115}$$

$$m = \ell, \ell - 1, ..., -\ell, \;\; m = 1, 2, 3, ...$$

where one can see again the mutation of the eigenvalues caused by the surrounding medium.

Applications to particle physics then require specific realizations of the isounit $\hat{I}$, such as the simple assumption of expressions (4) used in Ref. [75]

$$\rho = |\hat{I}| =\approx |e^{\gamma}|, \tag{116}$$

where ρ is a function of all possible or otherwise needed local variables of the medium.

3.5.3. Isotopies of non-relativistic spin-orbit coupling. As one can see, isoeigenvalues (115) do not allow a representation of the constrained hadronic angular momentum of the electron when compressed inside the proton (Figure 9).

In view of this insufficiency, Santilli conducted in ref. [?] study of the eigenvalues of the combined spin and angular momentum of the electron in the indicated interior conditions.

We consider then the *total hadronic momentum*

$$J_{tot} = L_{\ell} \hat{\otimes} J_s, \tag{117}$$

with corresponding basis $|\hat{Y}\hat{\otimes}\hat{u} >$ and isoexpectation values, Eqs. (2.34), Ref. [75],

$$\begin{gathered} J_{3,tot}|\hat{Y}\hat{\otimes}\hat{u} >= (\rho_m(\ell) \pm \tfrac{m(s)}{n_1 n_2})|\hat{Y}\hat{\otimes}\hat{u} > \\ J^{\hat{2}}_{tot} \star |\hat{Y}\hat{\otimes}\hat{u} >= (\rho\ell \pm \tfrac{s}{n_1 n_2})(\rho\ell \pm \tfrac{s}{n_1 n_2} + 1)|\hat{Y}\hat{\otimes}\hat{u} > \\ \ell = 0, 1, 2, 3, ... \;\; s = 0, \tfrac{1}{2}1, \tfrac{3}{2}, ..., \\ m(\ell) = \ell, \ell - 1, ..., -\ell, \;\; m(s) = s, s - 1, ..., -s, \end{gathered} \tag{118}$$

Following a laborious journey initiated in 1977, isoeigenvalues (118) finally permitted Santilli to achieve the desired solution for $\ell = 1$ and $s = \frac{1}{2}$, Eq. (2.36), Ref. [75],

$$\rho = \frac{1}{2}\frac{1}{n_1 n_2}, \tag{119}$$

for which *the total hadronic angular momentum of the electron in the synthesis of the neutron is identically null,* $J_{tot} = 0$*, and the spin of the neutron coincides with that of the proton.*

More detailed studies pertaining to electric and magnetic dipoles excluded the alternative $J = 1$ of eigenvalues (118), as well as total hadronic angular momenta of the electron other than zero.

The preceding studies permitted a quantitative non-relativistic representation of the spin of the neutron in its synthesis from the hydrogen atom. A representation of the remaining characteristics of the neutron (mass, radius, charge, dipole moments, etc.) is reviewed in Section 4.5.

3.5.3. Isotopies of relativistic spin-orbit couplings. The hadronic spin $\hat{S} = S\hat{I}$ is a realization of the $\hat{SU}(2)$ isosubalgebra of $\hat{\mathcal{P}}(3.1)$ with generators (57), while the hadronic angular momentum $\hat{L} = L\hat{I}$ is a realization of the isorotational $\hat{SO}(3)$ isosubalgebra. Their relativistic formulation on iso-Minkowskian isospace (107) has been first derived in Eqs. (6.4a) (6.4b), Ref. [61] and are given by

$$S_k = \frac{1}{2}\epsilon_{kij}\hat{\gamma}_i \star \hat{\gamma}_j,$$
$$L_k = \epsilon_{kij}(\epsilon_{kij} r_i \star p_j), \tag{120}$$

where $\hat{\gamma}_k$ are the Dirac-Santilli isomatrices.

We then have the irregularisocommutation rules

$$[S_i \hat{,} S_j] = \epsilon_{kij} m_k^2 \hat{S}_k,$$
$$[L_i \hat{,} L_j] = \epsilon_{ijk} m_k^2 L_k \tag{121}$$

and isoeigenvalues, Eqs, (6.4d) Ref. [61]

$$\hat{S}_3 \star |\hat{\psi} >= \pm\frac{1}{2}\frac{1}{m_1 m_2}|\hat{\psi} >,$$
$$\hat{S}^{\hat{2}} \star |\hat{\psi} >= \frac{1}{4}(m_1^{-2}m_2^{-2} + m_2^{-2}m_3^{-2} + m_3^{-2}m_1^{-2})|\hat{\psi} >,$$
$$\hat{L}_3 \star |\hat{\psi} >= \pm m_1 m_2|\hat{\psi} >,$$
$$\hat{L}^{\hat{2}} \star |\hat{\psi} >= (m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2)|\hat{\psi} > \tag{122}$$

The most salient difference between relativistic isoeigenvalues (122) and their non-relativistic counterparts (155) is that *the former admit fractional hadronic angular momenta while the latter do not.*

In fact, for the following values admitted by a homogeneous and isotropic medium [61]

$$m_1 = m_2 = m_3 = \frac{1}{\sqrt{2}}, \tag{123}$$

isoeigenvalues (122) become

$$\hat{L}_3 \star |\hat{\psi} >= \pm\frac{1}{2}|\hat{\psi} >,$$
$$\hat{L}^{\hat{2}} \star |\hat{\psi} >= \frac{3}{4}|\hat{\psi} > \tag{124}$$

Consequently, *isoeigenvalues (122) permit a quantitative representation of the hadronic angular momentum of the electron as being constrained to be equal to the proton spin* [60] [61] (Figure zzzz).

In this case too, the total hadronic angular momentum of the electron is null because the only stable hadronic spin-orbit coupling is in singlet, and the spin of the electron can be assumed in first good approximation not to be mutated since the electron is about $2,000$-times lighter than the proton.

Hadronic spins, hadronic angular momenta and hadronic spin-orbit couplings were studied in detail Chapter 6 of in Ref. [29] resulting in Lemma 6.12.1 here reproduced without proof:

LEMMA 3.2: When immersed within hadrons or nuclei with spin $1/2$, an elementary particle having spin $1/2$ in vacuum can only have a null total hadronic angular momentum.

As we shall see in Section 4.6, the above configuration of the synthesis of the neutron from the hydrogen is an apparent verification of the EPR argument.

3.6. Realization of hidden variables.

As recalled in Section 1.1, the conventional quantum mechanical realization of the Lie symmetry $SU(2)$ does not allow a consistent representation of hidden variables λ [3] [4].

It is easy to see that, despite the local isomorphism $\hat{SU}(2) \approx SU(2)$, the Lie-Santilli isosymmetry $\hat{SU}(2)$ does indeed allow explicit and concrete realizations of hidden variables thanks to the degree of freedom permitted by the isotopic element (1) in the structure of the Lie-Santilli isoproduct (2) with realizations of the isotopic element of type (3).

In this section, we review the explicit and concrete realization of *regular hidden variables,* namely, realizations that can be derived via non-unitary transforms of theLie algebra $su(2)$, and then review *irregular hidden variables,* namely, realizations that do not admit such a simple derivation.

Regular and irregular realizations of hidden variables have been first identified by Santilli in Ref. [57] of 1993, and then used for the proof of the EPR argument [10] reviewed in Section 3.7.

Realizations of regular hidden variables are easily provided by Pauli-Santilli isomatrices (83) with the identifications

$$n_1^2 = \lambda_1, \;\; n_2^2 = \lambda_2. \tag{125}$$

yielding the desired realization, Eqs. (3.9), Ref. [57],

$$\hat{\sigma}_1 = (\lambda_1\lambda_2)\begin{pmatrix} 0 & \lambda_1^{-1} \\ \lambda_2^{-1} & 0 \end{pmatrix}, \;\; \hat{\sigma}_2 = (\lambda_1\lambda_2)\begin{pmatrix} 0 & -i\lambda_1^{-1} \\ i\lambda_2^{-1} & 0 \end{pmatrix},$$
$$\hat{\sigma}_3 = (\lambda_1\lambda_2)\begin{pmatrix} \lambda_2^{-1} & 0 \\ 0 & -\lambda_1^{-1} \end{pmatrix}. \tag{126}$$

verifying isocommutation rules

$$[\hat{\sigma}_i \hat{,} \hat{\sigma}_j] = i\epsilon_{ijk}\hat{\sigma}_k, \tag{127}$$

and isoeigenvalue isoequations

$$\hat{\sigma}_3 \star |\hat{b}> = \pm(\lambda_1\lambda_2)|\hat{b}>$$
$$\hat{\sigma}^{\hat{2}} = 3(\lambda_1\lambda_2)^2|\hat{b}> \tag{128}$$

We consider now the particular case of Eq. (3), i.e.,

$$Det.\hat{T} = 1, \;\; n_1^2 = 1/n_2^2 = \lambda, \tag{129}$$

derivable via the basic non-unitary transformation

$$\hat{T} = (UU^{\dagger})^{-1} = \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & \lambda \end{pmatrix}. \tag{130}$$

In this case, isomatrices (83) become (Eqs. (3.9) of [57])

$$\hat{\sigma}_1(\lambda) = \begin{pmatrix} 0 & \lambda^{-1} \\ \lambda & 0 \end{pmatrix}, \quad \hat{\sigma}_2(\lambda) = \begin{pmatrix} 0 & -i\lambda^{-1} \\ i\lambda & 0 \end{pmatrix},$$
$$\hat{\sigma}_3(\lambda) = \begin{pmatrix} \lambda & 0 \\ 0 & -\lambda^{-1} \end{pmatrix}. \tag{131}$$

It is an instructive exercise for the interested reader to verify that the above realization of the regular Pauli-Santilli isomatrices verifies isocommutation rules with the same stricture constants of the $SU(2)$ algebra

$$[\hat{\sigma}_i(\lambda)\hat{,}\hat{\sigma}_j(\lambda)] = i2\epsilon_{ijk}\hat{\sigma}_k(\lambda), \tag{132}$$

and admit conventional eigenvalues

$$\hat{\sigma}_3(\lambda) \star |\hat{b}>= \pm|\hat{b}>$$
$$\hat{\sigma}(\lambda)^{\hat{2}} = 3|\hat{b}> . \tag{133}$$

Consequently. we have the following property [57]:

LEMMA 3.3. Regular Pauli-Santilli isomatrices provide an explicit and concrete realization of regular hidden variables directly in the spin $1/2$ *algebra.*

Note that, besides being positive-definite, hidden variables have an unrestricted functional dependence on all needed local variables, Eqs. (66).

An example of irregular hidden variables is provided by the $\hat{SU}(2)$ component of the spinorial covering of the Lorentz-Poincaré-Santilli isosymmetry.

To illustrate this realization, introduce *three* additional hidden variables for the characterization of isospace (107)

$$m_\mu = \lambda_\mu, \quad \mu = 1, 2, 3, 4. \tag{134}$$

Realization (108) then implies the following irregular Dirac-Santilli isomatrices

$$\hat{\gamma}_k(\lambda) = \tfrac{1}{\gamma_k} \begin{pmatrix} 0 & \hat{\sigma}_k(\lambda) \\ -\hat{\sigma}_k & 0 \end{pmatrix},$$
$$\hat{\gamma}_4 = \tfrac{i}{m_4} \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{135}$$

where $\hat{\sigma}_k$ are the *regular or irregular Pauli-Santilli isomatrices,* with isocommutation rules

$$[S_i(\lambda)\hat{,}S_j(\lambda)] = \epsilon_{ijk}\frac{1}{\lambda_k}S_k, \tag{136}$$

and isoeigenvalues

$$S_3 \star |\hat{\psi}>= \pm\frac{1}{2}\frac{1}{\sqrt{\lambda_1\lambda_2}}|\hat{\psi}>,$$
$$S^{\hat{2}} \star |\hat{\psi}>= \frac{1}{4}(\frac{1}{\sqrt{\lambda_1\lambda_2}} + \frac{1}{\sqrt{\lambda_2\lambda_3}} + \frac{1}{\sqrt{\lambda_3\lambda_1}})|\hat{\psi}> \tag{137}$$

Consequently, we have the following property [61]

LEMMA 3.4: The axioms of Dirac's equation admit up to five generally different, regular or irregular hidden variables.

Additional realizations of irregular hidden variables can be found in Eqs. (3.11) of Ref. [57] or can be easily derived from the preceding realization of the Pauli-Santilli isomatrices.

3.7. Apparent admission of classical counterparts.

As it is well known,. Bell's inequality [3] [4], von Neumann's theorem [5], and the theory of local realism at large (see review [6] with a comprehensive literature) are generally assumed to be evidence of the impossibility of "completing" quantum mechanics into a broader theory, with ensuing rejection of the EPR argument [1].

Following decades of preparatory works reviewed in Paper I [9] and in the preceding sections of this paper , Santilli proved in Ref. [10] of 1998 that:

1) Bell's inequality, von Neumann's theorem and related studies are indeed valid, but under the *tacit* assumption of representing particles as being point-like, with ensuing sole admission of linear, local and potential interactions (exterior dynamical problems).

2) Bell's inequality, von Neumann's theorem and related studies are *inapplicable* (rather than being violated) for extended particles within physical media, due to the presence of additional non-liner, non-local and non-potential interactions (interior dynamical systems).

3) The latter systems represented with the axiom-preserving "completion" of 20th century appleid mathematics into isomathematics and the ensuing "completion" of quantum mechanics into hadronic mechanics [28]-[30] verify Statement 2 and admit well defined classical counterparts.

To review the preceding advances, consider two quantum mechanical particles with spin $1/2$ denoted 1 and 2 which verify the $SU(2)$ spin symmetry.

Assume that, as a result of some interaction, the two particles have antiparallel spins represented in the Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$. The total state in $ca\hat{l}H$ is then given by

$$|S_{1-2}>= \frac{1}{\sqrt{2}}(|S_{1\uparrow}> \times|S_{2\downarrow}> -|S_{1\downarrow}> \times|S_{2\uparrow}>) \tag{138}$$

with conventional l normalization

$$< S_{1-2}|S_{1-2}>= 1, \tag{139}$$

where $\times$ is the conventional associative product.

Let a_1, b_1 and a_2, b_2 be unit vectors along the z-axis of a conventional Euclidean space $E(r, \delta, I)$ for particle 1 and 2, respectively. Introduce the quantum mechanical probability

$$P(a_1, b_1) =< S_{1-2}|(\sigma_1 \otimes a_1) \times (\sigma_2 \otimes b_1)|S_{1-2}>= -a_1 \otimes b_1, \tag{140}$$

where $\otimes$ is the conventional scalar product,

Then, Bell's inequality can be written [4] (see Ref. [6] for numerous equivalent formulations)

$$D^{QM}_{Bell} = Max|P(a_1, b_1) - P(a_1, b_2) + P(a_2, b_1) + P(a_2, b_2)| \leq 2. \tag{141}$$

and implies the following property:

LEMMA 3.5: Particles in vacuum verifying the Lie symmetry $SU(2)$ admit no classical counterparts.

PROOF: The classical counterpart of Bell's inequality is given by

$$D^{Classical}_{Max} = Max|a_1 \otimes b_1 - a_1 \otimes b_2| + |a_2 \otimes b_1 + a_2 \otimes b_2| = 2\sqrt{2}, \tag{142}$$

Butt the quantum mechanical value of D^{QM}_{Bell} is *always* smaller than its classical counterpart $D^{Classical}_{Max}$,

$$D^{QM}_{Bell} < D^{Classical}_{Max}, \tag{143}$$

by therefore establishing the impossibility for an $SU(2)$-invariant system to admit identical classical images. Q. E. D.

Santilli [10] has shown that inequality (141) is inapplicable for the same particles when they are in interior dynamical conditions., e.g., when they are in the core of a star or, at the limit, when they are in the interior of a gravitational collapse.

Considers two extended particles also denoted 1 and 2. Suppose that said particles verify the regular $\hat{SU}(2)$ isosymmetry with spin $1/2$ (Section 3.3), thus implying the elaboration via isomathematics (Section I-3) and the verification of the isotopic branch of hadronic mechanics (Section I-4).

Suppose that the two extended particles with spin $1/2$ are characterized by the following isotopic elements:

$$1) \begin{array}{c} Particle\ 1: \ \hat{T}_1 = Diag(\lambda_1, 1/\lambda_1), \\ Particle\ 2: \ \hat{T}_2 = Diag(\lambda_2, 1/\lambda_2), \end{array} \tag{144}$$

with realization (83) of the Pauli-Santilli isomatrices.

Suppose that, due to preceding interactions, the two extended particles are in single overlapping/entanglement thus having opposite spins.

Let $\hat{I}_1$ and $\hat{I}_2$ be the isounits for particles 1 and 2, respectively. The systems of the assumed two isoparticles is then characterized by the total isounit

$$\hat{I}_{tot} = \hat{I}_1 \times \hat{I}_2 = \frac{1}{\hat{T}_{tot}} = \frac{1}{\hat{T}_1 \times \hat{T}_2} \tag{145}$$

In this case, the total isostate on the Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ [18] over the isofield of isocomplex isonumbers $ca\hat{l}C$ [19] is given by

$$|\hat{S}_{1-2}> = \frac{1}{\sqrt{2}}(|\hat{S}_{1\uparrow} > \hat{\times}|\hat{S}_{2\downarrow} > -|\hat{S}_{1\downarrow} > \times|\hat{S}_{2\uparrow} >) \tag{146}$$

The lack of validity of inequality (141) for irregular isorepresentations of $\hat{SU}(2)$ is evident (e.g., because of the anomalous spin isoeigenvalues) and, as such, it is ignored.

A significant aspect of Ref. [10] is the proof of the inapplicability of inequality (141), not only for regular isorepresentation of $\hat{SU}(2)$, but also when such isorepresentations are isounimodular, Eqs. (144).

Let a_1, b_1, a_2, b_2 be unit vectors along the z-axis of an iso-Euclidean isospace. Introduce the isoprobability (Eq. (32.39), page 99, Ref. [29])

$$\begin{aligned} \hat{P}(a,b) =< \hat{S}_{1-2}| \star (\hat{\Sigma}_1 \hat{\otimes}_1 a) \times (\hat{\Sigma}_2 \hat{\otimes}_2 b) |\hat{S}_{1-2} > \hat{I}_{tot} = \\ =< \hat{S}_{1-2}| \star (\hat{\sigma}_1 \otimes a) \times (\hat{\sigma}_2 \otimes b) |\hat{S}_{1-2} > \hat{I}_{tot} \end{aligned} \tag{147}$$

with isonormalization (here referred to individual diagonal elements of isotopic elements and isounits)

$$< \hat{S}_{1-2}| \star |\hat{S}_{1-2} > =< \hat{S}_{1-2}|\hat{T}_{tot}|\hat{S}_{1-2} > = \hat{I}_{tot} \tag{148}$$

where: $\star$ is the total isoproduct; $\hat{\otimes}_k,\ k = 1, 2,$ is the isoscalar isoproduct; and we have used simplifications of the type

$$\hat{\Sigma}_1 \hat{\otimes}_1 a = (\hat{\sigma}_1 \hat{I}_1)(\hat{T}_1 \otimes) a = \hat{\sigma}_1 \otimes a. \tag{149}$$

An isotopy of the conventional case yields the following isobasis, Eq. (6.5) of Ref. [10],

$$|S_{1-2} > = \frac{1}{2} \left\{ \begin{pmatrix} \lambda_1^{-1/2} \\ 0 \end{pmatrix} \begin{pmatrix} o \\ \lambda_2^{1/2} \end{pmatrix} - \begin{pmatrix} 0 \\ \lambda_2^{1/2} \end{pmatrix} \begin{pmatrix} \lambda_1^{-1/2} \\ 0 \end{pmatrix} \right\} \tag{150}$$

The appropriate use of products and isoproducts then yield expression (5.6) Ref. [10], i.e.,

$$\begin{aligned} < \hat{S}_{1-2}|\hat{T}_{tot}(\hat{\sigma}_1 \otimes_1 a) \times (\hat{\sigma}_2 \otimes_2 b)\hat{T}_{tot}|\hat{S}_{1-2} > = \\ = -a_x b_x - a_y b_y - \tfrac{1}{2}(\lambda_1 \lambda_2^{-1} + \lambda_1^{-1} \lambda_2) a_z b_z \end{aligned} \tag{151}$$

The continuation of the isotopy of the conventional case, yields the main result, Eq. (5.8) of Ref. [10], which provides the following *isotopic "completion" of Bell's inequality,*

$$\begin{aligned} \hat{D}^{HM}_{Max} = D^{HM}_{Max} \hat{I}_{tot} = \\ Max|\hat{P}(a_1, b_1) - \hat{P}(a_1, b_2) + \hat{P}(a_2, b_1) + \hat{P}(a_2, b_2)| = \\ = [\tfrac{1}{2}(\lambda_1 \lambda_2^{-1} + \lambda_1^{-1} \lambda_2) D^{QM}_{Bell}\ \hat{I}_{tot}. \end{aligned} \tag{152}$$

with consequential:

LEMMA 3.6. Extended particles within physical media that are invariant under the Lie-Santilli isosymmetry $\hat{SU}(2)$ admit identical classical counterparts.

PROOF: Isoinequality (141) establishes the lack of universal validity of Bell's inequality (128) because the factor $\frac{1}{2}(\lambda_1 \lambda_2^{-1} + \lambda_1^{-1} \lambda_2)$ can have values *bigger* than one, thus implying

$$D^{HM}_{Max} \ \geq \ D^{QM}_{Bll}\,. \tag{153}$$

Consider then a classical iso-Euclidean isospace $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ representing motion of classical extended particles 1 and 2 within physical media [29] with isometric elements

$$\hat{\delta}_{11} = 1,\ \ \hat{\delta}_{22} = 1,\ \ \hat{\delta}_{33} = \frac{1}{2}(\lambda_1 \lambda_2^{-1} + \lambda_1^{-1} \lambda_2) = 2, \tag{154}$$

in which case

$$D_{Max}^{HM} \equiv htD_{Max}^{Classical}, \quad (155)$$

by therefore establishing that systems of extended particles within physical media verifying the $\hat{SU}(2)$ isosymmetry admits an identical classical counterpart along the EPR argument. Q.E.D.

It is an instructive exercise for the interested reader to prove ghat the above lemma also holds for different isorenormalizations, e.g., Eqs. (171) of next section, with the understanding that different isorenormalizations imply different isobasis and different hidden variable terms in Eqs. (151).

Note the crucial role of hidden variables for the proof of Lemma 3.6. It is an instructive exercise for interested readers to prove that Lemma 3.6 holds for any other regular, isounimodular isorepresentation of the isotopic $\hat{SU}(2)$ symmetry in terms of hidden variables presented in Section 3.3.

The proof of the lack of applicability of von Neumann's theorem [5] for extended particles n interior conditions is elementary. Recall that von Neumann's theorem is based on the uniqueness of the eigenvalues E of a Hermitean operator $H,\ H|\psi >= E|\psi >$ under unitary transformation on $\mathcal{H}$,

$$UH|\psi > U^\dagger = UE|\psi > U^\dagger = EU|\psi > U^\dagger,\ \ UU^\dagger = U^\dagger U = I \quad (156)$$

under the tacit assumption of point particles in vacuum.

By contrast, when the same particles is in interior conditions, it is subjected to an infinite number of different physical different interactions with the medium represented by the isotopic element $\hat{T}$ with ensuing isoeigenvalue equation (Section I-4), [9],

$$abel1H \star |\hat{\psi}_{\hat{T}}r >= HT|\hat{\psi}_{\hat{T}} >= E_{\hat{T}}|\hat{p}si_{\hat{T}} > \quad (157)$$

thus establishing that a given quantum mechanical operator H representing the energy of an extended particle in interior conditions has an infinite number of *generally different* isoeigenvalues $E_{\hat{T}}$ depending on the infinite number of different interior conditions.

Note that, for each given $\hat{T}$ the isoeigenvalue $E_{\hat{T}}$ is invariant under isounitary isotransformations (Section I-3-9).

3.8. Apparent admission of classical determinism.

Consider a point-like particle in empty space represented in the 3-dimensional Euclidean space $E(r, \delta, I)$, where r represents coordinates, $\delta = Diag.(1, 1, 1)$ represents the Euclidean metric and $I = Dian(1, 1, 1,)$ represents the space unit.

Let the operator representation of said point-like particle be done in a Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$ with states $\psi(r)$ and familiar normalization

$$< \psi(r)|\ |\psi(r) >= \int_{-\infty}^{+\infty} \psi(r)^\dagger \psi(r) dr = 1. \quad (158)$$

As it is well known, the primary objections against the EPR argument [2]- [6] were based on Heisenberg uncertainty principle according to which *the position r and the momentum p of said particle cannot both be measured exactly at the same time.*

By introducing the *standard deviations* Δr and Δp, the uncertainty principle is generally written in the form

$$\Delta r \Delta p \ \geq \ \frac{1}{2}\hbar, \quad (159)$$

which is easily derivable via the vacuum expectation value of the canonical commutation rule

$$\Delta r\Delta p \ \geq \ |\frac{1}{2i} < \psi| \ [r,p] \ |\psi > | = \frac{1}{2}\hbar. \tag{160}$$

Standard deviations have the known form (see, e.g., Ref. [?]) with $\hbar = 1$

$$\begin{aligned} \Delta r &= \sqrt{< \psi(r)|[\ r - (< \psi(r)|\ r\ |\psi(r)>)]^2|\psi(r)>}, \\ \Delta p &= \sqrt{< \psi(p)|\ [p - (< \psi(p)|\ p\ |\psi(p)>)]^2|\psi(p)>}, \end{aligned} \tag{161}$$

where $\psi(r)$ and $\psi(p)$ are the wavefunctions in coordinate and momentum spaces, respectively.

We consider now an extended particle, this time, in interior conditions, e.g., in the core of a star, classically represented by the *iso-Euclidean isospace* $\hat{E}(\hat{r},\hat{\delta},\hat{I})$ with isounit $\hat{I} = 1/\hat{T} > 0$, isocoordinates $\hat{r} = r\hat{I}$, isometric

$$\hat{\delta} = \hat{T}\delta, \tag{162}$$

and isotopic element (4)) under conditions (5).

For simplicity, we assume that the extended particle has no Hamiltonian interactions due to the dominance of the latter interactions over the former.

Consequently, we can represent the extended particle in the isospace $\hat{\mathcal{H}}$ over the isofield $\hat{\mathcal{C}}$ and introduce the time independent *isoplanewave* [17]

$$\begin{aligned} &\hat{\psi}(\hat{r}) = \hat{\psi}(\hat{r})\hat{I} = \\ &= \hat{N} \star (\hat{e}^{\hat{i}\star\hat{k}\star\hat{r}})\hat{I} = N(e^{ik\hat{T}\hat{r}})\hat{I}, \end{aligned} \tag{163}$$

where $\hat{N} = N\hat{I}$ is an *isonormalization isoscalar,* $\hat{k} = k\hat{I}$ is the *isowavenumber,* and the isoexponentiation is given by Eq. (I-22) [25].

The corresponding representation in isomomentum isospace is given by

$$\hat{\psi}(\hat{p}) = \hat{M} \star \hat{e}^{\hat{i}\star\hat{n}\star\hat{p}}, \tag{164}$$

where $\hat{M} = M\hat{I}$ is an isonormalization isoscalar and $\hat{n} = n\hat{I}$ is the isowavenumber in isomomentum isospace.

The *isopropability isofunction* is then given by (Ref. [29] page 99)

$$\hat{\mathcal{P}} = \hat{<}| \star |\hat{>} =< \hat{\psi}(\hat{r})|\ T\ |\hat{\psi}(\hat{r}) > I \tag{165}$$

that, written in terms of isointegrals (Ref. [28] page 354), becomes

$$\begin{aligned} &\hat{\int}_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \star \hat{\psi}(\hat{r}) \star \hat{d}\hat{r} = \\ &= \hat{\int}_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{\psi}(\hat{r})(dr + r\hat{T}d\hat{I}) \end{aligned} \tag{166}$$

where one should keep in mind that the isodifferential $\hat{d}\hat{r}$ given by Eqs. (I-29).

The *isoexpectation isovalues* of a Hermitean operator $\hat{Q}$ are then given by [29]

$$\hat{<}| \star \hat{Q} \star |\hat{>} =< \hat{\psi}(\hat{r})| \star \hat{Q} \star |\hat{\psi}(\hat{r}) >=$$
$$= \int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \star \hat{Q} \star \hat{\psi}(\hat{r}) \hat{d}\hat{r} = \tag{167}$$
$$= \int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{Q} \hat{\psi}(\hat{r}) \hat{d}\hat{r},$$

with corresponding expressions for the isoexpectation isovalues in isomomentum isospace.

Santilli then introduced apparently for the first time in Ref. [11] the *isotopic operator*

$$\hat{\mathcal{T}} = \hat{T}\hat{I} = I, \tag{168}$$

that, despite its seemingly irrelevant value, is indeed the correct operator formulation of the isotopic element for the "completion" of the isoproduct from its scalar form (1) to the isoscalar form

$$\hat{n}^{\hat{2}} = \hat{n} \star \hat{n} = \hat{n} \star \hat{\mathcal{T}} \star \hat{n} = n^2 \hat{I}. \tag{169}$$

In Sections 3.6, 3.7, we have shown that the Lie-Santilli isosymmetry $\hat{SU}(2)$ admits an explicit and concrete realization of hidden variables that allowed the construction of identical classical counterparts for interior dynamical systems.

Ref. [11] introduced the isoexpectation isovalue of the isotopic operator

$$\hat{<}| \star \hat{\mathcal{T}} \star |\hat{>} =< \hat{\psi}(\hat{r})| \star \hat{\mathcal{T}} \star |\hat{\psi}(\hat{r}) > \hat{I} =$$
$$= \int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{T} \hat{\psi}(\hat{r}) \hat{d}\hat{r} \tag{170}$$

and assumed the isonormalization (again, intended for diagonal matrix elements)

$$\hat{<}| \star \hat{\mathcal{T}} \star |\hat{>} =$$
$$= \int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{T} \hat{\psi}(\hat{r}) \hat{d}\hat{r} = \hat{T}. \tag{171}$$

Consider then the *isostandard isodeviation* for isocoordinates $\Delta\hat{r} = \Delta r \hat{I}$ and isomomenta $\Delta\hat{p} = \Delta p \hat{I}$, where Δr and Δp are the standard deviations in our space.

By using isocanonical isocommutation rules (I-81), we obtain the expression

$$\Delta\hat{r} \star \Delta\hat{p} = \Delta r \Delta p \hat{I} \;\approx\; \tfrac{1}{2}| < \hat{\psi}(\hat{r})| \star [\hat{r} \hat{,} \hat{p}] \star \hat{\psi}(\hat{r}) > | =$$
$$= \tfrac{1}{2}| < \hat{\psi}(\hat{r})| \hat{T} \, [\hat{r} \hat{,} \hat{p}] \, \hat{T} | \hat{\psi}(\hat{r}) > . \tag{172}$$

By eliminating the common isounit $\hat{I}$, Ref. [11] achieved the desired result here called *isodeterministic isoprinciple*

$$\Delta r \Delta p \approx \tfrac{1}{2}| < \hat{\psi}(\hat{r})| \star [\hat{r} \hat{,} \hat{p}] \star |\hat{\psi}(\hat{r}) >=$$
$$= \tfrac{1}{2}| < \hat{\psi}(\hat{r})| \hat{T} \, [\hat{r} \hat{,} \hat{p}] \, \hat{T} | \hat{\psi}(\hat{r}) >= \tag{173}$$
$$\int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{T} \hat{\psi}(\hat{r}) \hat{d}\hat{r} = T \;\ll\; 1$$

where the property $\Delta r \Delta p \ll 1$ follows from the fact that the isotopic element $\hat{T}$ has null value for gravitational collapse (Section I-4-11) values smaller than 1 established from the fitting of all experimental data dealing with hadronic media such as hadrons, nuclei and stars [30].

In this way, thanks to a laborious scientific journey initiated at Harvard University in late 1977, and than to contributions by numerous mathematicians, theoreticians and experimentalists [10]-[69], Santilli reached the following verification of the EPR argument [11]:

LEMMA 3.7 (ISODETERMINISTIC PRINCIPLE): The isostandard isodeviations for isocoordinates $\Delta \hat{r}$ and isomomenta $\Delta \hat{p}$, as well as their product, progressively approach classical determinism for extended particles in the interior of hadrons, nuclei, and stars, and achieve classical determinism at the extreme densities in the interior of gravitational collapse.

PROOF: Define the isostandard isodeviations via the following isotopy of quantum mechanical expressions (161) (where we ignore the common multiplication by the isounit)

$$\begin{aligned} \Delta r &= \sqrt{< \hat{\psi}(\hat{r})|[\, \hat{r} - < \hat{\psi}(\hat{r})| \star \hat{r} \star |\hat{\psi}(\hat{r}) >]^{\hat{2}} |\hat{\psi}(\hat{r}) >}, \\ \Delta p &= \sqrt{< \hat{\psi}(\hat{p})|\, [\hat{p} - < \hat{\psi}(\hat{p})| \star \hat{p} \star |\hat{\psi}(\hat{p}) >]^{\hat{2}} |\hat{\psi}(\hat{p}) >}, \end{aligned} \tag{174}$$

where the differentiation between the isotopic elements for isocoordinates and isomomenta is ignored for simplicity. But the isotopic element represents the interactions of the particle with the physical medium and tends toward null values for gravitational collapse, Eqs. (I-91) (I-92). Therefore, isosquare in expression (171) implies the expressions

$$\begin{aligned} \Delta r &= \sqrt{\hat{T} < \hat{\psi}(\hat{r})|[\, \hat{r} - < \hat{\psi}(\hat{r})| \star \hat{r} \star |\hat{\psi}(\hat{r}) >]^2 |\hat{\psi}(\hat{r}) >}, \\ \Delta p &= \sqrt{\hat{T} < \hat{\psi}(\hat{p})|\, [\hat{p} - < \hat{\psi}(\hat{p})| \star \hat{p} \star |\hat{\psi}(\hat{p}) >]^2 |\hat{\psi}(\hat{p}) >}, \end{aligned} \tag{175}$$

that approach indeed null value under the indicated limit conditions of gravitational collapse

$$\begin{aligned} Lim_{\hat{T}=0} \Delta r &= 0, \\ Lim_{\hat{T}=0} \Delta p &= 0, \end{aligned} \tag{176}$$

Q.E.D..

3.9. Isoparticles.

Relativistic, quantum mechanical bound states of point-particles in vacuum studied in the 20th century have been based on the notion of *elementary relativistic particles* which are technically intended as unitary irreducible representations of the spinorial covering of the Lorentz-Poincaré symmetry $\mathcal{P}$. Said particles are characterized as points on the Minkowskian space-time $M(x, \eta, I)$ and said representations are formulated on a Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$.

By recalling that massive points cannot sense contact/resistive forces, *the point-like character of the particles has the important consequence of solely admitting linear, local and action-at-a-distance/potential interactions that, in turn, can only affect kinematical characteristics, while leaving the intrinsic characteristics of spin, mass, charge, parity, etc. completely unchanged.*

Illustrations of the apparent proof of the EPR argument studied in Sections 3.7 and 3.8 require the use, this time, of bound states of extended particles in conditions of deep overlapping/entanglement.

In the latter case, *the extended character of mutually overlapping particles implies the emergence of additional on-linear, non-local and contact/non-potential interactions that, in turn, generally cause the alteration, (called mutation) of the intrinsic characteristics of particles, in additionl to changes of their kinematical characteristics.*

In the hope of preventing the rather instinctive interpretations of the illustrative examples of the next section with the 20th century notion of particles, we introduce the following:

DEFINITION 3.9.1. "Elementary relativistic isoparticle" refer defined by isounitary isoirreducible isorepresentations of the spinorial covering of the Lorentz-Poincaré-Santilli isosymmetry $\hat{\mathcal{P}}$ [61] (Section 2.11) formulated on an iso-Minkowskian isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$[22] with:

1) Space-time isocoordinates $\hat{x} = x\hat{I}$;
2) Hadronic linear momentum (I-79);
3) Hadronic angular momentum (121) with isocommutation rules (122) and isoeigenvalues (123);
4) Hadronic spin (94) with isocommutation rules (96) and isoeigenvalues (97); and
5) Relativistic isoequations (I-87) and (I-88).

The notion of *nonrelativistic isoparticle* is presented in detail in Ref. [70] and its knowledge is tacitly assumed when dealing with non-relativistic models.

4. ILLUSTRATIVE EXAMPLES

4.1. Foreword.

Being dimensionless, *Newtonian massive points cannot experience any resistive force.* By recalling that Newton's equation have been the foundations of physics for the past four centuries, 20th century mainstream physics has been developed without the notion of resistive force, with ensuing lack of treatment of the *pressure* exercised by a medium on a particle in its interior, contrary to clear evidence that a proton in the core of a star is exposed to extremely big pressures (Figure 9).

In this section, we shall illustrate the fact that, once admitted, interior pressure characterizes standard deviations $\Delta\hat{r}$ and $\Delta\hat{p}$ that, being *constrained* by said pressure, verify the isodeterministic principle of Lemma 3.9, by progressively approaching classical determinism with the increase of the pressure, up to the achievement of classical determinism for the interior of gravitational collapse as predicted by Einstein, Podolsky and Rosen [1].

A physically important notion emerging from the examples provided below is that *the EPR argument appears to be verified buy strong interactions* because, as indicated by Santilli in the 1978 Harvard University paper [16], contact non-Hamiltonian interactions responsible for the synthesis of the neutron and other hadrons are short range, strongly attractive and non-Hamiltonian (technically identified as *variationally non-selfadjoint interactions* [24]), thus providing a conceivable, first known, explicit and concrete representation of strong interactions.

The models outlined in this section were first proposed by Santilli in Ref. [16] in their time irreversible form, as requested for decaying bound states, thus being elaborated

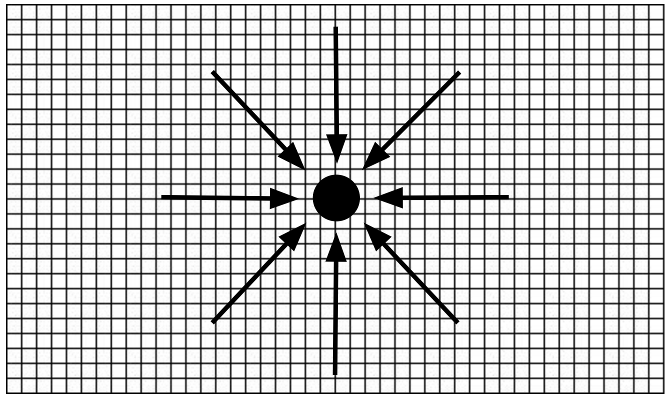

Figure 9: *In this figure, we present a conceptual rendering of a central notion needed for the study of the EPR argument, namely, the pressure exercised by hyperdense media on extended particles in their interior. Such a notion is absent in the mainstream physics of the 20th century due to the study of point-like particles that, as such, cannot experience any pressure or contact non-Hamiltonian interaction of any type.*

with the Lie- admissible genomathematics, in which case, the need for a "completion" oif quantum mechanics is beyond scientific doubt.

However, the objections against the EPR argument [2] - [6] have been formulated for conventional quantum axioms, thus implying the sole consideration of time-reversal invariant states. In this section, we illustrate the need for a "completion" of quantum mechanics also for reversal invariant systems, provided that they consist of extended particles in interior conditions.

Therefore, unstable strongly interacting particles are hereon studied for such a small period of time to allow their time-reversal invariant approximation.

4.2. Particles under pressure. One of the simplest possible illustrations of Lemma 3.9 is given by a particle in the center core of a star, thus being under extreme pressures π from the surrounding hadronic medium in all radial directions (Figure 9).

By ignoring particle reactions in first approximation, the conditions here considered can be rudimentarily represented for very short period of time by assuming that the function $\Gamma > 0$ of the isotopic element (2) is a constant linearly dependent on the pressure π, resulting in a realization of the isotopic element of the type

$$\hat{T} = e^{-w\pi} \ll 1, \ \ \hat{I} = e^{+w\pi} \gg 1, \tag{177}$$

where w is a positive constant.

Isodeterministic principle (173) for the considered particle is then given by

$$\Delta r \Delta p \approx \frac{1}{2} e^{-w\pi} \ \ll \ 1, \tag{178}$$

and tends to null values for diverging pressures.

The above example illustrates the consistency of isorenormalization (171) because a constant isotopic element verifies the isonormalization

$$\hat{<}\hat{\psi}(\hat{r})|\hat{T}|\hat{\psi}(\hat{r}) > \hat{I} =$$

$$T < \hat{\psi}(\hat{r})| \, |\hat{\psi}(\hat{r}) > \hat{I} = \quad (179)$$

$$< \hat{\psi}(\hat{r})| \, |\hat{\psi}(\hat{r}) > .$$

but not necessarily other isorenormalizations.

Note that we have considered an individual extended particle immersed in a hadronic medium, rather than the bound state of extended particles in condition of mutual penetration studied in the next sections.

Consequently, isotopic element (177) represents a *subsidiary constraint on standard deviations* caused, as indicated, by the pressure of the surrounding hadronic medium on the particle considered.

It is easy to see that, since $\Gamma(\pi) > 0$, more complex functional dependence on the pressure π continue to verify Lemma 3.9.

4.3. Non-relativistic hadronic bound states.

Due to the local character of the conventional differential calculus underlying its dynamical equations, quantum mechanical bound states solely admit *point-like particles* under linear, local and potential interactions (technically identified as *variationally self-adjoint interactions* [24]), as it is the case for the familiar Schrödinger equation for the bound state of two point-like particles with Coulomb potential $V(r)$ in a Euclidean space $E(r, \delta, I)$ formulated on a Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$ (where $\hbar = 1$)

$$i\frac{\partial}{\partial t}\psi(t,r) = H\psi(t,r) = \left[\frac{1}{m}\Sigma_k p_k p_k - V(r)\right]\psi(t,r) =$$

$$= \left[\frac{1}{m}\Sigma_k(-i\partial_k)(-i\partial_k) - V(r)\right]\psi(t,r) = \quad (180)$$

$$= \left[-\frac{1}{m}\Delta_r - V(r)\right]\psi(t,r) = E\psi(t,r).$$

By contrast, *hadronic bound states* are bound states of *extended particles* at mutual distances smaller or equal to the *hadronic horizon* (Figure 1.13)

$$R = \frac{1}{b} \approx 10^{-13} \, cm. \quad (181)$$

In such a region, bound states verify hadronic mechanics and are represented with isomathematics (Section I-3) and isomechanics (Section I-4) for the case of time-reversal invariant bound states, or genomathematics and genomechanics for time irreversible bound states (Section I-2).

By using the methods outlined in Paper I and in the preceding sections, hadronic bound states assumed to be stable in first approximation (thus being time reversal invariant) are characterized by the following main features:

1) The bound states occur between *isoparticles,* namely, isoirreducible isorepresentations of the isospinorial covering of the Galileo-Santilli isosymmetry $\hat{\mathcal{G}}$ for non-relativistic

treatments (Sections 2.5.1 and 3.9) or of the Lorentz-Poincaré-Santilli isosymmetry $\hat{\mathcal{P}}$ for relativistic treatments (Sections 2.5.11 and 3.9).

2) The representation of the extended character of the isoparticles is done with isoproducts (1) and isotopic elements (4), resulting in iso-Schrödinger equations of type (I-80), while the deep mutual penetration of the wavepackets and/or charge distributions of isoparticles generates novel, contact, non-linear, non-local and non-potential interactions represented by the exponent of the the isotopic element (4) and other means. Note that the latter interactions are short range, strongly attractive and non-Hamiltonian according to all studies conducted to date in the field, thus allowing an initial yet explicit and concrete realization of strong interactions [16].

3) By recalling that isosymmetries $\hat{\mathcal{G}}$ and $\hat{\mathcal{P}}$ are all *irregular* realizations of the Lie-Santilli isotheory (Sections I-2.7 and 2.5.4), a necessary condition for the invariance of hadronic dynamical equations under isosymmetries is that contact interactions *cannot* be derived via non-unitary transforms of quantum mechanical potentials, thus being basically *new* interactions. The physically equivalent property is that, as it is well known, *strong interaction cannot be derived via non-unitary or other known transformations of electromagnetic interactions* and this confirms the necessary irregular character of the Lie-Santilli theory and hadronic dyamical equations.

Tne notion of hadronic bound states was proposed, apparently for the first time, by Santilli in the 1978 Ref. [16] and thereafter was extensively studied and applied in various works by various authors (see the 2001 monograph [33], the 2011 independent review [44], and papers quoted therein).

We believed it is important to review the derivation of the basic non-relativistic and relativistic isoequations for hadronic bound states to show their apparent verification of the isodeterministic principle of Lemma 3.9.

The fundamental *non-relativistic, irregular isoequation of a time-reversal invariant hadronic bound state* of two isoparticles at mutual distances of the order of the hadronic horizon $R = 10^{-13}\ cm$ in an iso-Euclidean isospace $\hat{R}(\hat{r}, \hat{\delta}, \hat{I})$ formulated on a Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ over the isofield of isocomplex isonumbers $\hat{\mathcal{C}}$ can be written

$$\begin{aligned} i\frac{\hat{\partial}}{\hat{\partial}\hat{t}}\hat{\psi}(\hat{t},\hat{r}) &= \left[\tfrac{1}{m}\Sigma_k \hat{p}_k \star \hat{p}_k \pm \hat{V}(\hat{r}) - \hat{S}(\hat{\psi})\right] \star \hat{\psi}(\hat{t},\hat{r}) = \\ &= \left[\tfrac{1}{m}\Sigma_k \hat{p}_k \hat{T} \hat{p}_k \hat{T} \pm V(\hat{r}) - S(\hat{\psi})\right] \hat{\psi}(\hat{t},\hat{r}) = E\hat{\psi}(\hat{t},\hat{r}), \end{aligned} \tag{182}$$

where $\hat{V}(\hat{r}) = V(\hat{r})\hat{I}$; $\hat{S}(\hat{\psi}) = S(\hat{\psi})\hat{I}$ represents the novel short range, strongly attractive force; the value $-\hat{V}(\hat{r})$ occurs for bound states with opposite charge (as it is the case for the synthesis of hadrons reviewed below); and the value $+\hat{V}(\hat{r})$ occurs for isoparticles with the same charge (as occurring for valence electron bonds reviewed below).

Due to the large representational capabilities of isoequations (182), we use the following simplifying assumptions:

1) The isotime is equal to the conventional time, $\hat{t} = t\hat{I}_t = t,\ \hat{I}_t = 1$;

2) The orbits of the isoparticles, being extremely small, are assumed to be nearly constant circles, thus implying that the n_k characteristic quantities of then isotopic element (4) can be normalized to the sphere, $n_k = 1,\ k = 1, 2, 3$;

3) The isotopic element is assumed to be given by to the exponential term of Eq. (4) with realization of the non-linear, non-local and non-potential interactions of the type (Eq.

(4.7), page 170 Ref. [33])

$$\hat{T} = e^{-\Gamma} = e^{-N\psi/\hat{\psi}} \approx 1 - N\psi/\hat{\psi}, \tag{183}$$

where ψ behaves like the solution of quantum equation (180),

$$\psi(r) \approx N_1 e^{-br}, \tag{184}$$

, and $\hat{\psi}$ behaves like the solution of the hadronic equation expected to be of the type

$$\hat{\psi} \approx N_2(1 - e^{\frac{-br}{r}}). \tag{185}$$

where N_1 and N_2 are positive normalization constants.

We therefore have the following explicit form of the isotopic element

$$\hat{T} = e^{-N\psi/\hat{\psi}} = e^{-W_1 \frac{e^{-br}}{(1-e^{-br})/r}} \approx 1 - W_1 \frac{e^{-br}}{(1 - e^{-br})/r}, \tag{186}$$

exhibiting the Hulten potential

$$V_{hp} = W_2 \frac{e^{-br}}{1 - e^{-br}}, \tag{187}$$

directly in the exponent of the isotopic element, where W_1 and W_2 are normalization constants.

It should be recalled tat Santilli suggested the use of the Hulten potential tin he 1978 paper [16], Eq. (5.1.6), page 833, as an *initial yet explicit and concrete representation of strong interactions.*

Under the above simplifying assumptions, isotopic element (186) verifies the central condition for the validity of the isodeterministic principle inside the hadronic horizon (Lemma 3.9), in a way fully compatible with the validity of conventional uncertainties outside said hadronic horizon

$$\begin{aligned} |\hat{T}| \; &\ll \; 1, \\ Lim_{r \gg R} \hat{T} \; &= \; 1. \end{aligned} \tag{188}$$

Under the above simplified assumptions, and the use of the isolinear isomomentum (I-79), the projection of isodynamical equation (182) into our Euclidean space can be written in the form first derived in Eq. (5.1.9), page 833, Ref [16]

$$\begin{aligned} i\tfrac{\partial}{\partial t}\hat{\psi}(t,r) &= \left[\tfrac{1}{m}\Sigma_k \hat{p}_k \star \hat{p}_k \star \pm \tfrac{e^2}{r} - W_2 \tfrac{e^{-br}}{1-e^{-br}}\right] \hat{\psi}(t,r) = \\ &= \left[\tfrac{1}{m}\Sigma_k(-i\hat{I}\partial_k)(-i\hat{I}\partial_k) \pm \tfrac{e^2}{r} - W_2 \tfrac{e^{-br}}{1-e^{-br}}\right] \hat{\psi}(t,r) = \\ &= \left[-\tfrac{\hat{I}^2}{m}\Delta_r \pm \tfrac{e^2}{r} - W_2 \tfrac{e^{-br}}{1-e^{-br}}\right] \hat{\psi}(t,r) = \\ &= E\hat{\psi}(t,r). \end{aligned} \tag{189}$$

But the Hulten potential behaves like the Coulomb potential at short distances

$$V_{hp} \approx K \frac{1}{r}, \tag{190}$$

where K is a positive constant. Consequently, as noted in the originating paper [16], *the Hulten potential absorbs the Coulomb potential resulting in a short range strongly attractive force irrespective of whether the Coulomb force is attractive or repulsive.*

The dominance of the Hulten potential over repulsive Coulomb forces was identified, apparently for the first time, by A. O. E. Animalu and R. M. Santilli in the 1995 paper [?] which presented the first time an explicitly identified, attractive force between the two identical electrons of theCooper pair in superconductivity. *Such a 'charge independence' of the Hulten potential is here assumed as an additional support of the representation by strong interactions via contact, short range, non-Hamiltonian interactions.*

Property (190) implies the following simplified *non-relativistic, irregular isoequation for time-reversal invariant hadronic bound states*

$$i\frac{\partial}{\partial t}\hat{\psi}(t,r) = \left[-\frac{\hat{I}^2}{m}\Delta_r - W\frac{e^{-br}}{1-e^{-br}}\right]\hat{\psi}(t,r) = E\hat{\psi}(t,r) \tag{191}$$

where W is a positive constant renormalized following the absorption of the Coulomb potential.

The radial equation can then be written (Eq. (5.1.14a), page 836, Ref. [16])

$$\begin{gathered}\left[\tfrac{1}{r^2}(\tfrac{d}{dr}r^2\tfrac{d}{dr}) + \bar{m}(E_{Hulten} + W\tfrac{e^{-br}}{1-e^{-br}}\right] = 0,\\ \bar{m} = m/\rho, \quad \rho \approx |\hat{I}^2|.\end{gathered} \tag{192}$$

Its solutions were studied in all details (including boundary conditions) in Ref. [*loc. cit.*], pages 837-841, and were reduced to the numeric values of two positive constants k_1 and k_2 which are the solution of the following equations (Ref. [16], Eq. (

$$\begin{gathered}k_1[k_1 - (k_2-1)^3] = \tfrac{1}{2c}E_{tot}R,\\ \tfrac{(k_2-1)^3}{k_1} = \tfrac{9\times10^6}{3\pi c}\tfrac{R}{\tau}\end{gathered} \tag{193}$$

where E_{tot} is the total energy of the hadronic bound state, R is the hadronic horizon assumed to be equal to its charge radius, and τ is its meanlife.

The energy spectrum results to be the typical *finite* spectrum of the Hulten potential

$$|E_{Hulten}| = \frac{1}{4R^2\bar{m}}(k_2\frac{1}{n} - n)^2 \tag{194}$$

where n is a positive integer.

Since we have ignored Coulomb interactions and have solely assumed *contact* interactions represented with the exponent of the isotopic element (4), *the value of E_{Hulten} is expected to be zero*

$$|E_{Hulten}| = \frac{1}{4R^2\bar{m}}(k_2\frac{1}{n} - n)^2 = 0, , \tag{195}$$

because contact interactions do not carry potential energy (this is classically the case for a ballon moved by winds in our atmosphere).

Therefore, *the Hulten potential for consistent hadronic bound states is expected to admit one single energy level, the ground state.* This is due to the fact that all possible excited states imply radial distances bigger than the hadronic horizon R, with consequential recovering of quantum mechanics.

In particular, property (195) is solely possible for

$$k_1 > 0, \; k_2 \geq 1. \tag{196}$$

The absence of a spectrum of energies was called in Section 6 of Ref. [16] the *hadronic suppression of quantum mechanical energy spectra* in order to differentiate the *classification of hadrons into families* (which is characterized by energy spectra) from the *structure of each individual hadron of a given classification family.*

4.4. Relativistic hadronic bound states

The relativistic counterpart of Eqs. (188) was identified, apparently for the first time, in Ref. [61] and formulated in in the isoproduct of a real-valued iso-Minkowski isospace for the orbital motion and a complex valued iso-Euclidean isospace for the hadronic spin

$$\hat{S}_{tot} = \hat{M}(\hat{x}, \hat{\eta}, \hat{I}_{orb}) \star \hat{R}(\hat{z}\hat{\delta}, \hat{I}_{spin}), \tag{197}$$

resulting in the following *irregular extension of the Dirac-Santilli isoequation (I-88)* ,

$$\begin{aligned} &[\hat{\Omega}^{\mu\nu} \star \hat{\Gamma}_\mu \star \hat{\partial}_\nu + \hat{M} \star \hat{C} - \hat{V}_{hp}]\hat{\psi}(\hat{x}) = \\ &= (-i\hat{I}\hat{\eta}^{\mu\nu}\hat{\gamma}_\mu \partial_\nu + mC - V_{hp})\hat{\psi}(\hat{x}) = 0, \end{aligned} \tag{198}$$

where $\hat{S} = S\hat{I}_{orb}$ represents strong interactions, and the *Dirac-Santilli isogamma isomatrices* $\hat{\Gamma} = \hat{\gamma}\hat{I}$ are given by

$$\begin{aligned} \hat{\gamma}_k &= \frac{1}{n_k} \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \\ \hat{\gamma}_4 &= \frac{i}{n_4} \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}; \end{aligned} \tag{199}$$

where $\hat{\sigma}_k$ are the *irregular Pauli-Santilli isomatrices* studied in Section 3.4 with the following *anti-isocommutation rules*

$$\begin{aligned} \{\hat{\gamma}_\mu \hat{,} \hat{\gamma}_\nu\} &= \hat{\gamma}_\mu \hat{T} \hat{\gamma}_\nu + \hat{\gamma}_\nu \hat{T} \hat{\gamma}_\mu = \\ &= 2\hat{\eta}_{\mu\nu}. \end{aligned} \tag{200}$$

where $\hat{\eta}$ is the isometric of the orbital iso-Minkowskian isospace.

4.5. Isodeterminism in the structure of mesons.

4.5.1. Insufficiencies of quark conjectures. While the classification of hadrons into families received a rather general support since tits initiation by M. Gell-Mann in the 1960s [?], the conjecture that the hypothetical quarks are the actual physical constituents of hadrons has been controversial since their inception. These problematic aspects were reviewed in detail in the 1979 paper [?] (written at the Department of Mathematics of Harvard University under DOE support), and can be summarized as follows:

1) The quantum mechanical classification of point-like particles permitted by the $SU(3)$ model and, more recently, by the standard model, can be assumed to achieve a satisfactory *classification* of hadrons into families.

2) Quarks purely mathematical representations of a purely mathematical internal unitary symmetry defined on a purely mathematical complex-valued internal space and, as such, quarks cannot be the actual physical constituents of hadrons for numerous insufficiencies or sheer inconsistencies, such as:

2A) By recalling that quarks have to be point-like as a necessary condition to maintain the validity of quantum mechanics in the interior of hadrons, the ensuing conception of the hyperdense hadrons as ideal spheres with point-particles in their interior is not realistic;

2B) Quarks cannot be physical particles in our spacetime because they cannot be defined as unitary irreducible representations of the Poincaré symmetry (Section 3.9);

2C) Quarks cannot be rigorously confined inside hadrons (i.e,. confined with a rigorously proved, identically null probability of tunnel effect) due to the uncertainty principle;

2D) Quarks have not been directly detected under hadron collisions at the extremely big energies achieved at CERN and at other particle physics laboratories;

2E) The wavepackets of all particles are of the same order of magnitude of the size of all hadrons. Hence, the hyperdense character of hadrons is due to the total mutual penetration of the wavepackets of their constituents, resulting in unavoidable non-linear, non-local and non-Hamiltonian internal interactions under which the SU(3) and other symmetries cannot be consistently defined (Sections 3, 4).

3) History has thought that the study of atoms (as well as of other natural systems) required *two* different yet compatible models, one for the *classification* of atoms into family, and a different model for the *structure* of each atom of a given classification family. Particularly significant is the fact that the classification of atoms could be achieved via the use of *pre-existing mathematics,* while the structure of atoms required *new mathematics,* such as the Hilbert spaces, that are completely unnecessary for the classification problem.

In order to resolve the insufficiencies or sheer inconsistencies of the conjecture that quarks are physical particles, Santilli [*loc. cit.*] suggested to follow the teaching of history, and study hadrons via *two* different models, a conventional model for the *classification* of hadrons and a different yet compatible model for the *structure* of each individual hadron of a given classification multiplet.

In particular, the classification of hadrons can be effectively done via quantum mechanics because individual hadrons can be well approximated as being point- like particles in vacuum. By contrast, the structure of hadrons requires a necessary "completion" of quantum mechanics into a covering theory suggested beginning with the title of Ref. [?] in view of the unavoidable internal non-linear, non-local and non-potential interactions.

4.5.2. Hadronic structure model of mesons. A primary aim of papers [15] [16] [?] and monographs [24] [25] of 1978-1979 was the "completion" of quantum mechanics (qm) into the covering hadronic mechanics (hm) for the specific intent of achieving a representation of *all* characteristics of mesons as hadronic bound states of actual, massive, physical particles produced free in the spontaneous decays with the lowest mode, as illustrated in Table 1, page 429, Ref. [?] (reproduced in Figure 10), according to the following new structure models for the octet of mesons:

$$\pi^0 = (\tilde{e}^-, \tilde{e}^+)_{hm}, \tag{201}$$

$$\pi^\pm = (\tilde{\pi}^0, \tilde{e}^\pm)_{hm}, \tag{202}$$

Table I. The Technical Origin of Some of the Controversies in Hadron Physics[a]

$\{\pi^0 : 1^-, 0^-, +, 135, 0.8 \times 10^{-16}, \sim 1\,F\}$, $\{\pi^\pm : 1^-, 0^-, ..., 139, 2.6 \times 10^{-8}, \sim 1\,F\}$
$\{K^\pm : \frac{1}{2}, 0^-, ..., 439, 1.2 \times 10^{-8}, \sim 1\,F\}$, $\{K_S^0 : \frac{1}{2}, 0^-, ..., 498, 0.9 \times 10^{-10}, \sim 1\,F\}$
$\{K_L^0 : \frac{1}{2}, 0^-, ..., 498, 5.1 \times 10^{-8}, \sim 1\,F\}$, $\{\eta : 0^+, 0^-, +, 549, \Gamma = 0.8\ \text{keV}, \sim 1\,F\}$

$\pi^0 \to \gamma\gamma$, 98.8%	$K^\pm \to e\pi^0\nu\gamma$, 3.7×10^{-4}
$\pi^0 \to \gamma e^+e^-$, 1.15%	$K^\pm \to \pi e^+e^-$, 2.6×10^{-7}
$\pi^0 \to \gamma\gamma\gamma$, $<5 \times 10^{-6}$	$K^\pm \to \pi^\mp e^\pm e^\pm$, $<1.5 \times 10^{-8}$
$\pi^0 \to e^+e^-e^+e^-$, 3.3×10^{-5}	$K^\pm \to \pi\mu^+\mu^-$, $<2.4 \times 10^{-6}$
$\pi^0 \to \gamma\gamma\gamma\gamma$, $<6 \times 10^{-5}$	$K^\pm \to \pi\gamma\gamma$, $<3.5 \times 10^{-5}$
$\pi^0 \to e^+e^-$, $<2 \times 10^{-6}$	$K^\pm \to \pi\gamma\gamma\gamma$, $<3.0 \times 10^{-4}$
$\pi^\pm \to \mu\nu$, 100%	$K^\pm \to \pi\nu\nu$, $<0.6 \times 10^{-6}$
$\pi^\pm \to e\nu$, 1.2×10^{-4}	$K^\pm \to \pi\gamma$, $<4 \times 10^{-6}$
$\pi^\pm \to \mu\nu\gamma$, 1.2×10^{-4}	$K^\pm \to e\pi^\mp\mu^\pm$, $<2.8 \times 10^{-8}$
$\pi^\pm \to \pi^0 e\nu$, 1.02×10^{-8}	$K^\pm \to e\pi^\pm\mu^\mp$, $<1.4 \times 10^{-8}$
$\pi^\pm \to e\nu\gamma$, 3×10^{-8}	$K^\pm \to \mu\nu\nu\nu$, $<6 \times 10^{-6}$
$\pi^\pm \to e^+e^-\nu$, $<3.4 \times 10^{-8}$	$K_S^0 \to \pi^+\pi^-$, 68.7%
$K^\pm \to \mu\nu$, 63.6%	$K_S^0 \to \pi^0\pi^0$, 31.3%
$K^\pm \to \pi\pi^0$, 21.05%	$K_S^0 \to \mu^+\mu^-$, $<3.2 \times 10^{-7}$
$K^\pm \to \pi\pi^-\pi^+$, 5.6%	$K_S^0 \to e^+e^-$, $<3.4 \times 10^{-4}$
$K^\pm \to \pi\pi^0\pi^0$, 1.7%	$K_S^0 \to \pi^+\pi^-\gamma$, 2.0×10^{-3}
$K^\pm \to \mu\pi^0\nu$, 3.2%	$K_S^0 \to \gamma\gamma$, $<0.4 \times 10^{-4}$
$K^\pm \to e\pi^0\nu$, 4.8%	$K_L^0 \to \pi^0\pi^0\pi^0$, 21.4%
$K^\pm \to \mu\nu\gamma$, 5.8×10^{-3}	$K_L^0 \to \pi^+\pi^-\pi^0$, 12.2%
$K^\pm \to e\pi^0\pi^0\nu$, 1.8×10^{-5}	$K_L^0 \to \pi\mu\nu$, 27.1%
$K^\pm \to \pi\pi^\pm e^\pm\nu$, 3.7×10^{-5}	$K_L^0 \to \pi e\nu$, 39.0%
$K^\pm \to \pi\pi^\pm e^\pm\nu$, $<5 \times 10^{-7}$	$K_L^0 \to \pi e\nu\gamma$, 1.3%
$K^\pm \to \pi\pi^\pm\mu^\pm\nu$, 0.9×10^{-5}	$K_L^0 \to \pi^+\pi^-$, 0.2%
$K^\pm \to \pi\pi^\pm\mu^\pm\nu$, $<3.0 \times 10^{-6}$	$K_L^0 \to \pi^0\pi^0$, 0.09%
$K^\pm \to e\nu$, 1.5×10^{-5}	$K_L^0 \to \pi^+\pi^-\gamma$, 6.0×10^{-5}
$K^\pm \to e\nu\gamma$, 1.6×10^{-5}	$\eta \to \gamma\gamma$, 38.0%
$K^\pm \to \pi\pi^0\gamma$, 2.7×10^{-4}	$\eta \to \pi^0\gamma\gamma$, 3.1%
$K^\pm \to \pi\pi^+\pi^-\gamma$, 1×10^{-4}	$\eta \to 3\pi^0$, 29.9%
$K^\pm \to \mu\pi^0\nu\gamma$, $<6 \times 10^{-5}$	$\eta \to \pi^+\pi^-\pi^0$, 23.6%
$K_L^0 \to \pi^0\gamma\gamma$, $<2.4 \times 10^{-4}$	$\eta \to \pi^+\pi^-\gamma$, 4.9%
$K_L^0 \to \gamma\gamma$, 4.9×10^{-4}	$\eta \to e^+e^-\gamma$, 0.5%
$K_L^0 \to e\mu$, $<2.0 \times 10^{-9}$	$\eta \to \pi^0 e^+e^-$, <0.04%
$K_L^0 \to \mu^+\mu^-$, 1.0×10^{-8}	$\eta \to \pi^+\pi^-$, <0.15%
$K_L^0 \to \mu^+\mu^-\gamma$, $<7.8 \times 10^{-6}$	$\eta \to \pi^+\pi^-e^+e^-$, 0.1%
$K_S^0 \to \mu^+\mu^-\pi^0$, $<5.7 \times 10^{-5}$	$\eta \to \pi^+\pi^-\pi^0\gamma$, $<6 \times 10^{-4}$
$K_L^0 \to e^+e^-$, $<2.0 \times 10^{-9}$	$\eta \to \pi^+\pi^-\gamma\gamma$, <0.2%
$K_L^0 \to e^+e^-$, $<2.8 \times 10^{-9}$	$\eta \to \mu^+\mu^-$, 2.2×10^{-5}
$K_L^0 \to \pi^0\pi^\pm e^\pm\nu$, $<2.2 \times 10^{-3}$	

[a] In this table we have listed most of the experimental data on the octet of light mesons from the particle data[100] with only one addition, the inclusion of the (approximate) charge radius of the particles. The data are divided into two groups: those for total

Figure 10: *A reproduction of Table 1, page 429, Ref. [?] used to identify the physical constituents of mesons in the 'massive' particles produced free in the spontaneous decays, generally those with the lowest mode. The "completion" of quantum mechanics into hadronic mechanics, indicated since the title of Ref. [?], then becomes mandatory for any quantitative treatment of the indicated structure model.*

$$K^0 = (\tilde{\pi}^0, \tilde{\pi}^0)_{hm}, \quad (203)$$
$$K^{\pm} = (\tilde{\pi}^0, \tilde{\pi}^{\pm})_{hm}, \quad (204)$$
$$K_s = (\tilde{\pi}^-, \tilde{\pi}^+)_{hm}, \quad (205)$$
$$K_L = (\tilde{K}_-, \tilde{\pi}^+)_{hm}, \quad (206)$$

where the structural presence of positrons allows the representation of the very short meanlives of mesons, while all other characteristics can be numerically represented via the hadronic bound states of Section 4.3.

Structure models (201) - (206) are incompatible with quantum mechanics because the rest energy of all particles are *bigger* than the sum of the rest energies of the constituents (thus requiring *positive binding energies,* with ensuing *mass excesses* that are anathema for quantum mechanics as discussed in Section I.3); quantum mechanical elaborations of the models cannot account for the spin of the particles; and for other reasons.

Therefore, a necessary condition to prevent misrepresentations is that models (201) - (2-6) are treated with hadronic mechanics and, most importantly, their constituents are *isoparticles and anti-isoparticles* (Section 3.9) hereon denoted with an upper tilde. Therefore, the elementary constituents of the π^0 model (201) are *isoelectrons* $\tilde{e}^-$ *and anti-isoelectrons* $\tilde{e}^+$, respectively (called *eletons* in Ref. [16]); the constituents of the $\pi^{\pm}$ model, Eq. (201) are the *iso-meson* $\tilde{\pi}^0$ and isoelectrons or isopositrons; etc.

The physical constituents of mesons are assumed to be the massive particles generally emitted in the spontaneous decays with the lowest mode via tunnel effects after which isoparticles return to assume conventional features plus possible secondary effects with the emissions of massless particles.

Note that *all models (201)-(206) are two-bod hadronic bound states,* thus being fully represented by Eqs. (192)-(193). Note that *models (201)-(206) have a kind of pyramidal /bootstrap structures,* since a given meson appears in the synthesis of subsequent heavier mesons.Note finally that *models (201)-(206) imply the increase of the number of "elementary" constituents with the increase of the rest energy,* since the π^0 has only two elementary constituents while K_L has eight elementary constituents.

Since different structure models are characterized by numerically different isounits, the compatibility of the above hadronic structure model of mesons with their classification is readily achieved at the higher level of the *hyperstructural branch of hadronic mechanics* [29] [42], with the following total multivalued *hyperunit*

$$\hat{I}_{tot} = \{\hat{I}_{\pi^0}, \hat{I}_{\pi^{\pm}}, \hat{I}_{K^0}, \hat{I}_{K^{\pm}}, \hat{I}_S, \hat{I}_{K_L}\}. \quad (207)$$

4.5.3. Hadronic structure model of the π^0 meson. The characteristics of the π^0 meson are given by:'

1. Rest energy $E = 134.96 MeV$,
2) Meanlife $\tau = 0.828 \times 10^{-16}\ s$,
3) Charge radius $R = 10^{-13}\ cm$,
4) Null charge and spin.
5) Null electric and magnetic moments,
6) Negative parity; and
7) Primary decay

$$\pi^0 \rightarrow \gamma + \gamma, \ \ 98.85\,\%, \quad (208)$$

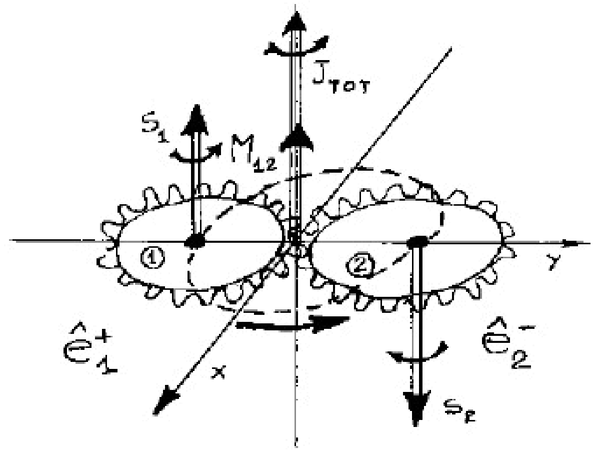

Figure 11: *A reproduction of the figure used in Section 5, Ref. [16] to illustrate the hadronic structure model of the π^0 known in the literature as"the gear model" because intended to illustrate the strongly "attractive" character of the contact non-Hamiltonian forces for singlet coupling and their strongly "repulsive" character for triplet couplings.*

and they are *all* numerically and exactly represented by hadronic bound state (192)-(193)) as a "compressed" form of the positronium (P), resulting in a hadronic bound state (201) of one isoelectron and one isopositron (Section 5, page 828 on, of Ref. [16])

$$P = (e^-, e^+)_{qm} \;\; \rightarrow \;\; \pi^0 = (e^-, e^+)_{hm}, \tag{209}$$

with numeric solution of the parametric equation (193)

$$k_1 = 0.34, \;\; k_2 = 1 + 4.27 \times 10^{-2} \tag{210}$$

The above results confirm all expectations indicated in preceding sections, namely,

1) Hadronic spectrum (194) admits one and only one energy level, the π, since all excited states are those of the positronium;

2) The presence of the antiparticle e^+ in the π^0 structure (201) explains its very low meanlife as well as the main decay (208);

3) Isodeterministic conditions (188) are verified by model (201), as a result of which standard deviations Δr and Δp have individual values smaller than one (Lemma 3.9).

4.5.4. Hadronic structure model of $\mu^\pm$ and $\pi^\pm$. The main characteristics of the muons $\mu^\pm$ are:

1) Rest energy $105,658\ MeV$;
2) Charge radius $R = 10^{-13}\ cm$;
3)) Meanlife $\tau = 2.19703 \times 10^{-6}\ s$;
4) Spin$1/2$ and elementary charge;
5) Tunnel effect decay

$$\mu^\pm \;\rightarrow\; e^- + e^\pm + e^+. \tag{211}$$

The increase of the number of elementary constituents of π^0 by one unit yields a *restricted three-isoparticle model* whose orbital motion is conventionally quantized, thus

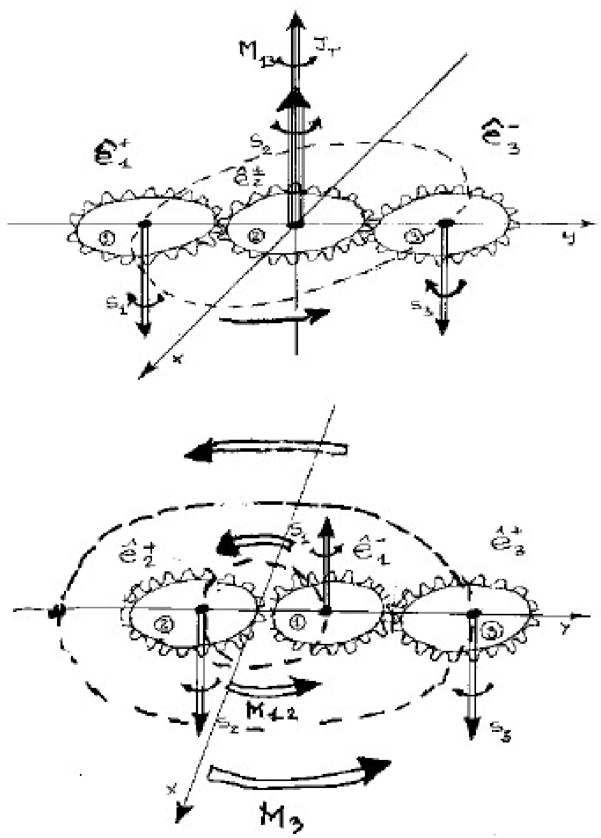

Figure 12: *A reproduction of the figure used in Section 5, Ref. [16], to illustrate the hadronic structure model of the* $\pi^{\pm}$ *mesons, Eq. (202). The top model represents the hadronic structure model* $\mu^{\pm} = (e^{-}, e^{\pm}, e^{+})_{hm}$*, while the bottom vie represents the hadronic structure model l* $\pi^{\pm} = (\pi^{0}, e^{\pm})_{hm}$*. Both models have three elementary constituents. The main difference is that the orbital motion of the top model follows has conventional values resulting in a weakly interacting particle with spin* $1/2$*, while the orbital motion of the bottom model follows irregular hadronic values (Section 3.4), resulting in a strongly interacting particle with spin zero.*

yielding a hadronic bound state with total angular momentum

$$J_{tot} = s_1 + s_2 + s_3 + L_{1-2} = 1/2 + 1/2 - 1/2 + 0 = 1/2. \tag{212}$$

Consequently, themodel was suggested in Section 5, Ref. [16] as providing the hadronic structure of muons

$$\mu^{\pm} = (\tilde{e}^-, e^{\pm}, \tilde{e}^+)_{hm} \equiv (\pi^0, e^{\pm})_{hm}. \tag{213}$$

with the representation of all indicated characteristics of the muons via solutions of the k_1 and k_2 parameters for Eqs. (193)

$$k_1 = 0.93, \;\; k_2 = 1 + 8.47 \times 10^{-2} \tag{214}$$

verifying conditions (188)

The main characteristics of the $\pi^{\pm}$ mesons are:

1) Rest energy $139.570\ MeV$;
2) Charge radius $R = 10^{-15}\ cm$;
3) Meanlife $\tau = 2.603 \times 10^{-8}\ s$;
4) Spin $J = 0$ and elementary charge;
5) Tunnel effect decay

$$\pi^{\pm} = (\tilde{e}^-, \tilde{e}^{\pm}, \tilde{e}^+)_{hm} \equiv (\tilde{\pi}^0, \tilde{e}^{\pm})_{hm}. \tag{215}$$

The above features are all represented by the hadronic structure model of the $\pi^{\pm}$ mesons first proposed in Ref. [16]

$$\pi^{\pm} = (\tilde{e}^-, \tilde{e}^{\pm}, \tilde{e}^+)_{hm} \equiv (\tilde{\pi}^0, \tilde{e}^{\pm})_{hm}, \tag{216}$$

with solutions for the k_1 and k_2 parameters

$$k_1 = 0.34, \;\; k_2 = 1 + 3.67 \times 10^{-3} \tag{217}$$

verifying conditions (188).

The main recent advance since the 1978 proposal [16] is the capability of a consistent representation of the total angular momentum $J = 0$ of the $\pi^{\pm}$ meson thanks to the irregular $SU(2)$ isosymmetry showing that the hadronic angular momentum of the $\tilde{e}^-, \tilde{e}^+$ pair compressed inside $\tilde{e}^{\pm}$ is $1/2$ with total angular momentum (Sections 3,4 and .3.5m Section 3.5.3 in particular).

$$J_{tot} = s_1 + s_2 + s_3 + L_{1-2} = -1/2 + 1/2 - 1/2 - 1/2 = 0. \tag{218}$$

With reference to Figures zzzz and zzz, model (216) is a "compressed" form of the $\mu^{\pm}$ model (213). The orbital motion of the peripheral electrons is unrestricted in model (213), yielding a total angular momentum $1/2$, Eq. (212). By contrast, the peripheral isoelectrons and isopositron in model (216) are *constrained to orbit inside the central isoelectron or isopositron* $\tilde{e}^{\pm}$, thus being forced to have the orbital value $1/2$ equal to the spin of the isoparticle since orbital values different than $1/2$ would imply extreme resistive forces with ensuing excessive instabilities.

4.5.4. Hadronic structure models of the remaining mesons. It is an instructive exercise for the interested reader to work out the hadronic structure models of the remaining mesons, Eqs. (203)-(206) with the hadronic suppression of the energy spectrum (195) permitted by solutions (195) with the increase of the k_1 value and the decrease of the k_2 value with the increase of the rest energy (see Section 6.2, Ref. [44] for an independent review).

4.6. Isodeterminism in the structure of baryons.

4.6.1. Nonrelativistic representation of the neutron synthesis. Santilli has conducted decades of mathematical, theoretical, experimental and industrial research on the most fundamental synthesis in nature, that of the neutron as a "compressed" hydrogen atom (H) in the core of stars [?] according to hadronic mechanics (See Refs. (I-85] to [I-95] and independent review in monograph [44])

$$H = (e^-, p^+)_{qm} \ \rightarrow \ n = (\tilde{e}^-, \tilde{p}^+)_{hm}. \tag{219}$$

In this section, we limit ourselves to recall that:

1) The excess $0.782\ MeV$ rest energy of the neutron over the sum of the rest energies of the proton and the electrons, Eqs. (96), cannot be represented via relative $e - p$ kinetic energy (or other conventional means) because the cross section of the $e - p$ scattering at $1\ MeV$ is virtually null, thus preventing any synthesis (see Section I-4). The only known way of representing the $0.782\ MeV$ excess energy in the neutron synthesis is via the EPR "completion" of quantum mechanics into the covering hadronic mechanics [28]-[30], [44].

2) By recalling that there is no energy available in the neutron synthesis for the production of a neutrino, the need to achieve the total angular momentum $1/2$ of the neutron from a hadronic hound state of two particles each having spin $1/2$ has stimulated systematic studies on regular and irregular Lie-Santilli isosymmetries [25] [37] (Sections 3, 4).

3) The need to understand the origin of the missing $0.782\ MeV$ energy is stimulating research expected to last for centuries on the ether as a universal substratum with extremely high density for the characterization and propagation of electromagnetic waves and elementary particles, which view renders possible the transmission of the missing $0.782\ MeV$ energy from the ether to the neutron via a *longitudinal impulse* known as the *etherino* (denoted with the symbol 'a from the Latin *aether*) [?].

The main characteristics of the neutron are the following:

1) Rest energy $939.565\ MeV$;

2) Charge radius $R = 10^{-15}\ cm$;

3) Meanlife $\tau = 881\ s(about\ 15\ m)$;

4) Spin $1/2$;

5) Elementary charge;

6) Anomalous magnetic moment $\mu_n = -1.9 \frac{e}{2m_p c}$ and null electric dipole moment;

7) Tunnel effect decay

$$n \ \rightarrow \ p^+ + e^- + \nu \ (or\ a?). \tag{220}$$

Experimental tests on the neutron synthesis initiated in the 1960s by don Carlo Borghi [?] and confirmed by subsequent tests [I-90] - [I-95], have systematically indicated the existence of an unstable intermedia state called *neutroid* (indicated with the symbol $\tilde{n}$) which is completely unidentified by neutron detectors while causing nuclear transmutations typically triggered by neutron irradiation.

Hadronic mechanics suggests the following representation of the structure of the neutroid

$$\tilde{n} = (\tilde{e}^-_{\downarrow}, p^+_{\uparrow})_{hm} \tag{221}$$

consisting of (Figure zzz) an isoelectron $\tilde{e}^-$ and spin s_1 in singlet contact coupling with a standard proton p^+ with spin s_2 and orbital motion L_{1-2} in the ground state. These assumptions result in a particle with rest energy of about $940\ MeV$, meanlife $\tau \approx 5\ s$, charge radius $R \approx 10^{-13}\ cm$, spin $J = 0$, and decay $\tilde{n} \rightarrow e^- + p^+$.

Once absorbed by a nucleus $N(A, Z, J)$, the neutroid is transformed by strong interactions into a neutron, resulting in a generally untabulated unstable nucleus according to reactions of the type,

$$\tilde{n} + N(A, Z, J) \ \rightarrow \ N(A + 1, Z, J) + \nu \ (or\ a?) \tag{222}$$

The above features explain the triggering by undetectable neutroids of conventional nuclear transmutations typically triggered by neutron irradiations.

Hadronic bound state (222) is clearly impossible for quantum mechanics, but readily possible for hadronic mechanics due to the combination of strongly attractive Coulomb and contact forces at mutual distances of the order of $10^{-13}\ cm$ with isotopic elements of the simple type (177) verifying isodeterministic Lemma 3.9.

Hadronic mechanics represents all characteristics of the neutron as a $compressed\ neutroid$ much similar to the structure of the $\pi^{\pm}$ as compressed , $\mu^{\pm}$ (Section 4.5.4), resulting in the structure

$$n = (\tilde{e}^-_{\downarrow}, \tilde{p}^+_{\uparrow}), \tag{223}$$

in which both the electron and the proton are mutated into corresponding isoparticles $\tilde{e}^-,\ \tilde{p}^+$ under the assumption that the electron is totally compressed inside the proton according to Rutherford [?].

The representation of the rest energy, charge radius, meanlife, charge and tunnel effect decay of the neutron have been first achieved in Ref. [75] Eqs. (2.19), page 521, via hadronic two-body bound state 192), with solutions of Eqs (193)

$$k_1 = 2.6, \ \ k_2 = 1 + 0.81 \times 10^{-8} \tag{224}$$

verifying conditions (188) for the validity of isodeterminism inside the neutron and the validity of conventional uncertainties in its outside.

The representation of the spin $1/2$ of the neutron was also achieved for the first time in Ref. [75], Eqs. (2.22)-(2.37), thanks to the appearance of an $orbital$ motion of the isoelectron when otally "compressed" inside the proton (which orbital motion is completely nonexistent for quantum mechanics).

The representation of the spin $1/2$ is additionally permitted by the isotopies of spin orbit couplings for which the hadronic angular momentum of the isoelectron in constrained to be equal to the spin of the isoproton as a necessary condition to avoid strong instabilities (Figure zzz) according to constraint (119) for which

$$J^n_{tot} = s_p + s_e + L_e = 1/2 - 1/2 + 1/2 = 1/2. \tag{225}$$

The representation of the anomalous magnetic moment of the neutron was also represented for the first time in Ref. [75], Eqs. (2.39)-(2.41), page 526, via the contribution to the total angular momentum of the orbital motion of the isoelectron inside the isoproton

$$\begin{aligned} \mu_n = -1.9\tfrac{e}{2m_pc} = \mu_p + \mu_e^{orb} + \mu_e^{intr} = \\ = 2.7\tfrac{e}{2m_pc} - 4.6\tfrac{e}{2m_pc}. \end{aligned} \tag{226}$$

The null value of the electric dipole moment was proved essentially along conventional lines.

4.6.2. Relativistic representation of the neutron synthesis. The relativistic representation of *all* characteristics of the neutron was first achieved in Refs. [60] [61] (see independent review [44], Section 6.3, page 342 on)via the isosymmetry of the irregular Dirac-Santilli isoequation (198), namely, the isotopy $\hat{\mathcal{P}}(3.1)$ of the spinorial covering of the Poincaré symmetry (Section 2.5.11) and can be outlined as follows.

Recall that the non-potential hadronic representation of strong interactions (Section 4.3) implies the "absorption" of the Coulomb interactions by the Hulten potential, as essentially implied by the charge independence of strong interactions.

This feature permits ignoring Coulomb binding energies in first approximation, resulting in a *weakly bounded* relativistic hadronic structure of the neutron, namely, a state with a very small binding energy which is typical of all non-potential interactions.

Let structure (221) of the neutroid be represented in the iso-Minkowski isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ with isometric (19). Assume in first approximation that the isoproton is perfectly spherical for which $n_1 = n_2 = n_3 = 1$ and assume that the density of the region of neutroid-proton overlapping is close to that of the vacuum, thus implying the value $n_4 = 1$ resulting in an isometric of the type

$$\hat{\eta} = \eta e^{-K} = Diag.(1, 1, 1, -1)e^{-K}, \;\; K > 0. \tag{227}$$

The relativistic representation of all characteristics of the neutroid then follows via a simply isotopy of the relativistic treatment of the hydrogen atom.

In the relativistic treatment of the structure of the neutron, Eq. (223), the isoproton cannot any longer be assumed to be perfectly spherical and the density of the overlapping region becomes dominant, resulting in values of the characteristic quantities $n_\mu \neq 1, \; \mu = 1, 2, 3, 4$.

In Refs. [60] [61], Santilli assumes that the value of the characteristic quantity n_4 representing the density of the neutron is equal to the corresponding value obtained from the fit of experimental data via hadronic mechanics of the Bose-Einstein correlation [?] [?].,

$$n_4 = 0.62, \;\; b_4 = \frac{1}{n)4} = 1.62 \tag{228}$$

where $b_4 = 1/n_4$ is the notation used in Refs. [61].

The Lorentz-Santilli isotransforms (42) then imply the following *isorenormalization of the rest energy of the electron,* namely a renormalization caused by non-potential interactions (Ref. [61], Eqs. (7.1), page 191)

$$E_e = m_e c^2 = 0.511 \; MeV \;\; \rightarrow \;\; E_{\tilde{e}} = m_e \frac{c^2}{n_4^2} = 1.341 \; MeV. \tag{229}$$

As one can see, the above isorenormalization removes the missing $0.782 \; MeV$ energy in the neutron synthesis when represented on isospaces over isofields, thus rendering consistent isorelativistic equations.

II should be stressed that the above isorenormalization continues to be based on the etherino for the delivery of the missing energy to the neutron. Other hypotheses are welcome, except the use of the neutrino its cross section with electrons and protons is essentially null, thus preventing any plausible delivery of energy.

The relativistic representation of the spin of the neutron in synthesis (223) was first achieved in Refs. [60] [61]. Recall from Figure zzz that the spin s_1 of isoelectron is opposite to the spin s_2 of the isoproton. The relativistic spin-orbit coupling implies the *constraint* that the orbital angular momentum L_1 of the isoelectron inside the isoproton be equal to the isoproton spin,

$$L_1 = s_2. \tag{230}$$

The above identity is manifestly impossible for the spinorial covering of the Poincaré symmetry $\mathcal{P}(3.1)$ and relativistic quantum mechanics, but it is indeed possible for the covering isosymmetry $\hat{\mathcal{P}}(3.1)$. In fact, with reference to Section 3.5.3, Eqs. (122), identity (230) implies the conditions (Ref. [61], Eqs. (7.2), page 192)

$$\begin{gathered}\hat{L}_3 \star |\hat{\psi}>= \pm n_1 n_2 |\hat{\psi}>= \\ = \hat{S}_3 \star |\hat{\psi}>= \tfrac{1}{2}\tfrac{1}{n_1 n_2}|\hat{\psi}>, \\ \hat{L}^{\hat{2}} \star |\hat{\psi}>= (n_1^2 n_2^2 + n_2^2 n_3^2 + n_3^2 n_1^2)|\hat{\psi}>= \\ = \hat{S}^{\hat{2}} \star |\hat{\psi}>= \tfrac{1}{4}(n_1^{-2} n_2^{-2} + n_2^{-2} n_3^{-2} + n_3^{-2} n_1^{-2})|\hat{\psi}>,\end{gathered} \tag{231}$$

admitting the simple solution

$$n_k^2 = n = \frac{1}{\sqrt{2}} = 0.706, \quad b_k^2 = \frac{1}{n_k^2} = \sqrt{2} = 1.415, \quad k = 1, 2, 3, \tag{232}$$

where $b_k^2 = 1/n_k^2$ is the notation used in Ref. [61].

The relativistic representaiton of the anomalius magnetic moment of the neutrion was also achueved f or the first time in ref. [61], Eqs. (7.4), page 192. The representation is again permitted by the contribution due to the orbital motion of the isoelectron inside the isoproton, and it is given by non-relativistic expression (226) with a more accurate representation of the orbital contribution.

It should be indicated that the non-relativistic and relativistic structure models of the neutron presented in this section are mere approximation of a much more complex reality in which all characteristics of the constituents are mutated, thus including the charge.

4.6.3. Structure and classification of the remaining baryons. A central requirement for the consistency of model (223) is that *the excited states of the neutron are the conventional states of the hydrogen atom.* This illustrates again the hadronic suppression of quantum mechanical energy spectra, since the latter are typical for the classification, rather than the structure of hadrons.

By recalling the condition that the number of elementary constituents of hadrons increases with the increase of the rest energy, the *hadronic structure of the remaining baryons* is reducible to the two isoparticle structures derived from the spontaneous decays generally with the lowest mode, much along the structure of mesons, resulting in the following structure models of the octet of baryons [?] (see [44], Section 6.3.J, page 366 for an independent review)

$$
\begin{aligned}
& p^+(938\ MeV) = stable, \\
& n(939\ MeV) = (\tilde{p}^+, \tilde{e}^-)_{hm}, \\
& \Lambda(1115\ MeV) = (\tilde{p}^+, \tilde{\pi}^-)_{hm}, \\
& \Sigma^+(1189\ MeV) = (\tilde{p}^+, \tilde{\pi}^0)_{hm}, \\
& \Sigma^0(1192\ MeV) = (\tilde{n}, \tilde{\pi}^0)_{hm}, \\
& \Sigma^-(1197\ MeV) = (\tilde{n}, \tilde{\pi}^-)_{hm}, \\
& \Sigma^0(1314\ MeV) = (\tilde{\Lambda}, \tilde{\pi}^0, \\
& \Xi^-(1321\ MeV) = (\tilde{\Lambda}, \tilde{\pi}^-)_{hm}.
\end{aligned}
\tag{233}
$$

by keeping in mind that numerous alternative internal exchanges of isoparticles are possible while keep constant the total rest energy.

It is an instructive exercise for the interested reader to see that all the above models verify condition (186) for the lack of hadronic excite states, as well as conditions (188) for the validity of isodeterministic Lemma 3.9 and related rapid convergence of isoseries.

The compatibility of hadronic structure models (233) with the $SU(3)$-color or more recent classifications is achieved at the hyperstructural level [29] [42] with the following ordered hyperunit [?]

$$\hat{I}_{tot} = \{\hat{I}_p, \hat{I}_n, \hat{I}_\Lambda, \hat{I}_{\Sigma^+}, \hat{I}_{\Sigma^0}, \hat{I}_{\sigma^-}, \hat{I}_{\Xi^0}, \hat{I}_{\Xi^-}\} \tag{234}$$

4.4.zzzzzz. Isdeterminism in the pseudoproton synthesis.
4.4.zzzz. Isodeterminism in chemical valence bonds

0.50cm

4.4.zzzzz Isoeterminism in gravitational collapse.

To provide a gravitational illustration, recall that isotopic element (2) contains as particular cases all possible symmetric metrics in $(3+1)$-dimensions, thus including the Riemannian metric [17].

We then consider the 3-dimensional sub-case of isotopic element (2) and factorize the space component of the Schwartzchild metric $g_s(r)$ according to isotopic rule introduced in Refs. [?] [?]

$$g_s(r) = \hat{T}(r)\delta, \tag{235}$$

where δ is the Euclidean metric.

We reach in this way the following realization of the isotopic element

$$\hat{T} = \frac{1}{1 - \frac{2M}{r}} = \frac{r}{r - 2M}, \tag{236}$$

where M is the gravitational mass of the body considered, with ensuing isodeterministic isoprinciple

$$\Delta\hat{r}\Delta\hat{p} \approx \hat{T} = \frac{r}{r-2M} \Rightarrow_{r\to 0} = 0, \tag{237}$$

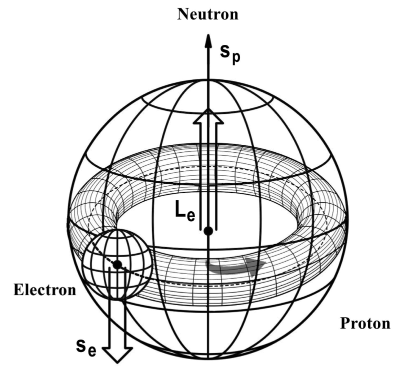

Figure 13: *This picture provides a conceptual rendering of Rutherford's synthesis of the neutron from the hydrogen in the core of stars via the "compression" of the electron inside the hyperdense proton in singlet coupling in which case the electron is "constrained" to orbit inside the proton with an angular momentum equal to the proton spin, since other configurations would imply extreme resistive forces, as shown non-relativistically in Ref. [75] and relativistically in Ref. [61]. It is evident that this configuration cannot be formulated via quantum mechanics, thus suggesting a "completion" of conventional spin and angular momentum studied in Sections 3.4 and 3.5.*

which confirms the statement in page 190 of Ref. [10], on the possible recovering of full classical determinism in the interior of gravitational collapse (see Ref. [?], Chapter 6 in particular, for a penetrating critical analysis of black holes).

It should perhaps be indicated that Refs. [?] [?] introduced the factorization of a full Riemannian metric $g(x),\ x = (r, t)$ in $(3 + 1)$-dimensions

$$g(x) = \hat{T}_{gr}(x)\eta, \tag{238}$$

where $\hat{T}_{gr}$ is the *gravitational isotopic element*, and η is the Minkowski metric $\eta = Diag.(1, 1,-1, -1)$.

Refs. [?] [?] then reformulated the Riemannian geometry via the transition from a formulation over the field of real numbers $\mathcal{R}$ to that over the isofield of isoreal isonumbers $\hat{\mathcal{R}}$ where the *gravitational isounit* is evidently given by

$$\hat{I}_{gr}(x) = 1/\hat{T}_{gr}(x). \tag{239}$$

The above reformulation turns the Riemannian geometry into a new geometry called *iso-Minkowskian isogeometry,* which is locally isomorphic to the *Minkowskian* geometry, while maintaining the mathematical machinery of the Riemannian geometry (covariant derivative, connection, geodesics, etc.) us fully maintained, although reformulated in terms of the isodifferential isocalculus [?].

The apparent advantages of the *identical* iso-Minkowskian reformulation of Riemannian metrics and Einstein's field equations (see, e.g., Eqs. (2.9), page 390 of Ref. [?]) are:

1) The achievement of a consistent operator form gravity in terms of *relativistic hadronic mechanics* [?] whose axioms are those of quantum mechanics, only subjected to a broader realization;

2) The achievement of a universal *symmetry* of *all* non-singular Riemannian metrics, which symmetry is locally isomorphic to the Lorentz-Poincaré symmetry, today known

as the *Lorentz-Poincaré-Santilli (LPS) isosymmetry* [?], and it is notoriously impossible on a conventional Riemannian space over the reals;

3) The achievement of clear compatibility of Einstein's field equation with 20th century sciences, such as a clear compatibility of general relativity with special relativity via the simple limit $\hat{I}_{gr} = I$ implying the transition from the universal LPS isosymmetry to the Poincaré symmetry of special relativity with ensuing recovering of conservation and other special relativity laws [?] [?]; the achievement of axiomatic compatibility of gravitation with electroweak interactions thanks to the replacement of curvature into the new notion of isoflatness with the ensuing, currently impossible, foundations for a grand unification [?]; and other intriguing advances.

3. Concluding remarks.

In this paper, we have continued the study of the EPR argument [1] conducted in Ref. [10] and preceding works, with particular reference to the study of the uncertainties for extended particles immersed within hyperdense medias with ensuing linear and non-linear, local and non-local and Hamiltonian as well as non-Hamiltonian interactions.

This study has been conducted via the use of isomathematics and isomechanics characterized by the isotopic element $\hat{T}$ of Eq. (1) which represents the non-linear, non-local and non-Hamiltonian interactions of the particles with the medium [15] [17] [18].

The main result of this paper is that the standard deviations of coordinates and momenta for particles within hyperdense media are characterized by the isotopic element that, being always very small, $\hat{T} \ll 1$, reduces the uncertainties in a way inversely proportional to a non-linear increase of the density, pressure, temperature, and other characteristics of the medium, while admitting the value $\hat{T} = 0$ under extreme/limit conditions with ensuing recovering of full determinism as predicted by A. Einstein, B. Podolsky and N. Rosen [1].

We can, therefore, tentatively summarize the content of this paper with the following:

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