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A STRUCTURE MODEL OF THE ELEMENTARY CHARGE

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Abstract

I formulate the hypothesis that the electrostatic interaction between two electrons, two positrons, or one electron and one positron is explicitly dependent on time according to a pulsating behaviour with period $T = 0.829 \times 10^{-20}$ sec. The hypothesis is motivated by the need to bypass a number of technical difficulties for a deeper understanding of the process $e^+ + e^- \to 2\gamma$, such as the fact that current relativistic treatments, including retarded and advanced potentials, of one electron and one positron at rest with respect to each other and at small mutual distances produce a null total electromagnetic field, while the total field of two photons is not null. An initial treatment of the hypothesis is presented at the level of the time-dependent perturbation theory. It is shown that, upon selection of a suitable functional form, the time dependence is ignorable for periods of time large compared to 10^{-20} sec, as well as for the electrostatic interactions of one electron with a cluster of elementary charges (e.g. a proton). A generalization of the Coulomb law is thus achieved in such a way as to reproduce the conventional law either for large values of time or for large collections of elementary charges. The generalization is based on a structure model of the electron charge, in the sense that the charge emerges as derived, via time averages, from structure data of the electron. A generalization of Rutherford's cross section is also identified in which, again, the departures from the conventional form become ignorable for large values of time or for large collections of elementary charges. The possibilities for experimental tests of the prediction of the paper are briefly indicated via the use of (a) resonance effects; (b) more accurate measurements of cross sections; and (c) suitable refinements of the experimental information on the positronium.

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1. INTRODUCTION

Since the time of my graduate studies in theoretical physics I have been fascinated, on one side, by the beauty and maturity of our description of the electromagnetic waves, while, on the other side, I have experienced uneasiness in accepting as of final character the current description of the electromagnetic field of the electron, despite its unequivocal physical value.

My uneasiness was due to a number of technical difficulties which are created by the rather profound differences between the potentials of the electrons and of the photons, e.g.

$$V_{e\ell} = \pm \frac{e^2}{r} \quad , \qquad V_{ph} = E \sin (k \cdot r + \omega t) \tag{1.1}$$

in attempting a deeper understanding of the annihilation process

$$e^+ + e^- \rightarrow 2\gamma \tag{1.2}$$

This process can be viewed as a form of transmutation * of one type of physical quantities (electrons and positrons) into another (photons), and vice versa. With the understanding that the process is expected to result considerably more complex of what currently acknowledged, one can study, as a first rudimentary step, the following three, separate, primary aspects.

(1) **The transmutation of kinematical quantities** such as the linear momentum, energy, and angular momentum;

(2) **the transmutation of structural quantities,** such as the preservation of the wave packet structure; and

(3) **the transmutation of electromagnetic quantities** , that is, the transformation of the electromagnetic field of the electron--positron pair into that of the photons.

The currently available descriptions of process (1.2), say, that via quantum electrodynamics, are fully satisfactory with respect to the transmutation of kinematical quantities (which is trivially achieved via conservation laws), as well as structural quantities (which is expressible via the preservation of the wave structure), but they are not equally satisfactory for that of electromagnetic character. In fact, any form of transmutation of the electromagnetic field of the electron--positron pair into that of the photons is precluded by the current descriptions, including weaker forms of correlations. At the level of these introductory remarks, it is sufficient to note, for instance, that the photon potential is explicity dependent on time, while that of the electrons is not.

* The term "transmutation" will be used in this paper in the meaning of complete transformation of one physical quantity into another without leaving residues . If the transmutation is not complete, the system is open (i.e. nonconservative), as it is the case, say, for particle reactions under external fields.

This situation can be put in quantitative terms via relativistic techniques, including the use of advanced and retarded potentials, by computing and comparing the total electromagnetic field of the left and right hand sides of reaction (1.2). One can then see that we have a **null total electromagnetic field** for two opposite charges at rest with respect to each other and at very small mutual distance, while we have a **non-null total electromagnetic field** for two photons. The situation, originating at the semiclassical level and persisting upon quantization, is interpreted here as an indication of the insufficiency of the current description of the electromagnetic field of the electron, and of the existence of yet deeper levels of treatment of structural character. The understanding is that no insufficiency exists for our current description of the photons.

Owing to these difficulties, I initiated a study aiming at a structurally more informative description of process (1.2). These studies were interrupted soon after my graduation, apart from one timid and unpublished note I wrote while at the Center for Theoretical Studies of the University of Miami, Florida in 1967 -- 1968 entitled "Time--dependence hypothesis in electron interactions".

In this paper I would like to report these studies as available in my unpublished notes without any significant advance. As the reader will see, the studies are incomplete and elementary . I essentially decided to release my studies in the hope that they are reinspected and eventually continued by interested readers. Expectedly, the analysis is nonrelativistic and often semiclassical.

2. A POSSIBLE GENERALIZATION OF THE COULOMB LAW.

Soon after its appearance in the celebrated three memoires published by CHARLES AUGUSTIN DE COULOMB in 1785, the Coulomb law

$$F = \frac{Q\,Q'}{r^2} \tag{2.1}$$

proved to be one of the true foundations of physics. Its validity for macroscopic charged bodies is impressive, apart from known limitations at very close proximities of the bodies.

It was therefore rather natural for Sir JOHN JOSEPH THOMSON to assume the same law for his celebrated studies of 1897 on the static interactions of elementary electric charges

$$F = \pm \frac{e^2}{r^2} \tag{2.2}$$

As a result of authoritative endorsements, such as that by Sir ERNEST RUTHERFORD, the preservation of the original Coulomb law for the elementary charges became established, as it is still the case today.

Intriguingly, microscopic law (2.2) implies macroscopic law (2.1), but the vice versa is not necessarily the case on both conceptual and technical grounds. In fact, there can exist fluctuations or anomalies or departures from law (2.2) which average out at the macroscopic level in such a way to reproduce law (2.1) identically.

An analysis of annihilation process (1.2) indicates that these conceivable fluctuations can occur not only in space, but perhaps more importantly in time. At any rate, if one desires to represent the transmutation of the electromagnetic field of the electrons into that of the photons, the assumption of an explicit time dependent for all fields is rather compelling.

These remarks lead me to the study of the possible existence of a generalization of the Coulomb law for elementary charges only (that is, for the electron-electron or electron-positron and positron-positron pairs only) exhibiting an explicit dependence not only in space but also in time, i.e.

$$F_{gen} = f\ (\ t,\ r,\ \dots\ ) \tag{2.3}$$

and verifying the following primary conditions.

Condition 1. The space and time average of law (2.3) must recover law (2.2) identically, i.e.,

$$\frac{1}{RT} \int_0^R dr \int_0^T dt\ f \equiv \pm \frac{e^2}{r^2} \tag{2.4}$$

Condition 2 The superposition of law (2.3) for a sufficiently large number of elementary charges must recover law (2.1) identically, i.e.,

$$\sum_{i,j=1}^{N} f_{ij} \equiv \frac{Q\, Q'}{r^2} \qquad N \gg 1 \tag{2.5}$$

Condition 3. The generalized law must be compatible with or be able to improve the representation of available experimental data on electrons and positrons.

Permit me to report below the results of my studies along the steps I originally followed.

The study of the joint generalization of the Coulomb law in space and time soon resulted to be a quite arduous task. Besides, Feynman had indicated at the Solvay Conference of 1962 that a modification of the space dependence of law (2.2) of the type

$$F = \pm \frac{e^2}{r\,(1+\epsilon)^2} \tag{2.6}$$

is incompatible with experimental data, unless $\epsilon \ll 10^{-10}$.

By no mean Feyman's point closes the problem of the space dependence. In fact, the point holds under the tacit assumption that the law has no explicit time dependence. Clearly, when a possible time dependence is introduced, the problem of the admissible space

dependence should be re-examined "ab initio".

Nevertheless, to avoid excessive initial complexities, I decided to study a simplified version of law (2.3), that in which the space dependence is the conventional one, and I write

$$F_{gen} = \pm \frac{S(\,t\,)}{r^2} \tag{2.7}$$

Next, I initiated the study of the time dependence which is admissible by conditions 1, 2, and 3. I attempted to draw all necessary information, again, from annihilation process (1.2). First, I assumed that the type of time dependence is the same as that of the wave packet of the electrons. I therefore assumed a periodic time dependence of the type

$$F_{gen} = \pm \frac{S(\,\omega t\,)}{r^2} \tag{2.8}$$

where

$$\omega = 2\pi\nu = \frac{m_{e\ell}\, c^2}{\hbar} = 7.570 \times 10^{20}\ \text{sec}^{-1} \tag{2.9}$$

is the characteristic frequency of the electron at rest.

A first objection I thought is that an explicit time dependence of the type (2.8) could imply the instability of the orbit of the electron in the hydrogen atom. This objection was disposed of by Condition 2. In fact, by recalling that the proton has a rather complex charge structure, Condition 2 implies the elimination of the time dependence in the electron-proton interactions.

Additional objections were circumvented by the property that the electromagnetic coupling constant is rather large by particle standards. As a result, we have sufficient grounds for a possible explicit time dependence which may have escaped experimental and theoretical detection until now simply because not looked for.

On these admittedly vague grounds, I proceeded to study whether the problem admits a solution. Condition 1 now assumes the simplified form

$$\frac{1}{T} \int_0^T dt\ S(\,\omega t)) = e^2 \qquad ,\ T \gg 10^{-20}\ \text{sec} \quad . \tag{2.10}$$

A first simple solution is given by

$$S(\,\omega t\,) = 2e^2 \sin^2 \omega t \tag{2.11}$$

on account of the property

$$\frac{2}{T}\int_0^T dt\,\sin^2\omega t = \frac{2}{n\pi}\int_0^{n\pi} dx\,\sin^2 x = 1 \quad . \tag{2.12}$$

A second solution is

$$S(\omega t) = e^2\pi\,\theta(\sin\omega t)\,\sin\omega t \quad , \tag{2.13}$$

where θ is the step function, with property

$$\frac{\pi}{T}\int_0^T dt\,\theta(\sin\omega t)\,\sin\omega t = \frac{1}{2n}\sum_{k=1}^{n}\int_{(2k-2)\pi}^{(2k-1)\pi} dx\,\sin x = 1 \tag{2.14}$$

A third solution is given by

$$S(\omega t) = e^2\left(1 + \frac{1}{W}\sin\omega t\cos\omega t\right) . \tag{2.15}$$

where W is a free dimensionless parameter to be determined via additional data. Condition 1 is now verified on account of the property

$$\frac{1}{WT}\int_0^T dt\,\sin\omega t\cos\omega t = \tag{2.16}$$

$$= \frac{1}{2WT}\int_0^T dt\,\sin 2\omega t = \frac{1}{4Wn\pi}\int_0^{n\pi} dx\,\sin x = 0 \quad ,$$

where we have used the value

$$T = n\pi\omega \quad , \qquad n >> 1 \tag{2.17}$$

Additional solutions can be identified by the interested reader.

Thus, within the context of condition (2.7) only, the following time-dependent generalizations of the Coulomb law for elementary charges are conceivable

$$F_{gen} = \pm\frac{e^2}{r^2}\,2\sin^2\omega t \quad , \tag{2.18a}$$

$$F_{gen} = \pm\frac{e^2}{r^2}\,\pi\,\theta(\sin\omega t)\,\sin\omega t \quad , \tag{2.18b}$$

$$F_{gen} = \pm\frac{e^2}{r^2}\left(1 + \frac{1}{W}\sin\omega t\cos\omega t\right) \tag{2.18c}$$

The models possess the following common features (Fig. 1).

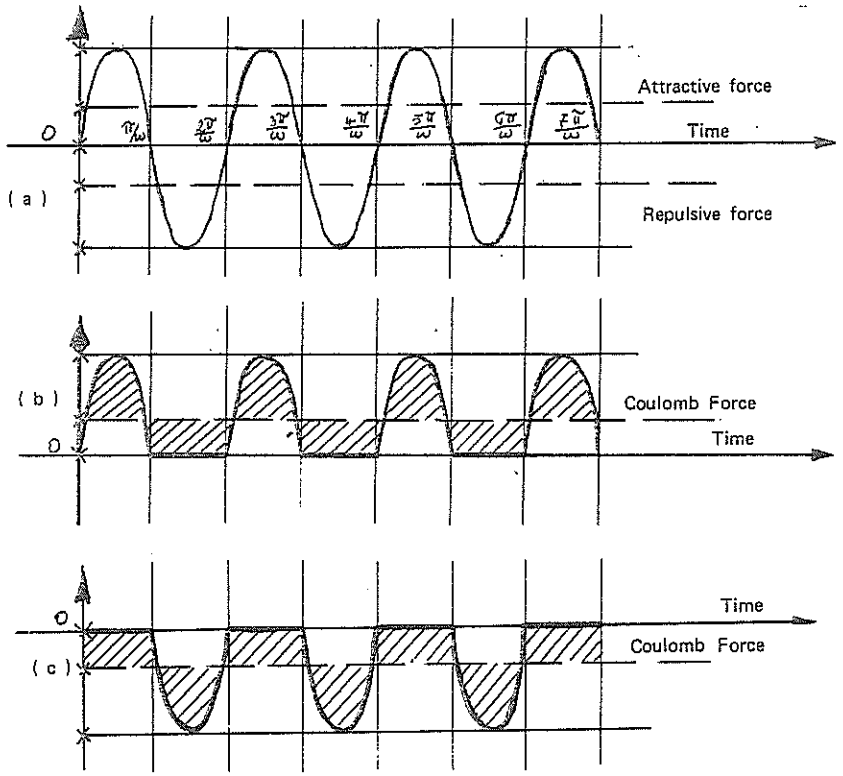

Figure 1. In these diagrams I illustrate the main ideas for one model of structure of the elementary charge, model (2.18b). The free electric field of the electron and/or of the positron is conceived as being inclusive of both attractive and repulsive forces. This is represented here via a monochromatic, oscillatory, force field (Fig. a). The oscillations are interpreted as the excitation of space and its geometry, which is assumed to underlie the structure of the particle as well as the propogation of its force field. The elementary nature of the particle is interpreted via the monochromaticity of the oscillation. The electrostatic interactions between two elementary charges is then interpreted as producing the separation of the attractive and repulsive forces, depending on the sign of the charges considered. This results in a generalization of the Coulomb Law (Figs. b and c) which exhibits an explicit time dependence of pulsating character, and which reproduces the conventional law (1) via a time average; or (2) for periods of time that are large compared to the characteristic frequency of pulsation; or (3) for a large collection of interactions. In this way, the static, perennial, abstract notion of charge is reduced to a notion profoundly dynamical in origin and behaviour, that is, to a structure model. Notice that the case considered here is not the only one conceivable. For instance, model (2.18a) achieves attractive and repulsive forces via a different mechanism, that of multiplication of the sinus dependence of each field. A still further mechanism occurs for model (2.18c). All models, however, present in common the pulsating character of the electrostatic force, which, upon a suitable mathematical representation, appears to be in agreement with the available data. The possibility of an experimental verification of the prediction is briefly considered in Section 3 for subsequent more detailed study.

(a) The time dependence is of periodic type with periodicity given by the characteristic frequency of the electron at rest.

(b) While the conventional Coulomb law has a constant behaviour in time at a fixed distance, generalized laws (2.18) have a sort of pulsating behaviour in time.

(c) The time dependence is ignorable for periods of time greater than 10^{-20} sec.

The last property indicates the compatibility of generalized laws (2.18) with Condition 2. In actuality, this compatibility is of (at least) two-fold nature. First, at a macroscopic level, electric forces are studied for periods of time much greater than 10^{-20} sec. Second, different pairs of electric charges generally imply different phases in laws (2.18). The pulsating effects are then generally not in phase. When summed up over a large number of elementary pairs, the pulsating effects become undetectable even for periods of time of the order of 10^{-20} sec.

Laws (2.18) also have considerable differences. The most significant is that under time inversion. It should be stressed in this respect that, owing to the extremely small periods of time under consideration here, it is premature to assume a stand either in favour or against the symmetry under time inversions. After all, T-violating effects in weak interactions could well have an origin more profound than that currently acknowledged.

Also, models (2.18) are different on dynamical grounds in the sense, for instance, that model (2.18c) possesses an additional parameter W permitting the recovering of the conventional law for diverging values, while this feature is absent for the other models.

The study of Condition 3 is predictably more involved. Without any claim of completeness I studied the conditions for models (2.18) to recover the data of the nonrelativistic perturbation theory.

Consider the transition probability for the time-dependent case

$$W = \frac{1}{T}\int |a_k^{(s)}|^2 \rho(k)\, d E_k \ , \qquad s = 0, 1, 2, \ldots \tag{2.19}$$

where (for $\hbar = c = 1$)

$$\begin{aligned} a_k^{(0)} &= \delta_{km} \ , \\ a_k^{(1)} &= -i \int_0^T dt\ H_{km}(t)\, e^{i\omega_{km}t} \ , \\ a_k^{(2)} &= -i \int_0^T dt\ H_{km}(t)\, a_m^{(1)}\, e^{i\omega_{km}t} \\ &\cdots\cdots\cdots \\ &\cdots\cdots\cdots \end{aligned} \tag{2.20}$$

and $\rho(k)$ is a density of final states considered as a slow varying function of the energy.

By assuming that the initial and final states can be represented by free waves

$$U_m(r) = \frac{1}{L^{3/2}} e^{ik\cdot r} \tag{2.21}$$

we have

$$H_{km}(t) = s(t) \int dv\, u_k^* \frac{1}{r} U_n = H_{km}^0\, s(t) \tag{2.22}$$

where

$$H_{km}^0 = \frac{\pi e^2}{L^3 k^2} \operatorname{cosec}^2 \tfrac{1}{2}\theta \tag{2.23}$$

is the well known transition matrix element for the time-independent potential e^2/r.

At first order, the transition probability is then given by

$$W = (H_{km}^0)^2 \rho(k) \frac{1}{T} \int_{-\infty}^{+\infty} \left| \int_0^T s(t)\, e^{i\omega_{km}t}\, dt \right|^2 d\omega_{km} \tag{2.24}$$

By recalling the known relations

$$\begin{aligned} \rho(k) &= \frac{\mu L^3}{8\pi^3} k \sin\theta\, d\theta\, d\varphi \ , \\ \sigma(\theta, \varphi) \sin\theta\, d\theta\, d\varphi &= \frac{\mu L^3}{k} W \end{aligned} \tag{2.25}$$

the differential cross section for a static interaction is given by

$$\sigma(\theta) = \left(\frac{e^2}{2\mu v^2}\right)^2 \operatorname{cosec}^4 \tfrac{1}{2}\theta \frac{1}{2\pi T} \int_{-\infty}^{+\infty} \left| \int_0^T s(t)\, e^{i\omega_{km}t}\, dt \right|^2 d\omega_{km} \tag{2.26}$$

with self-evident departure from Rutherford cross section

$$\sigma_R(\theta) = \left(\frac{e^2}{2\mu v^2}\right)^2 \operatorname{cosec}^4 \tfrac{1}{2}\theta \tag{2.27}$$

Thus, Condition 3 can be expressed for the case at hand

$$\frac{1}{2\pi T} \int_{-\infty}^{+\infty} \left| \int_0^T s(t)\, e^{i\omega_{km}t}\, dt \right|^2 d\omega_{km} \cong 1 \ , \quad T \gg 10^{-20} \text{ sec} \tag{2.28}$$

In fact, by putting $s = 1$, we recover Rutherford formula on account of the property

$$\frac{1}{2\pi T} \int_{-\infty}^{+\infty} \left| \int_0^T e^{i\omega_{km}t}\, dt \right|^2 d\omega_{km} = \frac{2}{\pi T} \int_{-\infty}^{+\infty} \frac{\sin^2 \frac{1}{2}\omega_{km}t}{\omega_{km}^2}\, d\omega_{km} = 1 \tag{2.29}$$

By proceeding along similar lines, I obtain the following condition for the second-order contribution

$$\frac{1}{2\pi T}\int_{-\infty}^{+\infty}\Big|\sum_n H^0_{kn}H^0_{nm}\int_0^T s(t)\,e^{i\omega_{kn}t}\int_0^t s(q)\,e^{i\omega_{nm}q}\,dq\,dt\Big|^2\,d\omega_{km}$$

$$\cong\Big|\sum_n\frac{H^0_{kn}H^0_{nm}}{E_n-E_m}\Big|^2\,,\quad T\gg 10^{-20}\ \text{sec} \tag{2.30}$$

And for the third order contribution, I have

$$\frac{1}{2\pi T}\int_{-\infty}^{+\infty}\Big|\sum_{n,n'}H^0_{kn}H^0_{nn'}H^0_{n'm}\ \times$$

$$\times\int_0^T s(t)\,e^{i\omega_{kn}t}\int_0^q s(q)\,e^{i\omega_{nn'}q}\int_0^r s(r)\,e^{i\omega_{n'm}r}\,dr\,dq\,dt\Big|^2\,d\omega_{km}$$

$$\cong\Big|\sum_{n,n'}\frac{H^0_{kn}H^0_{nn'}H^0_{n'm}}{(E_{n'}-E_n)(E_n-E_m)}\Big|^2\,,\quad T\gg 10^{-20}\ \text{sec} \tag{2.31}$$

For model (2.18a), Condition (2.28) reads

$$\frac{1}{2\pi T}\int_{-\infty}^{+\infty}\Big|\int_0^T s(t)\,e^{i\omega_{km}t}\,dt\Big|^2\,d\omega_{km}=$$

$$=\frac{2}{\pi T}\int_{-\infty}^{+\infty}d\omega_{km}\,\frac{\sin^2 \tfrac{1}{2}\omega'_{km}T}{\omega_{km}^2\,[1-(\omega_{km}/\omega)^2]^2}= \tag{2.32}$$

$$=\frac{1}{\pi}\int_{-\infty}^{+\infty}dx\,\frac{\sin^2 x}{x^2\,[1-(x/n\pi)^2]^2}\xrightarrow{n\to\infty}\frac{1}{\pi}\int_{-\infty}^{+\infty}dx\,\frac{\sin^2 x}{x^2}=1$$

where the following property is used

$$\int_0^T dt\,s(t)\,e^{i\omega_{km}t}=2\int_0^T dt\sin^2\omega t\,e^{i\omega_{km}t}=$$

$$=\int_0^T e^{i\omega_{km}t}\,dt-\tfrac{1}{2}\int_0^T e^{i(\omega_{km}+2\omega)t}\,dt-$$

$$-\tfrac{1}{2}\int_0^T e^{i(\omega_{km}-2\omega)t}\,dt=i\,\frac{(1-e^{i\omega_{km}T})}{\omega_{km}\,[1-(\omega_{km}/\omega)^2]} \tag{2.33}$$

Thus, Condition (2.28) is verified for model (2.18a). This occurrence was seen also by proving the convergence of integral (2.32) to 1, as well as by using computer assistance to show that the integral is well approximated by 1 beginning from values of time of the order of 10^{-18} sec.

Notice that

$$\frac{1}{\pi}\lim_{T/2\to\infty}\frac{1}{T/2}\,\frac{\sin^2\omega_{km}T/2}{\omega_{km}^2\,[1-(\omega_{km}/\omega)^2]^2}=$$

$$=\delta(\omega_{km})\,\frac{1}{[1-(\omega_{km}/\omega)^2]^2} \tag{2.34}$$

and

$$\lim_{T/2\to\infty}\frac{1}{\pi T}\int_{-\infty}^{+\infty}\Big|\int_0^T\sin^2\omega t\,e^{i\omega_{km}t}\,dt\Big|^2\,d\omega_{km}=$$

$$=\int\delta(\omega_{km})\,\frac{d\,\omega_{km}^2}{[1-(\omega_{km}/\omega)^2]^2}=1 \tag{2.35}$$

Thus, we recover the conservation of the energy level ($E_k = E_m$) for sufficiently large values of time. This can be recovered by considering also a sufficient collection of interactions, by therefore achieving the stability of the electron orbit of the hydrogen atom, as desired. In regard to the positronium, however, the situation is different because the orbits are now unstable. This property was seen as promising, rather than contradictory, because the positronium is, after all, unstable. What we may have here is an interpretation of the instability via arguments perhaps more fundamental than those currently used.

The higher order contributions (2.30) and (2.31) were also studied for model (2.18a) resulting in a behaviour consistent with the desired features. These calculations are omitted here for brevity.

The repetition of the analysis for other models will be left to the interested reader, jointly with the study of additional profiles (Møller and Bhabha scattering, electron–photon interaction, etc.).

The reader should keep in mind that models (2.18) represent a crude first approximation of the desired generalization of the Coulomb law, for which several implementations are conceivable. That which affects most directly the calculations is based on a differentiation of the characteristic frequencies of the two elementary particles considered, as expected from relativistic generalizations (because in this case the characteristic frequency of the electron depends on the speed).

We now close the section with a few remarks on the problem of the elementary charge. As indicated in the Introduction, the problematic aspects for an interpretation of reaction (1.2) via transmutation of fields are focused on the notion of charge. The model presented in this section indicates the possibility that what is currently considered as the "elementary" charge may, in the final analysis, be subjected to a structure model. This should not be surprising if one reflects a moment with an open mind on the fact that the currently avail–

able description of the photon is of structural character, while we do not possess at this time a structure model of the electron at rest. In fact, Dirac's equation provides a quite effective dynamical description of the electron, but it is not a structure equation, and certainly has no sufficient physical information for electrons at rest.

A possible initial model can be studied along the following assumptions.

(1) Space (and its geometry) can be considered as the fundamental physical entity which permits the existence and propogation not only of the photons (as generally accepted), but also of the electrons (as not so generally accepted).

(2) Electrons and positrons are conceived as oscillatory excitation of space which are "elementary" in the sense of being monochromatic; and

(3) The oscillatory structure is propogated by space, creating the radial and periodic force field (the electric field)

$$\mathbf{F} = \frac{\hbar \omega R}{\pi} \nabla \left(\frac{1}{r}\right) \sin (\omega t + a) \tag{2.36}$$

with a corresponding nonradial and rotational force field (the magnetic field) which is not considered here.

The main property of free force field (2.36) is that it contains both attractive and repulsive forces. The separation and selection of only one type of force can then be associated with the interaction of two elementary fields of this type, e.g., via the use of the distribution

$$\delta \pm (w) = \mp \frac{P}{2 \pi i w} + \tfrac{1}{2} \delta (w) \tag{2.37}$$

which results in the generalized Coulomb law

$$F = \frac{\hbar \omega R}{\pi} \nabla (\tfrac{1}{2}) \int_{-\infty}^{+\infty} e^{-iw \sin (\omega t + a)} \delta \pm (w) \sin (\omega t + a) \, dw \tag{2.38a}$$

The imposition of condition (2.10) for $T = 2 \pi n / \omega$, $n \gg 1$ yields the intriguing result whereby the amplitude of oscillation R must coincide with the classical radius of the electron

$$R = R_0 = 2.8 \times 10^{-13} \text{ cm} \tag{2.38b}$$

From law (2.38) one then obtains the desired structure model of the elementary electric charge

$$e = \left[\frac{\hbar \omega R_0}{\pi} \int_{-\infty}^{+\infty} e^{-iw \sin (\omega t + a)} \delta \pm (w) \sin (\omega t + a) \, dw \right]^{1/2} \tag{2.39}$$

The separation of the attractive and repulsive forces can also be achieved via models of type (2.18a). In this case, we would have the charge model

$$e = \left[\frac{2 \hbar \omega R_0}{\pi} \right]^{1/2} \sin \omega t \tag{2.40}$$

Intriguingly, the transition from electrons to positrons could be done in this case according to the old idea of reversing the direction of time. Other models are conceivable along similar lines, and they will not be studied here. Models (2.39) and (2.40) verify Conditions 1 and 2. The study of Condition 3 will be left to the interested readers.

Notice that the time average of the structure model for the elementary charge is relativistic invariant, as desirable for the electromagnetic interactions (although no necessarily for the strong).

As side philosophical comments, notice that the assumption of space as the fundamental physical entity underlying particles and their fields is reminiscent of Fresnel's views. Intriguingly, the known objection of the "ethereal wind" is eliminated by the model. In fact, according to the early view of space, there was a drastic distinction between electromagnetic waves and matter. The former were a manifestation of space, including their propagation, while the latter was not. Once all particles, and thus, matter, are reduced to a suitable collection of space excitation, the logical foundation of the objection ceases to exist because, when massive bodies are moved in space, we simply have the transmission of space excitation from one region to another.

Along more recent lines, the model is reminiscent of contemporary views of space and its geometry, such as Weeler's geometrodynamics. As a matter of fact, one can start from Weeler's view of the electron as space excitation, build a geometric model on the propagation of the central force field by space itself, and then use the structure model of charge proposed in this paper. What I am attempting to express here is the expectation that the proposed model can be suitably geometrized.

As a final comment, the reader should keep in mind that the compatibility of the model with experimental data may also be due to Heisenberg's uncertainty principle for the electromagnetic interactions. In fact, the pulsating behaviour of the model may, in the final analysis, result to be compatible with the uncertainty Δr of the electron coordinates.

3. CONCLUDING REMARKS

The content of this paper can be summarized in terms of the following.

HYPOTHESIS. The electrostatic interaction between two electrons, or two positrons, or one electron and one positron, has an explicit time dependence of pulsating type with periods of the order of the characteristic period of the electron at rest

$$T = \frac{2 \pi}{\omega} = 0.829 \times 10^{-20} \text{ sec.}$$

A number of preliminary considerations have been presented to the effect that, upon suitable technical treatments, the hypothesis may result to be compatible with currently available experimental data on the electron phenomenology.

Clearly, a possibility of this type is expected to be resolved experimentally in due time. Without entering into a detailed treatment, I would like to indicate the following lines of conceivable experiments.

1. **Experiments of resonance type.** Pulsating behaviours are typical phenomena which can be enhanced via resonance effects. The enhancement then constitutes rather clear experimental evidence of the pulsating behaviours themselves. With the understanding that the technical difficulties of realization are expected to be considerable (due to the high frequencies involved), the experiments in this case are conceptually simple, and consist, for instance, of subjecting an electron beam to a periodically varying electric field varying with a frequency suitable to activate resonance effects.

2. **Experiments via the use of cross sections.** As clearly indicated in equations (2.26) and (2.27), the hypothesis submitted in this paper implies a change of the cross sections involving electrostatic interactions of single electron pairs. Therefore, the hypothesis may be subjectable to experimental verification. An experimental difficulty is due to the fact that possible deviations are expected in very small periods of time, as indicated, for instance, in equation (2.28).

3. **New experiments on the positronium.** The hypothesis implies significant differences in the energy levels and orbits between the hydrogen atom and the positronium. In particular, the stability of the orbits of the former is confirmed, while the orbits of the latter are expected to be unstable. Conceivably, these differences are experimentally measurable via a refinement of available data on the positronium.

The connection of the structure model of the elementary charge presented in this paper and the studies on Lie–admissible formulations will be presented at some later time. At this moment, I would like to limit myself to the indication that the model of this paper is expected to be of some usefulness for the current studies on the old idea that the neutron is a bound state of one proton and one electron, although realized this time via extended particles under conditions of complete wave overlapping, nonlocal, non–Hamiltonian forces, and Lie–admissible generalizations of conventional Lie formulations. As familiar to researchers in the field, these latter studies are based on the idea that the intrinsic characteristics of particles change in the transition from the physical conditions under which they have been measured until now (long range electromagnetic interactions), to the rather peculiar physical conditions of wave overlappings and mutual penetration with other particles. In particular, the alteration (Lie–admissible mutation) of the spin of the electron is crucial to achieve a consistent structure model of the neutron as one proton and one electron in complete mutual penetration. However, an alteration of all intrinsic characteristics of particles , thus including the charge, is expected under the physical conditions and forces considered.

The point I would like to express is that, when the notion of static and perennial value

of the "elementary" charge is replaced by a dynamical structure model, its dependence on the physical conditions considered becomes rather natural, and so becomes its possible Lie–admissible mutation. This latter possibility calls for a Lie–admissible generalization of conventional (Abelian and non–Abelian) gauge models for nonlocal, non–Lagrangian, and gauge–noninvariant forces due to wave overlapping as expected, say, for the strong interactions. This line is under study by colleagues in Lie–admissible formulations, and I shall not consider it here.