Draft dated Deccember 24, 2007

HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY

Volume I:

Iso-, Geno-, Hyper-Formulations for Matter and Their Isoduals for Antimatter

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International Academic Press

U. S. Library of Congress
Cataloguing in publication data:
Santilli, Ruggero Maria, 1935 –
Foundations of Hadronic Mathematics
Mechanics and Chemistry
Volume I:
Iso-, Geno-, and Hyper-Formulations for
matter and their Isoduals for Antimatter
with
Bibliography and
Index
Additional data sup-plied on request

ISBN xxxxxxxxxxxxx

INTERNATIONAL ACADEMIC PRESS

This volume is dedicated to the memory of

Professor Grigorios Tsagas

in recognition of his pioneering work on the Lie-Santilli isotheory.

Contents

Chapter 1

SCIENTIFIC IMBALANCES OF THE TWENTIETH CENTURY

1.1 THE SCIENTIFIC IMBALANCE CAUSED BY ANTIMATTER

1.1.1 Needs for a Classical Theory of Antimatter

The first large scientific imbalances of the 20-th century studied in this monograph is that caused by the treatment of *matter* at all possible levels, from Newtonian to quantum mechanics, while *antimatter* was solely treated at the level of *second quantization* [1].

Besides an evident lack of scientific democracy in the treatment of matter and antimatter, the lack of a consistent *classical* treatment of antimatter left open a number of fundamental problems, such as the inability to study whether a faraway galaxy or quasar is made up of matter or of antimatter, because such a study requires first a classical representation of the gravitational field of antimatter, as an evident pre-requisite for the quantum treatment (see Figure 1.1).

It should be indicated that classical studies of antimatter simply cannot be done by merely reversing the sign of the charge, because of inconsistencies due to the existence of only one quantization channel. In fact, the quantization of a classical antiparticle solely characterized by the reversed sign of the charge leads to a *particle* (rather than a charge conjugated antiparticle) with the wrong sign of the charge.

It then follows that the treatment of the gravitational field of suspected antimatter galaxies or quasars cannot be consistently done via the Riemannian geometry in which there is a simple change of the sign of the charge, as rather popularly done in the 20-th century, because such a treatment would be structurally inconsistent with the quantum formulation.

At any rate, the most interesting astrophysical bodies that can be made up of antimatter are *neutral.* In this case general relativity and its underlying Rieman-

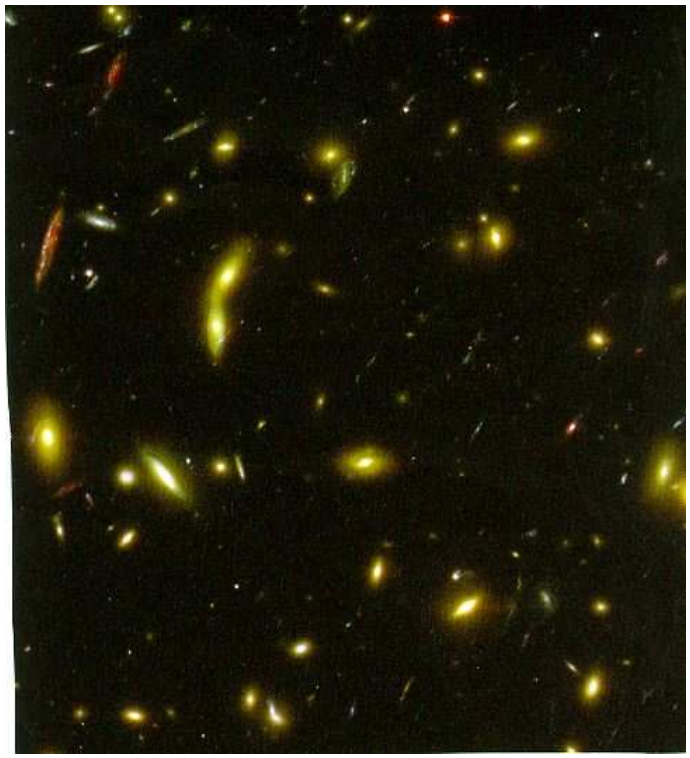

Figure 1.1. An illustration of the first major scientific imbalance of the 20-th century studied in this monograph, the inability to conduct classical quantitative studies as to whether faraway galaxies and quasars are made-up of matter or of antimatter. In-depth studies have indicated that the imbalance was not due to insufficient physical information, but instead it was due to the lack of a mathematics permitting the classical treatment of antimatter in a form compatible with charge conjugation at the quantum level.

nian geometry can provide no difference at all between matter and antimatter stars due to the null total charge. The need for a suitable new theory of antimatter then becomes beyond credible doubt.

As we shall see in Chapter 14, besides all the above insufficiencies, the biggest imbalance in the current treatment of antimatter occurs at the level of grand unifications, since all pre-existing attempts to achieve a grand unification of electromagnetic, weak and gravitational interactions are easily proved to be inconsistent under the request that the unification should hold not only for matter, as universally done until now, but also for antimatter. Hence, prior to venturing judgments on the need for a new theory of antimatter, serious scholars are suggested to inspect the entire scientific journey including the iso-grand-unification of Chapter 14.

1.1.2 The Mathematical Origin of the Imbalance

The origin of this scientific imbalance was not of physical nature, because it was due to the *lack of a mathematics suitable for the classical treatment of antimatter in such a way as to be compatible with charge conjugation at the quantum level.*

Charge conjugation is an *anti-homomorphism.* Therefore, a necessary condition for a mathematics to be suitable for the classical treatment of antimatter is that of being anti-homomorphic, or, better, anti-isomorphic to conventional mathematics.

Therefore, the classical treatment of antimatter requires *numbers, fields, functional analysis, differential calculus, topology, geometries, algebras, groups, symmetries, etc. that are anti-isomorphic to their conventional formulations for matter.*

The absence in the 20-th century of such a mathematics is soon established by the lack of a formulation of trigonometric, differential and other elementary functions, let alone complex topological structures, that are anti-isomorphic to the conventional ones.

In the early 1980s, due to the absence of the needed mathematics, the author was left with no other alternative than its construction along the general guidelines of hadronic mechanics, namely, the construction of the needed mathematics from the physical reality of antimatter, rather than adapting antimatter to pre-existing insufficient mathematics.[1]

After considerable search, the needed new mathematics for antimatter resulted in being characterized by the most elementary and, therefore, most fundamental possible assumption, that of a *negative unit,*

$$-1, \tag{1.1.1}$$

and then the reconstruction of the entire mathematics and physical theories of matter in such a way as to admit -1 as the correct left and right unit at all levels.

In fact, such a mathematics resulted in being anti-isomorphic to that representing matter, applicable at all levels of study, and resulting in being equivalent to charge conjugation after quantization.[2]

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[1]In the early 1980s, when the absence of a mathematics suitable for the classical treatment of antimatter was identified, the author was (as a theoretical physicist) a member of the Department of Mathematics at Harvard University. When seeing the skepticism of colleagues toward such an absence, the author used to suggest that colleagues should go to Harvard's advanced mathematics library, select any desired volume, and open any desired page at random. The author then predicted that the mathematics presented in that page resulted in being fundamentally inapplicable to the classical treatment of antimatter, as it did indeed result to be the case without exceptions. In reality, the entire content of advanced mathematical libraries of the early 1980s did not contain the mathematics needed for a consistent classical treatment of antimatter.

[2]In 1996, the author was invited to make a 20 minutes presentation at a mathematics meeting held in Sicily. The presentation initiated with a transparency solely containing the number -1 and the statement

1.1.3 Outline of the Studies on Antimatter

Recall that "science" requires a mathematical treatment producing numerical values that can be confirmed by experiments. Along these lines, Chapter 2 is devoted, first, to the presentation of the new mathematics suggested by the author for the classical treatment of antimatter under the name of *isodual mathematics* with Eq. (1.1.1) as its fundamental *isodual left and right unit.*

The first comprehensive presentation was made by the author in monograph [94]. The first is, however, in continuous evolution, thus warranting an update.

Our study of antimatter initiates in Chapter 2 where we present the classical formalism, proposed under the name of *isodual classical mechanics* that begins with a necessary reformulation of Newton's equations and then passes to the needed analytic theory.

The operator formulation turned out to be *equivalent,* but not identical, to the quantum treatment of antiparticles, and was submitted under the name of *isodual quantum mechanics.*

Following these necessary foundational studies, Chapter 2 includes the detailed verification that the new *isodual theory of antimatter* does indeed verify *all* classical and particle experimental evidence.

In subsequent chapters we shall then study some of the predictions of the new isodual theory of antimatter, such as antigravity, a causal time machine, the isodual cosmology in which the universe has null total characteristics, and other predictions that are so far reaching as to be at the true edge of imagination.

All these aspects deal with point-like antiparticles. The study of extended, nonspherical and deformable antiparticles (such as the antiproton and the antineutron) initiates in Chapter 3 for reversible conditions and continues in the subsequent chapters for broader irreversible and multi-valued conditions.

1.2 THE SCIENTIFIC IMBALANCE CAUSED BY NONLOCAL-INTEGRAL INTERACTIONS

1.2.1 Foundations of the Imbalance

The second large scientific imbalance of the 20-th century studied in this monograph is that caused by the *reduction of contact nonlocal-integral interactions*

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that such a number was assumed as the basic left and right unit of the mathematics to be presented. Unfortunately, this first transparency created quite a reaction by most participants who bombarded the author with questions advancing his presentation, questions often repeated with evident waste of precious time without the author having an opportunity to provide a technical answer. This behavior continued for the remaining of the time scheduled for the talk to such an extent that the author could not present the subsequent transparencies proving that numbers with a negative unit verify all axioms of a field (see Chapter 2). The case illustrates that the conviction of absolute generality is so engraved among most mathematicians to prevent their minds from admitting the existence of *new* mathematics.

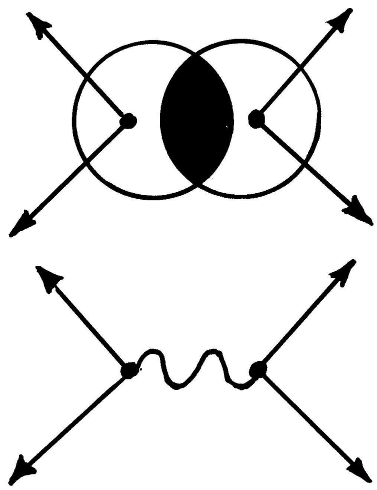

Figure 1.2. A first illustration of the second major scientific imbalance of the 20-th century studied in this monograph, the abstraction of extended hyperdense particles, such as protons and neutrons, to points, with consequential ignorance of the nonlocal and nonpotential effects caused by the deep overlapping of the hyperdense media in the interior of said particles. As we shall see, besides having major scientific implications, such as a necessary reformulation of Feynman's diagrams, the quantitative treatment of the nonlocal and nonpotential effects of this figure permits truly momentous advances, such as the conversion of divergent perturbative series into convergent forms, as well as the prediction and industrial development of basically new, clean energies and fuels.

among extended particles to pre-existing action-at-a-distance local-differential interactions among point-like particles (see Figure 1.2).

It should be indicated that there exist numerous definitions of "nonlocality" in the literature, a number of which have been adapted to be compatible with pre-existing doctrines. The notion of nonlocality studied by hadronic mechanics is that specifically referred to *interactions of contact type not derivable from a potential and occurring in a surface, as for the case of resistive forces, or in a volume, as for the case of deep mutual penetration and overlapping of the wavepackets and/or charge distributions of particles.*

The imbalance was mandated by the fact (well known to experts to qualify as such) that *nonlocal-integral and nonpotential interactions are structurally incompatible with quantum mechanics and special relativity*, beginning with its local-differential topology, because the interactions here considered cause the catastrophic collapse of the mathematics underlying special relativity, let alone the irreconcilable inapplicability of the physical laws.

In fact, the local-differential topology, calculus, geometries, symmetries, and other mathematical methods underlying special relativity permit the sole consistent description of *a finite number of point-like particles moving in vacuum (empty space).* Since points have no dimension and, consequently, cannot experience collisions or contact effects, the only possible interactions are at-a-distance, thus being derivable from a potential. The entire machinery of special relativity then follows. For systems of particles at large mutual distances for which the above setting is valid, such as for the structure of the hydrogen atom, special relativity is then *exactly valid.*

However, classical point-like particles do not exist; hadrons are notoriously extended; and even particles with *point-like charge,* such as the electron, do not have "point-like wavepackets". As we shall see, the representation of particles and/or their wavepackets as they really are in nature, that is, extended, generally nonspherical and deformable, cause the existence of contact effects of nonlocal-integral as well as zero-range nonpotential type that are beyond any hope of quantitative treatment via special relativity.

This is the case for all systems of particles at short mutual distances, such as the structure of hadrons, nuclei and stars, for which special relativity is *inapplicable* (rather than "violated") because not conceived or intended for the latter systems. The understanding is that the *approximate character* remains beyond scientific doubt.

Well known organized academic interests on Einsteinian doctrines then mandated the abstraction of nonlocal-integral systems to point-like, local-differential forms as a necessary condition for the validity of special relativity. This occurrence caused a scientific distortion of simply historical proportions because, while the existence of systems for which special relativity is fully valid is beyond doubt, the assumption that all conditions in the universe verify Einsteinian doctrines is a scientific deception for personal gains.

In Section 1.1 and in Chapter 2, we show the structural inability of special relativity to permit a *classical* representation of antimatter in a form compatible with charge conjugation. In this section and in Chapter 3, we show the inability of special relativity to represent extended, nonspherical and deformable particles or antiparticles and/or their wavepackets under nonlocal-integral interactions at short distances.

In Section 1.3 and in Chapter 4, we show the irreconcilable inapplicability of special relativity for all possible, classical and operator irreversible systems of particles and antiparticles. The widely ignored theorems of catastrophic inconsistencies of Einstein's gravitation are studied in Section 1.4 and in Chapter 3.

A primary purpose of this monograph is to show that the political adaptation of everything existing in nature to special relativity, rather than constructing new relativities to properly represent nature, prevents the prediction and quantitative treatment of new clean energies and fuels so much needed by mankind. In fact, new clean energies are permitted precisely by contact, nonlocal-integral and nonpotential effects in hadrons, nuclei and stars that are beyond any dream of treatment via special relativity.

Therefore, *the identification of the limits of applicability of Einsteinian doctrines and the construction of new relativities are nowadays necessary for scientific accountability vis-a-vis society, let alone science.*

Needless to say, due to the complete symbiosis of special relativity and relativistic quantum mechanics, the inapplicability of the former implies that of the latter, and vice-versa. In fact, quantum mechanics will also emerge from our studies as being only *approximately valid* for system of particles at short mutual distances, such as for hadrons, nuclei and stars, for the same technical reasons implying the lack of exact validity of special relativity.

The resolution of the imbalance due to nonlocal interactions is studied in Chapter 3.

1.2.2 Exterior and Interior Dynamical Problems

The identification of the scientific imbalance here considered requires the knowledge of the following fundamental distinction:

DEFINITION 1.2.1: Dynamical systems can be classified into:

EXTERIOR DYNAMICAL SYSTEMS, consisting of particles at sufficiently large mutual distances to permit their point-like approximation under sole action-at-a-distance interactions, and

INTERIOR DYNAMICAL PROBLEMS, consisting of extended and deformable particles at mutual distances of the order of their size under action-at-a-distance interactions as well as contact nonpotential interactions.

Interior and exterior dynamical systems of antiparticles are defined accordingly.

Typical examples of exterior dynamical systems are given by planetary and atomic structures. Typical examples of interior dynamical systems are given by the structure of planets at the classical level and by the structure of hadrons, nuclei, and stars at the operator level.

The distinction of systems into exterior and interior forms dates back to Newton [2], but was analytically formulated by Lagrange [3], Hamilton [4], Jacobi[3][5] and others (see also Whittaker [6] and quoted references). The distinction was

[3]Contrary to popular belief, the celebrated Jacobi theorem was formulated precisely for the general analytic equations with external terms, while all reviews known to this author in treatises on mechanics of the 20-th century present the reduced version of the Jacobi theorem for the equations without external terms. Consequently, the reading of the original work by Jacobi [5] is strongly recommended over simplified versions.

still assumed as fundamental at the beginning of the 20-th century, but thereafter the distinction was ignored.

For instance, Schwarzschild wrote two papers in gravitation, one of the *exterior gravitational problem* [7], and a second paper on the *interior gravitational problem* [8]. The former paper reached historical relevance and is presented in all subsequent treatises in gravitation of the 20-th century, but the same treatises generally ignore the second paper and actually ignore the distinction into gravitational exterior and interior problems.

The reasons for ignoring the above distinction are numerous, and have yet to be studied by historians. A first reason is due to the widespread abstraction of particles as being point-like, in which case all distinctions between interior and exterior systems are lost since all systems are reduced to point-particles moving in vacuum.

An additional reason for ignoring interior dynamical systems is due to the great successes of the planetary and atomic structures, thus suggesting the reduction of all structures in the universe to exterior conditions.

In the author's view, the primary reason for ignoring interior dynamical systems is that they imply the inapplicability of the virtual totality of theories constructed during the 20-th century, including classical and quantum mechanics, special and general relativities, etc., as we shall see.

The most salient distinction between exterior and interior systems is the following. Newton wrote his celebrated equations for a system of n point-particle under an arbitrary force not necessarily derivable from a potential,

$$m_a \times \frac{dv_{ak}}{dt} = F_{ak}(t, r, v), \tag{1.2.1}$$

where: $k = 1, 2, 3$; $a = 1, 2, 3, ..., n$; t is the time of the observer; r and v represent the coordinates and velocities, respectively; and the conventional associative multiplication is denoted hereon with the symbol $\times$ to avoid confusion with numerous additional inequivalent multiplications we shall identify during our study.

Exterior dynamical systems occur when Newton's force F_{ak} is entirely derivable from a potential, in which case the system is entirely described by the sole knowledge of a Lagrangian or Hamiltonian and the *truncated Lagrange and Hamilton analytic equations, those without external terms*

$$\frac{d}{dt}\frac{\partial L(t, r, v)}{\partial v_a^k} - \frac{\partial L(t, r, v)}{\partial r_a^k} = 0, \tag{1.2.2a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t, r, p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t, r, p)}{\partial r_a^k}, \tag{1.2.2b}$$

$$L = \frac{1}{2} \times m_a \times \mathbf{v}_a^2 - V(t, r, v), \tag{1.2.2c}$$

$$H = \frac{\mathbf{p}_a^2}{2 \times m_a} + V(t, r, p), \tag{1.2.2d}$$

$$V = U(t, r)_{ak} \times v_a^k + U_o(t, r); \tag{1.2.2e}$$

where: $\mathbf{v}$ and $\mathbf{p}$ represent three-vectors; and the convention of the sum of repeated indices is hereon assumed.

Interior dynamical systems when Newton's force F_{ak} is partially derivable from a potential and partially of contact, zero-range, nonpotential types thus admitting additional interactions that simply cannot be represented with a Lagrangian or a Hamiltonian. For this reason, Lagrange, Hamilton, Jacobi and other founders of analytic dynamics presented their celebrated equations with *external terms representing precisely the contact, zero-range, nonpotential forces among extended particles.* Therefore, the treatment of interior systems requires the *true Lagrange and Hamilton analytic equations, those with external terms*

$$\frac{d}{dt}\frac{\partial L(t, r, v)}{\partial v_a^k} - \frac{\partial L(t, r, v)}{\partial r_a^k} = F_{ak}(t, r, v), \tag{1.2.3a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t, r, p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t, r, p)}{\partial r_a^k} + F_{ak}(t, r, p), \tag{1.2.3b}$$

$$L = \frac{1}{2} \times m_a \times \mathbf{v}_a^2 - V(t, r, v), \tag{1.2.3c}$$

$$H = \frac{\mathbf{p}_a^2}{2 \times m_a} + V(t, r, p), \tag{1.2.3d}$$

$$V = U(t, r)_{ak} \times v_a^k + U_o(t, r), \tag{1.2.3e}$$

$$F(t, r, v) = F(t, r, p/m). \tag{1.2.3f}$$

Comprehensive studies were conducted by Santilli in monographs [9] (including a vast historical search) on the necessary and sufficient conditions for the existence of a Lagrangian or a Hamiltonian known as the *conditions of variational selfadjointness.* These studies permitted a rigorous separation of all acting forces into those derivable from a potential, or variationally selfadjoint (SA) forces, and those not derivable from a potential, or variationally nonselfadjoint (NSA) forces according to the expression

$$F_{ak} = F_{ak}^{SA}(t, r, v) + F_{ak}^{NSA}(t, r, v, a, ...). \tag{1.2.4}$$

In particular, the reader should keep in mind that, while selfadjoint forces are of Newtonian type, *nonselfadjoint forces are generally non-Newtonian,* in the sense

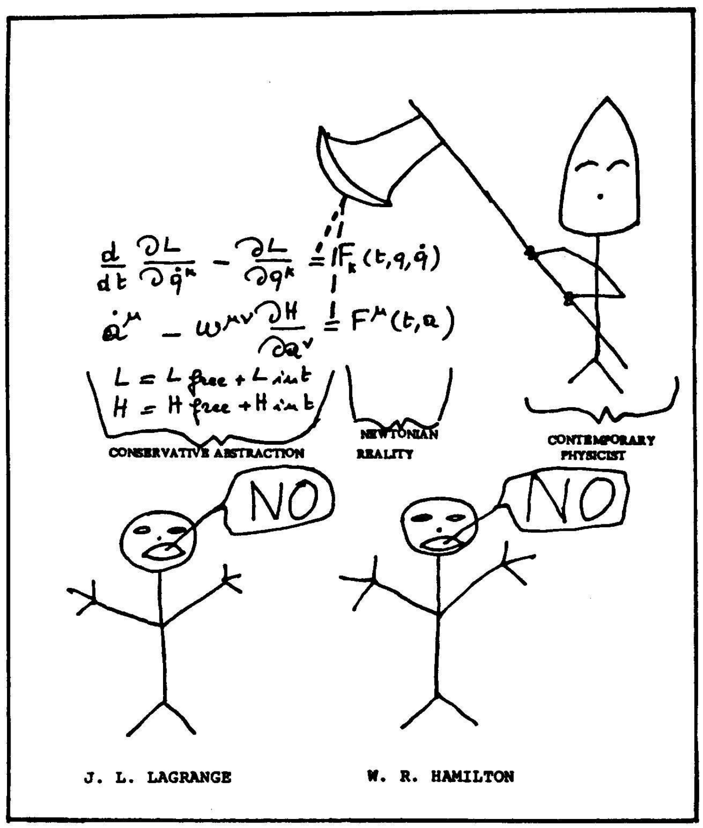

Figure 1.3. A reproduction of a "vignetta" presented by the author in 1978 to the colleagues at the Lyman Laboratory of Physics of Harvard University as part of his research under his DOE contract number DE-ACO2-80ER-10651.A001 to denounce the truncation of the external terms in Lagrange's and Hamilton's equations that was dominating physical theories of the time for the clear intent of maintaining compatibility with Einsteinian doctrines (since the latter crucially depend on the truncation depicted in this figure). The opposition by the Lyman colleagues at Harvard was so great that, in the evident attempt of tryinmg to discourage the author from continuing the research on the true Lagrange's and Hamilton's equations, the Lyman colleagues kept the author without salary for one entire academic year, even though the author was the recipient of a DOE grant and he had two children in tender age to feed and shelter. Most virulent was the opposition by the Lyman colleagues to the two technical memoirs [39,50] presented in support of the "vignetta" of this figure, for the evident reason that they dealt with a broadening of Einsteinian doctrines beginning with their title, and then continuing with a broadening of algebras, symmetries, etc.. But the author had no interest in a political chair at Harvard University, was sole interested in pursuing *new* scientific knowledge, and continued the research by dismissing the fierce opposition by his Lyman colleagues as ascientific and asocial (the episode is reported with real names in book [93] of 1984 and in the 1,132 pages of documentation available in Ref. [94]). As studied in details in these two volumes, the proper mathematical treatment of the true, historical, analytic equations, those with external terms, permits indeed the advances opposed by the Lyman colleagues, namely, the achievement of coverings of Einsteinian doctrines, that, being invariant (as shown later on), will indeed resist the test of time, while permitting the prediction and industrial development of new clean energies and fuels, thus confirming a societal, let alone scientific need for their serious study (se Footnote 1 of Volume II and subsequent footnotes for details).

of having an unrestricted functional dependence, including that on accelerations a and other non-Newtonian forms.[4]

As we shall see, nonselfadjoint forces generally have a nonlocal-integral structure that is usually reduced to a local-differential form via power series expansions in the velocities.

For instance, the contact, zero-range, resistive force experienced by a missile moving in our atmosphere is characterized by an integral over the surface of the missile and it is usually approximated by a power series in the velocities, e.g. $F^{NSA} = k_1 \times v + k_2 \times v^2 + k_3 \times v^3 + \ldots$ (see Figure 1.3).

Moreover, the studies of monographs [9] established that, for the general case in three dimensions, *Lagrange's and Hamilton's equations without external terms can only represent in the coordinates of the experimenter exterior dynamical systems, while the representation of interior dynamical systems in the given coordinates* (t, r) *of the experimenter require the necessary use of the true analytic equations with external terms.*

Whenever exposed to dynamical systems not entirely representable via the sole knowledge of a Lagrangian or a Hamiltonian, a rather general attitude is that of transforming them into an equivalent purely Lagrangian or Hamiltonian form. these transformations are indeed mathematically possible, but they are physically insidious.

It is known that, under sufficient continuity and regularity conditions and under the necessary reduction of nonlocal external terms to local approximations such as that in Eq. (1.2.4), the *Darboux's theorem* of the symplectic geometry or, equivalently, the *Lie-Koening theorem* of analytic mechanics assure the existence of coordinate transformations

$$\{r, p\} \rightarrow \{r'(r, p), p'(r, p)\}, \tag{1.2.5}$$

under which nonselfadjoint systems (1.2.2) can be turned into a selfadjoint form (1.2.1), thus eliminating the external terms.

However, coordinate transforms (1.2.5) are *necessarily nonlinear.* Consequently, the new reference frames are *necessarily noninertial.* Therefore, the elimination of the external nonselfadjoint forces via coordinate transforms cause the necessary loss of Galileo's and Einstein's relativities.

Moreover, it is evidently impossible to place measuring apparata in new coordinate systems of the type $r' = exp(k \times p)$, where k is a constant. For these reasons, the use of Darboux's theorem or of the Lie-Koening theorem was strictly prohibited in monographs [9,10,11]. Thus, to avoid misrepresentations, the following basic assumption is hereon adopted:

[4]There are serious rumors that a famous physicist from a leading institution visited NASA in 1998 to propose a treatment of the trajectory of the space shuttle during re-entry via (the truncated) Hamiltonian mechanics, and that NASA engineers kindly pushed that physicist through the door.

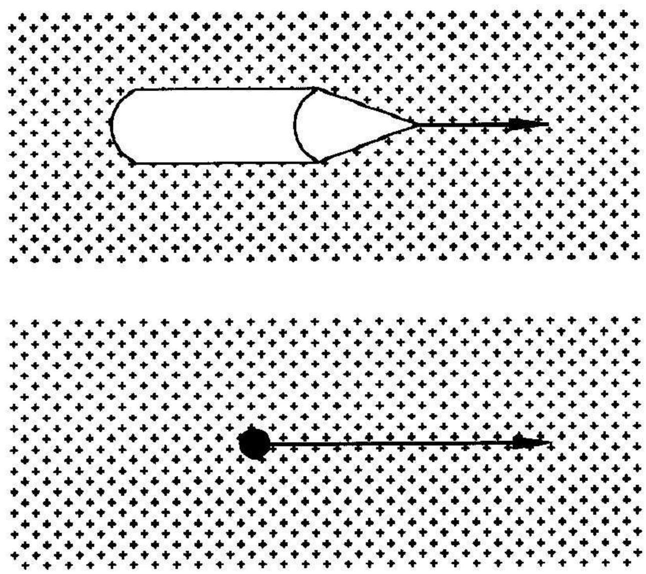

Figure 1.4. Another illustration of the major scientific imbalance studied in this monograph. The top view depicts a typical Newtonian system with nonlocal and nonpotential forces, such as a missile moving in atmosphere, while the bottom view depicts its reduction to point-like constituents conjectured throughout the 20-th century for the evident purpose of salvaging the validity of quantum mechanics and Einsteinian doctrines. However, the consistency of such a reduction has now been disproved by theorems, thus confirming the necessity of nonlocal and nonpotential interactions at the primitive elementary level of nature.

ASSUMPTION 1.2.1: The sole admitted analytic representations are those in the fixed references frame of the experimenter without the use of integrating factors, called direct analytic representations.

Only *after* direct representations have been identified, the use of the transformation theory may have physical relevance. Due to its importance, the above assumption will also be adopted throughout this monograph.

As an illustration, the admission of integrating factors within the fixed coordinates of the experimenter does indeed allow the achievement of an analytic representation without external terms of a restricted class of nonconservative systems, resulting in Hamiltonians of the type $H = e^{f(t,r,\ldots)} \times p^2/2 \times m$. This Hamiltonian has a fully valid *canonical meaning* of representing the time evolution. However, this Hamiltonian loses its meaning as representing the energy of the system. The quantization of such a Hamiltonian then leads to a plethora of illusions, such as the belief that the uncertainty principle for energy and time is still valid while, for the example here considered, such a belief has no sense because H does not represent the energy (see Refs. [9b] for more details).

Under the strict adoption of Assumption 1.2.1, all these ambiguities are absent because *H will always represent the energy,* irrespective of whether conserved or nonconserved, thus setting up solid foundations for correct physical interpretations.

1.2.3 General Inapplicability of Conventional Mathematical and Physical Methods for Interior Dynamical Systems

The impossibility of reducing interior dynamical systems to an exterior form within the fixed reference frame of the observer causes the loss for interior dynamical systems of all conventional mathematical and physical methods of the 20-th century.

To begin, the presence of irreducible nonselfadjoint external terms in the analytic equations causes the loss of their derivability from a variational principle. In turn, the lack of an action principle and related Hamilton-Jacobi equations causes the lack of any possible quantization, thus illustrating the reasons why the voluminous literature in quantum mechanics of the 20-th century carefully avoids the treatment of analytic equations with external terms.

By contrast, *one of the central objectives of this monograph is to review the studies that have permitted the achievement of a reformulation of Eqs. (1.2.3) fully derivable from a variational principle in conformity with Assumption 1.2.1, thus permitting a consistent operator version of Eqs. (1.2.3) as a covering of conventional quantum formulations.*

Recall that Lie algebras are at the foundations of all classical and quantum theories of the 20-th century. This is due to the fact that the brackets of the time evolution as characterized by Hamilton's equations,

$$\frac{dA}{dt} = \frac{\partial A}{\partial r_a^k} \times \frac{dr_a^k}{dt} + \frac{\partial A}{\partial p_{ak}} \times \frac{dp_{ak}}{dt} =$$

$$= \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ak}} - \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ak}} = [A, H], \tag{1.2.6}$$

firstly, verify the conditions to characterize an *algebra* as currently understood in mathematics, that is, the brackets $[A, H]$ verify the right and left scalar and distributive laws,

$$[n \times A, H] = n \times [A, H], \tag{1.2.7a}$$

$$[A, n \times H] = [A, H] \times n, \tag{1.2.7b}$$

$$[A \times B, H] = A \times [B, H] + [A, H] \times B, \tag{1.2.7c}$$

$$[A, H \times Z] = [A, H] \times Z + H \times [A, Z], \tag{1.2.7d}$$

and, secondly, the brackets $[A, H]$ verify the *Lie algebra axioms*

$$[A, B] = -[B, A], \tag{1.2.8a}$$

$$[[A, B], C] + [[B, C], A] + [[C, A], B] = 0. \tag{1.2.8b}$$

The above properties then persist following quantization into the operator brackets $[A, B] = A \times B - B \times A$, as well known.

When adding external terms, the resulting new brackets,

$$\begin{aligned} \frac{dA}{dt} &= \frac{\partial A}{\partial r_a^k} \times \frac{dr_a^k}{dt} + \frac{\partial A}{\partial p_{ak}} \times \frac{dp_{ak}}{dt} = \\ &= \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ak}} - \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ak}} + \frac{\partial A}{\partial r_a^k} \times F_a^k = \\ &= (A, H, F) = [A, H] + \frac{\partial A}{\partial r_a^k} \times F_a^k, \end{aligned} \tag{1.2.9}$$

violate the right scalar law (1.2.7b) and the right distributive law (1.2.7d) and, therefore, the brackets (A, H, F) do not constitute any algebra at all, let alone violate the basic axioms of the Lie algebras [9b].

The loss of the Lie algebras in the brackets of the time evolution of interior dynamical systems in their historical treatment by Lagrange, Hamilton, Jacobi and other founders of analytic dynamics, causes the loss of all mathematical and physical formulations built in the 20-th century.

The loss of basic methods constitutes the main reason for the abandonment of the study of interior dynamical systems. In fact, external terms in the analytic equations were essentially ignored through the 20-th century, by therefore adapting the universe to analytic equations (1.2.2) today known as the *truncated analytic equations.*

By contrast, *another central objective of this monograph is to review the studies that have permitted the achievement of a reformulation of the historical analytic equations with external terms,that is not only derivable from an action principle as indicated earlier, but also characterizes brackets in the time evolution that, firstly, constitute an algebra and, secondly, that algebra results in being a covering of Lie algebras.*

1.2.4 Inapplicability of Special Relativity for Dynamical Systems with Resistive Forces

The scientific imbalance caused by the reduction of interior dynamical systems to systems of point-like particles moving in vacuum, is indeed of historical proportion because it implied the belief of the exact applicability of special relativity and quantum mechanics for all conditions of particles existing in the universe, thus implying their applicability under conditions for which these theories were not intended for.

A central scope of this monograph is to show that the imposition of said theories to interior dynamical systems causes the suppression of new clean energies and fuels already in industrial, let alone scientific, development, thus raising serious problems of scientific ethics and accountability.

At the classical level, the "inapplicability" (rather then the "violation") of (the Galilean and) special relativities for the description of an interior system such as a missile in atmosphere (as depicted in Figure 1.4) is beyond credible doubt, as any expert should know to qualify as such, because said relativities can only describe systems with action-at-a-distance potential forces, while the force acting on a missile in atmosphere are of contact-zero-range nonpotential type.

Despite this clear evidence, the resiliency by organized academic interests on conventional relativities knows no boundaries. As indicated earlier, when faced with the above evidence, a rather general posture is, that the resistive forces are "illusory" because, when the missile in atmosphere is reduced to its elementary point-like constituents all resistive forces "disappear."

Such a belief is easily proved to be nonscientific by the following property that can be proved by a first year graduate student in physics:

THEOREM 1.2.1 [9b]: A classical dissipative system cannot be consistently reduced to a finite number of quantum particles under sole potential forces and, vice-versa, no ensemble of a finite number of quantum particles with only potential forces can reproduce a dissipative classical system under the correspondence or other principles.

Note that the above property causes the inapplicability of conventional relativities for the description of the *individual constituents* of interior dynamical systems, let alone their description as a whole.

Rather than adapting nature to pre-existing organized interests on Einsteinian doctrines, the scope of this monograph is that of adapting the theories to nature, as requested by scientific ethics and accountability.

1.2.5 Inapplicability of Special Relativity for the Propagation of Light within Physical Media

Another case of manipulation of scientific evidence to serve organized academic interests on conventional relativities is the propagation of light within physical media, such as water.

As it is well known, light propagates in water at a speed C much smaller than the speed c in vacuum and approximately given by the value

$$C = \frac{c}{n} = \frac{2}{3} \times c << c, \quad n = \frac{3}{2} >> 1. \tag{1.2.10}$$

It is well known that electrons can propagate in water at speeds bigger than the local speed of light, and actually approaching the speed of light in vacuum. In fact, the propagation of electrons faster than the local speed of light is responsible for the blueish light, called *Cerenkov light,* that can be seen in the pools of nuclear reactors.

It is well known that *special relativity was built to describe the propagation of light IN VACUUM, and certainly not within physical media.* In fact, the setting of a massive particle traveling faster than the local speed of light is in violation of the basic axioms of special relativity.

To salvage the principle of causality it is then often assumed that *the speed of light "in vacuum" is the maximal causal speed "within water".* However, in this case there is the violation of the axiom of relativistic addition of speeds, because *the sum of two speeds of light in water does not yield the speed of light,* as required by a fundamental axiom of special relativity,

$$V_{tot} = \frac{C + C}{1 + \frac{C^2}{c^2}} = \frac{12}{13} \times c \neq C. \tag{1.2.11}$$

Vice-versa, if one assumes that *the speed of light "in water" C is the maximal causal speed "in water",* the axiom of relativistic compositions of speeds is verified,

$$V_{tot} = \frac{C + C}{1 + \frac{C^2}{C^2}} = C, \tag{1.2.12}$$

but there is the violation of the principle of causality evidently due to the fact that ordinary massive particles such as the electron (and not hypothetical *tachyons*) can travel faster than the local causal speed.

Again, the resiliency by organized interests on established relativities has no boundaries. When faced with the above evidence, a general posture is that, *when light propagating in water is reduced to photons scattering among the atoms constituting water, all axioms of special relativities are recovered in full.* In fact, according to this belief, photons propagate in vacuum, thus recovering the conventional maximal causal speed c, while the reduction of the speed of light is due to the scattering of light among the atoms constituting water.

The nonscientific character of the above view is established by the following evidence known to experts to qualify as such:

1) Photons are neutral, thus having a high capability of penetration within electrons clouds, or, more technically, the scattering of photons on atomic electron

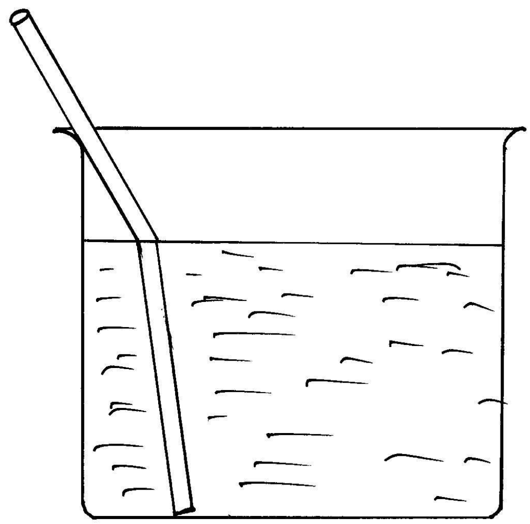

Figure 1.5. A further visual evidence of the lack of applicability of Einstein's doctrines within physical media, the refraction of light in water, due to the decrease of its speed contrary to the axiom of the "universal constancy of the speed of light". Organized academic interests on Einsteinian doctrines have claimed throughout the 20-th century that this effect is "illusory" because Einsteinian doctrines are recovered by reducing light to the scattering of photons among atoms. The political nature of the argument, particularly when proffered by experts, is established by numerous experimental evidence reviewed in the this section.

clouds (called *Compton scattering*) is rather small. Explicit calculations (that can be done by a first year graduate student in physics via quantum electrodynamics) show that, in the most optimistic of the assumptions and corrections, said scattering can account for only 3% of the reduction of the speed of light in water, thus leaving about 30% of the reduction quantitatively unexplained. Note that the deviation from physical reality is of such a magnitude that it cannot be "resolved" via the usual arbitrary parameters "to make things fit."

2) The reduction of speed occurs also for radio waves with one meter wavelength propagating within physical media, in which case the reduction to photons has no credibility due to the very large value of the wavelength compared to the size of atoms. The impossibility of a general reduction of electromagnetic waves to photon propagating within physical media is independently confirmed by the existence of vast experimental evidence on *non-Doppler's effects* reviewed in Chapter 9 indicating the existence of contributions outside the Doppler's law even when adjusted to the local speed.

3) There exist today a large volume of experimental evidence reviewed in Chapter 5 establishing that light propagates within hyperdense media, such as those in the interior of hadrons, nuclei and stars, at speed much bigger than the speed in vacuum,

$$C = \frac{c}{n} >> c, \quad n << 1. \tag{1.2.13}$$

in which case the reduction of light to photons scattering among atoms loses any physical sense (because such propagation can never reach the speed c, let alone speeds bigger than c).

In conclusion, experimental evidence beyond credible doubt has established that *the speed of light C is a local quantity dependent on the characteristics in which the propagation occurs, with speed $C = c$ in vacuum, speeds $C << c$ within physical media of low density and speeds $C >> c$ within media of very high density.*

The variable character of the speed of light then seals the lack of universal applicability of Einsteinian doctrines, since the latter are notoriously based on the philosophical assumption of "universal constancy of the speed of light".

1.2.6 Inapplicability of the Galilean and Poincaré symmetries for Interior Dynamical Systems

By remaining at the classical level, the inapplicability of Einsteinian doctrines within physical media is additionally established by the dramatic dynamical differences between the structure of a planetary system such as our Solar system, and the structure of a planet such as Jupiter.

The planetary system is a *Keplerian system,* that is, a system in which the heaviest component is at the center (actually in one of the two foci of elliptical orbits) and the other constituents orbit around it without collisions. By contrast, planets absolutely do not constitute a Keplerian system, because they do not have a Keplerian center with lighter constituents orbiting around it (see Figure 1.6).

Moreover, for a planetary system isolated from the rest of the universe, the total conservation laws for the energy, linear momentum and angular momentum are verified for each individual constituent. For instance, the conservation of the intrinsic and orbital angular momentum of Jupiter is crucial for its stability. On the contrary, for the interior dynamical problem of Jupiter, conservation laws hold only globally, while no conservation law can be formulated for individual constituents.

For instance, in Jupiter's structure we can see in a telescope the existence in Jupiter's atmosphere of *interior vortices with variable angular momentum,* yet always in such a way to verify total conservation laws. We merely have internal exchanges of energy, linear and angular momentum but always in such a way that they cancel out globally resulting in total conservation laws.

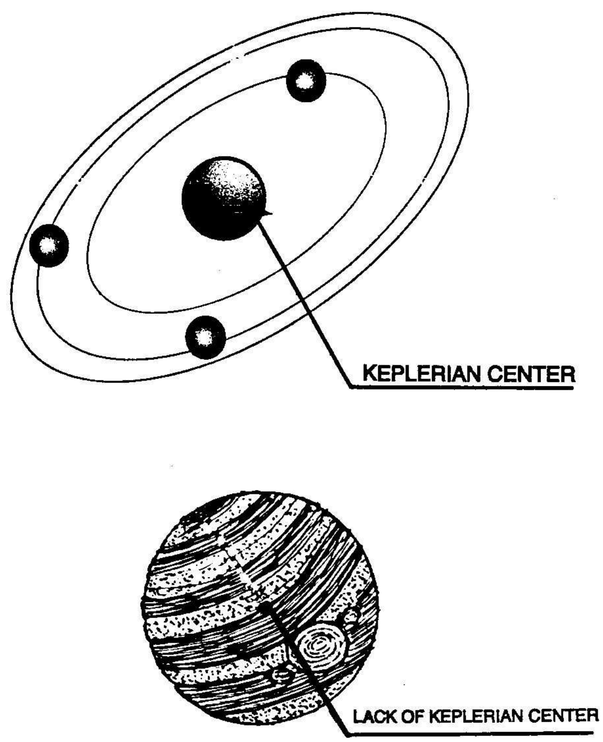

Figure 1.6. Another illustration of the second major scientific imbalance studied in this monograph, the dramatic structural differences between exterior and interior dynamical systems, here represented with the Solar system (top view) and the structure of Jupiter (bottom view). Planetary systems have a Keplerian structure with the exact validity of the Galilean and Poincaré symmetries. By contrast, interior systems such as planets (as well as hadrons, nuclei and stars) do not have a Keplerian structure because of the lack of the Keplerian center. Consequently, the Galilean and Poincaré symmetries cannot possibly be exact for interior systems in favor of covering symmetries and relativities studied in this monograph.

In the transition to particles the situation remains the same as that at the classical level. For instance, *nuclei do not have nuclei* and, therefore, nuclei are not Keplerian systems.

Similarly, the Solar system is a Keplerian system, but the Sun is not. At any rate, any reduction of the structure of the Sun to a Keplerian system directly implies the belief in the perpetual motion within a physical medium, because electrons and protons could move in the hyperdense medium in the core of a star with conserved angular momenta, namely, a belief exiting all boundaries of credibility, let alone of science.

The above evidence establishes beyond credible doubt the following:

THEOREM 1.2.2 [10b]: Galileo's and Poincaré symmetries are inapplicable for classical and operator interior dynamical systems due to the lack of Keplerian structure, the presence of contact, zero-range, non-potential interactions, and other reasons.

Note the use of the word "inapplicable", rather than "violated" or "broken". This is due to the fact that, as clearly stated by the originators of the basic spacetime symmetries (rather than their followers of the 20-th century), Galileo's and Poincaré symmetries were not built for interior dynamical conditions.

Perhaps the biggest scientific imbalance of the 20-th century has been the abstraction of hadronic constituents to point-like particles as a necessary condition to use conventional spacetime symmetries, relativities and quantum mechanics for interior conditions. In fact, such an abstraction is at the very origin of the conjecture that the undetectable *quarks* are the physical constituents of hadrons (see Section 1.2.7 for details)..

Irrespective of whether we consider quarks or other more credible particles, all particles have a wavepacket of the order of $1\,\mathrm{F} = 10^{-13}\,\mathrm{cm}$, that is, a wavepacket of the order of the size of all hadrons. Therefore, *the hyperdense medium in the interior of hadrons is composed of particles with extended wavepackets in conditions of total mutual penetration.* Under these conditions, the belief that Galileo's and Poincaré symmetries are *exactly* valid in the interior of hadrons implies the exiting from all boundaries of credibility, let alone of science.

The inapplicability of the fundamental spacetime symmetries then implies the inapplicability of Galilean and special relativities as well as of quantum nonrelativistic and relativistic mechanics. We can therefore conclude with the following:

COROLLARY 1.2.2A [10b]: Classical Hamiltonian mechanics and related Galilean and special relativities are not exactly valid for the treatment of interior classical systems such as the structure of Jupiter, while nonrelativistic and relativistic quantum mechanics and related Galilean and special relativities are not exactly valid for interior particle systems, such as the structure of hadrons, nuclei and stars.

Another important scope of this monograph is to show that the problem of the exact spacetime symmetries applicable to interior dynamical systems is not a mere academic issue, because it carries a direct societal relevance. In fact, we shall show that broader spacetime symmetries specifically built for interior systems predict the existence of new clean energies and fuels that are prohibited by the spacetime symmetries of the exterior systems.

As we shall see in Section 1.2.7, Chapter 6 and Chapter 12, the assumption that the undetectable quarks are physical constituents of hadrons *prohibits* possible new energy based on processes occurring in the interior of hadrons (rather than in the interior of their ensembles such as nuclei). On the contrary, the assumption of hadronic constituents that can be fully defined in our spacetime and can be produced free under suitable conditions, directly implies new clean energies.

1.2.7 The Scientific Imbalance Caused by Quark Conjectures

One of the most important objectives of this monograph, culminating in the presentation of Chapter 12, is to show that *the conjecture that quarks are physical particles existing in our spacetime constitutes one of the biggest threats to mankind because it prevents the orderly scientific process of resolving increasingly cataclysmic environmental problems.*

It should be clarified in this respect, as repeatedly stated by the author in his writings that *the unitary, Mendeleev-type, SU(3)-color classification of hadron into families can be reasonably considered as having a final character* (see e.g., Ref. [99] and papers quoted therein), in view of the historical capability of said classification to predict several new particles whose existence was subsequently verified experimentally. All doubts herein considered solely refer to the joint use of the same classification models as providing the structure of each individual element of a given hadronic family (for more details, see memoirs [100,101] and preprint [102] and Chapter 6).

Far from being alone, this author has repeatedly expressed the view that quarks cannot be physical constituents of hadrons existing in our spacetime for numerous independent reasons.

On historical grounds, the study of nuclei, atoms and molecules required *two different models,* one for the *classification* and a separate one for the *structure* of the individual elements of a given SU(3)-color family. Quark theories depart from this historical teaching because of their conception to represent with one single theory both the classification and the structure of hadrons.

As an example, the idea that the Mendeleev classification of atoms could jointly provide the structure of each individual atom of a given valence family is outside the boundary of science. The Mendeleev classification was essentially achieved via *classical theories,* while the understanding of the atomic structure required the construction of *a new theory,* quantum mechanics.

Independently from the above dichotomy classification vs structure, it is well known by specialists, but rarely admitted, that *quarks are purely mathematical quantities, being purely mathematical representations of a purely mathematical* *unitary symmetry defined in a purely mathematical complex-valued unitary space without any possibility, whether direct or implied, of being defined in our spacetime (representation technically prohibited by the O'Rafearthaigh theorem).*

It should be stressed that, *as purely mathematical objects, quarks are necessary for the consistency of SU(3)-color theories.* Again, quarks are the fundamental representations of said Lie symmetry and, as such, their existence is beyond doubt. All problems emerge when said mathematical representation of a mathematical symmetry in the mathematical unitary space is assumed as characterizing physical particles existing in our spacetime.

It follows that the conjecture that quarks are physical particles is afflicted by a plethora of major problematic aspects today known to experts as *catastrophic inconsistencies of quark conjectures,* such as:

1) No particle possessing the peculiar features of quark conjectures (fraction charge, etc.), has ever been detected to date in any high energy physical laboratory around the world. Consequently, a main consistency requirement of quark conjectures is that quarks cannot be produced free and, consequently, they must be "permanently confined" in the interior of hadrons. However, it is well known to experts that, despite half a century of attempts, *no truly convincing "quark confinement" inside protons and neutrons has been achieved,* nor can it be expected on serious scientific grounds by assuming (as it is the case of quark conjectures) that quantum mechanics is identically valid inside and outside hadrons. This is due to a pillar of quantum mechanics, Heisenberg's uncertainty principle, according to which, given any manipulated theory appearing to show confinement for a given quark, a graduate student in physics can always prove the existence of a *finite probability for the same quark to be free outside the hadron, in catastrophic disagreement with physical reality.* Hence, the conjecture that quarks are physical particles is afflicted by catastrophic inconsistencies in its very conception [100].

2) It is equally well known by experts to qualify as such that *quarks cannot experience gravity* because quarks cannot be defined in our spacetime, while gravity can only be formulated in our spacetime and does not exist in mathematical complex-unitary spaces. Consequently, if protons and neutrons were indeed formed of quarks, we would have the catastrophic inconsistency that all quark believers should float in space due to the absence of gravity [101].

3) It is also well known by experts that *"quark masses" cannot possess any inertia* since they are purely mathematical parameters that cannot be defined in our spacetime. A condition for any mass to be physical, that is, to have inertia, is that it has to be the eigenvalue of a Casimir invariant of the Poincaré symmetry, while quarks cannot be defined via said symmetry because of their hypothetical fractional charges and other esoteric assumptions. This aspect alone implies numerous catastrophic inconsistencies, such as the impossibility of having the

energy equivalence $E = mc^2$ for any particle composed of quarks, against vast experimental evidence to the contrary.

4) Even assuming that, because of some twist of scientific manipulation, the above inconsistencies are resolved, it is known by experts that quark theories have failed to achieve a representation of all characteristics of protons and neutron, with catastrophic inconsistencies in the representation of spin, magnetic moment, means lives, charge radii and other basic features [102].

5) It is also known by experts that the application of quark conjectures to the structure of nuclei has multiplied the controversies in nuclear physics, while resolving none of them. As an example, the assumption that quarks are the constituents of the protons and the neutrons constituting nuclei has failed to achieve a representation of the main characteristics of the simplest possible nucleus, the deuteron. In fact, quark conjectures are afflicted by the catastrophic inconsistencies of being unable to represent the spin 1 of the deuteron (since they predict spin zero in the ground state while the deuteron has spin 1), they are unable to represent the anomalous magnetic moment of the deuteron, they are unable to represent the deuteron stability, they are unable to represent the charge radius of the deuteron, and when passing to larger nuclei, such as the zirconium, the catastrophic inconsistencies of quark conjectures can only be defined as being embarrassing [102].

In summary, while the final character of the SU(3)-color classification of hadrons into families has reached a value beyond scientific doubt, the conjecture that quarks are the actual physical constituents of hadrons existing in our spacetime is afflicted by so many and so problematic aspects to raise serious issues of scientific ethics and accountability, particularly in view of the ongoing large expenditures of public funds in the field.

On a personal note the author remembers some of the seminars delivered by the inventor of quarks, Murray Gell Mann, at Harvard University in the early 1980s, at the end of which there was the inevitable question whether Gell Mann believed or not that quarks are physical particles. Gell Mann's scientific caution (denoting a real scientific stature) is still impressed in the author's mind because he routinely responded with essentially the viewpoint outlined here, namely, Gell Mann stressed the mathematical necessity of quarks, while avoiding a firm posture on their physical reality. It is unfortunate that such a serious scientific position by Murray Gell-Manns was replaced by his followers with nonscientific positions mainly motivated by money, power and prestige.

Subsequently, quark conjectures have become a real "scientific business", as established by claim proffered by large high energy physics laboratories to have "discovered that and that quark". while in reality they had discovered a new particle predicted by SU(3)-color classification.

The decay of scientific ethics in the field is so serious, and the implications for mankind so potentially catastrophic (due to the suppression by quark conjectures as physical particles of possible new clean energies studied in Volume II) that, in the author's view, quark conjectures have been instrumental in the creation of the current scientific obscurantism of potentially historical proportions (see the *Open Denunciation of the Nobel Foundation for Heading an Organized Scientific Obscurantism* available in the web site http://www.scientificethics.org/Nobel-Foundation.htm).

1.2.8 The Scientific Imbalance Caused by Neutrino Conjectures

Another central objective of this monograph is to show that *neutrino conjectures constitute a political obstacle of potentially historical proportions against the orderly prediction and development of much needed new clean energies of "hadronic type", that is, new energies originating in the structure of individual hadrons, rather than in their collection as occurring in nuclei.*

Moreover, we shall show that *neutrino conjectures constitute an additional political obstacle also of potentially historical proportions against the study of one of the most important scientific problems in history, the interplay between matter and the universal substratum needed for the existence and propagation of electromagnetic waves and elementary particles.*

To prevent misrepresentations by vociferous (yet self-destructing) organized interests in the field, it should be stressed up-front that, as it is the case for quark conjectures, *neutrino conjectures of are necessary for the "current" treatment of weak interactions.* Therefore, a large scientific imbalance emerges only for the *political use and interpretation* of neutrino conjectures that has been dominant in the 20-th century and remains dominant to this day, namely, the use and interpretation of neutrino conjectures conceived and implemented in a capillary way for the continuation of the dominance of Einsteinian doctrines for all of physics.

Most distressing are contemporary claims of "neutrino detections" (denounced technically in Volume II) when the originator of neutrinos, Enrico Fermi, is on record by stressing that "neutrinos cannot be detected." Hence, the scientifically correct steatment would be the "detection of physical particles predicted by neutrino conjectures." As it was the case for Murray Gell-M ann, it is unfortunate that the scientific caution by Enrico Fermi was replaced by his followers with political postures essentially aiming at money, prestige and power.

In this subsections we shall show the political character of neutrino conjectures via a review the historical objections against the belief that the current plethora of neutrinos constitute actual physical particles in our spacetime. Alternative theoretical interpretations can be presented only in Chapter 6 with

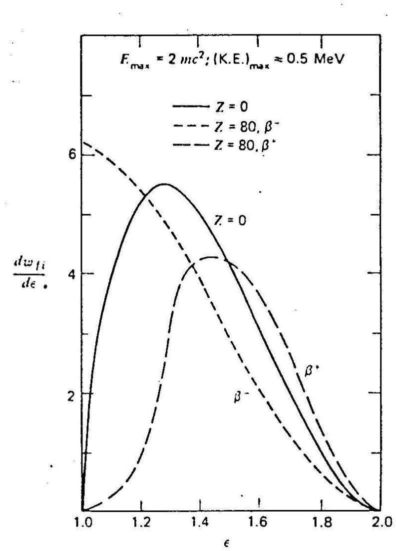

Figure 1.7. A view of the historical "bell shaped" curve representing the variation of the energy of the electron in nuclear beta decays (see, e.g., Ref. [13]). As soon as the apparent "missing energy" by the electron was detected in the early part of the 20-th century, it was claimed to be experimental evidence on the existence of a new particle with spin 1/2, charge zero and mass zero called by Fermi the "little neutron" or "neutrino".

industrial applications in Chapter 12 following the prior study and verification of *new mathematics* that is notoriously needed for true new vistas in science.

As it is well known, Rutherford [104] submitted in 1920 the conjecture that hydrogen atoms in the core of stars are compressed into a new particle he called the *neutron* according to the synthesis $(p^+, e^-) \rightarrow n$.

The existence of the neutron was subsequently confirmed experimentally in 1932 by Chadwick [105]. However, numerous objections were raised by the leading physicists of the time against Rutherford's conception of the neutron as a bound state of one proton p^+ and one electron e^-.

Pauli [106] first noted that Rutherford's synthesis violates the angular momentum conservation law because, according to quantum mechanics, a bound state of two particles with spin 1/2 (the proton and the electron) must yield a particle with integer spin and cannot yield a particle with spin 1/2 and charge zero such as the neutron. Consequently, Pauli conjectured the existence of a new neutral particle with spin 1/2 that is emitted in synthesis $(p^+, e^-) \rightarrow n$. or in similar radioactive processes so as to verify the angular momentum conservation law.

Fermi [107] adopted Pauli's conjecture, coined the name *neutrino* (meaning in Italian a "little neutron") and presented the first comprehensive theory of the underlying interactions (called "weak"), according to which the neutron synthesis should be written $(p^+, e^-) \rightarrow n + \nu$, where ν is the neutrino, in which case the inverse reaction (the spontaneous decay of the neutron) reads $n \rightarrow p^+ + e^- + \bar{\nu}$, where $\bar{\nu}$ is the *antineutrino.*

Despite the scientific authority of historical figures such as Pauli and Fermi, the conjecture on the existence of the neutrino and antineutrino as physical particles was never universally accepted by the entire scientific community because of: the impossibility for the neutrinos to be directly detected in laboratory; the neutrinos inability to interact with matter in any appreciable way; and the existence of alternative theories that do not need the neutrino conjecture (see Refs. [108-110] and literature quoted therein, plus the alternative theory presented in Chapter 6).

By the middle of the 20-th century there was no clear experimental evidence acceptable by the scientific community at large confirming the neutrino conjecture beyond doubt, except for experimental claims in 1959 that are known today to be basically flawed on various grounds, as we shall see below and in Chapter 6.

In the last part of the 20-th century, there was the advent of the so-called *unitary SU(3) theories* and related quark conjectures studied in the preceding subsection. In this way, neutrino conjectures became deeply linked to and their prediction intrinsically based on quark conjectures.

This event provided the first fatal blow to the credibility of the neutrino conjectures because serious physics cannot be done via the use of conjectures based on other conjectures.

In fact, the marriage of neutrino and quark conjectures within the standard model has requested the *multiplication of neutrinos,* from the neutrino and antineutrino conjectures of the early studies, to six different hypothetical particles, the so called *electron, muon and tau neutrinos and their antiparticles.* In the absence of these particles the standard model would maintain its meaning as classification of hadrons, but would lose in an irreconcilable way the joint capability of providing also the structure of each particle in a hadronic multiplet.

In turn, the multiplication of the neutrino conjectures has requested the *additional conjecture that the electron, muon and tau neutrinos have masses* and, since the latter conjecture resulted in being insufficient, there was the need for the *additional conjecture that neutrinos have different masses,* as necessary to salvage the structural features of the standard model. Still in turn, the lack of resolution of the preceding conjectures has requested the *yet additional conjecture that neutrinos oscillate,* namely, that "they change flavor" (transform among themselves back and forth).

In addition to this rather incredible litany of sequential conjectures, each conjecture being voiced in support of a preceding unverified conjecture, all conjectures being crucially dependent on the existence of quarks as physical particles despite their proved lack of gravity and physical masses, by far the biggest con-

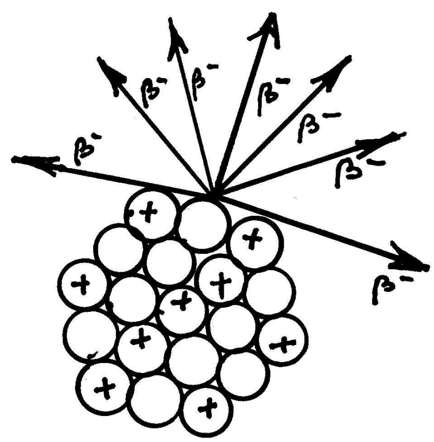

Figure 1.8. A schematic illustration of the fact that the electron in beta decays can be emitted in different directions. When the energy in the beta decay is computed with the inclusion of the Coulomb interactions between the expelled (negatively charged) electron and the (positively charged) nucleus at different expulsion directions, the nucleus acquires the "missing energy," without any energy left for the hypothetical neutrino. As we shall see in Chapter 6, rather than being a disaster, the occurrence is at the foundation of a possible basically new scientific horizon with implications sufficient to require studies over the entire third millennium.

troversies have occurred in regard to experimental claims of neutrino detection voiced by large collaborations.

To begin, both neutrinos and quarks cannot be directly detected as physical particles in our spacetime. Consequently, all claims on their existence are indirect, that is, based on the detection of actual physical particles predicted by the indicated theories. This occurrence is, per se, controversial. For instance, controversies are still raging following announcements by various laboratories to have "discovered" one or another quark, while in reality the laboratories discovered physical particles predicted by a Mendeleev-type classification of particles, the same classification being admitted by theories that require no quarks at all as physical particles, as we shall indicate in Chapter 6.

In the 1980s, a large laboratory was built deep into the Gran Sasso mountain in Italy to detect neutrinos coming from the opposite side of Earth (since the mountain was used as a shield against cosmic rays). Following the investment of large public funds and five years of tests, the Gran Sasso Laboratory released no evidence of clear detection of neutrino originated events.

Rather than passing to a scientific caution in the use of public funds, the failure of the Gran Sasso experiments to produce any neutrino evidence stimulated massive efforts by large collaborations involving hundred of experimentalists from various countries for new tests requiring public funds in the range of hundred of millions of dollars.

The increase in experimental research was evidently due to the scientific stakes, because, as well known by experts but studiously omitted, *the lack of verification of the neutrino conjectures would imply the identification of clear limits of validity of Einsteinian doctrines and quantum mechanics.*

These more recent experiments resulted in claims that, on strict scientific grounds, should be considered "experimental beliefs" by any serious scholars for numerous reasons, such as:

1) The predictions are based on a litany of sequential conjectures none of which is experimentally established on clear ground;

2) The theory contain a plethora of unrestricted parameters that can essentially fit any pre-set data (see next subsection);

3) The "experimental results" are based on extremely few events out of hundreds of millions of events over years of tests, thus being basically insufficient in number for any serious scientific claim;

4) In various cases the "neutrino detectors" include radioactive isotopes that can themselves account for the selected events;

5) The interpretation of the experimental data via neutrino and quark conjectures is not unique, since there exist nowadays other theories representing exactly the same events without neutrino and quark conjectures (including a basically new scattering theory of nonlocal type indicated in Chapter 3 and, more extensively, in monograph [10b]).

To understand the scientific scene, the serious scholar (that is, the scholar not politically aligned to the preferred "pet theories" indicated in the Preface) should note that *neutrino and quark conjectures have requested to date the expenditure of over one billion dollars of public funds in theoretical and experimental research with the result of increasing the controversies rather than resolving any of them.*

Therefore, it is now time for a moment of reflection: scientific ethics and accountability require that serious scholars in the field exercise caution prior to venturing claims of actual physical existence of so controversial and directly unverifiable conjectures.

Such a moment of reflection requires the re-inspection of the neutrino conjecture at its foundation. In fact, it is important to disprove the neutrino conjecture as originally conceived, and then disprove the flavored extension of the conjecture as requested by quark conjectures.

As reported in nuclear physics textbooks (see, e.g., Ref. [13]), the energy experimentally measured as being carried by the electron in beta decays is a bell-shaped curve with a maximum value of 0.782 MeV, that is the difference in value between the mass of the neutron and that of the resulting proton in the

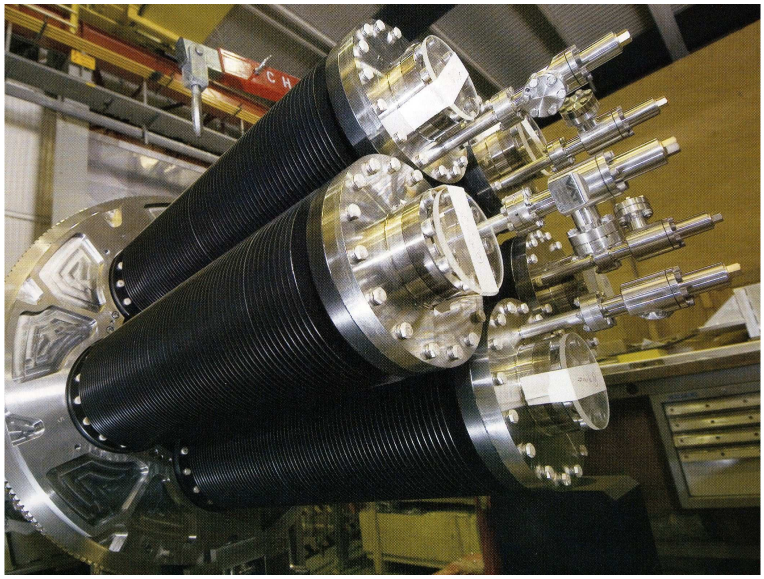

Figure 1.9. A picture of one of the "neutrino detectors" currently under construction at CERN for use to attempt "experimental measurements" of neutrinos (which one?) at the Gran Sasso Laboratory in Italy. The picture was sent to the author by a kind colleague at CERN and it is presented here to have an idea of the large funds now feverishly obtained from various governments by organized interests on Einsteinian doctrines in what can only be called their final frantic attempts at salvage the large litany of unverified and unverifiable quark, neutrino and other conjectures needed to preserve the dominance of Einstein doctrines in physics. For an understanding of the potential immense damage to mankind, we suggest the reader to study this monograph up to and including Chapter 12 on the necessity of abandoning these clearly sterile trends to achieve new clean energies.

neutron decay. As soon as the "missing energy" was identified, it was instantly used by organized interests in Einsteinian doctrines as evidence of the neutrino hypothesis for the unspoken yet transparent reasons that, in the absence of the neutrino conjectures, Einsteinian doctrines would be grossly inapplicable for the neutron decay.

As it is equally well known, the scientific community immediately accepted the neutrino interpretation of the "missing energy" mostly for academic gain, as it must be the case whenever conjectures are adopted without the traditional scientific process of critical examinations.

It is easy to see that the neutrino interpretation of the "missing energy" is fundamentally flawed. In fact, the electron in beta decays is negatively charged, while the nucleus is positively charged. Consequently, *the electron in beta decays experiences a Coulomb attraction from the original nucleus.*

Moreover, such an attraction is clearly dependent on the angle of emission of the electron by a decaying peripheral neutron. The maximal value of the energy occurs for radial emissions of the electron, the minimal value occurs for tangential emissions, and the intermediate value occur for intermediate directions of emissions, resulting in the experimentally detected bell-shaped curve of Figure 1.7.

When the calculations are done without political alignments on pre-existing doctrines, it is easy to see that the "missing energy" in beta decays is entirely absorbed by the nucleus via its Coulomb interaction with the emitted electron. Consequently, *in beta decays there is no energy at all available for the neutrino conjecture,* by reaching in this way a disproof of the conjecture itself at its historical origination.

Supporters of the neutrino conjecture are expected to present as counter-arguments various counter-arguments on the lack of experimental evidence for the nucleus to acquire said "missing energy." Before doing so, said supporters are suggested to exercise scientific caution and study the new structure models of the neutron without the neutrino conjecture (Chapter 6), as well as the resulting new structure models of nuclei (Chapter 7) and the resulting new clean energies (Chapter 12). Only then, depending on the strength of their political alignment, they may eventually realize that, in abusing academic authority to perpetrate unproved neutrino conjectures they may eventually be part of real crimes against mankind.

The predictable conclusion of this study is that theoretical and experimental research on neutrino and quark conjectures should indeed continue. However, theoretical and experimental research on theories without neutrino and quark conjectures and their new clean energies should be equally supported to prevent a clear suppression of scientific democracy on fundamental needs of mankind, evident problems of scientific accountability, and a potentially severe judgment by posterity.

For technical details on the damage caused to mankind by the current lack of serious scientific caution on neutrino conjectures, interested readers should study Volume Ii and inspect the *Open Denunciation of the Nobel Foundation for Heading an Organized Scientific Obscurantism* available in the web site http://www.-scientificethics.org/Nobel-Foundation.htm.

1.2.9 The Scientific Imbalance in Experimental Particle Physics

Another central objective of this monograph is to illustrate the *existence at the dawn of the third millennium of a scientific obscurantism of unprecedented proportions, caused by the manipulation of experimental data via the use of experimentally unverified and actually unverifiable quark conjectures, neutrino conjectures and other conjectures complemented by a variety of ad hoc parameters* *for the unspoken, but transparent and pre-meditated intent of maintaining the dominance of Einsteinian doctrines in physics.*

At any rate, *experimental data are elaborated via the conventional scattering theory that, even though impeccable for electromagnetic interactions among point-like particles, is fundamentally insufficient for a serious representation of the scattering among extended, nonspherical and hyperdense hadrons* (Figure 1.2 and Chapter 3).

As a matter of fact, serious scholars and, above all, future historians, should focus their main attention on the fact that the climax of unscientific conduct by organized interests on Einsteinian doctrines occurs primarily in the *manipulation of experiments,* beginning with the control of the conditions of funding, then following with the control of the conduction of the experiments and, finally, with the control of the theoretical elaboration of the data to make sure that the orchestrated compliance with Einsteinian doctrines occurs at all levels.

Among an unreassuringly excessive number of cases existing in the literature, some of which are reviewed in Chapter 6, a representative case is that of the *Bose-Einstein correlation* in which protons and antiprotons collide at high energy by annihilating each other and forming the so-called "fireball", that, in turn, emits a large number of unstable particles whose final product is a number of correlated mesons (see, e.g., review [7] and Figure 1.7).

The simplest possible case is that of the *two-points correlation function*

$$C_2 = \frac{P(p_1, p_2)}{P(p_1) \times P(p_2)}, \tag{1.2.14}$$

where p_1 and p_2 are the linear momenta of the two mesons and the P's represent their probabilities.

By working out the calculations via unadulterated axioms of relativistic quantum mechanics one obtains expressions of the type

$$C_2 = 1 + A \times e^{-Q_{12}} - B \times e^{-Q_{12}}, \tag{1.2.15}$$

where A and B are normalization parameters and Q_{12} is the momentum transfer. This expression is dramatically far from representing experimental data, as shown in Chapter 5.

To resolve the problem, supporters of the universal validity of quantum mechanics and special relativity then introduce *four arbitrary parameters of unknown physical origin and motivation* called "chaoticity parameters" c_μ, $\mu = 1, 2, 3, 4$, and expand expression (1.2.15) into the form

$$C_2 = 1 + A \times e^{-Q_{12}/c_1} + B \times e^{-Q_{12}/c_2} + C \times e^{-Q_{12}/c_3} - D \times e^{-Q_{12}/c_4}, \tag{1.2.16}$$

which expression does indeed fit the experimental data, as we shall see. However, the claim that quantum mechanics and special relativity are exactly valid is a scientific deception particularly when proffered by experts.

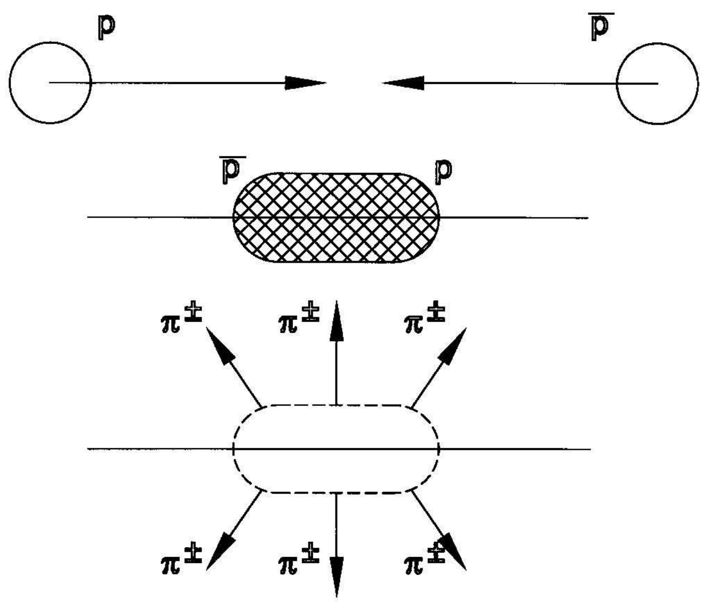

Figure 1.10. A schematic view of the Bose-Einstein correlation originating in proton-antiproton annihilations, for which the predictions of relativistic quantum mechanics are dramatically far from experimental data from unadulterated first principles. In order to salvage the theory and its underlying Einsteinian doctrines, organized interests introduce "four" ad hoc parameters deprived of any physical meaning or origin, and then claim the exact validity of said doctrines. The scientific truth is that these four arbitrary parameters are in reality a direct measurement of the deviation from the basic axioms of relativistic quantum mechanics and special relativity in particle physics.

As we shall see in technical details in Chapter 5, the quantum axiom of expectation values (needed to compute the probabilities) solely permit expression (1.2.15), since it deals with Hermitian, thus diagonalized operators of the type

$$< \psi_{\times}\psi_2| \times P \times |\psi_1 \times \psi_2 >= P_{11} + P_{22}, \tag{1.2.17}$$

while *the representation of a correlation between mesons 1 and 2 necessarily requires a structural generalization of the axiom of expectation value in such a form to admit off-diagonal elements for Hermitian operators,* for instance of the type

$$< \psi_{\times}\psi_2| \times T \times P \times T \times |\psi_1 \times \psi_2 >= P_{11} + P_{12} + P_{21} + P_{22}, \tag{1.2.18}$$

where T is a 2×2-dimensional nonsingular matrix with off-diagonal elements (and P remains diagonal).

The scientific deception occurs because quantum mechanics and special relativity are claimed to be exactly valid for the Bose-Einstein correlation when experts, to qualify as such, know that the representation requires a structural modification of the basic axiom of expectation values as well as for numerous additional reasons, such as:

1) The Bose-Einstein correlation is necessarily due to contact, nonpotential, nonlocal-integral effects originating in the deep overlapping of the hyperdense charge distributions of protons and antiprotons inside the fireball;

2) The mathematical foundations of quantum mechanics (such as its topology), let alone its physical laws, are inapplicable for a meaningful representation of said nonlocal and nonpotential interactions as outlined in preceding sections; and

3) Special relativity is also inapplicable, e.g., because of the inapplicability of the basic Lorentz and Poincaré symmetries due to lack of a Keplerian structure, the *approximate* validity of said theories remaining beyond scientific doubt.

Admittedly, there exist a number of semiphenomenological models in the literature capable of a good agreement with the experimental data. Scientific deception occurs when these models are used to claim the exact validity of quantum mechanics and special relativity since the representation of experimental data requires necessary structural departures from basic quantum axioms.

Of course, the selection of the appropriate generalization of quantum mechanics and special relativity for an exact representation of the Bose-Einstein correlation is open to scientific debate. Scientific deception occurs when the need for such a generalization is denied for personal gains.

As we shall see, relativistic hadronic mechanics provides an *exact and invariant* representation of the experimental data of the Bose-Einstein correlation at high and low energies via unadulterated basic axioms, by providing in particular a direct representation of the shape of the $p - \bar{p}$ fireball and its density, while recovering the basic invariant under a broader realization of the Poincaré symmetry.

An in depth investigation of all applications of quantum mechanics and special relativity at large reveals that they have provided an *exact* and*invariant* representation *from unadulterated basic axioms* of *all* experimental data of the hydrogen atom, as well as of physical conditions in which the mutual distances of particles is much bigger than the size of the charge distribution (for hadrons) or of the wavepackets of particles (for the case of the electron).

1.2.10 The Scientific Imbalance in Nuclear Physics

There is no doubt that quantum mechanics and special relativity permitted historical advances in also nuclear physics during the 20-th century, as illustrated, for instance, by nuclear power plants. However, any claim that quantum mechanics and special relativity are *exactly* valid in nuclear physics is a scientific deception, particularly when proffered by experts, because of the well known inability of these theories to achieve an exact and invariant representation of numerous nu-

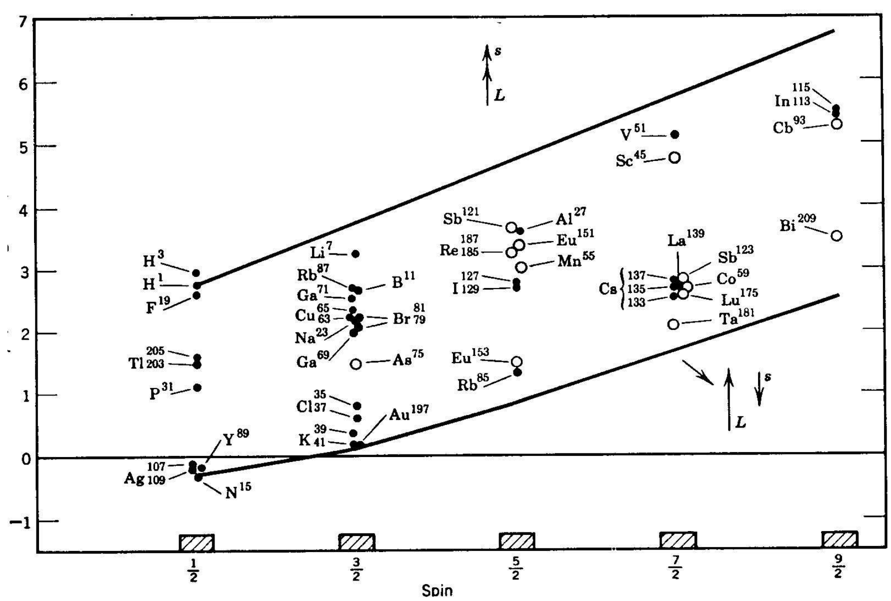

Figure 1.11. The first historical experimental evidence on the lack of exact validity of quantum mechanics in nuclear physics was given by data on nuclear magnetic moments that do not follow quantum mechanical predictions, and are instead comprised between certain minimal and maximal values, called the Schmidt Limits [13], without any possible quantum treatment. The additional suppression of the impossibility for the Galilean and Poincaré symmetries to be exact in nuclear physics due to the lack of a Keplerian center (see next figure), have essentially rendered nuclear physics a religion without a serious scientific process.

clear data despite one century of attempts and the expenditure of large public funds.

To resolve the insufficiencies, the use of arbitrary parameters of unknown physical origin and motivation was first attempted, semiphenomenological fits were reached and quantum mechanics and special relativity were again claimed to be exact in nuclear physics, while in the scientific reality the used parameters are a direct representation of *deviations* from the basic axioms of the theories as shown in detail in Chapter 5.

Subsequently, when the use of arbitrary parameters failed to achieve credible representations of nuclear data (such as nuclear magnetic moments as indicated below), organized academic interests claimed that "the deviations are resolved by deeper theories such as quark theories". At that point nuclear physics left the qualification of a true science to become a scientific religion.

Besides a plethora of intrinsic problematic aspects or sheer inconsistencies (such as the impossibility for quarks to have gravity mentioned earlier), quark theories failed to achieve any credible representation even of the spin of individual nucleons, let alone achieve exact representations of experimental data for their bound states.

Admittedly, the deviations here considered are at times small. Nevertheless, as we shall see in Chapter 6, small deviations directly imply new clean energies that cannot be even conceived, let alone treated, via quantum mechanics. Therefore, we have a societal duty to conduct serious investigations on broader mechanics specifically conceived for nuclear physics.

The first evidence on the lack of exact character of quantum mechanics in nuclear physics dates back to the birth of nuclear physics in the 1930s where it emerged that experimental values of nuclear magnetic moments could not be explained with quantum mechanics, because, starting with small deviations for small nuclei, the deviations then increased with mass, to reach deviations for large nuclei, such as the Zirconium so big to escape any use of unknown parameters "to fix things" (see Figure 1.8).

Subsequently, it became clear that quantum mechanics and special relativity could not explain the simplest possible nucleus, the deuteron, despite vast efforts. In fact, quantum mechanics missed about 1% of the deuteron magnetic moment despite all possible relativistic corrections, as well as the questionable assumptions that the ground state of the deuteron is a mixture of various states in a way manifestly against experimental evidence.

Next, quantum mechanics and special relativity were unable to represent the spin of the deuteron, an occurrence well known to experts in the field but carefully undisclosed. The axioms of quantum mechanics require that the ground state of two particles with spin 1/2 (such as the proton and the neutron) must have spin zero (*anti-parallel or singlet coupling*), while the case with spin 1 (*parallel spin or triplet coupling*) is unstable, as a first year graduate student in physics can prove.

By contrast, the deuteron has spin 1, thus remaining fundamentally unexplained by quantum mechanics and special relativity to this day.[5] Additionally, quantum mechanics has been unable to represent the stability of the neutron, its charge radius, and numerous other data.

Perhaps the most distressing, yet generally undisclosed, insufficiency of quantum mechanics and special relativity in nuclear physics has been the failure to understand and represent nuclear forces. Recall that a necessary condition for the applicability of quantum mechanics is that *all* interactions must be derivable from a potential.

The original concept that nuclear forces were of central type soon resulted in being disproved by nuclear reality, thus requiring the addition of non-central, yet

[5]As we shall see in Chapter 6, the correct interpretation of the spin 1 of the deuteron has implications so deep to require a revision of the very notion of neutron.

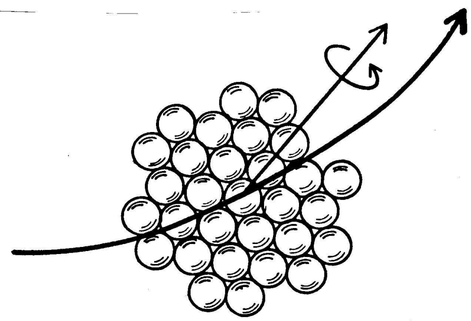

Figure 1.12. A visual evidence of the impossibility for quantum mechanics to be exactly valid in nuclear physics: the fact that "nuclei do not have nuclei." Consequently, the Galilean and Poincaré symmetries, as well as nonrelativistic and relativistic quantum mechanics, cannot possibly be exact for the nuclear structure since said symmetries demand the heaviest constituent at the center. The above occurrence establishes the validity of covering symmetries for interior systems without Keplerian centers, which symmetries are at the foundation of the covering hadronic mechanics.

still potential forces. The insufficiency of this addition requested the introduction of exchange, van der Waals, and numerous other potential forces. As of today, after about one century of adding new potentials to the Hamiltonian, we have reached the unreassuring representation of nuclear forces via some twenty or more different potentials in the Hamiltonian [13]

$$H = \Sigma_{k=1,2,\ldots,n} \frac{p_k^2}{2 \times m_k} + V_1 + V_2 + V_3 + V_4 + V_5 + V_6 +$$
$$+ V_7 + V_8 + V_9 + V_{10} + V_{11} + V_{12} + V_{13} + V_{14} +$$
$$+ V_{15} + V_{16} + V_{17} + V_{18} + V_{19} + V_{20} + \ldots\ldots\ldots \qquad (1.2.19)$$

and we still miss a credible understanding and representation of the nuclear force!

It is evident that this process cannot be kept indefinitely without risking a major condemnation by posterity. The time has long come to stop adding potentials to nuclear Hamiltonians and seek fundamentally new approaches and vistas.

In the final analysis, an inspection of nuclear volumes establishes that nuclei are generally composed of nucleons in conditions of partial mutual penetration, as illustrated in Figure 1.9. By recalling that nucleons have the largest density measured in laboratory until now, the belief that all nuclear forces are of action-at-a-distance, potential type, as *necessary* to preserve the validity of quantum mechanics and special relativity, is pure academic politics deprived of scientific value.

As we shall see in Chapter 7, a central objective of hadronic mechanics is that of truncating the addition of potentials and re-examining instead the nuclear force from its analytic foundations, by first separating potential nonpotential forces, and then examining in details each of them.

In summary, the lack of exact character of quantum mechanics and special relativity in nuclear physics is beyond scientific doubt. The open scientific issue is the selection of the appropriate generalization, but not its need.

As we shall see in Chapter 6, the covering hadronic mechanics and isospecial relativity resolve the fundamental open problems of nuclear physics by permitting the industrial development of new clean energies based on light natural and stable elements without the emission of dangerous radiations and without the release of radioactive waste.

1.2.11 The Scientific Imbalance in Superconductivity

The condition of superconductivity in the 20-th century can be compared to that of atomic physics prior to the representation of the structure of the atom.

Recall that individual electrons cannot achieve a superconducting state because their magnetic fields interact with electromagnetic fields of atoms by creating in this way what we call *electric resistance.* Superconductivity is instead reached by deeply correlated-bonded pairs of electrons in singlet couplings, called *Cooper pairs.* In fact, these pairs have an essentially null total magnetic field (due to the opposite orientations of the two fields), resulting in a substantial decrease of electric resistance.

There is no doubt that quantum mechanics and special relativity have permitted the achievement of a good description of an "ensemble" of Cooper pairs, although each Cooper pair is necessarily abstracted as a point, the latter condition being necessary from the very structure of the theories.

However, it is equally well known that quantum mechanics and special relativity have been unable to reach a final understanding and representation of the structure of *one* Cooper pair, trivially, because electrons repel each other according to the fundamental Coulomb law.

The failure of basic axioms of quantum mechanics and special relativity to represent the *attractive* force between the two *identical* electrons of the Cooper pairs motivated the hypothesis that the attraction is caused by the exchange of a new particle called *phonon.* However, phonons certainly exist in sounds, but they have found no verification at all in particle physics, thus remaining purely conjectural to this day.

In reality, as we shall see in Chapter 7, the interactions underlying the Cooper pairs are of purely contact, nonlocal and integral character due to the mutual penetration of the wavepackets of the electrons, as depicted in Figure 1.10. As such, they are very similar to the interactions responsible for Pauli's exclusion principle in atomic structures.

Under these conditions, the granting of a potential energy, as necessary to have phonon exchanges, is against physical evidence, as confirmed by the fact that any representation of Pauli's exclusion principle via potential interactions cause sizable deviations from spectral lines.

Therefore, the belief that quantum mechanics and special relativity provide a complete description of superconductivity is pure academic politics deprived of scientific content.

Superconductivity is yet another field in which the exact validity of quantum mechanics and special relativity has been stretched in the 20-th century well beyond its limit for known political reasons. At any rate, superconductivity has exhausted all its predictive capacities, while all advances are attempted via empirical trials and errors without a guiding theory.

As it was the case for particle and nuclear physics, the lack of exact character of quantum mechanics and special relativity in superconductivity is beyond doubt. Equally beyond doubt is the need for a deeper theory.

As we shall see in Chapter 7, the covering hadronic mechanics and isospecial relativity provide a quantitative representation of the structure of the Cooper pair in excellent agreement with experimental data, and with basically novel predictive capabilities, such as the industrial development of a new electric current, that is characterized by correlated electron pairs in single coupling, rather than electrons.

1.2.12 The Scientific Imbalance in Chemistry

There is no doubt that quantum chemistry permitted the achievement of historical discoveries in the 20-th century. However, there is equally no doubt that the widespread assumption of the exact validity of quantum chemistry caused a large scientific imbalance with vast implications, particularly for the alarming environmental problems.

After about one century of attempts, quantum chemistry still misses a historical 2% of molecular binding energies when derived from axiomatic principles without *ad hoc* adulterations (see below). Also, the deviations for electric and magnetic moments are embarrassing not only for their numerical values, but also because they are wrong even in their sign [14], not to mention numerous other insufficiencies outlined below.

It is easy to see that the reason preventing quantum chemistry from being exactly valid for molecular structures is given by contact, nonlocal-integral and nonpotential interactions due to deep wave-overlappings in valence bonds that, as such, are beyond any realistic treatment by local-differential-potential axioms, such as those of quantum chemistry (Figure 1.10).

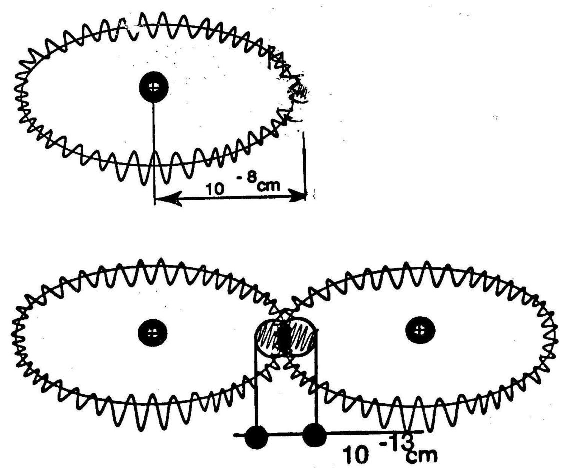

Figure 1.13. A schematic view of the fundamental conditions studied in this monograph, the deep overlapping of the extended wavepackets of electrons in valence bonds and Cooper pairs according to a singlet coupling as required by Pauli's principle. Recall that, for quantum mechanics and special relativity, electrons are points and, therefore, the conditions of this figure have no meaning at all. However, said point character can only be referred to the charge structure of the electron, since "point-like wavepackets" do not exist in nature. For the covering hadronic mechanics, superconductivity and chemistry, the point-like charge structure of the electrons remains, with the additional presence of the contact nonpotential interactions due to the overlapping of the extended wavepackets represented via a nonunitary structure. As shown in Chapters 8, 9 and 11, the treatment of the latter interactions via hadronic mechanics and chemistry has permitted the achievement, for the first time in scientific history, of an "exact and invariant" representations of molecular data from first axioms without ad hoc adulterations.

Recall that quantum mechanics achieved an exact and invariant representation of all experimental data of *one* hydrogen atom. Nevertheless, quantum mechanics and chemistry miss 2% of the binding energy of *two* hydrogen atoms coupled into the hydrogen molecule (Figure 1.11).

The only possible explanation is that in the hydrogen atom all interactions are of action-at-a-distance potential type due to the large mutual distances of the constituents with respect to the size of their wavepackets. By contrast, in the hydrogen molecule we have the mutual penetration of the wavepackets of valence electrons with the indicated contact, nonlocal-integral and nonpotential interactions at short mutual distances that are absent in the structure of the hydrogen atom.

Alternatively and equivalently, the nuclei of the two hydrogen atoms of the H_2 molecule cannot possibly be responsible for said 2% deviation. Therefore, the deviation from basic axioms can only originate in the valence bond.

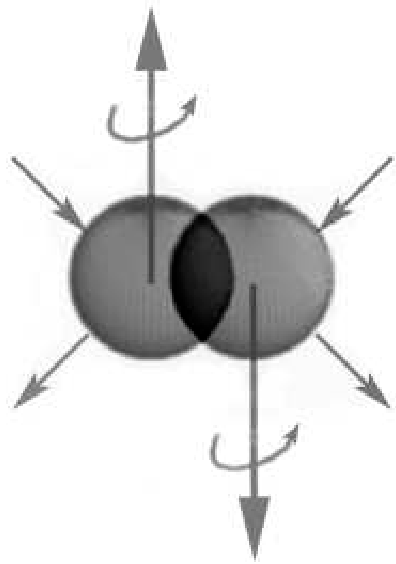

Figure 1.14. A first clear evidence of the lack of exact validity of quantum chemistry. The top view depicts one hydrogen atom for which quantum mechanics resulted in being exactly valid. The bottom view depicts two hydrogen atoms coupled into the H_2 molecule in which case quantum chemistry has historically missed a 2% of the binding energy when applied without adulteration of basic axioms "to fix things" (such as via the used of the screening of the Coulomb law and then claim that quantum chemistry is exact). Since nuclei do not participate in the molecular bond, the origin of the insufficiency of quantum mechanics and chemistry rests in the valence bond.

By no means the above insufficiencies are the only ones. Quantum chemistry is afflicted by a true litany of limitations, insufficiencies or sheer inconsistencies that constitute the best kept secret of the chemistry of the 20-th century because known to experts (since they have been published in refereed journals), but they remain generally ignored evidently for personal gains.

We outline below the insufficiencies of quantum chemistry for the simplest possible class of systems, those that are *isolated from the rest of the universe*, thus verifying conventional *conservation laws* of the total energy, total linear momentum, etc., and are *reversible* (namely, their time reversal image is as physical as the original system).

The most representative systems of the above class are given by *molecules,* here generically defined as aggregates of atoms under a valence bond. Despite undeniable achievements, quantum chemical models of molecular structures have the following fundamental insufficiencies studied in detail in monograph [11]:

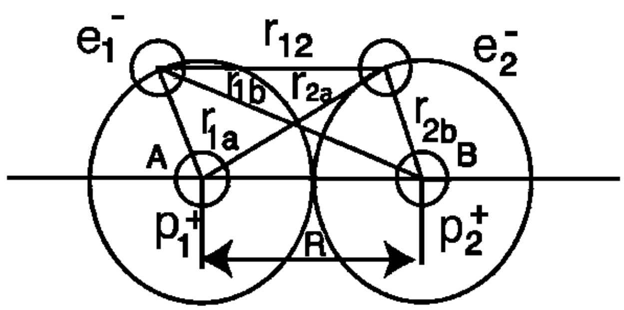

Figure 1.15. A schematic view of the fact that the total Coulomb force among the atoms of a molecular structure is identically null. As a consequence, conventional Coulomb interactions cannot provide credible grounds for molecular bonds. At the same time, existing chemical conjectures, such as the exchange and van der Waals forces, are weak, as known from nuclear physics. These facts establish that the chemistry of the 20-th century is like nuclear physics before the discovery of the strong interactions, because chemistry missed the identification of an attractive force sufficiently strong to represent molecular structure. As we shall see in Chapter 8, hadronic chemistry will indeed provide, for the first time in scientific history, the numerical identification of the missed "attractive strong attractive valence force" as being precisely of contact, nonlocal and nonpotential type. The achievement of an exact representation of molecular data is then consequential.

1: Quantum chemistry lacks a sufficiently strong molecular binding force. After 150 years of research, chemistry has failed to identify to this day the *attractive force* needed for a credible representation of valence bonds. In the absence of such an attractive force, names such as "valence" are pure nomenclatures without quantitative meaning.

To begin, the average of all Coulomb forces among the atoms constituting a molecule is identically null. As an example, the currently used Schrödinger equation for the H_2 molecule is given by the familiar expression [15],

$$(-\frac{\hbar^2}{2\mu_1}\nabla_1^2 - \frac{\hbar^2}{2\mu_2}\nabla_2^2 - \frac{e^2}{r_{1a}} - \frac{e^2}{r_{2a}} - \frac{e^2}{r_{1b}} - \frac{e^2}{r_{2b}} + \frac{e^2}{R} + \frac{e^2}{r_{12}})|\psi >= E|\psi >, \quad (1.2.20)$$

which equation contains the Coulomb attraction of each electron by its own nucleus, the Coulomb attraction of each electron from the nucleus of the other atom, the Coulomb repulsion of the two electrons, and the Coulomb repulsion of the two protons.

It is easy to see that, in semiclassical average, the two attractive forces of each electron from the nucleus of the other atom are compensated by the average of the two repulsive forces between the electrons themselves and those between the protons, under which Eq. (1.2.20) reduces to two independent *neutral* hydrogen atoms *without* attractive interaction, as depicted in Fig. 1.2.12,

$$\left[\left(-\frac{\hbar^2}{2\mu_1}\nabla_1^2 - \frac{e^2}{r_{1a}}\right) + \left(-\frac{\hbar^2}{2\mu_2}\nabla_2^2 - \frac{e^2}{r_{2a}}\right)\right]|\psi\rangle = E|\psi\rangle. \tag{1.2.21}$$

In view of the above occurrence, quantum chemistry tries to represent molecular bonds via *exchange, van der Waals and other forces* [15]. However, the latter forces were historically introduced for *nuclear structures* in which they are known to be *very weak*, thus being insufficient to provide a true representation of molecular bonds.

It is now part of history that, due precisely to the insufficiencies of exchange, van der Waals and other forces, nuclear physicists were compelled to introduce the *strong nuclear force*. As an illustration, calculations show that, under the currently assumed molecular bonds, the molecules of a three leaf should be decomposed into individual atomic constituents by a weak wind of the order of 10 miles per hour.

To put it in a nutshell, after about one century of research, *quantum chemistry still misses in molecular structures the equivalent of the strong force in nuclear structures.*

As we shall see in Chapter 8, one of the objectives of hadronic chemistry is precisely to introduce the missing force, today known as the *strong valence force,* that is, firstly, ATTRACTIVE, secondly, sufficiently STRONG, and, thirdly, INVARIANT. The exact and invariant representation of molecular data will then be a mere consequence.

2: Quantum chemistry admits an arbitrary number of atoms in the hydrogen, water and other molecules. This inconsistency is proved beyond scientific doubt by the fact that the exchange, van der Waals, and other forces used in current molecular models were conceived in nuclear physics for the primary purpose of admitting a large number of constituents.

When the same forces are used for molecular structures, they also admit an arbitrary number of constituents. As specific examples, when applied to the structure of the hydrogen or water molecule, any graduate student in chemistry can prove that, under exchange, van der Waals and other forces of nuclear type, the hydrogen, water and other molecules admit an *arbitrary* number of hydrogen atoms (see Figure 1.13).

Rather than explaining the reason why nature has selected the molecules H_2 and H_2O as the sole possible, current molecular models admit "molecules" of the type H_5, H_{23}, H_7O, H_2O_{121}, $H_{12}O_{15}$, etc., in dramatic disagreement with experimental evidence.

3: Quantum chemistry has been unable to explain the correlation of valence electrons solely into pairs. Experimental evidence clearly establishes that the valence correlations only occur between *electron pairs* in singlet coupling. By contrast, another known insufficiency of quantum chemistry is the intrinsic inability to restrict correlations to valence pairs.

This insufficiency is then passed to orbital theories, that work well at semi-empirical levels but remain afflicted by yet unresolved problems, eventually resulting in deviations of the prediction of the theory from experimental data that generally grow with the complexity of the molecule considered.

The inability to restrict correlations to valence pairs also provides an irrefutable additional confirmation that quantum chemistry predicts an arbitrary number of constituents in molecular structures.

As we shall see in Chapter 8, thanks to the advent of the new strong valence bond, the covering quantum chemistry does indeed restrict valence bonds strictly and solely to electron pairs. The resolution of inconsistency 2 will then be a mere consequence.

4: The use in quantum chemistry of "screened Coulomb potentials" violates basic quantum principles. The inability by quantum chemistry to achieve an exact representation of binding energies stimulated the adulteration of the basic Coulomb law into the so-called *screened Coulomb law* of the type

$$F = \pm f(r) \times \frac{e^2}{r}, \tag{1.2.22}$$

that did indeed improve the representation of experimental data.

However, the Coulomb law is a fundamental invariant of quantum mechanics, namely, the law remains invariant under all possible unitary transforms

$$F = \pm \frac{e^2}{r} \rightarrow U \times (\pm \frac{e^2}{r}) \times U^\dagger = \pm \frac{e^2}{r}, \tag{1.2.23a}$$

$$U \times U^\dagger = I. \tag{1.2.23b}$$

Therefore, any structural deviation from the Coulomb law implies deviations from the basic quantum axioms.

It then follows that the only possibility of achieving screened Coulomb laws is via the use of *nonunitary transforms* of the type

$$F = \pm \frac{e^2}{r} \rightarrow W \times (\pm \frac{e^2}{r}) \times W^\dagger = \pm e^{A \times r} \times \frac{e^2}{r}, \tag{1.2.24a}$$

$$W \times W^\dagger = e^{A \times r} \neq I. \tag{1.2.24b}$$

Therefore, by their very conception, *the use of screened Coulomb laws implies the exiting from the class of equivalence of quantum chemistry.* Despite that,

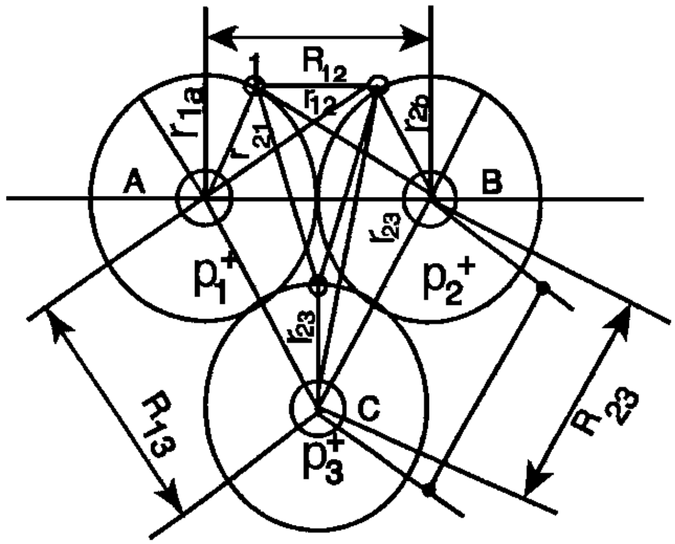

Figure 1.16. A schematic view of the fact that quantum chemistry predicts an arbitrary number of atoms in molecules because the exchange, van der Waals, and other bonding forces used in chemistry were identified in nuclear physics for an arbitrary number of constituents. Consequently, quantum chemistry is basically unable to explain the reasons nature has selected the molecules H_2, H_2O, CO_2, etc. as the sole possible molecular structures, and other structures such as H_5, H_{23}, H_7O, HO_{21}, $H_{12}O_{15}$, etc. cannot exist. As we shall see in Chapter 8, the "strong valence force" permitted by hadronic chemistry can only occur among "pairs" of valence electrons, thus resolving this historical problem in a quantitative way.

organized academic interests have continued to claim that screened Coulomb laws belong to quantum chemistry, thus exiting from the boundaries of science.

Irrespective from the above, a first year graduate student in chemistry can prove that *screened Coulomb laws cause the abandonment of the very notion of quantum in favor of the continuous emission or absorption of energy.* In fact, quantized emissions and absorptions of photons crucially depend on the existence of quantized orbits that, in turn, solely exist for unadulterated Coulomb potentials, as well known.

This insufficiency establishes the need to generalize quantum chemistry into a covering theory since the Coulomb law is indeed insufficient to represent molecular data. Rather than adapting a theory to adulterated basic axioms, it is scientifically more appropriate to build a new theory based on the needed broader axioms.

As we shall see in Chapter 8, the covering hadronic chemistry has been conceived to have a *nonunitary structure* as an evident necessary condition for novelty. In so doing, quantum chemistry naturally admits all infinitely possible screened Coulomb laws of type (1.2.22). However, such screenings are solely admitted in the nonlocal-integral region of deep wave-overlappings of valence electrons that are of the order of $1\,\mathrm{F} = 10^{-13}\,\mathrm{cm}$, while recovering the conventional Coulomb law automatically for all distances greater that 1 F.

This conception permits the achievement of an exact representation of molecular binding energies while preserving in full the quantum structure of the individual atoms.

5: Quantum chemistry cannot provide a meaningful representation of thermodynamical reactions. The missing 2% in the representation of binding energies is misleadingly small, because it corresponds to about 1,000 Kcal/mole while an ordinary thermodynamical reaction (such as that of the water molecule) implies an average of 50 Kcal/mole. No scientific calculation can be conducted when the error is of about twenty times the quantity to be computed.[6]

As we shall see in Chapter 8, our covering hadronic chemistry does indeed permit exact thermochemical calculations because it has achieved exact representations of molecular characteristics.

6: Computer usage in quantum chemical calculations requires excessively long periods of time. This additional, well known insufficiency is notoriously due to the slow convergence of conventional quantum series, an insufficiency that persists to this day despite the availability of powerful computers.

As we shall also see in Chapter 8, our covering hadronic chemistry will also resolve this additional insufficiency because the mechanism permitting the exact representation of molecular characteristics implies a fast convergent lifting of conventional slowly convergent series.

7: Quantum chemistry predicts that all molecules are paramagnetic. This inconsistency is a consequence of the most rigorous discipline of the 20-th century, quantum electrodynamics, establishing that, under an external magnetic field, the orbits of peripheral atomic electrons must be oriented in such a way to offer a magnetic polarity opposite to that of the external field (a polarization that generally occurs via the transition from a three-dimensional to a toroidal distribution of the orbitals).

According to quantum chemistry, atoms belonging to a molecule preserve their individuality. Consequently, quantum electrodynamics predicts that the periph-

[6]The author received a request from a U. S. public company to conduct paid research on certain thermochemical calculations. When discovering that the calculations had to be based on quantum chemistry due to political needs by the company to be aligned with organized academic interests, the author refused the research contract on grounds that it would constitute a fraud of public funds, due to the excessively large error of all thermochemical calculations when based on quantum chemistry.

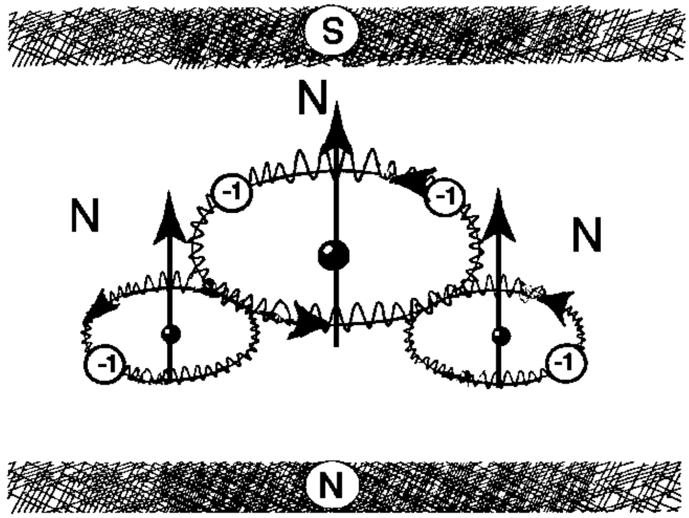

Figure 1.17. A schematic view of the prediction by quantum chemistry that water is paramagnetic, in dramatic disagreement with experimental evidence. In fact, quantum chemistry does not restrict the correlation of valence bonds to pairs. As a result, the individual valence electrons of the water molecule remain essentially independent. Quantum electrodynamics then demands the capability to polarize all valence electrons under an external magnetic field, resulting in the net magnetic polarity of this figure, and the consequential paramagnetic character of the water (as well as of all) molecules. As we shall see in Chapter 8, hadronic chemistry resolves this additional historical problem because our "strong valence force" deeply correlates valence electron pairs, thus permitting a global polarization of a molecule only in special cases, such as those with unbounded electrons.

eral atomic electrons of a molecule must acquire polarized orbits under an external magnetic field.

As a result, quantum chemistry predicts that the application of an external magnetic field, to hydrogen $H-H$, water $H-O-H$ and other molecules imply their acquisition of a net total, opposite polarity, $H_\uparrow - H_\uparrow$, $H_\uparrow - O_\uparrow - H_\uparrow$, etc., which polarization is in dramatic disagreement with experimental evidence.

The above inconsistency can also be derived from its inability to restrict the correlation solely to valence pairs. By contrast, the strong valence bond of the covering hadronic chemistry eliminates the independence of individual atoms in a molecular structure, by correctly representing the diamagnetic or paramagnetic character of substances.

No serious advance in chemistry can occur without, firstly, the admission of the above serious insufficiencies and/or inconsistencies, secondly, their detailed study, and, thirdly, their resolution via a covering theory.

Most importantly, we shall show in Chapter 10 that no resolution of the now alarming environmental problems is possible without a resolution of the above serious inconsistencies of quantum chemistry.

1.2.13 Inconsistencies of Quantum Mechanics, Superconductivity and Chemistry for Underwater Electric Arcs

Submerged electric arcs among carbon-base electrodes are known to permit the production of cost competitive and clean burning gaseous fuels via a highly efficient process since the primary source of energy is carbon combustion by the arc, the electric current used by the arc being a comparatively smaller energy. As such, submerged electric arcs have particular relevance for the main objectives of hadronic mechanics, as studied in Chapter 10 (see also monograph [11]).

An understanding of the motivations for the construction of hadronic mechanics, superconductivity and chemistry requires a knowledge of the fact that, contrary to popular beliefs, submerged electric arcs provide undeniable evidence of the following deviations from established doctrines:

1) When the liquid feedstock is distilled water and the electrodes are given by essentially pure graphite, quantum mechanics and chemistry predict that the produced gas is composed of 50% H_2 and 50% CO. However, CO is combustible in atmosphere and its exhaust is given by CO_2. Therefore, in the event said prediction was correct, the combustion exhaust of the gas should contain about 42% of CO_2. Numerous measurements conducted by an EPA accredited automotive laboratory [11] have established that the combustion exhaust contains about 4%-5% CO_2 without an appreciable percentage of unburned CO. Consequently, *the error of quantum mechanics and chemistry is of about ten times the measured value,* the error being in defect.

2) For the same type of gas produced from distilled water and carbon electrodes, quantum mechanics and chemistry predict that the thermochemical processes underlying the formation of the gas release about 2,250 British Thermal Units (BTU) per standard cubic feet (scf) (see Ref. [11]). In reality, systematic measurements have established that the heat produced is of the order of 250 BTU/scf. Therefore, *the error of quantum mechanics and chemistry is again of the order of ten times the measured quantity,* the error being this time in excess. Note that deviation 1) is fully compatible with deviation 2). In fact, the primary source of heat is the production of CO. Therefore, the production of 1/10-th of the heat predicted confirms that the CO is about 1/10-th the value predicted by quantum mechanics and chemistry.

3) Again for the case of the gas produced from distilled water and graphite electrodes, quantum mechanics and chemistry predict that no oxygen is present in the combustion exhaust, since the prediction is that, under the correct stochio-

metric ratio between atmospheric oxygen and the combustible gas, the exhaust is composed of 50% H_2O and 50% CO_2. In reality, independent measurements conducted by an EPA accredited automotive laboratory have established that, under the conditions here considered, the exhaust contains about 14% of breathable oxygen. Therefore, in this case the error of quantum mechanics and chemistry if about fourteen times the measured value.

4) Quantum mechanics and chemistry predict that the H_2 component of the above considered gas has the conventional specific weight of 2.016 atomic mass units (amu). Numerous measurements conducted in various independent laboratories have established instead that the hydrogen content of said gas has the specific weight of 14.56 amu, thus implying it a seven-fold deviation from the prediction of conventional theories.

5) Numerous additional deviations from the prediction of quantum mechanics and chemistry also exist, such as the fact that the gas has *a variable energy content, a variable specific weight, and a variable Avogadro number* as shown in Chapters 8 and 10, while conventional gases have *constant energy content, specific weight and Avogadro number,* as it is well known.

Above all *the most serious deviations in submerged electric arc occurs for Maxwell's electrodynamics,* to such an extent that any industrial or governmental research in the field based on Maxwell's electrodynamics is a misuse of corporate or public funds. At this introductory level we restrict ourselves to the indication of the axial attractive force between the electrodes and other features structurally incompatible with Maxwell's electrodynamics.

Needless to say, structural incompatibilities with Maxwell's electrodynamics automatically imply structural incompatibilities with special relativity due to the complete symbiosis of the two theories.

Note the re-emergence of the distinction between exterior and interior problems also in regard to Maxwell's electrodynamics. In fact, an arc in vacuum constitutes an exterior problem, while an arc within a liquid constitutes an interior problem. The impossibility of conducting serious industrial research via Maxwell's electrodynamics for submerged electric arcs can then be derived from the inapplicability of special relativity in the conditions considered.

The departures also extend to quantum superconductivity because the initiation of submerged electric arcs causes the collapse of the electric resistance, from very high value (as it is the case for distilled water) down to fractional Ohms. As a consequence, a submerged electric arc has features reminiscent of superconductivity. But the arc occurs at about 10,000 times the maximal temperature predicted by quantum superconductivity. The limitations of the theory is then beyond credible doubt, the only open scientific issues being the selection of the appropriate generalization.

In summary, under the above deviations, any use of quantum mechanics, superconductivity and chemistry for the study of submerged electric arcs exits the boundaries of scientific ethics and accountability. The departures of experimental evidence from old doctrines are just too big to be removed via arbitrary parameters "to fix things", thus mandating the construction of suitable covering theories.

1.3 THE SCIENTIFIC IMBALANCE CAUSED BY IRREVERSIBILITY

1.3.1 The Scientific Imbalance in the Description of Natural Processes

Numerous basic events in nature, including particle decays, such as

$$n \rightarrow p^{+} + e^{-} + \bar{\nu}, \tag{1.3.1}$$

nuclear transmutations, such as

$$C(6,12) + H(1,2) \rightarrow N(7,14), \tag{1.3.2}$$

chemical reactions, such as

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O, \tag{1.3.3}$$

and other processes are called *irreversible* when their images under time reversal, $t \rightarrow -t$, are prohibited by causality and other laws. Systems are instead called *reversible* when their time reversal images are as causal as the original ones, as it is the case for planetary and atomic structures when considered isolated from the rest of the universe.

Yet another large scientific imbalance of the 20-th century has been the treatment of *irreversible* systems via the formulations developed for *reversible* systems, such as Lagrangians and Hamiltonian mechanics, quantum mechanics and chemistry and special relativity. In fact, all these formulations are strictly reversible, in the sense that all their basic axioms are fully reversible in time, by causing in this way limitations in virtually all branches of science.

The imbalance was compounded by use of the *truncated Lagrange and Hamilton equations* (see Section 1.2.2) based on conventional Lagrangians or Hamiltonians,

$$L = \Sigma_{k=1,2,\ldots,n} \frac{1}{2} \times m_k \times v_k^2 - V(r), \tag{1.2.4a}$$

$$H = \Sigma_{a=1,2,..,n} \frac{\mathbf{p}_a^2}{2 \times m_a} + V(r), \tag{1.3.4b}$$

under the full awareness that *all known potentials (such as those for electric, magnetic, gravitational and other interactions), and therefore, all known Hamiltonians, are reversible.*

This additional scientific imbalance was dismissed by academicians with vested interests in reversible theories with unsubstantiated statements, such as "irreversibility is a macroscopic occurrence that disappears when all bodies are reduced to their elementary constituents".

The underlying belief is that mathematical and physical theories that are so effective for the study of one electron in a reversible orbit around a proton are tacitly believed to be equally effective for the study of the same electron when in irreversible motion in the core of a star with the local *nonconservation* of energy, angular momentum, and other characteristics.

Along these lines a vast literature grew during the 20-th century on the dream of achieving compatibility of quantum mechanics with the evident irreversibility of nature at all levels, most of which studies were of manifestly political character due to the strictly reversibility of all methods used for the analysis.

These academic beliefs have been disproved by the following:

THEOREM 1.3.1 [10b]: A classical irreversible system cannot be consistently decomposed into a finite number of elementary constituents all in reversible conditions and, vice-versa, a finite collection of elementary constituents all in reversible conditions cannot yield an irreversible macroscopic ensemble.

The property established by the above theorems dismisses all nonscientific beliefs on irreversibility, and identify the real needs, the construction of formulations that are *structurally irreversible,* that is, irreversible for all known reversible potentials, Lagrangians or Hamiltonians, and are applicable at all levels of study, from Newtonian mechanics to second quantization.

The historical origin of the above imbalance can be outlined as follows. One of the most important teaching in the history of science is that by Lagrange [2], Hamilton [3], and Jacobi [4] who pointed out that *irreversibility originates from contact nonpotential interactions not representable with a potential,* for which reason they formulated their equations with *external terms*, as in Eqs. (1.2.3).

In the planetary and atomic structures, there is no need for external terms, since all acting forces are of potential type. In fact, these systems admit an excellent approximation as being made-up of *massive points moving in vacuum without collisions* (exterior dynamical problems). In these cases, the historical analytic equations were "truncated" with the removal of the external terms.

In view of the successes of the planetary and atomic models, the main scientific development of the 20-th century was restricted to the "truncated analytic equa-

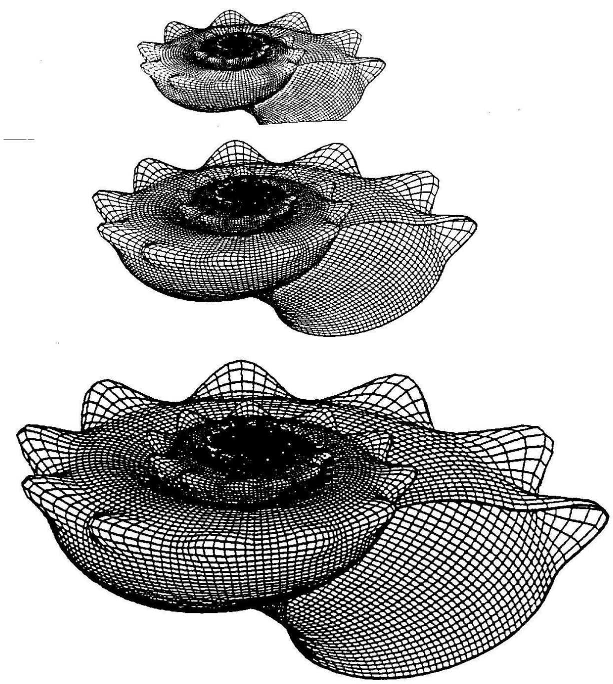

Figure 1.18. A pictorial view of the impossibility for quantum mechanics to be exactly valid in nature: the growth of a seashell. In fact, quantum mechanics is structurally irreversible, in the sense that all its axioms, geometries and symmetries, potentials, etc., are fully reversible in time, while the growth of a seashell is structurally irreversible. The need for an irreversible generalization of quantum mechanics is then beyond credible doubt, as studied in detail in Chapter 4.

tions", without any visible awareness that they are not the equations conceived by the founders of analytic mechanics.

Therefore, the origin of the scientific imbalance on irreversibility is the general dismissal by scientists of the 20-th century of the historical teaching by Lagrange, Hamilton and Jacobi, as well as academic interests on the truncated analytic equations, such as quantum mechanics and special relativity. In fact, as outlined earlier, the use of external terms in the basic analytic equations cause the inapplicability of the mathematics underlying said theories.

It then follows that no serious scientific advance on irreversible processes can be achieved without first identifying a structurally irreversible mathematics and then the compatible generalizations of conventional theories, a task studied in details in Chapter 4.

As we shall see, contrary to popular beliefs, the origin of irreversibility results in being at the ultimate level of nature, that of elementary particles in interior conditions. irreversibility then propagates all the way to the macroscopic level so as to avoid the inconsistency of Theorem 1.3.1.

1.3.2 The Scientific Imbalance in Astrophysics and Cosmology

Astrophysics and cosmology are new branches of science that saw their birth in the 20-th century with a rapid expansion and majestic achievements. Yet, these new fields soon fell pray to organized interests in established doctrines with particular reference to quantum mechanics, special relativity and gravitation, resulting in yet another scientific imbalance of large proportions.

To begin, all interior planetary or astrophysical problems are *irreversible,* as shown by the very existence of *entropy,* and known thermodynamical laws studiously ignored by supporters of Einsteinian doctrines. This feature, alone, is sufficient to cause a scientific imbalance of historical proportions because, as stressed above, irreversible systems cannot be credibly treated with reversible theories.

Also, quantum mechanics has been shown in the preceding sections to be inapplicable to all interior astrophysical and gravitational problems for reasons other than irreversibility. Any reader with an independent mind can then see the limitations of astrophysical studies for the interior of stars, galaxies and quasars based on a theory that is intrinsically inapplicable for the problems considered.

The imposition of special relativity as a condition for virtually all relativistic astrophysical studies of the 20-th century caused an additional scientific imbalance. To illustrate its dimensions and implications, it is sufficient to note that all calculations of astrophysical energies have been based on the relativistic mass-energy equivalence

$$E = m \times c^2, \tag{1.3.5}$$

namely, on the philosophical belief that the speed of light c is the same for all conditions existing in the universe (this is the well known "universal constancy of the speed of light").

As indicated earlier, this belief has been disproved by clear experimental evidence, particularly for the case of interior astrophysical media in which the maximal causal speed has resulted to be $C = c/n >> c$, $n << 1$, in which case the correct calculation of astrophysical energies is given by the equivalence principle of the isospecial relativity (see Chapter 3)

$$E = m \times C^2 = m \times c^2/n^2 >> m \times c^2, \ n << 1, \tag{1.3.6}$$

thus invalidating current view on the "missing mass", and others.

A further large scientific imbalance in astrophysics and cosmology was caused by the imposition of general relativity, namely, by one of the most controversial theories of the 20-th century because afflicted by problematic aspects and sheer inconsistencies so serious called catastrophic, as outlined in the next section.

It is hoped these preliminary comments are sufficient to illustrate the weakness of the scientific foundations of astrophysical studies of the 20-th century.

1.3.3 The Scientific Imbalance in Biology

By far one of the biggest scientific imbalances of the 20-th century occurred in biology because biological structures were treated via quantum mechanics in full awareness that the systems described by that discipline are dramatically different than biological structures.

To begin, quantum mechanics and chemistry are strictly *reversible,* while all biological structures and events are structurally *irreversible,* since biological structures such as a cell or a complete organism, admit a birth, then grow and then die.

Moreover, quantum mechanics and chemistry can only represent *perfectly rigid systems*, as well known from the fundamental rotational symmetry that can only describe "rigid bodies".

As a consequence, the representation of biological systems via quantum mechanics and chemistry implies that our body should be perfectly rigid, without any possibility of introducing deformable-elastic structures, because the latter would cause catastrophic inconsistencies with the basic axioms.

Moreover, another pillar of quantum mechanics and chemistry is the verification of total conservation laws, for which Heisenberg's equation of motion became established. In fact, the quantum time evolution of an arbitrary quantity A is given by

$$i \times \frac{dA}{dt} = [A, H] = A \times H - H \times A, \tag{1.3.7}$$

under which expression we have the conservation law of the energy and other quantities, e.g.,

$$i\ dH/dt = H \times H - H \times H \equiv 0. \tag{1.3.8}$$

A basic need for a scientific representation of biological structures is instead the representation of *the time-rate-of-variations of biological characteristics*, such as size, weight, density, etc. This identifies another structural incompatibility between quantum mechanics and biological systems.

When passing to deeper studies, the insufficiencies of quantum mechanics and chemistry emerge even more forcefully. As an example, quantum theories can well represent the *shape* of sea shells, but not their *growth in time*.

In fact, computer visualizations [16] have shown that, when the geometric axioms of quantum mechanics and chemistry (those of the Euclidean geometry) are imposed as being *exactly* valid, sea shells first grow in a deformed way, and then crack during their growth.

Finally, the ideal systems described with full accuracy by quantum mechanics, such as an isolated hydrogen atom or a crystal, are *eternal.* Therefore, the description via quantum theories implies that biological systems are eternal.

These occurrences should not be surprising to inquisitive minds, because the birth and growth, e.g., of a seashell is strictly *irreversible and nonconservative,* while the geometric axioms of quantum theories are perfectly *reversible and conservative*, as indicated earlier, thus resulting in a structural incompatibility, this time, at the geometric level without any conceivable possibility of reconciliation, e.g., via the introduction of unknown parameters "to fix things".

Additional studies have established that the insufficiencies of quantum mechanics and chemistry in biology are much deeper than the above, and invest the *mathematics* underlying these disciplines. In fact, Illert [16] has shown that a minimally correct representation of the growth in time of sea shells requires the *doubling of the Euclidean axes.*

However, sea shells are perceived by the human mind (via our three Eustachian tubes) as growing in our *three-dimensional* Euclidean space. As we shall see in Chapter 8, the only known resolution of such a dichotomy is that via *multi-valued irreversible mathematics*, that is, mathematics in which operations such as product, addition, etc., produce a *set of values*, rather than one single value as in quantum mechanics and chemistry.

At any rate, the belief that the simplistic mathematics underlying quantum mechanics and chemistry can explain the complexity of the DNA code, has no scientific credibility, the only serious scientific issue being the search for broader mathematics.

In conclusion, science will never admit "final theories". No matter how valid any given theory may appear at any point in time, its structural broadening for the description of more complex conditions is only a matter of time.

This is the fate also of quantum mechanics and chemistry, as well as special and general relativities that cannot possibility be considered as "final theories" for all infinitely possible conditions existing in the universe.

After all, following only a few centuries of research, rather than having reached a "final stage", science is only at its infancy.

1.4 THE SCIENTIFIC IMBALANCE CAUSED BY GENERAL RELATIVITY AND QUANTUM GRAVITY

1.4.1 Consistency and Limitations of Special Relativity

As it is well known, thanks to historical contributions by Lorentz, Poincaré, Einstein, Minkowski, Weyl and others, *special relativity* achieved a majestic axiomatical consistency.[7]

After one century of studies, we can safely identify the origins of this consistency in the following crucial properties:

1) Special relativity is formulated in the Minkowski spacetime over the field of real numbers;

2) All laws of special relativity are *invariant* (rather than covariant) under the fundamental *Poincaré symmetry*;

3) The Poincaré transformations and, consequently, all times evolutions of special relativity, are *canonical* at the classical level and *unitary* at the operator level with implications crucial for physical consistency.

Consequently, since canonical or unitary transforms conserve the unit by their very definition, *special relativity admits basic units and numerical predictions that are invariant in time.* After all, the quantities characterizing the dynamical equations are the *Casimir invariants* of the Poincaré symmetry.

As a result of the above features, special relativity has been and can be confidently applied to experimental measurements because the units selected by the experimenter do not change in time, and the numerical predictions of the theory can be tested at any desired time under the same conditions without fear of internal axiomatic inconsistencies.

It is well established at this writing that special relativity is indeed "compatible with experimental evidence" for the arena of its original conception, the classical and operator treatment of "point-like" particles and electromagnetic waves moving in vacuum. Despite historical results, it should be stressed that, as is the fate for all theories, *special relativity has numerous well defined limits of applicability,* whose identification is crucial for any serious study on gravitation, since

[7] It should be indicated that the name "Einstein's special relativity" is political, since a scientifically correct name should be "Lorentz-Poincaré-Einstein relativity." Also, it is appropriate to recall (as now reviewed in numerous books under testimonials by important eyewitnesses) that Einstein ended up divorcing his first wife Mileva Maric because she was instrumental in writing the celebrated paper on special relativity of 1905 and, for that reason, she had been originally listed as a co-author of that article, co-authorship that was subsequently removed when the article appeared in print. In fact, Einstein awarded his Nobel Prize money on that article to Mileva. Similarly, it should be recalled that Einstein avoided quoting Poincaré in his 1905 article following his consultation, and in documented knowledge that Poincaré had preceded him in various features of special relativity (see, e.g., the historical account by Logunov [96] or the instructive books [97,98]).

general relativity is known to be an extension of the special. Among the various limitations, we quote the following:

INAPPLICABILITY # 1: Special relativity is *inapplicable* for the *classical* treatment of antiparticles as shown in Section 1.1 and Chapter 2. This is essentially due to the existence of only one quantization channel. Therefore, the quantization of a *classical antiparticle* characterized by special relativity (essentially via the sole change of the sign of the charge) clearly leads to a quantum mechanical *particle* with the wrong sign of the charge, and definitely not to the appropriate charge conjugated antiparticle, resulting in endless inconsistencies.

INAPPLICABILITY # 2: Special relativity has also been shown to be inapplicable (rather than violated) for the treatment of both, particles and antiparticles when represented as they are in the physical reality, extended, generally non-spherical and deformable particles (such as protons or antiprotons), particularly when interacting at very short distances. In fact, these conditions imply the mutual penetration of the wavepackets and/or the hyperdense media constituting the particles, resulting in nonlocal, integro-differential and nonpotential interactions that cannot be entirely reduced to potential interactions among point-like constituents.

INAPPLICABILITY # 3: Special relativity is also afflicted by the historical inability to represent irreversible processes. This inapplicability has been identified in Section 1.3 in the reversibility of the mathematical methods used by special relativity, under which conditions the reversibility in time of its basic axioms is a mere consequence.

INAPPLICABILITY # 4: An additional field of clear inapplicability of special relativity is that for all biological entities, since the former can only represent perfectly rigid and perfectly reversible, thus eternal structures, while biological entities are notoriously deformable and irreversible, having a finite life.

INAPPLICABILITY # 5: In addition, serious scholars should keep in mind that the biggest limitation of special relativity may well result to be the forgotten universal medium needed for the characterization and propagation not only of electromagnetic waves, but also of elementary particles, since truly elementary particles such as the electron appear to be pure oscillations of said universal medium. Rather than being forgotten, the issue of the *privileged reference frame* and its relationship to reference frames of our laboratory settings appears to be more open than ever.

1.4.2 The Scientific Imbalance Caused by General Relativity on Antimatter, Interior Problems, and Grand Unifications

As indicated above, special relativity has a majestic axiomatic structure with clear verifications in the field of its original conception. By contrast, it is safe to state that *general relativity* (see, e.g., monograph [17]) has been the most controversial theory of the 20-th century for a plethora of inconsistencies that have grown in time, rather than being addressed and resolved.

We now address some of the inconsistencies published by numerous scholars in refereed technical journals, yet generally ignored by organized interests on Einsteinian doctrines, which inconsistencies are so serious to be known nowadays as being "catastrophic". The apparent resolution of the inconsistencies will be presented in Chapters 3, 4, 5, 13, and 14.

Let us begin with the following basic requirement for any *classical* theory of gravitation to be consistent:

REQUIREMENT 1: Any consistent classical theory of antimatter must allow a consistent representation of the *gravitational field of antimatter.* General Relativity does not verify this first requirement because, in order to attempt a compatibility of classical and quantum formulations, antimatter requires negative-energies, while general relativity solely admit positive-definite energies, as well known.

Even assuming that this insufficiency is somewhat bypassed, general relativity can only represent antimatter via the reversal of the sign of the charge. But the most important astrophysical bodies expected to be made up of antimatter are neutral. This confirms the structural inability of general relativity to represent antimatter in a credible way.

REQUIREMENT 2: Any consistent classical theory of antimatter must be able to represent *interior gravitational problems.* General relativity fails to verify this second requirement for numerous reasons, such as the inability to represent the *density* of the body considered, its *irreversible* condition, e.g., due to the increase of entropy, the *locally varying speed of light,* etc.

REQUIREMENT 3: Any consistent classical theory of gravitation must permit a grand unifications with other interactions. It is safe to state that this requirement too is not met by general relativity since all attempts to achieve a grand unification have failed to date since Einstein times (see Chapter 12 for details).

REQUIREMENT 4: Any consistent classical theory of gravitation must permit a consistent operator formulation of gravity. This requirement too has not been met by general relativity, since its operator image, known as *quantum gravity* [18] is afflicted by additional independent inconsistencies mostly originating from its unitary structure as studied in the next section.

REQUIREMENT 5: Any consistent classical theory of gravitation must permit the representation of the *locally varying nature of the speed of light.* This requirement too is clearly violated by general relativity.

The above insufficiencies are not of marginal character because they caused serious imbalances in most branches of quantitative sciences.

As an illustration, the first insufficiency prevented any study whatever as to whether a far-away galaxy or quasar is made up of matter or of antimatter. The second insufficiency created a form of religion related to the so-called "black holes", since before claiming their existence, gravitational singularities must evidently come out of *interior* gravitational problems and definitely not from theoretical abstractions solely dealing with exterior gravitation. The third insufficiency has been responsible for one of the longest list of failed attempts in grand unification without addressing the origin of the failures in the gravitational theory itself. The fourth insufficiency prevented throughout the entire 20-th century a consistent quantum formulation of gravity with large implications in particle physics. The fifth insufficiency cause cosmological models that can only be qualified as scientific beliefs, rather than quantitative theories based on sound physical foundations.

It is hoped that even the most representative members of organized interests on Einsteinian doctrines will admit that any additional support for said interests is now counterproductive, since it has already passed the mark for a severe condemnation by posterity.

It is time to provide a scientific identification of the basic insufficiencies of general relativity and initiate systematic studies for their resolution.

1.4.3 Catastrophic Inconsistencies of General Relativity due to Lack of Sources

There exist subtle distinctions between "general relativity", "Einstein's Gravitation", and "Riemannian" formulation of gravity. For our needs, we here define *Einstein's gravitation* of a body with null electric and magnetic moments as the reduction of exterior gravitation in vacuum to pure geometry, namely, gravitation is solely represented via curvature in a Riemannian space $\mathcal{R}(x, g, R)$ with spacetime coordinates $x = \{x^\mu\}$, $\mu = 1, 2, 3, 0$ and nowhere singular real-valued and symmetric metric $g(x)$ over the reals R, with field equations [19,20][8]

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = 0, \tag{1.4.1}$$

in which, as a central condition to have Einstein's gravitation, *there are no sources for the exterior gravitational field in vacuum for a body with null total electromagnetic field (null total charge and magnetic moment).*

For our needs, we define as *general relativity* any description of gravity on a Riemannian space over the reals with Einstein-Hilbert field equations with a

[8]The dubbing of Eqs. (1.4.1) as "Einstein's field equations" is political since it is known, or it should be known by "expert" in the field to qualify as such, that Hilbert independently published the same equations, and that Einstein consulted Hilbert without quotation his work in his gravitational paper of 1916, as done by Einstein in other cases.

It is also appropriate to recall that the publication of his 1916 paper on gravitation caused Einstein the divorce from his second wife, Elsa Loewenstein, for essentially the same reason of his first divorce. In fact, unlike Einstein, Elsa was a true mathematician, had trained Einstein on the Riemannian geometry (a topic only for very few pure mathematics at that time), and was supposed to be a co-author of Einstein's 1916 paper, a co-authorship denied as it was the case for the suppression of co-authorship of his first wife Mileva for his 1905 paper on special relativity (see the instructive books [97,98]).

To avoid a scandal for the 1905 paper, Einstein donate to Mileva the proceeds of his Nobel Prize. However, he did not receive a second Nobel Prize to quite down his second wife Elsa. A scandal was then avoided for the 1916 paper via the complicity of the Princeton community, complicity that is in full force and effect to this day. Hence, Princeton can indeed be considered as being an academic community truly leading in new basic advances during Einstein's times. By contrast, Princeton is nowadays perceived as a "scientific octopus" with kilometric tentacles reaching all parts of our globe for the studious suppression, via the abuse of academic credibility, of any spark of advance over Einsteinian doctrines. In fact, no truly fundamental advance came out of Princeton since Einstein's times, thus leaving Einstein as the sole source of money, prestige and power.

The documentation of the actions by Princeton academicians to oppose, jeopardize and disrupt research beyond Einstein is vast and includes hundreds of researchers in all developed countries. It is their ethical duty, if they really care for scientific democracy and the human society, to come out and denounce publicly the serious misconducts by Princeton academicianns they had to suffer (for which denunciations I am sure that the *International Committee on Scientific Ethics and Accountability* will offer its website http://www.scientificethics.org).

In regard to the author's documented experiences, it is sufficient to report here for the reader in good faith the rejection by the Princeton academic community with offensive language of *all* requests by the author (when still naive) for delivering an informal seminar on the isotopic lifting of special relativity for the intent of receiving technical criticisms. There is also documentation that, when the unfortunate session chairman of the second *World Congress in Mathematics* of the new century, the president of the Institute for Advanced Studies in Princeton prohibited presentations on Lie-isotopic and Lie-admissible algebras not only by the author, but also by the late Prof. Grigorios Tsagas, then Chairman of the Mathematics Department of Aristotle University in Thessaloniki, Greece. This volume has been dedicated to the memory of Prof. Gr. Tsagas also in view of the vexations he had to suffer for his pioneering mathematical research from decaying U. S, academia.

The climax of putrescence in the Princeton academic community is reached by the mumbo-jambo research in the so called "controlled hot fusion" under more than one billion of public funds, all spent under the condition of compatibility with Einsteinian doctrines, and under clear the technical proofs of the impossibility of its success (see Volume II for technical details).

The author spares the reader the agony of additional documented episodes of scientific misconducts because too demeaning, and expresses the view that, with a few exceptions, the Princeton academic community is nowadays an enemy of mankind.

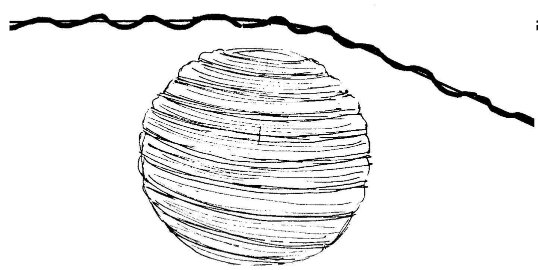

Figure 1.19. When the "bending of light" by astrophysical bodies was first measured, organized interests in Einsteinian doctrines immediately claimed such a bending to be an "experimental verification" of "Einstein's gravitation", and the scientific community accepted that claim without any critical inspection (for evident academic gains), according to an unreassuring trend that lasts to this day by being at the foundation of the current scientific obscurantism of potentially historical proportions. It can be seen by first year physics students that the measured bending of light is that predicted by the NEWTONIAN attraction. The representation of the same "bending of light" as being entirely due to curvature, as necessary in "Einstein's gravitation", implies its formulation in such a way to avoid any Newtonian contribution, with catastrophic inconsistencies in other experiments (see, e.g., next figure).

source due to the presence of electric and magnetic fields,

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times t_{\mu\nu}, \tag{1.4.2}$$

where k is a constant depending on the selected unit whose value is here irrelevant. For the scope of this section it is sufficient to assume that the *Riemannian description of gravity* coincides with general relativity according to the above definition.

In the following, we shall first study the inconsistencies of Einstein gravitation, that is, the inconsistencies in the entire reduction of gravity to curvature without source, and then study the inconsistency of general relativity, that is, the inconsistencies caused by curvature itself even in the presence of sources.

It should be stressed that a technical appraisal of the content of this section can only be reached following the study of the axiomatic inconsistencies of grand unified theories of electroweak and gravitational interactions whenever gravity is represented with curvature on a Riemannian space irrespective of whether with or without sources, as studied in Chapter 12.

THEOREM 1.4.1 [21]: Einstein's gravitation and general relativity at large are incompatible with the electromagnetic origin of mass established by quantum electrodynamics, thus being inconsistent with experimental evidence.

Proof. Quantum electrodynamics has established that the mass of all elementary particles, whether charged or neutral, has a primary electromagnetic origin, that is, all masses have a first-order origin given by the volume integral of the 00-component of the energy-momentum tensor $t_{\mu\nu}$ of electromagnetic origin,

$$m = \int d^4x \times t^{elm}_{oo}. \tag{1.4.3a}$$

$$t_{\alpha\beta} = \frac{1}{4\pi}(F^{\mu}_{\alpha} F_{\mu\beta} + \frac{1}{4} g_{\alpha\beta} F_{\mu\nu} F^{\mu\nu}), \tag{1.4.3b}$$

where $t_{\alpha\beta}$ is the *electromagnetic tensor,* and $F_{\alpha\beta}$ is the *electromagnetic field* (see Ref. [11a] for explicit forms of the latter with retarded and advanced potentials).

Therefore, quantum electrodynamics requires the presence of a *first-order source tensor* in the *exterior field equations* in vacuum as in Eqs. (1.4.2). Such a source tensor is absent in Einstein's gravitation (1.4.1) by conception. Consequently, Einstein's gravitation is incompatible with quantum electrodynamics.

The incompatibility of general relativity with quantum electrodynamics is established by the fact that the source tensor in Eqs. (1.4.2) is of *higher order in magnitude*, thus being ignorable in first approximation with respect to the gravitational field, while according to quantum electrodynamics said source tensor is of first order, thus not being ignorable in first approximation.

The inconsistency of both Einstein's gravitation and general relativity is finally established by the fact that, for the case when the total charge and magnetic moment of the body considered are null, Einstein's gravitation and general relativity allows no source at all. By contrast, as illustrated in Ref. [21], quantum electrodynamics requires a first-order source tensor even when the total charge and magnetic moments are null due to the charge structure of matter. **q.e.d.**

The first consequence of the above property can be expressed via the following:

COROLLARY 1.4.1A [21]: Einstein's reduction of gravitation in vacuum to pure curvature without source is incompatible with physical reality.

A few comments are now in order. As is well known, the mass of the electron is entirely of electromagnetic origin, as described by Eq. (3.3), therefore requiring a first-order source tensor in vacuum as in Eqs. (3.2). Therefore, Einstein's gravitation for the case of the electron is inconsistent with nature. Also, the electron has a point charge. Consequently, *the electron has no interior problem at all, in which case the gravitational and inertial masses coincide,*

$$m^{Grav.}_{Electron} \equiv m^{Iner}_{Electron}. \tag{1.4.4}$$

Next, Ref. [21] proved Theorem 1.4.1 for the case of a neutral particle by showing that the π^o meson also needs a first-order source tensor in the exterior gravitational problem in vacuum since its structure is composed of one charged particle and one charged antiparticle in high dynamical conditions.

In particular, the said source tensor has such a large value to account for the entire *gravitational mass* of the particle [21]

$$m_{\pi^o}^{Grav.} = \int d^4x \times t_{00}^{Elm}. \tag{1.4.5}$$

For the case of the interior problem of the π^o , we have the additional presence of short range weak and strong interactions representable with a new tensor $\tau_{\mu\nu}$. We, therefore, have the following:

COROLLARY 1.4.1B [21]: In order to achieve compatibility with electromagnetic, weak and strong interactions, any gravitational theory must admit two source tensors, a traceless tensor for the representation of the electromagnetic origin of mass in the exterior gravitational problem, and a second tensor to represent the contribution to interior gravitation of the short range interactions according to the field equations

$$G_{\mu\nu}^{Int.} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times (t_{\mu\nu}^{Elm} + \tau_{\mu\nu}^{ShortRange}). \tag{1.4.6}$$

A main difference of the two source tensors is that the electromagnetic tensor $t_{\mu\nu}^{Elm}$ is notoriously traceless, while the second tensor $\tau_{\mu\nu}^{ShortRange}$ is not. A more rigorous definition of these two tensors will be given shortly.

It should be indicated that, for a possible solution of Eqs. (1.4.6), various explicit forms of the electromagnetic fields as well as of the short range fields originating the electromagnetic and short range energy momentum tensors are given in Ref. [21].

Since both source tensors are positive-definite, Ref. [21] concluded that the interior gravitational problem characterizes the *inertial mass* according to the expression

$$m^{Iner} = \int d^4x \times (t_{00}^{Elm} + \tau_{00}^{ShortRange}), \tag{1.4.7}$$

with consequential general law

$$m^{Inert.} \geq m^{Grav.}, \tag{1.4.8}$$

where the equality solely applies for the electron.

Finally, Ref. [21] proved Theorem 1.4.1 for the exterior gravitational problem of a neutral massive body, such as a star, by showing that the situation is essentially the same as that for the π^o. The sole difference is that the electromagnetic field requires the sum of the contributions from *all* elementary constituents of the star,

$$m_{Star}^{Grav.} = \Sigma_{p=1,2,\ldots} \int d^4x \times t_{p00}^{Elem.}. \tag{1.4.9}$$

In this case, Ref. [21] provided methods for the approximate evaluation of the sum that resulted in being of first-order also for stars with null total charge.

When studying a charged body, there is no need to alter equations (3.6) since that particular contribution is automatically contained in the indicated field equations.

Once the incompatibility of general relativity at large with quantum electrodynamics has been established, the interested reader can easily prove the incompatibility of general relativity with quantum field theory and quantum chromodynamics, as implicitly contained in Corollary 1.4.1B.

An important property apparently first reached in Ref. [11a] in 1974 is the following:

COROLLARY 1.4.1C [21]: The exterior gravitational field of a mass originates entirely from the total energy-momentum tensor (3.3b) of the electromagnetic field of all elementary constituents of said mass.

In different terms, a reason for the failure to achieve a "unification" of gravitational and electromagnetic interactions initiated by Einstein himself is that the said interactions can be "identified" with each other and, as such, they cannot be unified. In fact, in all unifications attempted until now, the gravitational and electromagnetic fields preserve their identity, and the unification is attempted via geometric and other means resulting in redundancies that eventually cause inconsistencies.

Note that conventional electromagnetism is represented with the tensor $F_{\mu\nu}$ and related Maxwell's equations. When electromagnetism is identified with exterior gravitation, it is represented with the energy-momentum tensor $t_{\mu\nu}$ and related equations (1.4.6).

In this way, *gravitation results as a mere additional manifestation of electromagnetism.* The important point is that, besides the transition from the field tensor $F_{\mu\nu}$ to the energy-momentum tensor $T_{\mu\nu}$, there is no need to introduce a new interaction to represent gravity.

Note finally the irreconcilable alternatives emerging from the studies herein considered:

ALTERNATIVE I: Einstein's gravitation is assumed as being correct, in which case quantum electrodynamics must be revised in such a way to avoid the electromagnetic origin of mass; or

ALTERNATIVE II: Quantum electrodynamics is assumed as being correct, in which case Einstein's gravitation must be irreconcilably abandoned in favor of a more adequate theory.

By remembering that quantum electrodynamics is one of the most solid and experimentally verified theories in scientific history, it is evident that the rather widespread assumption of Einstein's gravitation as having final and universal character is non-scientific.

THEOREM 1.3.2 [22,10b]: Einstein's gravitation (1.4.1) is incompatible with the Freud identity of the Riemannian geometry, thus being inconsistent on geometric grounds.

Proof. The Freud identity [11b] can be written

$$R^\alpha_\beta - \frac{1}{2} \times \delta^\alpha_\beta \times R - \frac{1}{2} \times \delta^\alpha_\beta \times \Theta \ = \ U^\alpha_\beta + \partial V^{\alpha\rho}_\beta / \partial x^\rho \ = \ k \times (t^\alpha_\beta + \tau^\alpha_\beta), \quad (1.4.10)$$

where

$$\Theta \ = \ g^{\alpha\beta} g^{\gamma\delta} (\Gamma_{\rho\alpha\beta} \Gamma^\rho_{\gamma\beta} - \Gamma_{\rho\alpha\beta} \Gamma^\rho_{\gamma\delta}), \quad (1.4.11a)$$

$$U^\alpha_\beta \ = \ -\frac{1}{2} \frac{\partial \Theta}{\partial g^{\rho\alpha}_{|\rho}} g^{\gamma\beta} \uparrow_\gamma, \quad (1.4.11b)$$

$$V^{\alpha\rho}_\beta \ = \ \frac{1}{2} [g^{\gamma\delta} (\delta^\alpha_\beta \Gamma^\rho_{\alpha\gamma\delta} - \delta^\rho_\beta \Gamma^\rho_{\alpha\delta}) +$$

$$+ (\delta^\rho_\beta g^{\alpha\gamma} - \delta^\alpha_\beta g^{\rho\gamma}) \Gamma^\delta_{\gamma\delta} + g^{\rho\gamma} \Gamma^\alpha_{\beta\gamma} - g^{\alpha\gamma} \Gamma^\rho_{\beta\gamma}]. \quad (1.4.11c)$$

Therefore, the Freud identity requires two first order source tensors for the exterior gravitational problems in vacuum as in Eqs. (1.4.6) of Ref. [21]. These terms are absent in Einstein's gravitation (1.4.1) that, consequently, violates the Freud identity of the Riemannian geometry. **q.e.d.**

By noting that trace terms can be transferred from one tensor to the other in the r.h.s. of Eqs. (1.4.10), it is easy to prove the following:

COROLLARY 1.4.2A [10b]: Except for possible factorization of common terms, the t- and τ-tensors of Theorem 3.2 coincide with the electromagnetic and short range tensors, respectively, of Corollary 1.4.1B.

A few historical comments regarding the Freud identity are in order. It has been popularly believed throughout the 20-th century that the Riemannian geometry possesses only *four identities* (see, e.g., Ref. [17]). In reality, Freud [22] identified in 1939 a *fifth identity* that, unfortunately, was not aligned with Einstein's doctrines and, as such, the identity was ignored in virtually the entire literature on gravitation of the 20-th century, as it was also the case for Schwarzschild's interior solution [8].

However, as repeatedly illustrated by scientific history, structural problems simply do not disappear with their suppression, and actually grow in time. In fact, the Freud identity did not escape Pauli who quoted it in a footnote of his celebrated book of 1958 [24]. Santilli became aware of the Freud identity via an accurate reading of Pauli's book (including its important footnotes) and assumed the Freud identity as the geometric foundation of the gravitational studies presented in Ref. [10b].

Subsequently, in his capacity as Editor in Chief of *Algebras, Groups and Geometries,* Santilli requested the mathematician Hanno Rund, a known authority in Riemannian geometry [24], to inspect the Freud identity for the scope of ascertaining whether the said identity was indeed a new identity. Rund kindly accepted Santilli's invitation and released paper [26] of 1991 (the last paper prior to his departure) in which Rund confirmed indeed the character of Eqs. (3.10) as a genuine, independent, fifth identity of the Riemannian geometry.

The Freud identity was also rediscovered by Yilmaz (see Ref. [27] and papers quoted therein) who used the identity for his own broadening of Einstein's gravitation via an external *stress-energy tensor* that is essentially equivalent to the source tensor with non-null trace of Ref. [11a], Eqs. 1.4.6).

Despite these efforts, the presentation of the Freud identity to various meetings and several personal mailings to colleagues in gravitation, the Freud identity continues to remain vastly ignored to this day, with very rare exceptions (the indication by colleagues of additional studies on the Freud identity not quoted herein would be gratefully appreciated.)

Theorems 1.4.1 and 1.4.2 complete our presentation on the catastrophic inconsistencies of Einstein's gravitation due to the lack of a first-order source in the exterior gravitational problem in vacuum. These theorems, by no means, exhaust all inconsistencies of Einstein's gravitation, and numerous additional inconsistencies do indeed exist.

For instance, Yilmaz [27] has proved that Einstein's gravitation explains the 43" of the precession of Mercury, but cannot explain the basic Newtonian contribution. This result can also be seen from Ref. [21] because the lack of source implies the impossibility of importing into the theory the basic Newtonian potential. Under these conditions the representation of the Newtonian contribution is reduced to a religious belief, rather than a serious scientific statement.

For these and numerous additional inconsistencies of general relativity we refer the reader to Yilmaz [27], Wilhelm [28-30], Santilli [31], Alfvén [32,33], Fock [34], Nordensen [35], and large literature quoted therein.

1.4.4 Catastrophic Inconsistencies of General Relativity due to Curvature

We now pass to the study of the structural inconsistencies of general relativity caused by the very use of the Riemannian *curvature*, irrespective of the selected field equations, including those fully compatible with the Freud identity.

THEOREM 1.4.3 [36]: Gravitational theories on a Riemannian space over a field of real numbers do not possess time invariant basic units and numerical predictions, thus having serious mathematical and physical inconsistencies.

Proof. The map from Minkowski to Riemannian spaces is known to be *non-canonical,*

$$\eta = Diag.(1,1,1,-1) \ \rightarrow \ g(x) = U(x) \times \eta \times U(x)^\dagger, \tag{1.4.12a}$$

$$U(x) \times U(x)^\dagger \neq I. \tag{1.4.12b}$$

Thus, the time evolution of Riemannian theories is necessarily noncanonical, with consequential lack of invariance in time of the basic units of the theory, such as

$$I_{t=0} = Diag.(1cm, 1cm, 1cm, 1sec) \rightarrow I'_{t>0} = U_t \times I \times U_t^\dagger \ \neq I_{t=0}. \tag{1.4.13}$$

The lack of invariance in time of numerical predictions then follows from the known "covariance", that is, lack of time invariance of the line element. **q.e.d.**

As an illustration, suppose that an experimentalist assumes at the initial time $t = 0$ the units 1 cm and 1 sec. Then, all Riemannian formulations of gravitation, including Einstein's gravitation, predict that at the later time $t > 0$ said units have a different numerical value.

Similarly, suppose that a Riemannian theory predicts a numerical value at the initial time $t = 0$, such as the 43" for the precession of the perihelion of Mercury. One can prove that the same prediction at a later time $t > 0$ is numerically different precisely in view of the "covariance", rather than invariance as intended in special relativity, thus preventing a serious application of the theory to physical reality. We therefore have the following:

COROLLARY 1.4.3A [36]: Riemannian theories of gravitation in general, and Einstein's gravitation in particular, can at best describe physical reality at a fixed value of time, without a consistent dynamical evolution.

Interested readers can independently prove the latter occurrence from the *lack of existence of a Hamiltonian in Einstein's gravitation.* It is known in analytic mechanics (see, e.g., Refs. [17,24]) that Lagrangian theories not admitting an equivalent Hamiltonian counterpart, as is the case for Einstein's gravitation, are inconsistent under time evolution, unless there are suitable subsidiary constraints that are absent in general relativity.

It should be indicated that the inconsistencies are much deeper than that indicated above. For consistency, the Riemannian geometry must be defined on the field of numbers $R(n, +, \times)$ that, in turn, is fundamentally dependent on the basic unit I. But the Riemannian geometry does not leave time invariant the

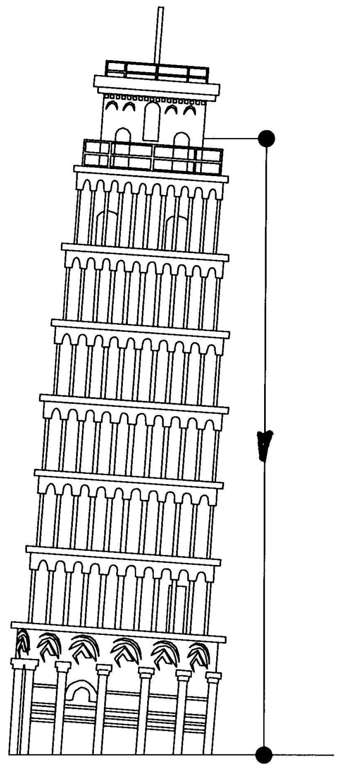

Figure 1.20. A conceptual rendering of the reason the author was unable to accept "Einstein's gravitation" as a correct theory since the time of his *high school studies,* the free fall of bodies under gravity that has to occur necessarily along a straight radial line, thus without any possible curvature. On technical terms, the free fall establishes the consistency need for any gravitational theory not only to incorporate the NEWTONIAN attraction in a clear and unambiguous way, but also in such a way that all contributions from curvature should disappear for the free fall in favor of the pure Newtonian attraction. The fact that evidence so incontrovertible continues to be denied by organized interests on Einsteinian doctrines and their vast followers, most holding chairs of high academic fame, confirm the existence of a scientific obscurantism of potentially historical proportions.

basic unit I due to its noncanonical character. The loss in time of the basic unit I then implies the consequential loss in time of the base field R, with consequential catastrophic collapse of the entire geometry [36].

In conclusion, not only is Einstein's reduction of gravity to pure curvature inconsistent with nature because of the lack of sources, but also the ultimate origin of the inconsistencies rests in the curvature itself when assumed for the representation of gravity, due to its inherent noncanonical character at the classical level with consequential nonunitary structure at the operator level.

Serious mathematical and physical inconsistencies are then unavoidable under these premises, thus establishing the impossibility of any credible use of general relativity, for instance, as an argument against the test on antigravity predicted for antimatter in the field of matter [5], as well as establishing the need for a profound revision of our current views on gravitation.

THEOREM 1.4.4. Gravitational experimental measurements do not verify general relativity uniquely.

Proof. All claimed "experimental verifications" of Einstein's gravitation are based on the PPN "expansion" (or linearization) of the field equations (such as the post-Newtonian approximation), that, as such, is not unique. In fact, Eqs. (1.4.1) admit a variety of inequivalent expansions depending on the selected parameter, the selected expansion and the selected truncation. It is then easy to show that the selection of an expansion of the same equations (3.1) but different from the PPN approximation leads to dramatic departures from experimental values. **q.e.d.**

THEOREM 1.4.5: General relativity is incompatible with experimental evidence because it does not represent the bending of light in a consistent, unique and invariant way.

Proof. Light carries energy, thus being subjected to a bending due to the conventional Newtonian gravitational attraction, while, general relativity predicts that the bending of light is entirely due to curvature (see, e.g., Ref. [17], Section 40.3). In turn, the absence of the Newtonian contribution causes other catastrophic inconsistencies, such as the inability to represent the free fall where curvature does not exist (Theorem 1.4.6 below). Assuming that consistency is achieved with yet unknown manipulations, the representation of the bending of light is not unique because bases on a nonunique PPN approximation having different parameters for different expansions. Finally, assuming that consistency and uniqueness are somewhat achieved, the representation is not invariant in time due to the noncanonical structure of general relativity.

THEOREM 1.4.6: General relativity is incompatible with experimental evidence because of the lack of consistent, unique and invariant representation of the free fall of test bodies along a straight radial line.

Proof. A consistent representation of the free fall of a test body along a straight radial line requires that the Newtonian attraction be represented by the field equations necessarily without curvature, thus disproving the customary belief needed to avoid Corollary 1.4.2.A that said Newtonian attraction emerges at the level of the post-Newtonian approximation. **q.e.d.**

The absence in general relativity at large, thus including Einstein's gravitation, of well defined contributions due to the Newtonian attraction and to the assumed curvature of spacetime, and the general elimination of the former in favor of the latter, causes other catastrophic inconsistencies, such as the inability to represent the base Newtonian contribution in planetary motion as shown by Yilmaz [47], and other inconsistencies [48-52].

A comparison between special and general relativities is here in order. Special relativity can be safely claimed to be "verified by experiments" because the said experiments verify numerical values uniquely and unambiguously predicted by special relativity. By contrast, no such statement can be made for general relativity since the latter does not uniquely and unambiguously predict given numerical values due, again, to the variety of possible expansions and linearization.

The origin of such a drastic difference is due to the fact that *the numerical predictions of special relativity are rigorously controlled by the basic Poincaré "invariance". By contrast, one of the several drawbacks of the "covariance" of general relativity is precisely the impossibility of predicting numerical values in a unique and unambiguous way, thus preventing serious claims of true "experimental verifications" of general relativity.*

By no means the above analysis exhausts all inconsistencies of general relativity, and numerous additional ones do indeed exist, such as that expressed by the following:

THEOREM 1.4.7 [36]: Operator images of Riemannian formulations of gravitation are inconsistent on mathematical and physical grounds.

Proof. As established by Theorem 1.4.3, classical formulations of Riemannian gravitation are noncanonical. Consequently, all their operator counterparts must be nonunitary for evident reasons of compatibility. But nonunitary theories are known to be inconsistent on both mathematical and physical grounds [36]. In fact, on mathematical grounds, nonunitary theories of quantum gravity (see, e.g., Refs. [2j,2k]) do not preserve in time the basic units, fields and spaces, while, on physical grounds, the said theories do not possess time invariant numerical predictions, do not possess time invariant Hermiticity (thus having no acceptable observables), and violate causality. **q.e.d**

The reader should keep in mind the additional well known inconsistencies of quantum gravity, such as the historical incompatibility with quantum mechanics, the lack of a credible PCT theorem, etc.

By no means, the inconsistencies expressed by Theorems 1.4.1 through 1.4.7 constitute all inconsistencies of general relativity. In the author's opinion, additional deep inconsistencies are caused by the fact that *general relativity does not possess a well defined Minkowskian limit,* while the admission of the Minkowski space as a tangent space is basically insufficient on dynamical grounds (trivially, because on said tangent space gravitation is absent).

As an illustration, we should recall the controversy on conservation laws that raged during the 20-th century [11]. Special relativity has rigidly defined total conservation laws because they are the Casimir invariants of the fundamental Poincaré symmetry. By contrast, there exist several definitions of total conservation laws in a Riemannian representation of gravity due to various ambiguities evidently caused by the absence of a symmetry in favor of covariance.

Moreover, none of the gravitational conservation laws yields the conservation laws of special relativity in a clear and unambiguous way, precisely because of the lack of any limit of a Riemannian into the Minkowskian space. Under these conditions, the compatibility of general relativity with the special reduces to personal beliefs outside a rigorous scientific process.

1.4.5 Concluding Remarks

In the author view, the most serious inconsistencies of general relativity are those of *experimental* character, such as the structural impossibility for the Riemannian geometry to permit unique and unambiguous numerical predictions due to the known large degrees of freedom in all PPN expansions; the necessary *absence* of curvature to represent consistently the free fall of bodies along a straight radial line; the gravitational deflection of light measured until now being purely *Newtonian* in nature; and others.

These inconsistencies are such to prevent serious attempts in salvaging general relativity. For instance, if the deflection of the speed of light is re-interpreted as being solely due to curvature without any Newtonian contribution, then general relativity admits other catastrophic inconsistencies, such as the inability to represent the Newtonian contribution of planetary motions pointed out by Yilmaz [27] and other inconsistencies such as those identified by Wilhelm [28-30] and other researchers.

When the inconsistencies of general relativity with experimental evidence are combined with the irreconcilable incompatibility of general relativity with unified field theory and the catastrophic axiomatic inconsistencies due to lack of invariance [11m], time has indeed arrived for the scientific community to admit the need for fundamentally new vistas in our representation of gravitation, without which research is turned from its intended thrilling pursue of "new" knowledge to a sterile fanatic attachment to "past" knowledge.

1.5 THE SCIENTIFIC IMBALANCE CAUSED BY NONCANONICAL AND NONUNITARY THEORIES

1.5.1 Introduction

When facing the limitations of special relativity and quantum mechanics for the representation of extended, nonspherical, deformable and hyperdense particles and antiparticles under linear and nonlinear, local and nonlocal as well as potential and nonpotential forces, a rather general attitude is that of attempting their generalization via the broadening into noncanonical and nonunitary structures, while preserving the mathematics of their original formulation.

Despite the widespread publication of papers on theories with noncanonical or nonunitary structures in refereed journals, including those of major physical societies, it is not generally known that these broader theories are afflicted by inconsistencies so serious to be also known as catastrophic.

Another basic objective of this monograph is the detailed identification of these inconsistencies because their only known resolution is that presented in the next chapters, that permitted by *new mathematics* specifically constructed from the physical conditions considered.

In fact, the broadening of special relativity and quantum mechanics into noncanonical and nonunitary forms, respectively, is necessary to exit form the class of equivalence of the conventional formulations. The resolution of the catastrophic inconsistencies of these broader formulations when treated via the mathematics of canonical and unitary theories, then leaves no other possibility than that of *broadening the basic mathematics.*

To complete the presentation of the foundations of the covering hadronic mechanics, in the next two sections we shall review the inconsistencies of noncanonical and nonunitary theories. The remaining sections of this chapter are devoted to an outline of hadronic mechanics so as to allow the reader to enter in a progressive way into the advanced formulations presented in the next chapters.

1.5.2 Catastrophic Inconsistencies of Noncanonical Theories

As recalled in Section 1.2, the research in classical mechanics of the 20-th century has been dominated by *Hamiltonian systems,* that is, systems admitting their complete representation via the *truncated Hamilton equations* (1.2.2), namely, the historical equations proposed by Hamilton in which the external terms have been cut out.

For the scope of this section, it is best to rewrite Eqs. (1.2.2) in the following unified form (see monographs [9] for details)[9]

$$b = (b^\mu) = (r, p) = (r^k, p_k), \tag{1.5.1a}$$

$$\frac{db^\mu}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t, b)}{\partial b^\nu}, \tag{1.5.1b}$$

$$H = K(p) + V(t, r, p), \tag{1.5.1c}$$

$$\mu = 1, 2, 3, ..., 6, \quad k = 1, 2, 3,$$

where H is the Hamiltonian, K is the kinetic energy, V is the potential energy, $\omega^{\mu\nu}$ is the canonical Lie tensor with explicit form

$$\omega^{\mu\nu} = \begin{pmatrix} 0 & I_{3\times3} \\ -I_{3\times3} & 0 \end{pmatrix} \tag{1.5.2}$$

and $I_{3\times3} = Diag(1, 1, 1)$ is the unit matrix.

In the above unified notation, the brackets of the time evolution can be written

$$\frac{dA}{dt} = [A, H] = \frac{\partial A}{\partial b^\mu} \times \omega^{\mu\nu} \times \frac{\partial H}{\partial b^\nu}, \tag{1.5.3}$$

and they characterize a *Lie algebra,* as well known.

The above equations have a *canonical structure,* namely, their time evolution characterizes a *canonical transformation*[10],

$$b^\mu \to b'^\mu(b), \tag{1.5.4a}$$

$$\omega^{\mu\nu} \to \frac{\partial b'^\mu}{\partial b^\rho} \times \omega^{\rho\sigma} \times \frac{\partial b'^\nu}{\partial b^\sigma} \equiv \omega^{\mu\nu}; \tag{1.5.4b}$$

and the theory possesses the crucial property of predicting the same numbers under the same conditions at different times, a property generically referred to as *invariance,* such as the invariance of the basic analytic equations under their own time evolution

$$\frac{db^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H(t, b)}{\partial b^\nu} = 0 \to$$
$$\to \frac{db'^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H(t', b')}{\partial b'^\nu} = 0. \tag{1.5.5}$$

---

[9] We continue to denote the conventional associative multiplication of numbers, vector fields, operators, etc. with the notation $A \times B$ rather than the usual form AB, because the new mathematics necessary to resolve the catastrophic inconsistencies studied in this chapter is based on various different generalizations of the multiplication. As a consequence, the clear identification of the assumed multiplication will soon be crucial for the understanding of the equations of this monograph.

[10] For several additional different but equivalent definitions of canonical transformations one may consult Ref. [54a], pages 187-188.

where the invariance is expressed by the preservation of the Lie tensor $\omega^{\mu\nu}$ and of the Hamiltonian H.

It is easy to predict that future research in classical mechanics will be dominated by *non-Hamiltonian systems,* that is, systems that cannot be entirely described by the Hamiltonian and require at least a second quantity for their complete description.

Alternatively, we are referring to systems with internal forces that are partly of potential type, represented by V, and partly of nonpotential type, thus requiring new quantities for their representation.

We are also referring to the transition from *exterior dynamical systems* recalled in Section 1.3 (systems of point-like particles moving in vacuum without collisions under sole action-at-a-distance potential interactions) to *interior dynamical systems* (extended, nonspherical and deformable particles moving within a resistive medium with action-at-a-distance potential forces plus contact, nonpotential, nonlocal, and integral forces).

As also recalled in Section 1.2, exterior dynamical systems can be easily represented with the truncated Hamilton equations, while the first representation of the broader non-Hamiltonian systems is given precisely by the historical analytic equations with external terms, Eqs. (1.3.2) that we now rewrite in the unified form

$$\frac{db^\mu}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} + F^\mu(t, b, \dot{b}, ...), \tag{1.5.6a}$$

$$F^\mu = (0, F_k), \quad \mu = 1, 2, ..., 6, \quad k = 1, 2, 3. \tag{1.5.6b}$$

Nevertheless, as also recalled in Section 1.3, the addition of the external terms creates serious structural problems since the brackets of the new time evolution

$$\frac{dA}{dt} = (A, H, F) = \frac{\partial A}{\partial b^\mu} \times \omega^{\mu\nu} \times \frac{\partial H}{\partial b^\nu} + \frac{\partial A}{\partial b^\mu} \times F^\mu, \tag{1.5.7}$$

violate the conditions to characterize an algebra (since they violate the right distributive and scalar laws), let alone violate all possible Lie algebras, thus prohibiting the studies of basic aspects, such as spacetime symmetries under nonpotential forces.

As experienced by the author, when facing the latter problems, a rather natural tendency is that of using coordinate transforms $b \to b'(b)$ to turn a systems that is non-Hamiltonian in the b-coordinates into a Hamiltonian form in the b'-coordinates,

$$\frac{db^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} - F^\mu(t, b, \dot{b}, ...) = 0 \to$$
$$\to \frac{db'^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H'(t,b')}{\partial b^\nu} = 0. \tag{1.5.8}$$

These transformations always exist under the necessary continuity and regularity conditions, as guaranteed by *Lie-Koening theorem* of analytic mechanics or the *Darboux Theorem* of the symplectic geometry) [9b].

This first attempt has no physical value because of excessive problems identified in Section 1.2, such as: the lack of physical meaning of quantum formulations in the b'-coordinates; the impossibility of placing a measuring apparatus in the transformed coordinates; the loss of all known relativities due to the necessarily nonlinear character of the transforms with consequential mapping of inertial into noninertial frames; and other problems.

The above problems force the restriction of analytic representations of non-Hamiltonian systems within the fixed coordinates of the experimenter, the so-called *direct analytic representations* of Assumption 1.2.1 [9].

Under the latter restriction, the second logical attempt for quantitative treatments of non-Hamiltonian systems is that of broadening conventional canonical theories into a noncanonical form at least admitting a consistent algebra in the brackets of the time evolution, even though the resulting time evolution of the broader equations cannot characterize a canonical transformation.

As an illustration of these second lines of research, in 1978 the author wrote for Springer-Verlag his first volume of *Foundations of Theoretical Mechanics* [9a] devoted to the integrability conditions for the existence of a Hamiltonian representation (the so-called *Helmholtz's conditions of variational selfadjointness*). The evident scope was that of identifying the limits of applicability of the theory within the fixed coordinates of the experimenter.

A main result was the proof that *the truncated Hamilton equations admit a direct analytic representation in three space dimensions only of systems with potential (variationally selfadjoint) forces,*[11] thus representing only a small part of what are generally referred to as *Newtonian systems.*

In this way, monograph [9a] confirmed the need to enlarge conventional Hamiltonian mechanics within the fixed frame of the experimenter in such a way to admit a direct representation of all possible Newtonian systems verifying the needed regularity and continuity conditions.

Along the latter line of research, in 1982 the author published with Springer-Verlag his second volume of *Foundations of Theoretical Mechanics* [9b] for the specifically stated objective of broadening conventional Hamiltonian mechanics in such a way to achieve *direct universality,* that is, the capability of representing *all* Newtonian systems (universality) in the fixed frame of the experimenter (direct universality), while jointly preserving not only an algebra, but actually the *Lie algebra* in the brackets of the time evolution.

[11] The truncated Hamilton equations admit analytic representations of nonconservative systems but only *in one dimension,* which systems are essentially irrelevant for serious physical applications.

These efforts gave birth to a broader mechanics called by the author *Birkhoffian mechanics* in honor of the discoverer of the basic equations, G. D. Birkhoff [37], which equations can be written in the unified form

$$\frac{db^\mu}{dt} = \Omega^{\mu\nu}(b) \times \frac{\partial B(t,b)}{\partial b^\nu}, \tag{1.5.9}$$

where $B(t,b)$ is called the *Birkhoffian* in order to distinguish it from the Hamiltonian (since B does not generally represent the total energy), and $\Omega^{\mu\nu}$ is a *generalized Lie tensor,* in the sense that the new brackets

$$\frac{dA}{dt} = [A,B]^* = \frac{\partial A}{\partial b^\mu} \times \Omega^{\mu\nu} \times \frac{\partial B}{\partial b^\nu}, \tag{1.5.10}$$

still verify the Lie algebra axioms (see Ref. [9b] for details).

Stated in different terms, the main efforts of monograph [54b] were to show that, under the necessary continuity and regularity properties, the historical Hamilton's equations with external terms always admit a reformulation within the fixed frame of the experimenter with a consistent Lie algebra in the brackets of the time evolution,

$$\frac{db^\mu}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} + F^\mu(t,b,...) \equiv \Omega^{\mu\nu}(b) \times \frac{\partial B(t,b)}{\partial b^\nu}. \tag{1.5.11}$$

In this case, rather than being represented with H and F, non-Hamiltonian systems are represented with B and Ω.

Monograph [9b] achieved in full the intended objective with the proof that *Birkhoffian mechanics is indeed directly universal for all possible well behaved local-differential Newtonian systems,* and admits the following *generalized canonical transformations,*

$$\Omega^{\mu\nu}(b) \rightarrow \frac{\partial b'^\mu}{\partial b^\rho} \times \Omega^{\rho\sigma}(b(b')) \times \frac{\partial b'^\nu}{\partial b^\sigma} \equiv \Omega^{\mu\nu}(b'). \tag{1.5.12}$$

Monograph [9b] concluded with the indication of the apparent full equivalence of the Birkhoffian and Hamiltonian mechanics, since the latter is admitted as a particular case of the former (when the generalized Lie tensor acquires the canonical form), both theories are derivable from a variational principle, and both theories admit similar transformation properties.

Since the generalized Lie tensor $\Omega^{\mu\nu}$ and related brackets $[A,B]^*$ are antisymmetric, we evidently have conservation laws of the type

$$\frac{dB}{dt} = [B,B]^* \equiv 0, \tag{1.5.13}$$

Consequently, Birkhoffian mechanics was suggested in monograph [54b] for the representation of *closed-isolated non-Hamiltonian systems* (such as Jupiter).

The representation of *open-nonconservative non-Hamiltonian systems* required the identification of a yet broader mechanics with a consistent algebra in the brackets of the time evolution, yet such that the basic brackets are not antisymmetric. The solution was reached in monographs [38] via the *Birkhoffian-admissible mechanics* with basic analytic equations

$$\frac{db^\mu}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} + F^\mu(t,b,...) \equiv S^{\mu\nu}(b) \times \frac{\partial B(t,b)}{\partial b^\nu}, \tag{1.5.14}$$

where the tensor $S^{\mu\nu}$ is *Lie-admissible* According to Santilli's [39] realization of Albert [40] abstract formulation, namely, in the sense that the generalized brackets of the time evolution

$$\frac{dA}{dt} = (A,B) = \frac{\partial A}{\partial b^\mu} \times S^{\mu\nu}(b) \times \frac{\partial B}{\partial b^\nu}, \tag{1.5.15}$$

verify all conditions to characterize an algebra, and their attached antisymmetric brackets

$$[A,B]^* = (A,B) - (B,A), \tag{1.5.16}$$

characterize a generalized Lie algebra as occurring in Birkhoffian mechanics.

The representation of the open-nonconservative character of the equations was then consequential, since the lack of antisymmetry of the brackets yields the correct *time rate of variation of the energy* $E = B$

$$\frac{dE}{dt} = (E,E) = F_k \times v^k, \tag{1.5.17}$$

and the same occurs for all other physical quantities.

Monographs [38] then proved the direct universality of Birkhoffian-admissible mechanics for all open-nonconservative systems, identified its transformation theory and provided the following elementary, yet universal realization of the Lie-admissible tensor S for $B = H$ representing the total *nonconserved* energy

$$S^{\mu\nu} = \begin{pmatrix} 0 & I \\ -I & F/(\partial H/\partial p) \end{pmatrix}. \tag{1.5.18}$$

Note that *the Birkhoffian-admissible mechanics is structurally irreversible,* in the sense of being irreversible for all possible energies and Birkhoffian functions since the basic Lie-admissible tensor is itself irreversible, $S(t,b) \neq S(-t,b)$, thus being particularly suited to represent irreversible systems.

However, studies conducted after the publication of monographs [9,38] revealed the following seemingly innocuous feature:

LEMMA 1.5.1 [11b]: *Birkhoffian and Birkhoffian-admissible mechanics are noncanonical theories, i.e., the generalized canonical transformations, are noncanonical,*

$$\omega^{\mu\nu} \rightarrow \frac{\partial b'^\mu}{\partial b^\rho} \times \omega^{\rho\sigma} \times \frac{\partial b'^\nu}{\partial b^\sigma} \equiv \Omega^{\mu\nu}(b') \neq \omega^{\mu\nu}. \tag{1.5.19}$$

It is important to understand that *Birkhoffian and Birkhoffian-admissible mechanics are mathematically attractive, but they are not recommended for physical applications, both classically as well as foundations of operator theories.*

The canonical Lie tensor has the well known explicit form (1.5.2). Therefore, the diagonal matrix $I_{3\times3}$ is left invariant by canonical transformations. But $I_{3\times3}$ is the *fundamental unit of the basic Euclidean geometry.* As such, it represents in an abstract and dimensionless form the basic units of measurement, such as

$$I_{3\times3} = Diag.(1\text{cm}, 1\text{cm}, 1\text{cm}). \tag{1.5.20}$$

By their very definition, noncanonical transformations do not preserve the basic unit, namely, they are transformations of the representative type (with arbitrary new values)

$$I_{3\times3} = Diag.(1\text{cm}, 1\text{cm}, 1\text{cm}) \rightarrow$$

$$\rightarrow U \times I_{3\times3} \times U^t = Diag.(3.127 \text{ cm},\ e^{-212} \text{ cm},\ \log 45 \text{ cm}), \tag{1.5.21a}$$

$$U \times U^t \neq I, \tag{1.5.21b}$$

where t stands for transposed. We, therefore, have the following important:

THEOREM 1.5.1 [53]: *Whether Lie or lie-admissible, all classical noncanonical theories are afflicted by catastrophic mathematical and physical inconsistencies.*

Proof. Noncanonical theories do not leave invariant under time evolution the basic unit. This implies the loss under the time evolution of the base field on which the theory is defined. Still in turn, the loss in time of the base field implies catastrophic mathematical inconsistencies, such as the lack of preservation in time of metric spaces, geometries, symmetries, etc., since the latter are defined over the field of real numbers.

Similarly, noncanonical theories do not leave invariant under time evolution the basic units of measurements, thus being inapplicable for consistent measurements. The same noncanonical theories also do not possess time invariant numerical predictions, thus suffering catastrophic physical inconsistencies. **q.e.d.**

In conclusion, the regaining of a consistent algebra in the brackets of the time evolution, as it is the case for Birkhoffian and Birkhoffian-admissible mechanics, is not sufficient for consistent physical applications because the theories remain noncanonical. In order to achieve a physically consistent representation of non-Hamiltonian systems, it is necessary that

1) The analytic equations must be derivable from a first-order variational principle, as necessary for quantization;

2) The brackets of the time evolution must characterize a consistent algebra admitting exponentiation to a transformation group, as necessary to formulate symmetries; and

3) The resulting theory must be invariant, that is, must admit basic units and numerical predictions that are invariant in time, as necessary for physical value.

Despite the large work done in monographs [9,38], the achievement of all the above conditions required the author to resume classical studies from their foundations.

These third efforts finally gave rise to the new *Hamilton-Santilli iso-, geno- and hypermechanics* [10b] that do verify all conditions 1), 2) and 3), thus being suitable classical foundations of hadronic mechanics, as reviewed in Chapter 3.

However, the joint achievement of conditions 1), 2) and 3) for non-Hamiltonian systems required the prior construction of *basically new mathematics,* [10a] today known as *Santilli's iso-, geno- and hyper-mathematics,* as also reviewed in Chapter 3.

This section would be grossly incomplete and potentially misleading without a study of requirement 1), with particular reference to the derivability of analytic equations from a "first-order" variational principle.

Classical studies of non-Hamiltonian systems are essential, not only to identify the basic methods for their treatment, but above all to identify quantization channels leading to unique and unambiguous operator formulations.

Conventional Hamiltonian mechanics provides a solid foundation of quantum mechanics because it is derivable from the variational principle that we write in the unified notation [9a]

$$\begin{aligned}\delta A^\circ = \delta \int [R^\circ_\mu(b) \times db^\mu - H \times dt] = \\ = \delta \int (p_k \times dr^k - H \times dt),\end{aligned} \tag{1.5.22}$$

where the functions R°_μ have the canonical expression

$$(R^\circ_\mu) = (p_k, 0), \tag{1.5.23}$$

under which expression the canonical tensor assumes the realization

$$\omega_{\mu\nu} = \frac{\partial R^\circ_\nu}{\partial b^\mu} - \frac{\partial R^\circ_\mu}{\partial b^\nu}, \tag{1.5.24a}$$

$$(\omega_{\mu\nu}) = (\omega^{\alpha\beta})^{-1}. \tag{1.5.24b}$$

As it is well known, the foundations for quantization are given by the *Hamilton-Jacobi equations* here expressed in the unified notation of Ref. [9a]

$$\frac{\partial A^\circ}{\partial t} = -H, \quad \frac{\partial A^\circ}{\partial b^\mu} = R^\circ_\mu, \tag{1.5.25}$$

that can be written explicitly in the familiar forms

$$\frac{\partial A^\circ}{\partial t} + H = 0, \tag{1.4.26a}$$

$$\frac{\partial A^\circ}{\partial r^k} - p_k = 0, \tag{1.5.26b}$$

$$\frac{\partial A^\circ}{\partial p_k} = 0, \tag{1.5.26c}$$

The use of the *naive quantization*

$$A^\circ \rightarrow -i \times \hbar \times \ell n\ \psi, \tag{1.5.27}$$

yields *Schrödinger's equations* in a unique and unambiguous way

$$\frac{\partial A^\circ}{\partial t} + H = 0 \rightarrow -i \times \bar{h}\frac{\partial \psi}{\partial t} - H \times \psi = 0, \tag{1.5.28a}$$

$$\frac{\partial A^\circ}{\partial r^k} = p_k \rightarrow -i \times \hbar \times \frac{\partial \psi}{\partial r^k} - p_k \times \psi = 0, \tag{1.5.28b}$$

$$\frac{\partial A^\circ}{\partial p_k} = 0 \rightarrow \frac{\partial \psi}{\partial p_k} = 0. \tag{1.4.28c}$$

The much more rigorous *symplectic quantization* yields exactly the same results and, as such, it is not necessary for these introductory notes.

A feature crucial for quantization is Eq. (1.5.26c) from which it follows that *the canonical action A° is independent from the linear momentum*, i.e.,

$$A^\circ = A^\circ(t, r). \tag{1.5.29}$$

an occurrence generally (but not universally) referred in the literature as characterizing a *first-order action functional.*

From the naive quantization it follows that, in the configuration representation, *the wave function originating from first-order action functionals is independent from the linear momentum* (and, vice-versa, in the momentum representation it is independent from the coordinates),

$$\psi = \psi(t, r), \tag{1.5.30}$$

which property is crucial for the axiomatic structure of quantum mechanics, e.g., for the correct formulation of Heisenberg's uncertainty principle, causality, Bell's inequalities, etc.

A serious knowledge of hadronic mechanics requires the understanding of the reason *Birkhoffian mechanics cannot be assumed as a suitable foundations for* *quantization.* Birkhoff's equations can indeed be derived from the variational principle (see monograph [9b] for details)

$$\delta A = \delta \int [R_\mu(b) \times db^\mu - B \times dt], \tag{1.5.31}$$

where the new functions $R_\mu(b)$ have the general expression

$$(R_\mu(b)) = (A_k(t,r,p), B^k(t,r,p)), \tag{1.5.32}$$

subject to the regularity condition that Det. $\Omega \neq 0$, under which Birkhoff's tensor assumes the realization

$$\Omega_{\mu\nu}(b) = \frac{\partial R_\nu}{\partial b^\mu} - \frac{\partial R_\mu}{\partial b^\nu}, \tag{1.5.33a}$$

$$(\Omega_{\mu\nu}) = (\Omega)^{\alpha\beta})^{-1}, \tag{1.5.33b}$$

with *Birkhoffian Hamilton-Jacobi equations* [9b]

$$\frac{\partial A}{\partial t} = -B, \quad \frac{\partial A}{\partial b^\mu} = R_\mu. \tag{1.5.34}$$

As one can see, Birkhoffian expressions (1.5.31)–(1.5.33) appear to be greatly similar to the corresponding Hamiltonian forms (1.4.22)–(1.4.26). Nevertheless, there is a fundamental structural difference between the two equations given by the fact that *the Birkhoffian action does indeed depend on the linear momenta,*

$$A = A(t,r,p), \tag{1.5.35}$$

a feature generally referred to as characterizing a *second-order action functional.*

As a consequence, *the "wavefunction" resulting from any quantization of Birkhoffian mechanics also depends on the linear momentum,*

$$\psi = \psi(t,r,p), \tag{1.5.36}$$

by characterizing an operator mechanics that is beyond our current technical knowledge for quantitative treatment, since such a dependence would require a dramatic restructuring of all quantum axioms.

In fact, the use of a naive quantization,

$$A(t,r,p) \rightarrow -i \times \hbar \times \ell n\ \psi(t,r,p), \tag{1.5.37}$$

characterizes the following maps

$$\frac{\partial A}{\partial t} + B = 0 \rightarrow -i \times \bar{h} \frac{\partial \psi}{\partial t} - B \times \psi = 0, \tag{1.5.38a}$$

$$\frac{\partial A}{\partial b^\mu} - R_\mu = 0 \rightarrow -i \times \hbar \times \frac{\partial \psi}{\partial b^\mu} - R_\mu \times \psi = 0. \tag{1.5.38b}$$

A first problem is that the latter equations are generally nonlinear and, as such, they cannot be generally solved in the r- and p-operators. This causes the emergence of an operator mechanics in which it is impossible to define basic physical quantities, such as the linear momentum or the angular momentum, with consequential lack of currently known physical relevance at this moment.

On more technical grounds, in the lifting of Hamiltonian into Birkhoffian mechanics, there is the replacement of the r-coordinates with the R-functions. In fact, the Birkhoffian action has the explicit dependence on the R-functions, $A = A[t, R(b)] = A'(t, r, p)$. As such, the Birkhoffian action can indeed be interpreted as being of first-order, but in the R-functions, rather than in the r-coordinates.

Consequently, a correct operator image of the Birkhoffian mechanics is given by the expressions (first derived in Ref. [11b])

$$i \times \hbar \times \frac{\partial \psi[t, R(b)]}{\partial t} = B \times \psi[t, R(b)], \qquad (1.5.39a)$$

$$-i \times \hbar \times \frac{\partial \psi[t, R(b)]}{\partial b^\mu} = R_\mu(b) \times \psi[t, R(b)]. \qquad (1.5.39b)$$

As we shall see in Chapter 3, the above equations characterize a *covering of hadronic (rather than quantum) mechanics,* in the sense of being structurally more general, yet admitting hadronic mechanics as a particular case.

Even though mathematically impeccable, intriguing, and deserving further studies, the mechanics characterized by Eqs. (1.5.39) is excessively general for our needs, and its study will be left to the interested reader.

The above difficulties identify quite precisely the first basic problem for the achievement of a physically consistent and effective formulation of hadronic mechanics, consisting in the need of constructing a new mathematics capable of representing CLOSED (that is, isolated) non-Hamiltonian systems via a first-order variational principle (as required for consistent quantization), admitting antisymmetric brackets in the time evolution (as required by conservation laws), and possessing time invariant units and numerical predictions (as required for physical value).

The need to construct a new mathematics is evident from the fact that no pre-existing mathematics can fulfill the indicated needs. As we shall see in Chapter 3, *Santilli's isomathematics* [10a] has been constructed precisely for and does indeed solve these specific problems.

The impossibility of assuming the *Birkhoffian-admissible mechanics* as the foundation of operator formulation for OPEN (that is, nonconservative) non-

Hamiltonian systems is clearly established by the fact that said mechanics *is not derivable from a variational principle.*[12]

The latter occurrence identifies a much more difficult task given by the *need to construct a yet broader mathematics capable of representing open non-Hamiltonian systems via a first-order variational principle (as required for consistent quantization), admitting non-antisymmetric brackets in the time evolution (as required by non-conservation laws), and possessing time invariant units and numerical predictions (as required by physical value).*

The lack of any pre-existing mathematics for the fulfillment of the latter tasks is beyond credible doubt. Rather than adapting nature to pre-existing mathematics, the author has constructed a yet broader mathematics, today known as *Santilli's genomathematics* [10a], that does indeed achieve all indicated objectives, as outlined in Chapter 4.

Readers interested in the depth of knowledge are suggested to meditate a moment on the implications of the above difficulties. In fact, these difficulties have caused the impossibility in the 20-th century to achieve a meaningful operator formulation of contact, nonconservative and nonpotential interactions.

A consequence has been the widespread belief that nonpotential interactions "do not exist" in the particle world, a view based on the lack of existence of their operator representation, with negative implications at all levels of knowledge, such as the impossibility of achieving a meaningful understanding of the origin of irreversibility.

As a consequence, the resolution of the difficulties in the quantization of nonpotential interactions achieved by hadronic mechanics implies a rather profound revision of most of the scientific views of the 20-th century, as we shall see in the subsequent chapters.

1.5.3 Catastrophic Inconsistencies of Nonunitary Theories

Once the limitations of quantum mechanics are understood (and admitted), another natural tendency is to exit from the class of equivalence of the theory via suitable generalizations, while keeping the mathematical methods used for quantum mechanics.

It is important for these studies to understand that these efforts are afflicted by catastrophic mathematical and physical inconsistencies equivalent to those suffered by classical noncanonical formulations based on the mathematics of canonical theories.

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[12]Because conventional variations δ can only characterize antisymmetric tensors of type $\omega_{\mu\nu}$ or $\Omega_{\mu\nu}$ and cannot characterize non-antisymmetric tensors such as the Lie-admissible tensor $S_{\mu\nu}$.

The author has dedicated his research life to the construction of axiomatically consistent and invariant generalizations of quantum mechanics for the treatment of nonlinear, nonlocal, and nonpotential effects because they are crucial for the prediction and treatment of new clean energies and fuels.

In this section we review the foundations of these studies with the identification, most importantly, of the failed attempts in the hope of assisting receptive colleagues in avoiding the waste of their time in the study of theories that are mathematically significant, yet cannot possibly have real physical value.

To begin, let us recall that a theory is said to be *equivalent to quantum mechanics* when it can be derived from the latter via any possible *unitary transform* on a conventional Hilbert space $\mathcal{H}$ over the field of complex numbers $C = C(c, +, \times)$,

$$U \times U^\dagger = U^\dagger \times U = I, \tag{1.5.40}$$

under certain conditions of topological smoothness and regularity hereon ignored for simplicity, where "$\times$" represents again the conventional associative product of numbers or matrices, $U \times U^\dagger \equiv UU^\dagger$.

As a consequence, *a necessary and sufficient condition for a theory to be inequivalent to quantum mechanics is that it must be outside its class of unitary equivalence*, that is, the new theory is connected to quantum mechanics via a *nonunitary transform*

$$U \times U^\dagger \neq I. \tag{1.5.41}$$

generally defined on a conventional Hilbert space $\mathcal{H}$ over C.

Therefore, true generalized theories must have a *nonunitary structure*, i.e., their time evolution must verify law (1.5.41), rather than (1.5.40).[13]Deformed brackets

During his graduate studies in physics at the University of Torino, Italy, and as part of his Ph. D. thesis, Santilli [41-43] published in 1967 the following (p, q)-*parametric deformation of the Lie product* $A \times B - B \times A$, the first in scientific records

$$\begin{aligned}(A, B) = p \times A \times B - q \times B \times A = \\ = m \times (A \times B - B \times A) + n \times (A \times B + B \times A) = \\ = m \times [A, B] + n \times \{A, B\},\end{aligned} \tag{1.5.42}$$

where $p = m + n, q = n - m$ and $p \pm q$ are non-null parameters.[14]

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[13]The reader should be aware that there exist in the literature numerous claims of "generalizations of quantum mechanics" although they have a unitary time evolution and, consequently, do not constitute true generalizations. All these "generalizations" will be ignored in this monograph because they will not resist the test of time.

[14]In 1985, Biedenharn [44] and MacFairlane [45] published their papers on the simpler q-deformations

$$A \times B - q \times B \times A$$

without a quotation of the origination of the broader form by the author [41] of 1967

$$p \times A \times B - q \times B \times A$$

Biedenharn was fully aware of origination [41] as established by the fact that Biedenharn had been part of a DOE research grant application jointly with the author and others, precisely on the latter deformations, application filed two years before the publication of paper [44] (see the full documentation in Refs. [93,94]). Unfortunately for him. Biedenharn was unable to quote origination [41] in his paper [44] for reasons explained below. Similarly, MacFairlane had been made aware of the (p, q)-deformations by the author himself years before paper [45] (see, again, the documentation in [93,94]), but was requested to abstein from proper quotation.

Ironically, by the time Biedenharn and MacFairlane published their papers, the author had already abandoned the field he initiated two decades earlier because of catastrophic inconsistencies studied in this section. the author met Biedenharn the last time prior to his departure at the *Wigner Symposium* held at Oxford University, England, in 1993. During that meeting Biedenharn confessed to the author that he had suppressed origination [41] of the q-deformations in his paper [44] because of "peer pressures from the Cantabridgean area." Biedenharn also confessed to the author that, following the publication of his paper [44], he became aware of the catastrophic inconsistencies of q-deformations, and confirmed that the "q-deformations have no physical value as treated so far."

Following the above behavior by Biedenharn and MacFairlane, the editors in the late 1980s and early 1990s of the American, British, Italian and other physical societies refused to quote paper [41] in the thousands of papers in the field, despite clear documentation of prior paternity. Because of these occurrences, the author acquired the dubbing of *the most plagiarized physicist of the 20-th century.* In reality, the author expressed his appreciation to both Biedenharn and MacFairlane because he did not want to have his name associated to thousands of papers *all* catastrophically inconsistent.

The author remembers Larry Biedenharn as a very brilliant scientist with a pleasant personality and a great potential for basic discoveries. Unfortunately, he was unable to avoid being controlled by organized interests in physics as a condition for an academic position. Consequently, he did indeed achieve a brilliant chair in physics at Duke University, but at the prize of being mainly remembered as an expert in the rotational, symmetry with some ethical overtone for plagiarisms. By contrast, the author trashed out any desire for a political chair at Harvard University as a necessary condition for freedom in basic research (see book [93] and the 1132 pages of documentation [94]).

The following episode illustrates the above lines. In the early 1980s, the author was working at the foundation of the isotopies of the Galilei and Einstein relativities, the lifting of the rotational symmetry to represent the transition from stationary orbits with the usual *conserved* angular momentum (exact $O(3)$ symmetry), to unstable orbits with *varying* angular momentum (exact $O(3)$-admissible symmetry), discussed in details in *Elements of Hadronic Mechanics,* Volume II, with a brief review in Chapters 3 and 4 of this volume. To proceed, the author phoned the biggest U. S. expert in the rotational symmetry, Larry Biedenharn, and asked to deliver an seminar at his department to hear his critical comments. With his innate courtesy, Biedenharn quickly agreed, and set the date of the seminar. The author and his family then drove for two days, from Cambridge, massachusetts, to Durham, North Carolina, for the meeting.

At the time of the seminar, the large lecture room at Duke University was empty (an occurrence often experienced by the author), with the sole exception of Larry Biedenharn and the chairman of the department (the author is unable to remember names of insignificant persons). Following routine presentations, the author's seminar lasted only a few seconds consisting in drawing in the blackboard a stable orbit of a satellite around Earth with exact $O(3)$ symmetry, and then drawing a decaying orbit of the same satellite during re-entry in Earth's atmosphere with "continuously decaying angular momentum and consequential breaking of the rotational symmetry." At the mere mention of this physical evidence, the department chairman went into a rage of nonscientific nonsense preventing the author from proffering any additional word for the unspoken but trivial reason that the breaking of the rotational symmetry implies the collapse of Einsteinian doctrines with consequential ;loss of money, prestige and power. In the middle of said rage, the author broke the chalk and left the room.

The author sensed Biedenharn's inner tragedy for, on one side, being sincerely interested in the topic while, on the other side, being forced to accept the control of his science to keep his academic job. For this reason, the author and his wife accepted the kind dinner invitation by the Biedenharns, but did

By remembering that the Lie product characterizes *Heisenberg's equations,* the above generalized product was submitted as part of the following *parametric generalization of Heisenberg's equations* in its finite and infinitesimal forms [41,42]

$$A(t) = U \times A(0) \times U^{\dagger} = e^{i\times H\times q\times t} \times A(0) \times e^{-i\times t\times p\times H}, \tag{1.5.43a}$$

$$i\ dA/dt = (A, H) = p\times A\times H - q\times H\times A, \tag{1.5.43b}$$

with classical counterpart studied in Ref. [43].

After an extensive research in European mathematics libraries (conducted prior to the publication of Ref. [41] with the results listed in the same publication), the brackets $(A, B) = p\times A\times B - q\times B\times A$ resulted to be *Lie-admissible* according to A. A. Albert [40], that is, the brackets are such that their attached antisymmetric product

$$[A\hat{,}B] = (A, B) - (B, A) = (p + q) \times [A, B], \tag{1.5.44}$$

characterizes a *Lie algebra.*

Jointly, brackets (A, B) are *Jordan admissible* also according to Albert, in the sense that their attached symmetric product,

$$\{A\hat{,}B\} = (A, B) + (B, A) = (p + q) \times \{A, B\}, \tag{1.5.45}$$

characterizes a *Jordan algebra.*

At that time (1967), only three articles on this subject had appeared in Lie- and Jordan-admissibility in the sole mathematical literature (see Ref. [41]).

In 1985, Biederharn [44] and MacFairlane [45] published their papers on the simpler q-deformations $A \times B - q \times B \times A$ without a quotation of the origination of the broader form $p \times A \times B - q \times B \times A$ by Santilli [41] in 1967.

Regrettably, Biedenharn and MacFairlane abstained from quoting Santilli's origination of twenty years earlier despite their documented knowledge of such an origination.

For instance, Biedenharn and Santilli had applied for a DOE grant precisely on the same deformations two years prior to Biedenharn's paper of 1985, and Santilli had personally informed MacFairlaine of said deformations years before his paper of 1985.

The lack of quotation of Santilli's origination of q-deformations resulted in a large number of subsequent papers by numerous other authors that also abstained from quoting said origination (see representative contributions [46-49]), for which

run away from Duke University as fast as possible early the following morning. Had Larry Biedenharn been able to cut out the organized scientific crime at his department (where "crime" is intended in the latin sense of damage to society for equivocal personal gains), he would have been remembered for a major structural advance in his field. The episode reinforced the soundedness of the author's decision to have trashed out Harvard University by the time of this episode as a necessary condition for freedom of scientific inquiries.

reason Santilli has been often referred to as the "most plagiarized physicist of the 20-th century".

Ironically, at the time Biedenharn and MacFairlane published their paper on q-deformations, Santilli had already abandoned them because of their catastrophic mathematical and physical inconsistencies studied in this Section.

In 1978, when at Harvard University, Santilli proposed the following *operator deformation of the Lie product* [Ref. [50], Eqs. (4.15.34) and (4.18.11)],

$$\begin{aligned}(A\hat{,}B) &= A \lhd B - B \rhd A = \\ &= A \times P \times B - B \times Q \times A = \\ &= (A{\times}T{\times}B - B{\times}T{\times}A) + (A{\times}W{\times}B + B{\times}W{\times}A) = \\ &= [A\hat{,}B] + \{A\hat{,}B\},\end{aligned} \tag{1.5.46}$$

where $P = T + W, Q = W - T$ and $P \pm Q$ are, this time, fixed non-null matrices or operators.

Evidently, product (1.5.46) remains jointly Lie-admissible and Jordan-admissible because the attached antisymmetric and symmetric brackets,

$$[A\hat{,}B] = (A\hat{,}B) - (B\hat{,}A) = A \times T \times B - B \times T \times A, \tag{1.5.47a}$$

$$\{A\hat{,}B\} = (A\hat{,}B) + (B\hat{,}A) = A \times W \times B + B \times W \times A, \tag{1.5.47b}$$

characterizes a *Lie-Santilli and Jordan-Santilli isoalgebra* (see Chapter 4 for details).

The reader should be aware that the following alternative versions of product (1.5.46),

$$P \times A \times B - Q \times B \times A, \tag{1.5.48a}$$

$$A \times B \times P - B \times A \times Q, \tag{1.5.48b}$$

do not constitute an algebra since the former (latter) violates the left (right) distributive and scalar laws [50].

The above operator deformations of the Lie product was also submitted in the original proposal [50] of 1978 as the fundamental equations of hadronic mechanics via the following broader *operator Lie-admissible and Jordan-admissible generalization of Heisenberg's equations* in its finite and infinitesimal forms[15]

$$A(t) = U{\times}A(0){\times}U^\dagger = e^{i\times H\times Q\times t}{\times}A(0){\times}e^{-i\times t\times P\times H}, \tag{1.5.49a}$$

[15]The author would like to be buried in Florida, the land he loved most, and have Eq. (1.5.49b) reproduced in his tombstone as follows:

Ruggero Maria Santilli
Sept. 8, 1935 - xxx, xx, xxxx

$$i\ dA/dt = A \lhd H - H \rhd A.$$

$$i\ dA/dt = (A\hat{,}H) = A \triangleleft H - H \triangleright A =$$
$$= A \times P \times H - H \times Q \times A, \tag{1.5.49b}$$
$$P = Q^\dagger, \tag{1.5.49c}$$

which equations, as we shall see in Chapter 4, are the fundamental equations of hadronic mechanics following proper mathematical treatment.

It is an instructive exercise for the reader interested in learning the foundation of hadronic mechanics to prove that:

1) Time evolutions (1.5.43) and (1.5.49) are *nonunitary,* thus being outside the class of unitary equivalence of quantum mechanics;

2) The application of a nonunitary transform $R \times R^\dagger \neq I$ to structure (1.5.43) yields precisely the broader structure (1.5.49) by essentially transforming the parameters p and q into the operators

$$P = p \times (R \times R^\dagger)^{-1}, \quad Q = q \times (R \times R^\dagger)^{-1}; \tag{1.5.50}$$

3) The application of additional nonunitary transforms $S \times S^\dagger \neq I$ to structure (1.5.50) preserves its Lie-admissible and Jordan-admissible character, although with different expressions for the P and Q operators.

The above properties prove the following:

LEMMA 1.5.2 [36]: *General Lie-admissible and Jordan-admissible laws (1.5.49) are "directly universal" in the sense of containing as particular cases all infinitely possible nonunitary generalizations of quantum mechanical equations ("universality") directly in the frame of the observer ("direct universality"), while admitting a consistent algebra in their infinitesimal form.*

The above property can be equally proven by noting that the product $(A\hat{,}B)$ is the most general possible "product" of an "algebras" as commonly understood in mathematics (namely, a vector space with a bilinear composition law verifying the right and left distributive and scalar laws).

In fact, the product $(A\hat{,}B)$ constitutes the most general possible combination of Lie and Jordan products, thus admitting as particular cases *all* known algebras, such as associative algebras, Lie algebras, Jordan algebras, alternative algebras, supersymmetric algebras, Kac-Moody algebras, *etc.*

Despite their unquestionable mathematical beauty, theories (1.5.43) and (1.5.49) possess the following catastrophic physical and mathematical inconsistencies:

Also, the author would like his coffin to be sufficiently heavy so as to avoid floating when Florida will be submerged by the now inevitable melting of the polar ice. The author wants Eq. (1.5.49b) in his tombstone because, in view of its direct universality, it will take centuries to achieve a broader description of nature equally invariant and equally based on the axioms of a field, particularly when said equation is formulated via the multi-valued hyperstructures of Chapter 5, Eqs. (5.3).

THEOREM 1.5.2 [36] *(see also Refs.* [51-58]*): All theories possessing a nonunitary time evolution formulated on conventional Hilbert spaces $\mathcal{H}$ over conventional fields of complex numbers $C(c, +, \times)$ do not admit consistent physical and mathematical applications because:*

1) They do not possess invariant units of time, space, energy, etc., thus lacking physically meaningful application to measurements;

2) They do not conserve Hermiticity in time, thus lacking physically meaningful observables;

3) They do not possess unique and invariant numerical predictions;

4) They generally violate probability and causality laws; and

5) They violate the basic axioms of Galileo's and Einstein's relativities.

Nonunitary theories are also afflicted by catastrophic mathematical inconsistencies.

The proof of the above theorem is essentially identical to that of Theorem 1.5,1 (see Ref. [36] for details). Again, the basic unit is not an abstract mathematical notion, because it embodies the most fundamental quantities, such as the units of space, energy, angular momentum, etc.

The nonunitary character of the theories here considered then causes the lack of conservation of the numerical values of such units with consequential catastrophic inapplicability of nonunitary theories to measurements.

Similarly, it is easy to prove that the condition of Hermiticity at the initial time,

$$(\langle\phi| \times H^\dagger) \times |\psi\rangle \equiv \langle\phi| \times (H \times |\psi\rangle), \quad H = H^\dagger, \tag{1.5.51}$$

is violated at subsequent times for theories with nonunitary time evolution when formulated on $\mathcal{H}$ over C. This additional catastrophic inconsistency (known as *Lopez's lemma* [52,53]), can be expressed by

$$[\langle\psi| \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times H \times U^\dagger] \times U|\psi\rangle =$$
$$= \langle\psi| \times U^\dagger \times [(U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times U|\psi\rangle] =$$
$$= (\langle\hat{\psi} \times T \times H'^\dagger) \times |\hat{\psi}\rangle = \langle\hat{\psi}| \times (\hat{H} \times T \times |\hat{\psi}\rangle), \tag{1.5.52a}$$

$$|\hat{\psi}\rangle = U \times |\psi\rangle, \quad T = (U \times U^\dagger)^{-1} = T^\dagger, \tag{1.5.52b}$$

$$H'^\dagger = T^{-1} \times \hat{H} \times T \neq H. \tag{1.5.52c}$$

As a result, nonunitary theories do not admit physically meaningful observables.

Assuming that the preceding inconsistencies can be by-passed with some manipulation, nonunitary theories still remain with additional catastrophic inconsistencies, such as the lack of invariance of numerical predictions.

To illustrate this additional inconsistency, suppose that the considered nonunitary theory is such that, at $t = 0$ sec, $U \times U^\dagger_{[t=0]} = 1$, at $t = 15$ sec, $U \times U^\dagger_{[t=15]} = 15$, and the theory predicts at time $t = 0$ sec, say, the eigenvalue of 2 eV,

$$H|_{t=0} \times |\psi >= 2\ eV \times |\psi > . \tag{1.5.53}$$

It is then easy to see that the same theory predicts under the same conditions the *different* eigenvalue 30 eV at $t = 15$ sec, thus having no physical value of any type. In fact, we have

$$U \times U^\dagger|_{t=0} = I, \quad U \times U^\dagger|_{t=15} = 15, \tag{1.5.54a}$$

$$\begin{gathered} U \times H \times |\psi\rangle = (U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi\rangle) = \\ = H' \times T \times |\hat{\psi}\rangle = U \times E \times |\psi\rangle = E \times (U \times |\psi\rangle) = E \times |\hat{\psi}\rangle, \\ H' = U \times H \times U^\dagger, \quad T = (U \times U^\dagger)^{-1}, \end{gathered} \tag{1.5.54b}$$

$$H' \times |\hat{\psi}\rangle\ |_{t=0} = 2\mathrm{C} \times |\hat{\psi}\rangle\ |_{t=0}, \quad T = 1\ |_{t=0}, \tag{1.4.54c}$$

$$\begin{gathered} H' \times |\hat{\psi}\rangle\ |_{t=15} = 2\mathrm{C} \times (U \times U^\dagger) \times |\hat{\psi}\rangle\ |_{t=15} = \\ = 30\mathrm{C} \times |\hat{\psi}\rangle\ |_{t=15}\ . \end{gathered} \tag{1.5.54d}$$

Probability and causality laws are notoriously based on the unitary character of the time evolution and the invariant decomposition of the unit.

Their violation for nonunitary theories is then evident. It is an instructive exercise for the reader interested in learning hadronic mechanics, superconductivity and chemistry to identify a specific example of nonunitary transforms for which the effect *precedes* the cause.

The violation by nonunitary theories of the basic axioms of Galileo's and Einstein's relativities is so evident to require no comment.

An additional, most fundamental inconsistency of the theories considered is their *noninvariance*, that can be best illustrated with the lack of invariance of the general Lie-admissible and Jordan-admissible laws (1.5.49).

In fact, under nonunitary transforms, we have, e.g., the lack of invariance of the Lie-admissible and Jordan-admissible product,

$$U \times U^\dagger \neq I \tag{1.5.55a}$$

$$\begin{gathered} U \times (A \hat{,} B) \times U^\dagger = U \times (A \lhd B - B \rhd A) \times U^\dagger = (U \times A \times U^\dagger) \times \\ \times [(U \times U^{-1}) \times (U \times P \times U^\dagger) \times (U \times U^\dagger)^{-1}] \times (U \times B \times U^\dagger) - \\ -(U \times B \times U^\dagger) \times [(U \times U^{-1}) \times (U \times Q \times U^\dagger) \times (U \times U^\dagger)^{-1}] \times \\ \times (U \times A \times U^\dagger) = A' \times P' \times B' - B' \times Q' \times A' = \\ = A' \lhd' B' - B' \rhd' A'. \end{gathered} \tag{1.5.55b}$$

The above rules confirm the preservation of a Lie-admissible structure under the most general possible transforms, thus confirming the direct universality of laws (1.4.49) as per Theorem 1.4.2. The point is that *the formulations are not invariant* because

$$P' = (U \times U^{-1}) \times (U \times Q \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \neq P, \tag{1.5.56a}$$

$$Q' = (U \times U^{-1}) \times (U \times Q \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \neq Q, \tag{1.5.56b}$$

that is, because *the product itself is not invariant.*

By comparison, the invariance of quantum mechanics follows from the fact that the associative product "$\times$" is not changed by unitary transforms

$$U \times U^{\dagger} = U^{\dagger} \times U = I, \tag{1.5.57a}$$

$$A \times B \rightarrow U \times (A \times B) \times U^{\dagger} =$$
$$= (U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger}) = A' \times B'. \tag{1.5.57b}$$

Therefore, generalized Lie-admissible and Jordan-admissible theories (1.5.49) are not invariant because the generalized products "$\triangleleft$" and "$\triangleright$" are changed by nonunitary transformations, including the time evolution of the theory itself. The same results also holds for other nonunitary theories, as the reader is encouraged to verify.

The mathematical inconsistencies of nonunitary theories are the same as those of noncanonical theories. Recall that mathematics is formulated over a given field of numbers. Whenever the theory is nonunitary, the first noninvariance is that of the basic unit of the field.

The lack of conservation of the unit then causes the loss of the basic field of numbers on which mathematics is constructed. It then follows that the entire axiomatic structure as formulated at the initial time, is no longer applicable at subsequent times.

For instance, the formulation of a nonunitary theory on a conventional Hilbert space has no mathematical sense because that space is defined over the field of complex numbers.

The loss of the latter property under nonunitary transforms then implies the loss of the former. The same result holds for metric spaces and other mathematics based on a field.

In short, the lack of invariance of the fundamental unit under nonunitary time evolutions causes the catastrophic collapse of the entire mathematical structure, without known exception.

The reader should be aware that the above physical and mathematical inconsistencies apply not only for Eqs. (1.5.49) but also for a large number of generalized theories, as expected from the direct universality of the former.

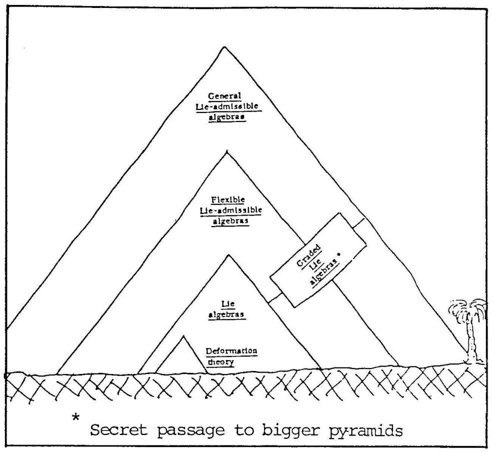

Figure 1.21. The reproduction of another "vignetta" presented by the author in 1978 to his colleagues at the Lyman Laboratory of Physics at Harvard University as part of his research under DOE (see Refs. [93,94] for details). This "vignetta" is a complement of that of Figure 1.3 on the need to maintain the external terms in the historical analytic equations because, when properly formulated, said equations yield covering, directly universal. Lie-admissible theories because Lie-admissible algebras contain as particular cases *all* algebras as defined in mathematics (universality) without the us of any transformation (direct universality). Finally, this "vignetta" was intended to illustrate that all theories preferred by the Lyman colleagues at the time, including symmetry breakings, supersymmetries, etc., were mere particular cases of the universal Lie-admissible formulations.

It is of the essence to identify in the following at least the most representative cases of physically inconsistent theories, to prevent their possible application (see Ref. [36] for details):

1) Dissipative nuclear theories [13] represented via an imaginary potential in non-Hermitian Hamiltonians,

$$H = H_0 = iV \neq H^\dagger \tag{1.5.58}$$

lose all algebras in the brackets of their time evolution (requiring a bilinear product) in favor of the triple system,

$$i \times \ dA/dt = A \times H - H^\dagger \times A = [A, H, H^\dagger]. \tag{1.5.59}$$

This causes the loss of nuclear notions such as "protons and neutrons" as conventionally understood, e.g., because the definition of their spin mandates the presence of a consistent algebra in the brackets of the time evolution.

2) Statistical theories with an external collision term C (see Ref. [59] and literature quoted therein) and equation of the density

$$i\ d\rho/dt = \rho \odot H = [\rho, H] + C, \quad H = H^\dagger, \tag{1.5.60}$$

violate the conditions for the product $\rho \odot H$ to characterize any algebra, as well as the existence of exponentiation to a finite transform, let alone violating the conditions of unitarity.

3) The so-called "q-deformations" of the Lie product (see, e.g., [64,65,66–69] and very large literature quoted therein)

$$A \times B - q \times B \times A, \tag{1.5.61}$$

where q is a non-null scalar, that are a trivial particular case of Santilli's (p, q)-deformations (1.4.42).

4) The so-called "k-deformations" [60-63] that are a relativistic version of the q-deformations, thus also being a particular case of general structures (1.4.42).

5) The so-called "star deformations" [64] of the associative product

$$A \star B = A \times T \times B, \tag{1.5.62}$$

where T is fixed, and related generalized Lie product

$$A \star B - B \star A, \tag{1.5.63}$$

are manifestly nonunitary and *coincide* with Santilli's Lie-isotopic algebras [50].

6) Deformed creation-annihilation operators theories [65,66].

7) Nonunitary statistical theories [67].

8) Irreversible black holes dynamics [68] with Santilli's Lie-admissible structure (1.4.46) [103,104].

9) Noncanonical time theories [6971].

10) Supersymmetric theories [104] with product

$$(A, B) = [A, B] + \{A, B\} =$$
$$= (A \times B - B \times A) + (A \times B + B \times A), \tag{1.5.64}$$

are an evident particular case of Santilli's Lie-admissible product (1.4.46) with $T = W = I$.

11) String theories (see ref. [58] and literature quoted therein) generally have a noncanonical structure due to the inclusion of gravitation with additional catastrophic inconsistencies when including supersymmetries.

12) The so-called squeezed states theories [73,74] due to their manifest nonunitary character.

13) All quantum groups (see, e.g., refs. [75-77]) with a nonunitary structure.

14) Kac-Moody superalgebras [78] are also nonunitary and a particular case of Santilli's Lie-admissible algebra (1.4.46) with $T = I$ and W a phase factor.

Numerous additional theories are also afflicted by the catastrophic inconsistencies of Theorem 1.5.2, such as quantum groups, quantum gravity, and other theories the reader can easily identify from the *departures* of their time evolution from the unitary law.

All the above theories have a nonunitary structure formulated via conventional mathematics and, therefore, are afflicted by the catastrophic physical and mathematical inconsistencies of Theorem 1.5.2.

Additional generalized theories were attempted via *the relaxation of the linear character of quantum mechanics* [56]. These theories are essentially based on eigenvalue equations with the structure

$$H(t, r, p, |\psi\rangle) \times |\psi\rangle = E \times |\psi\rangle, \tag{1.5.65}$$

(i.e., H depends on the wavefunction).

Even though mathematically intriguing and possessing a seemingly unitary time evolution, these theories also possess rather serious physical drawbacks, such as: they violate the superposition principle necessary for composite systems such as a hadron; they violate the fundamental Mackay imprimitivity theorem necessary for the applicability of Galileo's and Einstein's relativities and possess other drawbacks [36] so serious to prevent consistent applications.

Yet another type of broader theory is *Weinberg's nonlinear theory* [79] with brackets of the type

$$\begin{aligned} &A \odot B - B \odot A = \\ &= \frac{\partial A}{\partial \psi} \times \frac{\partial B}{\partial \psi^\dagger} - \frac{\partial B}{\partial \psi} \times \frac{\partial A}{\partial \psi^\dagger}, \end{aligned} \tag{1.5.66}$$

where the product $A \odot B$ is *nonassociative.*

This theory violates Okubo's No-Quantization Theorem [70], prohibiting the use of nonassociative envelopes because of catastrophic physical consequences, such as the loss of equivalence between the Schrödinger and Heisenberg representations (the former remains associative, while the latter becomes nonassociative, thus resulting in inequivalence).

Weinberg's theory also suffers from the absence of any unit at all, with consequential inability to apply the theory to measurements, the loss of exponentiation to a finite transform (lack of Poincaré-Birkhoff-Witt theorem), and other inconsistencies studied in Ref. [55].

These inconsistencies are not resolved by the adaptation of Weinberg's theory proposed by Jordan [80] as readers seriously interested in avoiding the publication of theories known to be inconsistent *ab initio* are encouraged to verify.

Several authors also attempted *the relaxation of the local-differential character of quantum mechanics* via the addition of "integral potentials" in the Hamiltonian,

$$V = \int d\tau \Gamma(\tau, \dots). \tag{1.5.67}$$

These theories are structurally flawed on both mathematical and physical grounds.

In fact, the nonlocal extension is elaborated via the conventional mathematics of quantum mechanics which, beginning with its topology, is strictly local-differential, thus implying fundamental *mathematical* inconsistencies. Nonlocal interactions are in general of contact type, for which the notion of a potential has no physical meaning, thus resulting in rather serious *physical* inconsistencies.

In conclusion, by the early 1980's Santilli had identified classical and operator generalized theories [103,104] that are directly universal in their fields, with a plethora of simpler versions by various other authors.

However, all these theories subsequently resulted in being mathematically significant, but having no physical meaning because they are noninvariant when elaborated with conventional mathematics.

As we shall see in Chapter 3 and 4, thanks to the construction of new mathematics, hadronic mechanics does indeed solve all the above inconsistencies. The clear difficulties in the solutions then illustrate the value of the result.

1.5.4 The Birth of Isomathematics, Genomathematics and their Isoduals

As it is well known, the basic equations of quantum mechanics, *Heisenberg's time evolution* of a (Hermitian) operator A ($\hbar = 1$),

$$i \times \frac{dA}{dt} = A \times H - H \times A = [A, H], \tag{1.5.68a}$$

$$H = p^2/2 \times m + V(r), \tag{1.5.68b}$$

can only represent the *conservation* of the total energy H (and other quantities) under action-at-a-distance interactions derivable from a potential $V(r)$,

$$i \times \frac{dH}{dt} = [H, H] = H \times H - H \times H \equiv 0. \tag{1.5.69}$$

Consequently, the above equations are basically insufficient to provide an operator representation of *closed non-Hamiltonian systems,* namely, systems of extended particles verifying conventional total conservation laws yet possessing internal potential; and nonpotential interactions, as it is the case for all interior problems, such as the structure of hadron, nuclei and stars.

The central requirement for a meaningful representation of closed, classical or operator interior systems of *particles* with internal contact interactions is the achievement of a *generalization of Lie's theory* in such a way to admit broader brackets, hereon denoted $[A \hat{,} B]$, verifying the following conditions:

1) The new brackets $[A \hat{,} B]$ must verify the distributive and scalars laws (3.9) in order to characterize an algebra.

2) Besides the Hamiltonian, the new brackets should admit a new Hermitian operator, hereon denoted with $\hat{T} = \hat{T}^\dagger$, and we shall write $[A \hat{,} B]_{\hat{T}}$, as a necessary condition for the representation of all non-Hamiltonian forces and effects.

3) The new brackets must be anti-symmetric in order to allow the conservation of the total energy under contact nonpotential internal interactions

$$i \times \frac{dH}{dt} = [H \hat{,} H]_{\hat{T}} \equiv 0. \tag{1.5.70}$$

For the case of *open,* classical or operator irreversible interior systems of *particles* there is the need of a *second generalization of Lie's theory* characterizing broader brackets, hereon denoted $(A \hat{,} B)$ verifying the following conditions:

1') The broader brackets (A, B) must also verify the scalar and distributive laws (3.9) to characterize an algebra;

2') The broader brackets must include *two* non-Hermitian operators, hereon denoted $\hat{P}$ and $\hat{Q}, \hat{P} = \hat{Q}^\dagger$ to represent the two directions of time, and the new brackets, denoted ${}_{\hat{P}}(A \hat{,} B)_{\hat{Q}}$, must be neither antisymmetric nor symmetric to characterize the time rate of variation of the energy and other quantities,

$$i \times \frac{dH}{dt} \; = \; {}_{\hat{P}}(H \hat{,} H)_{\hat{Q}} \; \neq \; 0; \tag{1.5.71}$$

3') The broader brackets must admit the antisymmetric brackets $[A \hat{,} B]$ and $[A, B]$ as particular cases because conservation laws are particular cases of nonconservation laws.

For the case of closed and open interior systems of *antiparticles,* it is easy to see that the above generalizations of Lie's theory will not apply for the same reason that the conventional Lie theory cannot characterize exterior systems of point-like antiparticles at classical level studied in Section 1.1 (due to the existence of only one quantization channel, the operator image of classical treatments of antiparticles can only yield particles with the wrong sign of the charge, and certainly not their charge conjugate).

The above occurrence requires a *third generalization of Lie's theory* specifically conceived for the representation of closed or open interior systems of antiparticles at *all* levels of study, from Newton to second quantization. As we shall see, the latter generalization is provided by the isodual map.

In an attempt to resolve the scientific imbalances of the preceding section, when at the Department of Mathematics of Harvard University, Santilli [39,50] proposed in 1978 an axiom-preserving generalization of conventional mathematics verifying conditions 1), 2) and 3), that he subsequently studied in various works (see monographs [9,10,11,38] and quoted literature).

The new mathematics is today known as *Santilli's isotopic and genotopic mathematics* or *isomathematics and genomathematics* for short [81-86], where the word "isotopic" or the prefix "iso" are used in the Greek meaning of preserving the original axioms, and the word "geno" is used in the sense of inducing new axioms.

Proposal [39] for the new isomathematics was centered in the generalization (called *lifting*) of the conventional, N-dimensional unit, $I = Diag.(1, 1, ..., 1)$ into an $N \times N$-dimensional matrix $\hat{I}$ that is nowhere singular, Hermitian and positive-definite, but otherwise possesses an unrestricted functional dependence on local coordinates r, velocities v, accelerations a, dimension d, density μ, wavefunctions ψ, their derivatives $\partial\psi$ and any other needed quantity,

$$I = Diag.(1,1,...,1) > 0 \rightarrow \hat{I}(r,v,a,d,\mu,\psi,\partial\psi,...) = \hat{I}^{\dagger} = 1/\hat{T} > 0 \qquad (1.5.72)$$

while jointly lifting the conventional associative product $A \times B$ among generic quantities A and B (numbers, vector fields, matrices, operators, etc.) into the form

$$A \times B \rightarrow A\hat{\times}B = A \times \hat{T} \times B, \qquad (1.5.73)$$

under which $\hat{I}$, rather than I, is the correct left and right unit,

$$I \times A = A \times I \equiv A \rightarrow \hat{I}\hat{\times}A = A\hat{\times}\hat{I} \equiv A, \qquad (1.5.74)$$

for all A of the set considered, in which case $\hat{I}$ is called *Santilli's isounit,* and $\hat{T}$ is called the *isotopic element.*

Eqs. (1.5.72)–(1.5.74) illustrate the isotopic character of the lifting. In fact, $\hat{I}$ preserves all topological properties of I; the isoproduct $A\hat{\times}B$ remains as associative as the original product $A \times B$; and the same holds for the preservation of the axioms for a left and right identity.

More generally, the lifting of the basic unit required, for evident reasons of consistency, a corresponding compatible lifting of *all* mathematics used by special relativity and quantum mechanics, with no exception known to this author, thus resulting in the new *isonumbers*, isospaces, isofunctional analysis, isodifferential calculus, isotopologies, isogeometries, etc. (for mathematical works see Refs. [10,11,38]).

Via the use of the above liftings, Santilli presented in the original proposal [39] a step-by-step isotopic (that is, axiom-preserving) lifting of all main branches of Lie's theory, including the isotopic generalization of universal enveloping associative algebras, Lie algebras, Lie groups and the representation theory. The new theory was then studied in various works and it is today known as the *Lie-Santilli isotheory* [81-86]. Predictably. from Eqs. (1.5.73) one can see that the new isobrackets have the form

$$[A\hat{,}B]_{\hat{T}} = A\hat{\times}B - B\hat{\times}A =$$
$$= A \times \hat{T} \times B - B \times \hat{T} \times A = [A\hat{,}B], \qquad (1.5.75)$$

where the subscript $\hat{T}$ shall be dropped hereon, whose verification of conditions 1), 2), 3) is evident.

The point important for these introductory lines is that *isomathematics does allow a consistent representation of extended, nonspherical, deformable and hyperdense particles under local and nonlocal, linear and nonlinear, and potential as well as nonpotential interactions.*

In fact, all conventional linear, local and potential interactions can be represented with a conventional Hamiltonian, while the shape and density of the particles and their nonlinear, nonlocal and nonpotential interactions can be represented with Santilli's isounits via realizations of the type

$$\hat{I} = \Pi_{k=1,2,\ldots,n} Diag(n_{k1}^2, n_{k2}^2, n_{k3}^2, n_{k4}^2) \times e^{\Gamma(\psi,\psi^\dagger)\times\int d^3r\psi^\dagger(r)_k\times\psi(r)_k}, \qquad (1.5.76)$$

where: the $n_{k1}^2, n_{k2}^2, n_{k3}^2$ allow to represent, for the first time, the actual, extended, nonspherical and deformable shapes of the particles considered (normalized to the values $n_k = 1$ for the perfect sphere); n_{k4}^2 allows to represent, also for the first time, the density of the interior medium (normalized to the value $n_4 = 1$ for empty space); the function $\Gamma(\psi, \psi^\dagger)$ represents the nonlinear character of the interactions; and the integral $\int d^3r\psi^\dagger(r)_k \times \psi(r)_k$ represents nonlocal interactions due to the overlapping of particles or of their wave packets.

When the mutual distances of the particles are much greater than 10^{-13}cm $= 1$ F, the integral in Eq. (1.5.76) is identically null, and all nonlinear and nonlocal effects are null. When, in addition, the particles considered are reduced to points moving in vacuum, all the n-quantities are equal to 1, generalized unit (1.3.22) recovers the trivial unit, and isomathematics recovers conventional mathematics identically, uniquely and unambiguously.

In the same memoir [39], in order to represent irreversibility, Santilli proposed a broader genomathematics based on the following differentiation of the product to the right and to the left with corresponding generalized units

$$A > B = A \times \hat{P} \times B, \quad \hat{I}^> = 1/\hat{P}; \qquad (1.5.77a)$$

$$A < B = A \times \hat{Q} \times B, \quad {}^<\hat{I} = 1/\hat{Q}, \qquad (1.5.77b)$$

$$\hat{I}^> = {}^< \hat{I}^\dagger, \qquad (1.5.77c)$$

where evidently the product to the right, $A > B$, represents motion forward in time and that to the left, $A < B$, represents motion backward in time. Since $A > B \neq A < B$, the latter mathematics represents irreversibility from the most elementary possible axioms.

The latter mathematics was proposed under a broader lifting called "genotopy" in the Greek meaning of inducing new axioms, and it is known today as *Santilli genotopic mathematics,* pr *genomathematics* for short [81-86].

It is evident that genoliftings (1.5.77) require a step by step generalization of all aspects of isomathematics, resulting in *genonumbers, genofields, genospaces, genoalgebras, genogeometries, genotopologies, etc.* [9b,10b,11,38a].

Via the use of the latter mathematics, Santilli proposed also in the original memoir [39] a genotopy of the main branches of Lie's theory, including a genotopic broadening of universal enveloping isoassociative algebras, Lie-Santilli isoalgebras, Lie-Santilli isogroup, isorepresentation theory, etc. and the resulting theory is today known as the *Lie-Santilli genotheory* with basic brackets

$$
\begin{aligned}
{}_{\hat{P}}(A\hat{,}B)_{\hat{Q}} = A < B - B > A = \\
= A \times P \times B - B \times Q \times A = (A\hat{,}B),
\end{aligned}
\tag{1.5.78}
$$

where the subscripts $\hat{P}$ and $\hat{Q}$ shall be dropped from now on.

It should be noted that the main proposal of memoir [39] is genomathematics, while isomathematics is presented as a particular case for

$$
(A\hat{,}B)_{\hat{P}=\hat{Q}=\hat{T}} = [A\hat{,}B]. \tag{1.5.79}
$$

as we shall see in Chapters 3 and 4, the *isodual isomathematics* and *isodual genomathematics* for the treatment of antiparticles are given by the isodual image (1.1.6) of the above iso- and geno-mathematics, respectively.

1.5.5 Hadronic Mechanics

Thanks to the prior discovery of isomathematics and genomathematics, in memoir [50] also of 1978 Santilli proposed a generalization of quantum mechanics for closed and open interior systems, respectively, under the name of *hadronic mechanics,* because hyperdense hadrons, such as protons and neutrons, constitute the most representative (and most difficult) cases of interior dynamical systems.

For the case of closed interior systems of particles, hadronic mechanics is based on the following *isotopic generalization of Heisenberg's equations* (Ref. [50], Eqs. (4.15.34) and (4.18.11))

$$
i \times \frac{dA}{dt} = [A\hat{,}H] = A\hat{\times}H - H\hat{\times}A. \tag{1.5.80}
$$

while for the broader case of open interior systems hadronic mechanics is based on the following *genotopic generalization of Heisenberg's equations* (Ref. [50], Eqs. (4.18.16))

$$
\begin{aligned}
i \times \frac{dA}{dt} = (A\hat{,}H) = A < H - H > A = \\
= A \times P \times H - H \times Q \times A.
\end{aligned}
\tag{1.5.81}
$$

The isodual images of Eqs. (1.5.80) and (1.5.81) for antiparticles as well as their multivalued hyperformulations significant for biological studies, were added more recently [88].

A rather intense scientific activity followed the original proposal [50], including five *Workshops on Lie-admissible Formulations* held at Harvard University from 1978 to 1982, fifteen *Workshops on Hadronic Mechanics*, and several formal conferences held in various countries, plus a rather large number of research papers and monographs written by various mathematicians, theoreticians and experimentalists, for an estimated total of some 15,000 pages of research published refereed journals (see the *General References on Hadronic Mechanics* at the end of this volume).

It should be indicated that, following the original proposal of 1978 [50], maturity on the basic new numbers of hadronic mechanics, the *iso-, geno- and hyper-numbers and their isoduals* was reached only in 1993 [87]; a correct mathematical formulation was reached only in 1996 [88] due to problems that had remained unsolved for years; and a fully invariant physical formulation was reached only in 1997 for invariant Lie-isotopic theories [89] and invariant Lie-admissible theories [89] (see also memoir [91] for a recent review).

The lapse of time between the original proposal of 1978 and the achievement of mathematical and physical maturity illustrates the difficulties to be resolved.

As a result of all these efforts, hadronic mechanics is today a rather diversified discipline conceived and constructed for quantitative treatments of all classical and operator systems of particles according to Definition 1.3.1 with corresponding isodual formulations for antiparticles.

It is evident that in the following chapters we can review only the most salient foundations of hadronic mechanics and have to defer the interested reader to the technical literature for brevity.

As of today, hadronic mechanics has experimental verifications and applications in particle physics, nuclear physics, atomic physics, superconductivity, chemistry, biology, astrophysics and cosmology, including numerous industrial applications outlined in monograph [92].

Hadronic mechanics can be classified into **sixteen different branches**, including: four branches of classical treatment of particles with corresponding four branches of operator treatment also of particles, and eight corresponding (classical and operator) treatments of antiparticles.

An effective classification of hadronic mechanics is that done via the main topological features of the assumed basic unit, since the latter characterizes all branches according to:

$I = 1 > 0$:
HAMILTONIAN AND QUANTUM MECHANICS
Used for the description of closed and reversible systems of point-like particles in exterior conditions in vacuum;

$I^d = -1 < 0$:
ISODUAL HAMILTONIAN AND ISODUAL QUANTUM MECHANICS
Used for the description of closed and reversible systems of point-like antiparticles in exterior conditions in vacuum;

$\hat{I}(r, v, ...) = \hat{I}^\dagger > 0$:

HADRONIC MECHANICS

MECHANICS AND THEIR ISODUALS

Newtonian Mechanics	Isodual Newtonian Mechanics
Hamiltonian mechanics	Isodual Hamiltonian Mechanics
Quantization	Isodual Quantization
Quantum mechanics	Isodual Quantum Mechanics
Special Relativity	Isodual Special Relativity

REPRESENTATION: isolated systems of point-like particles (mechanics) and antiparticles (isodual mechanics) under local, linear and potential forces.

ISOMECHANICS AND THEIR ISODUALS

Iso-Newtonian Mechanics	Isodual iso-Newtonian Mech.
Iso-Hamiltonian mechanics	Isodual iso-Hamiltonian Mech.
Isoquantization	Isodual Isoquantization
Isohadronic mechanics	Isodual isohadronic Mech.
Isospecial Relativity	Isodual Special Relativity

REPRESENTATION: Isolated, reversible and single-valued systems of extended particles (isomechanics) and antiparticles (isodual isomechanics) under internal, local and nonlocal, linear and nonlinear, potential and nonpotential forces.

GENOMECHANICS AND THEIR ISODUALS

Geno-Newtonian Mechanics	Isodual Geno-Newtonian Mech.
Geno-Hamiltonian mechanics	Isodual Geno-Hamiltonian Mech.
Genoquantization	Isodual Genoquantization
Genohadronic mechanics	Isodual Genohadronic Mechanics
Genospecial Relativity	Isodual Genospecial Relativity

REPRESENTATION: open, irreversible and single-valued systems of extended particles (genomechanics) and antiparticles (isodual genomechanics) under external, local and nonlocal, linear and nonlinear, potential and nonpotential forces.

HYPERMECHANICS AND THEIR ISODUALS

Hyper-Newtonian Mechanics	Isodual Hyper-Newtonian Mech.
Hyper-Hamiltonian mechanics	Isodual Hyper-Hamiltonian Mech.
Hyperquantization	Isodual Hyperquantization
Hyperhadronic mechanics	Isodual Hyperhadronic Mech.
Hyperspecial Relativity	Isodual Hyperspecial Relativity

REPRESENTATION: open, irreversible and multi--valued systems of extended particles (hypermechanics) and antiparticles (isodual hypermechanics) under external, local and nonlocal, linear and nonlinear, potential and nonpotential forces.

Figure 1.22. The structure of hadronic mechanics.

CLASSICAL AND OPERATOR ISOMECHANICS

Used for the description of closed and reversible systems of extended particles in interior conditions;

$\hat{I}^d(r^d, v^d, ...) = \hat{I}^{d\dagger} < 0$:

ISODUAL CLASSICAL AND OPERATOR ISOMECHANICS

Used for the description of closed and reversible systems of extended antiparticles in interior conditions;

$\hat{I}^>(r^>, v^>, ...) = (^<\hat{I})^\dagger$:

CLASSICAL AND OPERATOR GENOMECHANICS

Used for the description of open and irreversible systems of extended particles in interior conditions;

$\hat{I}^{d>}(r^{d>}, v^{d>}, ...\} = (^<\hat{I})^{d\dagger}$:

ISODUAL CLASSICAL AND OPERATOR GENOMECHANICS

Used for the description of open and irreversible systems of extended antiparticles in interior conditions;

$\hat{I}^> = (\hat{I}_1^>, \hat{I}_2^>, ...) = (^<\hat{I})^\dagger$:

CLASSICAL AND OPERATOR HYPERMECHANICS

Used for the description of multivalued open and irreversible systems of extended particles in interior conditions;

$\hat{I}^{d>} = \{\hat{I}_1^>, \hat{I}_2^>, ...\} = (^<\hat{I})^\dagger$:

ISODUAL CLASSICAL AND OPERATOR HYPERMECHANICS

Used for the description of multivalued open and irreversible systems of extended antiparticles in interior conditions.

In summary, a serious study of antiparticles requires its study beginning at the classical level and then following at all subsequent levels, exactly as it is the case for particles.

In so doing, the mathematical and physical treatments of antiparticles emerge as being deeply linked to that of particles since, as we shall see, the former are an anti-isomorphic image of the latter.

Above all, a serious study of antiparticles requires the admission of their existence in physical conditions of progressively increasing complexity, that consequently require mathematical and physical methods with an equally increasing complexity, resulting in the various branches depicted in Figure 5.

All in all, young minds of any age will agree that, rather than having reached a terminal character, our knowledge of nature is still at its first infancy and so much remains to be discovered.

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Chapter 2

ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.1 ELEMENTS OF ISODUAL MATHEMATICS

2.1.1 Isodual Unit, Isodual Numbers and Isodual Fields

The first comprehensive study of the isodual theory for point-like antiparticles has been presented by the author in monograph [34]. However, the field is subjected to continuous developments following its first presentation in papers [1] of 1985. Hence, it is important to review the most recent formulation of the isodual mathematics in sufficient details to render this monograph self-sufficient.

In this section, we identify only those aspects of isodual mathematics that are essential for the understanding of the physical profiles presented in the subsequent sections of this chapter. We begin with a study of the most fundamental elements of all mathematical and physical formulations, units, numbers and fields, from which all remaining formulations can be uniquely and unambiguously derived via simple compatibility arguments. To avoid un-necessary repetitions, we assume the reader has a knowledge of the basic mathematics used for the classical and operator treatment of matter, including a knowledge of the fields of real, complex and quaternionic numbers. The symbol † usec in this chapter denotes conventional Hermitean conjugation, namely, transpose t plus complex conjugation ^{c}. Hence, for real numbers n we have $n^\dagger = n$, for complex numbers a we have $a^\dagger = a^c$ and for qauternions q we have $q^\dagger = q^{tc}$.

DEFINITION 2.1.1: Let $F = F(a, +, \times)$ be a field (of characteristic zero), namely a ring with elements given by real number $a = n,\ F = R(n, +, \times)$, complex numbers $A = c,\ F = C(c, +, \times)$, or quaternionic numbers $a = q,\ F = Q(q, +, \times)$, *with conventional sum* $a + b$ *verifying the commutative law*

$$a + b = b + a = c \in F, \tag{2.1.1}$$

the associative law

$$(a + b) + c = a + (b + c) = d \in F, \tag{2.1.2}$$

conventional product $a \times b$ *verifying the associative law*

$$(a \times b) \times c = a \times (b \times c) = e \in F, \tag{2.1.3}$$

(but not necessarily the commutative law, $a \times b \neq b \times a$ *since the latter is violated by quaternions), and the right and left distributive laws*

$$(a + b) \times c = a \times c + b \times c = f \in F, \tag{2.1.4a}$$

$$a \times (b + c) = a \times b + a \times c = g \in F, \tag{2.1.4b}$$

left and right additive unit 0,

$$a + 0 = 0 + a = a \in F, \tag{2.1.5}$$

and left and right multiplicative unit I,

$$a \times I = I \times a = a \in F, \tag{2.1.6}$$

$\forall a, b, c \in F$. *Santilli's isodual fields (first introduced in* Refs. [1] *and then presented in details in* Ref. [2]*) are rings* $F^d = F^d(a^d, +^d, \times^d)$ *with elements given by isodual numbers*

$$a^d = -a^\dagger, \ a^d \in F, \tag{2.1.7}$$

with associative and commutative isodual sum

$$a^d +^d b^d = -(a + b)^\dagger = c^d \in F^d, \tag{2.1.8}$$

associative and distributive isodual product

$$a^d \times^d b^d = a^d \times (I^d)^{-1} \times b^d = c^d \in F^d, \tag{2.1.9}$$

additive isodual unit $0^d = 0$,

$$a^d +^d 0^d = 0^d +^d a^d = a^d, \tag{2.1.10}$$

and multiplicative isodual unit $I^d = -I^\dagger$,

$$a^d \times^d I^d = I^d \times^d a^d = a^d, \forall a^d, b^d \in F^d. \tag{2.1.11}$$

The proof of the following property is elementary.

LEMMA 2.1.1 [1,2]: *Isodual fields are fields, namely, if F is a field, its image F^d under the isodual map is also a field.*

The above lemma establishes the property (first identified in Refs. [1]) that *the axioms of a field do not require that the multiplicative unit be necessarily positive-definite, because the same axioms are also verified by negative-definite units.* The proof of the following property is equally simple.

LEMMA 2.1.2 [1,2]: *Fields F and their isodual images F^d are anti-isomorphic to each other.*

Lemmas 2.1.1 and 1.2.2 illustrate the origin of the name "isodual mathematics". In fact, to represent antimatter the needed mathematics must be a suitable "dual" of conventional mathematics, while the prefix "iso" is used in its Greek meaning of preserving the original axioms.

It is evident that for real numbers we have

$$n^d = -n, \tag{2.1.12}$$

while for complex numbers we have

$$c^d = (n_1 + i \times n_2)^d = -n_1 + i \times n_2 = -\bar{c}, \tag{2.1.13}$$

with a similar formulation for quaternions.

It is also evident that, for consistency, *all operations on numbers must be subjected to isoduality when dealing with isodual numbers.* This implies: the *isodual powers*

$$(a^d)^{n^d} = a^d \times^d a^d \times^d a^d \ldots \tag{2.1.14}$$

(n times, with n an integer); the *isodual square root*

$$a^{d(1/2)d} = -\sqrt{-a^\dagger}^\dagger, a^{d(1/2)d} \times^d a^{d(1/2)d} = a^d, \quad 1^{d(1/2)d} = -i; \tag{2.1.15}$$

the *isodual quotient*

$$a^d/^d b^d = -(a^\dagger/b^\dagger) = c^d, \;\; b^d \times^d c^d = a^d; \tag{2.1.16}$$

etc.

An important property for the characterization of antimatter is the following:

LEMMA 2.1.3. [2]: *isodual fields have a negative-definite norm, called isodual norm,*

$$|a^d|^d = |a^\dagger| \times I^d = -(aa^\dagger)^{1/2} < 0, \tag{2.1.17}$$

where $|\ldots|$ denotes the conventional norm.

For isodual real numbers we therefore have the isodual isonorm

$$|n^d|^d = -|n| < 0, \tag{2.1.18}$$

and for isodual complex numbers we have

$$|c^d|^d = -|\bar{c}| = -(c\bar{c})^{1/2} = -(n_1^2 + n_2^2)^{1/2}. \tag{2.1.19}$$

LEMMA 2.1.4 [2]: *All quantities that are positive-definite when referred to positive units and related fields of matter (such as mass, energy, angular momentum, density, temperature, time, etc.) become negative-definite when referred to isodual units and related isodual fields of antimatter.*

As recalled Chapter 1, antiparticles have been discovered in the *negative-energy solutions* of Dirac's equation and they were originally thought to evolve *backward in time* (Stueckelberg, Feynman, and others, see Refs. [1,2] of Chapter 1). The possibility of representing antiparticles via isodual methods is therefore visible already from these introductory notions.

The main novelty is that the conventional treatment of negative-definite energy and time was (and still is) referred to the conventional unit $+1$. This leads to a number of contradictions in the physical behavior of antiparticles.

By comparison, *negative-definite physical quantities of isodual theories are referred to a negative-definite unit $I^d < 0$.* This implies a mathematical and physical equivalence between *positive-definite quantities referred to positive-definite units, characterizing matter, and negative-definite quantities referred to negative-definite units, characterizing antimatter.* These foundations then permit a novel characterization of antimatter beginning at the *Newtonian* level, and then persisting at all subsequent levels.

DEFINITION 2.1.2 [2]: *A quantity is called isoselfdual when it coincides with its isodual.*

It is easy to verify that the imaginary unit is isoselfdual because

$$i^d = -i^\dagger = -\bar{i} = -(-i) = i. \tag{2.1.20}$$

This property permits a better understanding of the isoduality of complex numbers that can be written explicitly

$$c^d = (n_1 + i \times n_2)^d = n_1^d + i^d \times^d n_2^d = -n_1 + i \times n_2 = -\bar{c}. \tag{2.1.21}$$

The above property will be important to prove the equivalence of isoduality and charge conjugation at the operator level.

As we shall see, *isoselfduality is a new fundamental view of nature* with deep physical implications, not only in classical and quantum mechanics but also in cosmology. For instance we shall see that Dirac's gamma matrices are isoselfdual, thus implying a basically new interpretation of this equation that has remained unidentified for about one century. We shall also see that, when applied to cosmology, isoselfduality implies equal distribution of matter and antimatter in the universe, with identically null total physical characteristic, such as identically null total time, identically null total mass, etc.

We should also indicate that we have assumed the isoduality of the multiplication, $\times \rightarrow \times^d = \times(-1)\times = -\times$, but *not* that of the sum, $+ \rightarrow +^d = +(-1)+ = -$. This approach may not appear entirely motivated to the mathematically inclined reader because *fields are invariant under the above defined isoduality of the sum* due to the invariance of the additive unit, $0 \rightarrow 0^d \equiv 0$ (although fields are not invariant under the isoduality of the product due to the lack of invariance of the multiplicative unit, $1 \rightarrow 1^d = -1$).

The above decision is motivated by pragmatic, rather than mathematical arguments and, more specifically, for compatibility with the more general isofields and genofields, studied in the following chapters. In fact, at the latter broader levels, we have the loss of the invariance of the axioms of a field under these broader liftings of the sum. In turn, the loss of the field axioms cause the consequential inapplicability of the theory for physical applications as currently known, that is, based on "numbers" as rings verifying the axioms of a field, thus admitting a right and left, well defined, multiplicative unit representing the selected units of measurements.

It should also be stressed that, to avoid apparent inconsistencies, the isodual conjugation must be app;lied to all numbers and all their multiplications (or divisions). For instance, the isodual of a real numbers $n = n \times 1$ is given by $n^d \times^d 1^d = -n \times 1 = -n$ and not by $n^d \times 1^d =$ n.

We assume the reader is aware of the emergence here of *new numbers,* those with a negative unit, that have no connection with ordinary negative numbers and are the true foundations of the isodual theory of antimatter.

2.1.2 Isodual Functional Analysis

All conventional and special functions and transforms, as well as functional analysis at large, must be subjected to isoduality for consistent applications, resulting in the simple, yet unique and significant *isodual functional analysis*, studied by Kadeisvili [3], Santilli [4] and others.

We here mention the *isodual trigonometric functions*

$$\sin^d \theta^d \ = \ -\sin(-\theta), \ \ \cos^d \theta^d \ = \ -\cos(-\theta), \tag{2.1.22}$$

with related basic property

$$\cos^{d\,2d}\theta^d +^d \sin^{d\,2d}\theta^d \;=\; 1^d \;=\; -1, \tag{2.1.23}$$

the *isodual hyperbolic functions*

$$\sinh^d w^d \;=\; -\sinh(-w),\; \cosh^d w^d \;=\; -\cosh(-w), \tag{2.1.24}$$

with related basic property

$$\cosh^{d\,2d} w^d -^d \sinh^{d\,2d} w^d \;=\; 1^d \;=\; -1, \tag{2.1.25}$$

the *isodual logarithm* and the *isodual exponentiation* defined respectively by

$$\log^d n^d \;=\; -\log(-n), \tag{2.1.26a}$$

$$e_d^{X^d} = 1^d + X^d/^d 1!^d + X^{d2d}/^d 2!^d + \ldots = -e^X, \tag{2.1.26b}$$

etc. Interested readers can then easily construct the isodual image of special functions, transforms, distributions, etc.

2.1.3 Isodual Differential and Integral Calculus

Contrary to a rather popular belief, the differential calculus is indeed dependent on the assumed unit. This property is not so transparent in the conventional formulation because the basic unit is the trivial number +1. However, the dependence of the unit emerges rather forcefully under its generalization.

The *isodual differential calculus*, first introduced by Santilli in Ref. [5a], is characterized by the *isodual differentials*

$$d^d x^k \;=\; I^d \times dx^k \;=\; -dx^k,\;\; d^d x_k \;=\; -dx_k, \tag{2.1.27}$$

with corresponding *isodual derivatives*

$$\partial^d/^d\partial^d x^k \;=\; -\partial/\partial x^k,\;\; \partial^d/^d\partial^d x_k \;=\; -\partial/\partial x_k, \tag{2.1.28}$$

and related isodual properties.

Note that *conventional differentials are isoselfdual*, i.e.,

$$(dx^k)^d \;=\; d^d x^{kd} \equiv d\,x^k, \tag{2.1.29}$$

but *derivatives are not isoselfdual*,

$$[\partial f/\partial x^k]^d \;=\; -\partial^d f^d/^d\partial^d x^{kd}. \tag{2.1.30}$$

The above properties explain why the isodual differential calculus remained undiscovered for centuries.

Other notions, such as the *isodual integral calculus,* can be easily derived and shall be assumed as known hereon.

2.1.4 Lie-Santilli Isodual Theory

Let $\mathbf{L}$ be an n-dimensional Lie algebra in its regular representation with universal enveloping associative algebra $\xi(\mathbf{L})$, $[\xi(\mathbf{L})]^- \approx \mathbf{L}$, n-dimensional unit $I = Diag.(1, 1, \ldots, 1)$, ordered set of Hermitian generators $X = X^\dagger = \{X_k\}$, $k = 1, 2, \ldots, n$, conventional associative product $X_i \times X_j$, and familiar Lie's Theorems over a field $F(a, +, \times)$.

The *Lie-Santilli isodual theory* was first submitted in Ref. [1] and then studied in Refs. [4-7] as well as by other authors [23-31]. *The isodual universal associative algebra* $[\xi(\mathbf{L})]^d$ is characterized by the *isodual unit* I^d, *isodual generators* $X^d = -X$, and isodual associative product

$$X_i^d \times^d X_j^d = -X_i \times X_j, \tag{2.1.31}$$

with corresponding infinite-dimensional basis characterized by the *Poincaré-Birkhoff-Witt-Santilli isodual theorem*

$$I^d, X_i^d \times^d X_j^d, \quad i \le j; \quad X_i^d \times^d X_j^d \times X_k^d, \ i \le j \le k, \ldots \tag{2.1.32}$$

and related *isodual exponentiation* of a generic quantity A^d

$$e^{dA^d} = I^d + A^d/^d 1!^d + A^d \times^d A^d/^d 2!^d + \ldots = -e^{A^\dagger}, \tag{2.1.33}$$

where e is the conventional exponentiation.

The attached *Lie-Santilli isodual algebra* $\mathbf{L}^d \approx (\xi^d)^-$ over the isodual field $F^d(a^d, +^d, \times^d)$ is characterized by the *isodual commutators* [1]

$$[X_i^d,^d X_j^d] = -[X_i, X_j] = C_{ij}^{k^d} \times^d X_k^d. \tag{2.1.34}$$

with classical realizations given in Section 2.2.6.

Let G be a conventional, connected, n-dimensional Lie transformation group on a metric (or pseudo-metric) space $S(x, g, F)$ admitting $\mathbf{L}$ as the Lie algebra in the neighborhood of the identity, with generators X_k and parameters $w = \{w_k\}$.

The *Lie-Santilli isodual transformation group* G^d admitting the isodual Lie algebra $\mathbf{L}^d$ in the neighborhood of the isodual identity I^d is the n-dimensional group with generators $X^d = \{-X_k\}$ and parameters $w^d = \{-w_k\}$ over the isodual field F^d with generic element [1]

$$U^d(w^d) = e^{d^{i^d \times^d w^d \times^d X^d}} = -e^{i \times (-w) \times X} = -U(-w). \tag{2.1.35}$$

The *isodual symmetries* are then defined accordingly via the use of the isodual groups G^d and they are anti-isomorphic to the corresponding conventional symmetries, as desired. For additional details, one may consult Ref. [4,5b].

In this chapter we shall therefore use the *conventional Poincaré, internal and other symmetries* for the characterization of *matter*, and the *Poincaré-Santilli, internal and other isodual symmetries* for the characterization of *antimatter.*

2.1.5 Isodual Euclidean Geometry

Conventional (vector and) metric spaces are defined over conventional fields. It is evident that the isoduality of fields requires, for consistency, a corresponding isoduality of (vector and) metric spaces. The need for the isodualities of all quantities acting on a metric space (e.g., conventional and special functions and transforms, differential calculus, etc.) becomes then evident.

DEFINITION 2.1.3: Let $S = S(x, g, R)$ be a conventional N-dimensional metric or pseudo-metric space with local coordinates $x = \{x^k\}$, $k = 1, 2, \ldots, N$, nowhere degenerate, sufficiently smooth, real-valued and symmetric metric $g(x, \ldots)$ and related invariant

$$x^2 \ = \ (x^i \times g_{ij} \times x^j) \times I, \tag{2.1.36}$$

over the reals R. The isodual spaces, first introduced in Ref. [1] (see also Refs. [4,5] and, for a more recent account, Ref. [22]), *are the spaces $S^d(x^d,\, g^d,\, R^d)$ with isodual coordinates $x^d = x^d = -x^t$ (where t stands for transposed), isodual metric*

$$g^d(x^d, \ldots) \ = \ -g^\dagger(-x^\dagger, \ldots) \ = \ -g(-x^t, \ldots), \tag{2.1.37}$$

and isodual interval

$$\begin{aligned}(x-y)^{d^2\,d} \ &= \ [(x-y)^{id} \times^d g^d_{ij} \times^d (x-y)^{jd}] \times I^d = \\ &= \ [(x-y)^i \times g^d_{ij} \times (x-y)^j] \times I^d,\end{aligned} \tag{2.1.38}$$

defined over the isodual field $R^d \ = \ R^d(n^d, +^d, \times^d)$ with the same isodual isounit I^d.

The basic nonrelativistic space of our analysis is the three-dimensional *isodual Euclidean space* [1,9],

$$E^d(r^d, \delta^d, R^d) : r^d \ = \ \{r^{kd}\} \ = \ \{-r^k\} \ = \ \{-x, -y, -z\}, \tag{2.1.39a}$$

$$\begin{aligned}\delta^d \ &= \ -\delta \ = \ Diag.(-1, -1, -1), \\ I^d \ &= \ -I \ = \ Diag.(-1, -1, -1).\end{aligned} \tag{2.1.39b}$$

The *isodual Euclidean geometry* is the geometry of the isodual space E^d over R^d and it is given by a step-by-step isoduality of all the various aspects of the conventional geometry (see monograph [5a] for details).

By recalling that the norm on R^d is negative-definite, the *isodual distance* among two points on an isodual line is also negative definite and it is given by

$$D^d \ = \ D \times I^d \ = \ -D, \tag{2.1.40}$$

where D is the conventional distance. Similar isodualities apply to all remaining notions, including the notions of parallel and intersecting isodual lines, the Euclidean axioms, etc.

The *isodual sphere* with radius $R^d = -R$ is the perfect sphere on E^d over R^d and, as such, it has *negative radius* (Figure 2.1),

$$R^{d2d} = (x^{d2d} + y^{d2d} + z^{d2d}) \times I^d = \\ = (x^2 + y^2 + z^2) \times I = R^2. \qquad (2.1.41)$$

Note that the above expression coincides with that for the conventional sphere. This illustrates the reasons, following about one century of studies, the isodual rotational group and symmetry where identified for the first time in Ref. [1]. Note, however, that the latter result required the prior discovery of *new numbers,* those with a negative unit.

A similar characterization holds for other isodual shapes characterizing antimatter in our isodual theory.

LEMMA 2.1.5: The isodual Euclidean geometry on E^d over R^d is anti-isomorphic to the conventional geometry on E over R.

The group of isometries of E^d over R^d is the *isodual Euclidean group* $E^d(3) = \mathcal{R}^d(\theta^d) \times^d T^d(3)$ where $\mathcal{R}^d(\theta)$ is the isodual group of rotations first introduced in Ref. [1], and $T^d(3)$ is the isodual group of translations (see also Ref. [5a] for details).

2.1.6 Isodual Minkowskian Geometry

Let $M(x, \eta, R)$ be the conventional Minkowski spacetime with local coordinates $x = (r^k,\, t) = (x^\mu),\ k = 1,\, 2,\, 3,\ \mu = 1,\, 2,\, 3,\, 4,$ metric $\eta = Diag.(1,\, 1,\, 1, -1)$ and basic unit $I = Diag.(1,\, 1,\, 1,\, 1)$ on the reals $R = R(n,\, +,\, \times)$.

The *Minkowski-Santilli isodual spacetime*, first introduced in Ref. [7] and studied in details in Ref. [8], is given by

$$M^d(x^d, \eta^d, R^d) : x^d = \{x^{\mu d}\} = \{x^\mu \times I^d\} = \{-r, -c_o t\} \times I, \qquad (2.1.42)$$

with isodual metric and isodual unit

$$\eta^d = -\eta = Diag.(-1, -1, -1, +1), \qquad (2.1.43a)$$

$$I^d = Diag.(-1, -1, -1, -1). \qquad (2.1.43b)$$

The *Minkowski-Santilli isodual geometry* [8] is the geometry of isodual spaces M^d over R^d. The new geometry is also characterized by a simple isoduality of the conventional Minkowskian geometry as studied in details in memoir.

The fundamental symmetry of this chapter is given by the group of isometries of M^d over R^d, namely, the *Poincaré-Santilli isodual symmetry* [7,8]

$$P^d(3.1) = \mathcal{L}^d(3.1) \times T^d(3.1), \qquad (2.1.44)$$

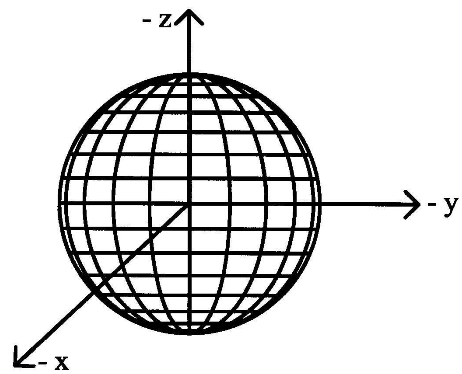

Figure 2.1. A schematic view of the isodual sphere on isodual Euclidean spaces over isodual fields. The understanding of the content of this chapter requires the knowledge that the isodual sphere and the conventional sphere coincide when inspected by an observer either in the Euclidean or in the isodual Euclidean space, due to the identity of the related expressions (2.1.36) and (2.1.38). This identity is at the foundation of the perception that antiparticles "appear" to exist in our space, while in reality they belong to a structurally different space coexisting within our own, thus setting the foundations of a "multidimensional universe" coexisting in the same space of our sensory perception. The reader should keep in mind that the isodual sphere is the idealization of the shape of an antiparticle used in this monograph.

where $\mathcal{L}^d(3.1)$ is the Lorentz-Santilli isodual group and $T^d(3.1)$ is the isodual group of translations.

2.1.7 Isodual Riemannian Geometry

Consider a Riemannian space $\Re(x, g, R)$ in $(3+1)$ dimensions [32] with basic unit $I = Diag.(1, 1, 1, 1)$, nowhere singular and symmetric metric $g(x)$ and related Riemannian geometry in local formulation (see, e.g., Ref. [27]).

The *Riemannian-Santilli isodual spaces* (first introduced in Ref. [11]) are given by

$$\begin{aligned} \Re^d(x^d, g^d, R^d): \;\; x^d &= \{-x^\mu\}, \\ g^d &= -g(x), \;\; g \in \Re(x, g, R), \\ I^d &= Diag.(-1, -1, -1, -1) \end{aligned} \tag{2.1.45}$$

with interval

$$\begin{aligned} x^{2d} &= [x^{dt} \times^d g^d(x^d) \times^d x^d] \times I^d = \\ &= [x^t \times g^d(x^d) \times x] \times I^d \in R^d, \end{aligned} \tag{2.1.46}$$

where t stands for transposed.

The *Riemannian-Santilli isodual geometry* [8] is the geometry of spaces $\Re^d$ over R^d, and it is also given by step-by-step isodualities of the conventional geometry, including, most importantly, the isoduality of the differential and exterior calculus.

As an example, an *isodual vector field* $X^d(x^d)$ on $\Re^d$ is given by $X^d(x^d) = -X^t(-x^t)$. The *isodual exterior differential* of $X^d(x^d)$ is given by

$$D^d X^{kd}(x^d) \ = \ d^d X^{kd}(x^d) + \Gamma^{dk}_{i\,j} \times^d X^{id} \times^d d^d x^{jd} \ = \ DX^k(-x), \tag{2.1.47}$$

where the Γ^d's are the components of the *isodual connection*. The *isodual covariant derivative* is then given by

$$X^{id}(x^d)_{|^{dk}} \ = \ \partial^d X^{id}(x^d)/^d \partial^d x^{kd} + \Gamma^{di}_{jk} \times^d X^{jd}(x^d) \ = \ -X^i(-x)_{|^k}. \tag{2.1.48}$$

The interested reader can then easily derive the isoduality of the remaining notions of the conventional geometry.

It is an instructive exercise for the interested reader to work out in detail the proof of the following:

LEMMA 2.1.6 [8]: *The isodual image of a Riemannian space* $\Re^d(x^d, g^d, R^d)$ *is characterized by the following maps*:

$$Basic \quad Unit$$

$$I \ \rightarrow \ I^d \ = \ -I,$$

$$Metric$$

$$g \ \rightarrow \ g^d \ = \ -g, \tag{2.1.49a}$$

$$Connection \quad Coefficients$$

$$\Gamma_{klh} \ \rightarrow \ \Gamma^d_{klh} \ = \ -\Gamma_{klh}, \tag{2.1.49b}$$

$$Curvature \quad Tensor$$

$$R_{lijk} \ \rightarrow \ R^d_{lijk} \ = \ -R_{lijk}, \tag{2.1.49c}$$

$$Ricci \quad Tensor$$

$$R_{\mu\nu} \ \rightarrow R^d_{\mu\nu} \ = \ -R_{\mu\nu}, \tag{2.1.49d}$$

$$Ricci \quad Scalar$$

$$R \ \rightarrow \ R^d \ = \ R, \tag{2.1.49e}$$

$$Einstein - Hilbert \quad Tensor$$

$$G_{\mu\nu} \ \rightarrow \ G^d_{\mu\nu} \ = \ -G_{\mu\nu}, \tag{2.1.49f}$$

$$Electromagnetic \quad Potentials$$

$$A_\mu \ \rightarrow \ A^d_\mu \ = \ -A_\mu, \tag{2.1.49g}$$

$$Electromagnetic \quad Field$$

$$F_{\mu\nu} \ \rightarrow \ F^d_{\mu\nu} \ = \ -F_{\mu\nu}, \tag{2.1.49h}$$

$$ElmEnergy - Momentum \quad Tensor$$

$$T_{\mu\nu} \ \rightarrow \ T^d_{\mu\nu} \ = \ -T_{\mu\nu}, \tag{2.1.49i}$$

In summary, the geometries significant for this study are: the *conventional Euclidean, Minkowskian and Riemannian geometries* used for the characterization of *matter*; and the *isodual Euclidean, Minkowskian and Riemannian geometries* used for the characterization of *antimatter*.

The reader can now begin to see the achievement of axiomatic compatibility between gravitation and electroweak interactions that is permitted by the isodual theory of antimatter. In fact, the latter is treated via negative-definite energy-momentum tensors, thus being compatible with the negative-energy solutions of electroweak interactions, therefore setting correct axiomatic foundations for a true grand unification studied in the next chapter.

2.2 CLASSICAL ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.2.1 Basic Assumptions

Thanks to the preceding study of isodual mathematics, we are now sufficiently equipped to resolve the scientific impasse caused by the absence of a classical theory of antimatter studied in Section 1.1.

As it is well known, the contemporary treatment of matter is characterized by *conventional mathematics*, here referred to ordinary numbers, fields, spaces, etc. with *positive units and norms*, thus having positive characteristics of mass, energy, time, etc.

In this chapter we study the *characterization of antimatter via isodual numbers, fields, spaces, etc., thus having negative-definite units and norms.* In particular, all characteristics of matter (and not only charge) change sign for antimatter when represented via isoduality.

The above characterization of antimatter evidently provides the correct conjugation of the charge at the desired classical level. However, by no means, the sole change of the sign of the charge is sufficient to ensure a consistent classical representation of antimatter. To achieve consistency, the theory must resolve the main problematic aspect of current classical treatments, the fact that their operator image is not the correct charge conjugate state (Section 2.1).

The above problematic aspect is indeed resolved by the isodual theory. The main reason is that, jointly with the conjugation of the charge, isoduality also conjugates *all* other physical characteristics of matter. This implies *two* channels of quantization, the conventional one for matter and a new *isodual quantization* for antimatter (see Section 2.3) in such a way that its operator image is indeed the charge conjugate of that of matter.

In this section, we study the physical consistency of the theory in its classical formulation. The novel isodual quantization, the equivalence of isoduality and charge conjugation and related operator issues are studied in the next section.

Beginning our analysis, we note that the isodual theory of antimatter resolves the traditional obstacles against negative energies and masses. In fact, *particles with negative energies and masses measured with negative units are fully equivalent to particles with positive energies and masses measured with positive units.* This result has permitted the elimination of sole use of second quantization for the characterization of antiparticles because antimatter becomes treatable at *all* levels, including second quantization.

The isodual theory of antimatter also resolves the additional, well known, problematic aspects of motion backward in time. In fact, *time moving backward measured with a negative unit is fully equivalent on grounds of causality to time moving forward measured with a positive unit.*

This confirms the plausibility of the first conception of antiparticles by Stueckelberg and others as moving backward in time (see the historical analysis in Ref. [1] of Chapter 1), and creates new possibilities for the ongoing research on the so-called "spacetime machine" studied in Chapter 5.

In this section, we construct the classical isodual theory of antimatter at the Newtonian, Lagrangian, Hamiltonian, Galilean, relativistic and gravitational levels; we prove its axiomatic consistency; and we verify its compatibility with available classical experimental evidence (that dealing with electromagnetic interactions only). Operator formulations and their experimental verifications will be studied in the next section.

2.2.2 Need for Isoduality to Represent All Time Directions

It is popularly believed that time has only two directions, the celebrated *Eddington's time arrows.* In reality, *time has four different directions* depending on whether motion is forward or backward and occurs in the future or in the past, as illustrated in Figure 2.2. In turn, the correct use of all four different directions of time is mandatory, for instance, in serious studies of bifurcations, as we shall see.

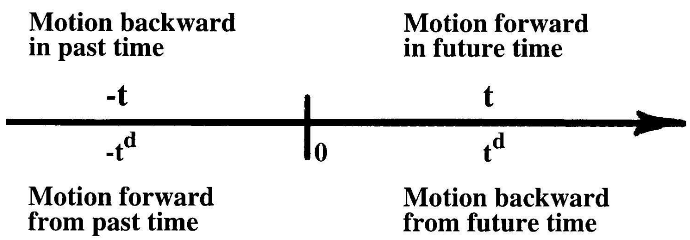

Figure 2.2. A schematic view of the "four different directions of time", depending on whether motion is forward or backward and occurs in the future or in the past. Due to the sole existence of one time conjugation, time reversal, the theoretical physics of the 20-th century missed two of the four directions of time, resulting in fundamental insufficiencies ranging from the lack of a deeper understanding of antiparticles to basic insufficiencies in biological structures and excessively insufficient cosmological views. It is evident that isoduality can indeed represent the two missing time arrows and this illustrates a basic need for the isodual theory.

It is evident that theoretical physics of the 20-th century could not explain all four directions of time, since it possessed only one conjugation, time reversal, and this explains the reason the two remaining directions of time were ignored.

It is equally evident that isoduality does indeed permit the representation of the two missing directions of time, thus illustrating its need.

We assume the reader is now familiar with the differences between time reversal and isoduality. Time reversal changes the direction of time while keeping the underlying space and units unchanged, while isoduality changes the direction of time while mapping the underlying space and units into different forms.

Unless otherwise specified, through the rest of this volume time t will be indicate *motion forward in future times,* $-t$ will indicate *motion backward in past times,* t^d will indicate motion backward from future times, and $-t^d$ will indicate *motion forward from past times.*

2.2.3 Experimental Verification of the Isodual Theory of Antimatter in Classical Physics

The experimental verification of the isodual theory of antimatter at the *classical* level is provided by the compliance of the theory with the only available experimental data, those on Coulomb interactions.

For that purpose, let us consider the Coulomb interactions under the customary notation that *positive (negative) forces represent repulsion (attraction)* when formulated in conventional Euclidean space.

Under such an assumption, the *repulsive* Coulomb force among two *particles* of negative charges $-q_1$ and $-q_2$ in Euclidean space $E(r,\, \delta,\, R)$ is given by

$$F \;=\; K \times (-q_1) \times (-q_2)/r \times r > 0, \tag{2.2.1}$$

where K is a positive constant whose explicit value (here irrelevant) depends on the selected units, the operations of multiplication $\times$ and division $/$ are the conventional ones of the underlying field $R(n,\, +,\, \times)$.

Under isoduality to $E^d(r^d,\, \delta^d,\, R^d)$ the above law is mapped into the form

$$F^d \;=\; K^d \times^d (-q_1)^d \times^d (-q_2)^d /^d r^d \times^d r^d \;=\; -F < 0, \tag{2.2.2}$$

where $\times^d = -\times$ and $/^d = -/$ are the isodual operations of the underlying field $R^d(n^d,\, +,\, \times^d)$.

But the isodual force $F^d = -F$ occurs in the isodual Euclidean space and it is, therefore, defined with respect to the unit -1. This implies that the reversal of the sign of a repulsive force measured with a negative unit also describes repulsion. As a result, isoduality correctly represents the *repulsive* character of the Coulomb force for two *antiparticles* with *positive* charges, a result first achieved in Ref. [9].

The formulation of the cases of two particles with positive charges and their antiparticles with negative charges is left to the interested reader.

The Coulomb force between a *particle* and an *antiparticle* can only be computed by *projecting the antiparticle in the conventional space of the particle or vice-versa*. In the former case we have

$$F \;=\; K \times (-q_1) \times (-q_2)^d/r \times r < 0, \tag{2.2.3}$$

thus yielding an *attractive* force, as experimentally established. In the projection of the particle in the isodual space of the antiparticle, we have

$$F^d \;=\; K^d \times^d (-q_1) \times^d (-q_2)^d /^d r^d \times^d r^d > 0. \tag{2.2.4}$$

But this force is now measured with the unit -1, thus resulting in being again *attractive*.

The study of Coulomb interactions of magnetic poles and other classical experimental data is left to the interested reader.

In conclusion, the isodual theory of antimatter correctly represents all available classical experimental evidence in the field.

2.2.4 Isodual Newtonian Mechanics

A central objective of this section is to show that the isodual theory of antimatter resolves the scientific imbalance of the 20-th century between matter and antimatter, by permitting the study of antimatter at *all* levels as occurring for matter. Such an objective can only be achieved by first establishing the existence of a

Newtonian representation of antimatter subsequently proved to be compatible with known operator formulations.

As it is well known, the Newtonian treatment of N *point-like particles* is based on a $7N$-dimensional representation space given by the Kronecker products of the Euclidean spaces of time t, coordinates r and velocities v (for the conventional case see Refs. [33,34]),

$$S(t,\, r,\, v) = E(t,\, R_t) \times E(r,\, \delta,\, R_r) \times E(v,\, \delta,\, R_v), \tag{2.2.5}$$

where

$$r = (r^k_a) = (r^1_a,\, r^2_a,\, r^3_a) = (x_a,\, y_a,\, z_a), \tag{2.2.6a}$$

$$v = (v_{ka}) = (v_{1a},\, v_{2a},\, v_{3a}) = (v_{xa},\, v_{ua},\, v_{za}) = dr/dt, \tag{2.2.6b}$$

$$\delta = Diag.(1,\, 1,\, 1),\ k = 1,\, 2,\, 3,\ a = 1,\, 2,\, 3, \ldots,\, N, \tag{2.2.6c}$$

and the base fields are trivially identical, i.e., $R_t = R_r = R_v$, since all units are assumed to have the trivial value $+1$, resulting in the trivial total unit

$$I_{tot} = I_t \times I_r \times I_v = 1 \times 1 \times 1 = 1. \tag{2.2.7}$$

The resulting basic equations are then given by the celebrated *Newton's equations for point-like particles*

$$m_a \times dv_{ka}/dt = F_{ka}(t,\, r,\, v),\ k = 1,\, 2,\, 3,\ a = 1,\, 2,\, 3, \ldots,\, N. \tag{2.2.8}$$

The basic space for the treatment of n antiparticles is given by the $7N$-dimensional *isodual space* [9]

$$S^d(t^d, r^d, v^d) \ = \ E^d(t^d, R^d_t) \times E^d(r^d, \delta^d, R^d) \times E^d(v^d, \delta^d, R^d), \tag{2.2.9}$$

with *isodual unit* and *isodual metric*

$$I^d_{Tot} \ = \ I^d_t \times I^d_r \times I^d_v, \tag{2.2.10a}$$

$$I^d_t \ = \ -1,\ I^d_r \ = \ I^d_v \ = Diag.(-1, -1, -1), \tag{2.2.10b}$$

$$\delta^d = Diag.(1^d, 1^d, 1^d) = Diag.(-1, -1, -1). \tag{2.2.10c}$$

We reach in this way the basic equations of this chapter, today known as the *Newton-Santilli isodual equations for point-like antiparticles,* first introduced in Ref. [4],[1]

$$m^d_a \times^d d^d v^d_{ka} /^d d^d t^d \ = \ F^d_{ka}(t^d, r^d, v^d), \tag{2.2.11}$$
$$k = x,\, y,\, z,\ a = 1,\, 2, \ldots,\, n,$$

[1]Note as necessary pre-requisites of the new Newton's equations, the prior discovery of isodual numbers, spaces and differential calculus.

whose experimental verification has been provided in the preceding section.

It is easy to see that the isodual formulation is anti-isomorphic to the conventional version, as desired, to such an extent that the two formulations actually coincide at the abstract, realization-free level.

Despite this axiomatic simplicity, the physical implications of the isodual theory of antimatter are rather deep. To begin their understanding, note that throughout the 20-th century it was believed that matter and antimatter exist in the same spacetime. In fact, as recalled earlier, charge conjugation is a map of our physical spacetime into itself.

One of the first physical implications of the Newton-Santilli isodual equations is that *antimatter exists in a spacetime co-existing, yet different than our own.* In fact, the isodual Euclidean space $E^d(r^d, \delta^d, R^d)$ co-exists within, but it is physically distinct from our own Euclidean space $E(r, \delta, R)$, and the same occurs for the full representation spaces $S^d(t^d, r^d, v^d)$ and $S(t, r, v)$.

The next physical implication of the Newton-Santilli isodual equations is the confirmation that *antimatter moves backward in time in a way as causal as the motion of matter forward in time* (again, because negative time is measured with a negative unit). In fact, the *isodual time* t^d is necessarily negative whenever t is our ordinary time. Alternatively, we can say that *the Newton-Santilli isodual equations provide the only known causal description of particles moving backward in time.*

Yet another physical implication is that *antimatter is characterized by negative mass, negative energy and negative magnitudes of other physical quantities.* As we shall see, these properties have the important consequence of eliminating the necessary use of Dirac's "hole theory."

The rest of this chapter is dedicated to showing that the above novel features are necessary in order to achieve a consistent representation of antimatter at all levels of study, from Newton to second quantization.

As we shall see, the physical implications are truly at the edge of imagination, such as: the existence of antimatter in a new spacetime different from our own constitutes the first known evidence of multi-dimensional character of our universe despite our sensory perception to the contrary; the achievement of a fully equivalent treatment of matter and antimatter implies the necessary existence of antigravity for antimatter in the field of matter (and vice-versa); the motion backward in time implies the existence of a causal spacetime machine (although restricted for technical reasons only to isoselfdual states); and other far reaching advances.

2.2.5 Isodual Lagrangian Mechanics

The second level of treatment of *matter* is that via the conventional *classical Lagrangian mechanics.* It is, therefore, essential to identify the corresponding formulation for *antimatter,* a task first studied in Ref. [4] (see also Ref. [9]).

A conventional (first-order) Lagrangian $L(t, r, v) = \frac{1}{2} \times m \times v^k \times v_k + V(t, r, v)$ on configuration space (2.2.5) is mapped under isoduality into the *isodual Lagrangian*

$$L^d(t^d, r^d, v^d) = -L(-t, -r, -v), \tag{2.2.12}$$

defined on isodual space (2.2.9).

In this way we reach the basic analytic equations of this chapter, today known as *Lagrange-Santilli isodual equations,* first introduced in Ref. [4]

$$\frac{d^d}{d^d t^d} d \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d v^{kd}} d - \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d r^{kd}} d = 0, \tag{2.2.13}$$

All various aspects of the *isodual Lagrangian mechanics* can then be readily derived.

It is easy to see that isodual equations (2.3.13) provide a *direct analytic representation* (i.e., a representation without integrating factors or coordinate transforms) of the isodual equations (2.2.11),

$$\frac{d^d}{d^d t^d} d \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d v^{kd}} d - \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d x^{kd}} d =$$

$$= m_k^d \times^d d^d v_k^d /^d d^d t^d - F_k^{d\,SA}(t, r, v) \ = \ 0. \tag{2.2.14}$$

The compatibility of the isodual Lagrangian mechanics with the primitive Newtonian treatment then follows.

2.2.6 Isodual Hamiltonian Mechanics

The *isodual Hamiltonian* is evidently given by [4,9]

$$H^d \ = \ p_k^d \times^d p^{dk} /^d 2^d \times^d m^d + V^d(t^d, r^d, v^d) \ = \ -H. \tag{2.2.15}$$

It can be derived from (nondegenerate) isodual Lagrangians via a simple isoduality of the Legendre transforms and it is defined on the $7N$-dimensional *isodual phase space (isocotangent bundle)*

$$S^d(t^d, r^d, p^d) \ = \ E^d(t^d, R_t^d) \times E^d(r^d, \delta^d, R^d) \times E^d(p^d, \delta^d, R^d). \tag{2.2.16}$$

The *isodual canonical action* is given by [4,9]

$$A^{\circ d} \ = \ \int_{t_1}^{t_2} (p_k^d \times^d d^d r^{kd} - H^d \times^d d^d t^d) \ =$$

$$= \int_{t_1}^{t_2} [R_\mu^{\circ d}(b^d) \times^d d^d b^{\mu d} - H^d \times^d d^d t^d], \qquad (2.2.17a)$$

$$R^\circ \ = \ \{p, 0\}, \ b = \{x, p\}, \ \mu = 1, \, 2, \ldots, \, 6. \qquad (2.2.17b)$$

Conventional variational techniques under simple isoduality then yield the fundamental canonical equations of this chapter, today known as *Hamilton-Santilli isodual equations* [4,24-31] that can be written in the disjoint r and p notation

$$\frac{d^d x^{kd}}{d^d t^d} = \frac{\partial^d H^d(t^d, x^d, p^d)}{\partial^d p_k^d}, \ \frac{d^d p_k^d}{d^d t^d} = -\frac{\partial^d H^d(t^d, x^d, p^d)}{\partial^d x^{dk}}, \qquad (2.2.18)$$

or in the unified notation

$$\omega_{\mu\nu}^d \times^d \frac{d^d b^{d\nu}}{d^d t^d} \ = \ \frac{\partial^d H^d(t^d, b^d)}{\partial^d b^{d\mu}}, \qquad (2.2.19)$$

where $\omega_{\mu\nu}^d$ is the *isodual canonical symplectic tensor*

$$(\omega_{\mu\nu}^d) \ = \ (\partial^d R_\nu^{\circ d} /^d \partial^d b^{d\mu} - \partial^d R_\mu^{\circ d} /^d \partial^d b^{d\nu}) = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \ = \ (\omega^{\mu\nu}). \qquad (2.2.20)$$

Note that isoduality maps the canonical symplectic tensor into the canonical Lie tensor, with intriguing geometric and algebraic implications.

The *Hamilton-Jacobi-Santilli isodual equations* are then given by [4,9]

$$\partial^d A^{\circ d} /^d \partial^d t^d + H^d \ = \ 0, \qquad (2.2.21a)$$

$$\partial^d A^{\circ d} /^d \partial^d x_k^d - p_k^d \ = \ 0, \ \partial^d A^{\circ d} /^d \partial^d p_k^d \equiv 0. \qquad (2.2.21b)$$

The *Lie-Santilli isodual brackets* among two isodual functions A^d and B^d on $S^d(t^d, \, x^d, \, p^d)$ then become

$$[A^d,^d B^d] \ = \ \frac{\partial^d A^d}{\partial^d b^{d\mu}} d \ \times^d \omega^{d\mu\nu} \times^d \frac{\partial^d B^d}{\partial^d b^{d\nu}} d \ = \ -[A, B], \qquad (2.2.22)$$

where

$$\omega^{d\mu\nu} \ = \ (\omega_{\mu\nu}) \qquad (2.2.23)$$

is the *Lie-Santilli isodual tensor* (that coincides with the conventional canonical tensor). The direct representation of isodual equations in first-order form is self-evident.

In summary, all properties of the isodual theory at the Newtonian level carry over at the level of isodual Hamiltonian mechanics.

2.2.7 Isodual Galilean Relativity

As it is well known, the Newtonian, Lagrangian and Hamiltonian treatment of matter are only the pre-requisites for the characterization of physical laws via basic relativities and their underlying symmetries. Therefore, no equivalence in the treatment of matter and antimatter can be achieved without identifying the relativities suitable for the *classical* treatment of antimatter.

To begin this study, we introduce the *Galilei-Santilli isodual symmetry* $G^d(3.1)$ [7,5,9,22-31] as the step-by-step isodual image of the conventional *Galilei symmetry* $G(3.1)$ (herein assumed to be known[2]). By using conventional symbols for the Galilean symmetry of a Keplerian system of N point particles with non-null masses m_a, $a = 1, 2, \ldots, n$, $G^d(3.1)$ is characterized by *isodual parameters and generators*

$$w^d \;=\; (\theta_k^d, r_o^{kd}, v_o^{kd}, t_o^d) = -w, \tag{2.2.24a}$$

$$J_k^d \;=\; \sum{}_{aijk} r_{ja}^d \times^d p_{ja}^k \;=\; -J_k \tag{2.2.24b}$$

$$P_k^d \;=\; \sum{}_a p_{ka}^d \;=\; -P_k, \tag{2.2.24c}$$

$$G_k^d \;=\; \sum{}_a (m_a^d \times^d r_{ak}^d - t^d \times p_{ak}^d), \tag{2.2.24d}$$

$$H^d \;=\; \frac{1}{2}^d \times^d \sum{}_a p_{ak}^d \times^d p_a^{kd} + V^d(r^d) \;=\; -H, \tag{2.2.24e}$$

equipped with the *isodual commutator*

$$[A^d,{}^d B^d] \;=\; \sum{}_{a,k}[(\partial^d A^d/^d\partial^d r_a^{kd}) \times^d (\partial^d B^d/^d\partial^d p_{ak}^d) -$$
$$-(\partial^d B^d/^d\partial^d r_a^{kd}) \times^d (\partial^d A^d/^d\partial^d p_{ak}^d)]. \tag{2.2.25}$$

In accordance with rule (2.1.34), the structure constants and Casimir invariants of the isodual algebra $G^d(3.1)$ are negative-definite. If $g(w)$ is an element of the (connected component) of the Galilei group $G(3.1)$, its isodual is characterized by

$$g^d(w^d) \;=\; e^{d - i^d \times^d w^d \times^d X^d} = -e^{i\times(-w)\times X} \;=\; -g(-w) \in G^d(3.1). \tag{2.2.26}$$

The *Galilei-Santilli isodual transformations* are then given by

$$t^d \;\to\; t'^d \;=\; t^d + t_o^d \;=\; -t', \tag{2.2.27a}$$

$$r^d \;\to\; r'^d \;=\; r^d + r_o^d \;=\; -r' \tag{2.2.27b}$$

[2]The literature on the conventional Galilei and special relativities and related symmetries is so vast as to discourage discriminatory quotations.

$$r^d \;\rightarrow\; r'^d \;=\; r^d + v_o^d \times^d t_o^d \;=\; -r', \qquad (2.2.27c)$$

$$r^d \;\rightarrow\; r'^d \;=\; R^d(\theta^d) \times^d r^d \;=\; -R(-\theta) \times r. \qquad (2.2.27d)$$

where $R^d(\theta^d)$ is an element of the *isodual rotational symmetry* first studied in the original proposal [1].

The desired classical nonrelativistic characterization of antimatter is therefore given by imposing the $G^d(3.1)$ invariance to the considered isodual equations. This implies, in particular, that the equations admit a representation via isodual Lagrangian and Hamiltonian mechanics.

We now confirm the classical experimental verification of the above isodual representation of antimatter already treated in Section 2.2.2. Consider a conventional, classical, massive *particle* and its *antiparticle* in exterior dynamical conditions in vacuum. Suppose that the particle and antiparticle have charge $-e$ and $+e$, respectively (say, an *electron* and a *positron*), and that they enter into the gap of a magnet with constant magnetic field **B**.

As it is well known, visual experimental observation establishes that particles and antiparticles under the same magnetic field have spiral trajectories of *opposite orientation*. But this behavior occurs for the *representation of both the particle and its antiparticle in the same Euclidean space*. The situation under isoduality is different, as described by the following:

LEMMA 2.2.1 [5a]: *The trajectories under the same magnetic field of a charged particle in Euclidean space and of the corresponding antiparticle in isodual Euclidean space coincide.*

Proof: Suppose that the particle has negative charge $-e$ in Euclidean space $E(r,\, \delta,\, R)$, i.e., the value $-e$ is defined with respect to the positive unit $+1$ of the underlying field of real numbers $R = R(n,\, +,\, \times)$. Suppose that the particle is under the influence of the magnetic field **B**.

The characterization of the corresponding antiparticle via isoduality implies the reversal of the sign of all physical quantities, thus yielding the charge $(-e)^d = +e$ in the isodual Euclidean space $E^d(r^d,\, \delta^d,\, R^d)$, as well as the reversal of the magnetic field $B^d = -B$, although now defined with respect to the negative unit $(+1)^d = -1$.

It is then evident that the trajectory of a particle with charge $-e$ in the field B defined with respect to the unit $+1$ in Euclidean space and that for the antiparticle of charge $+e$ in the field $-B$ defined with respect to the unit -1 in isodual Euclidean space coincide (Figure 2.3). **q.e.d.**

An aspect of Lemma 2.2.1, which is particularly important for this monograph, is given by the following:

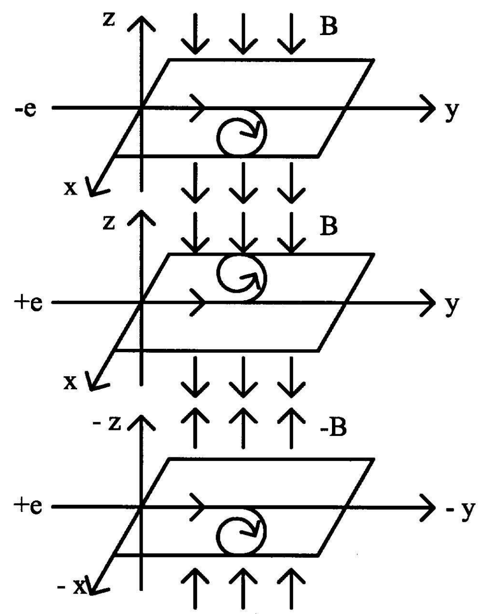

Figure 2.3. A schematic view of the trajectories of an electron and a positron with the same kinetic energy under the same magnetic field. The trajectories "appear" to be the reverse of each other when inspected by one observer, such as that in our spacetime (top and central views). However, when the two trajectories are represented in their corresponding spacetimes they coincide, as shown in the text (top and bottom views).

COROLLARY 2.2.1A: Antiparticles reverse their trajectories when projected from their own isodual space into our own space.

Lemma 2.2.1 assures that isodualities permit the representation of the correct trajectories of antiparticles as physically observed, despite their negative energy, thus providing the foundations for a consistent representation of antiparticles at the level of *first* quantization studied in the next section. Moreover, Lemma 2.2.1 tells us that the trajectories of antiparticles *appear* to exist in our space while in reality they belong to an independent space.

2.2.8 Isodual Special Relativity

We now introduce *isodual special relativity* for the classical relativistic treatment of point-like antiparticles (for the conventional case see Ref. [32]).

As it is well known, conventional special relativity is constructed on the fundamental 4-dimensional unit of the Minkowski space $I = Diag.(1, 1, 1, 1)$, representing the dimensionless units of space, e.g., (+1 cm, +1 cm, +1 cm), and the dimensionless unit of time, e.g., +1 sec, and constituting the basic unit of the conventional *Poincarè symmetry* $P(3.1)$ (hereon assumed to be known).

It then follows that *isodual special relativity* is characterized by the map

$$I \ = \ Diag.(\{1,\ 1,\ 1\},\ 1) > 0 \rightarrow$$

$$rightarrow I^d \ = \ Diag.(\{-1,\ -1,\ -1\},\ -1) < 0. \qquad (2.2.28)$$

namely, the antimatter relativity is based on *negative units of space and time*, e.g., $I^d = Diag.(-1\,\mathrm{cm}, -1\,\mathrm{cm}, -1\,\mathrm{cm}, -1\,\mathrm{sec})$. This implies the reconstruction of the entire mathematics of the special relativity with respect to the common, isodual unit I^d, including: the *isodual field* $R^d = R^d(n^d,\ +^d,\ \times^d)$ of *isodual numbers* $n^d = n \times I^d$; the *isodual Minkowski spacetime* $M^d(x^d, \eta^d, R^d)$ with isodual coordinates $x^d = x \times I^d$, isodual metric $\eta^d = -\eta$ and basic invariant over R^d

$$(x-y)^{d2d} \ = \ [(x^\mu - y^\mu) \times \eta^d_{\mu\nu} \times (x^\nu - y^\nu)] \times I^d \in R^d. \qquad (2.2.29)$$

This procedure yields the central symmetry of this chapter indicated in Section 2.2.6, today known as the *Poincaré-Santilli isodual symmetry* [7]

$$P^d(3.1) \ = \ \mathcal{L}^d(3.1) \times^d T^d(3.1), \qquad (2.2.30)$$

where $\mathcal{L}^d(3.1)$ is the *Lorentz-Santilli isodual symmetry*, $\times^d$ is the *isodual direct product* and $T^d(3.1)$ represents the *isodual translations.*

The algebra of the connected component $P_+^{\uparrow d}(3.1)$ of $P^d(3.1)$ can be constructed in terms of the isodual parameters $w^d = \{-w_k\} = \{-\theta, -v, -a\}$ and isodual generators $X^d = -X = \{-X_k\} = \{-M_{\mu\nu}, -P_\mu\}$. The isodual commutator rules are given by [7]

$$[M^d_{\mu\nu},^d M_{\alpha\beta}]^d \ =$$

$$= i^d \times^d (\eta^d_{\nu\alpha} \times^d M^d_{\mu\beta} - \eta^d_{\mu\alpha} \times^d M^d_{\nu\beta} - \eta^d_{\nu\beta} \times^d M^d_{\mu\alpha} + \eta^d_{\mu\beta} \times^d \hat{M}^d_{\alpha\nu}), \qquad (2.2.31a)$$

$$[M^d_{\mu\nu},^d p^d_\alpha] \ = \ i^d \times^d (\eta^d_{\mu\alpha} \times^d p^d_\nu - \eta^d_{\nu\alpha} \times^d p^d_\mu), \qquad (2.2.31b)$$

$$[p^d_\alpha, p^d_\beta]^d \ = \ 0. \qquad (2.3.31c)$$

The *Poincarè-Santilli isodual transformations* are given by[3]

$$x^{1d\prime} \ = \ x^{1d} = -x^1, \qquad (2.2.32a)$$

$$x^{2d\prime} \ = \ x^{2d} = -x^2, \qquad (2.2.32b)$$

[3]It should be indicated that, contrary to popular beliefs, the conventional Poincaré symmetry will be shown in Chapter 3 to be *eleven* dimensional, the 11-th dimension being given by a new invariant under change of the unit. Therefore, the isodual symmetry $P^d(3.1)$ is also 11-dimensional.

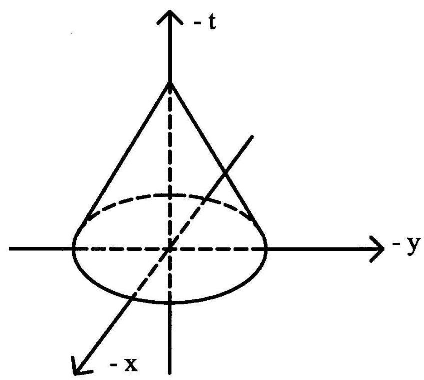

Figure 2.4. A schematic view of the "isodual backward light cone" as seen by an observer in our own spacetime with a time evolution reversed with respect to the "conventional forward light cone."

$$x^{3d\prime} = \gamma^d \times^d (x^{3d} - \beta^d \times^d x^{4d}) = -x^{3\prime}, \tag{2.2.32c}$$

$$x^{4d\prime} = \gamma^d \times^d (x^{4d} - \beta^d \times^d x^{3d}) = -x^{4\prime}, \tag{2.2.32d}$$

$$x^{d\mu\prime} = x^{d\mu} + a^{d\mu} = -x^{\mu\prime}, \tag{2.3.32e}$$

where

$$\beta^d = v^d/^d c_\circ^d = -\beta, \ \beta^{d2d} = -\beta^2, \ \gamma^d = -(1-\beta^2)^{-1/2}, \tag{2.2.33}$$

and the use of the isodual operations (quotient, square roots, etc.) is assumed.

The *isodual spinorial covering*

$$\mathcal{P}^d(3.1) = \mathcal{SL}^d(2.C^d) \times^d \mathcal{T}^d(3.1) \tag{2.2.34}$$

can then be constructed via the same methods.

The basic postulates of the isodual special relativity are also a simple isodual image of the conventional postulates [7]. For instance, the *maximal isodual causal speed in vacuum* is the speed of light in M^d, i.e.,

$$V^d_{max} = c_\circ^d = -c_\circ, \tag{2.2.35}$$

with the understanding that it is measured with a *negative-definite unit*, thus being fully equivalent to the conventional maximal speed c_o referred to a positive unit. A similar situation occurs for all other postulates.

The *isodual light cone* is evidently given by (Figure 2.4)

$$x^{d\,2\,d} = (x^{\mu d} \times^d \eta^d_{\mu\nu} \times^d x^{\nu d}) \times I^d =$$

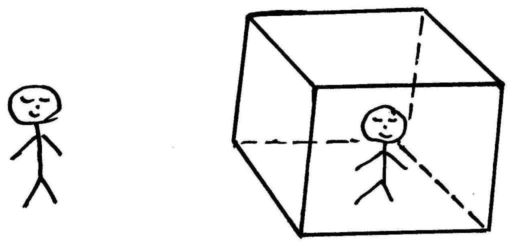

Figure 2.5. A schematic view of the "isodual cube," here defined as a conventional cube with two observers, an external observer in our spacetime and an internal observer in the isodual spacetime. The first implication of the isodual theory is that the same cube coexists in the two spacetimes and can, therefore, be detected by both observers. A most intriguing implication of the isodual theory is that each observer sees the other becoming younger. This occurrence is evident for the behavior of the internal observer with respect to the exterior one, since the former evolves according to a time opposite that of the latter. The same occurrence is less obvious for the opposite case, the behavior of the external observer with respect to the internal one, and it is due to the fact that the projection of our positive time into the isodual spacetime is indeed a motion backward in that spacetime.

$$= \ (-x \times x - y \times y - z \times z + t \times c_\circ^2 \times t) \times (-I) \ = \ 0. \tag{2.2.36}$$

As one can see, the above cone formally coincides with the conventional light cone, although the two cones belong to different spacetimes. The isodual light cone is used in these studies as *the cone of light emitted by antimatter in empty space (exterior problem).*

Note that the *two* Minkowskian metrics $\eta = Diag.(+1,+1,+1,-1)$ and $\eta = Diag.(-1,-1,-1,+1)$ have been popular since Minkowski's times, although both referred to the *same* unit I. We have learned here that these two popular metrics are connected by isoduality.

We finally introduce the *isodual electromagnetic waves* and related *isodual Maxwell's equations* [9]

$$F^d_{\mu\nu} \ = \ \partial^d A^d_\mu /^d \partial^d x^{\nu d} - \partial^d A^d_\nu /^d \partial^d x^{d\mu}, \tag{2.2.37a}$$

$$\partial^d_\lambda F^d_{\mu\nu} + \partial^d_\mu F^d_{\nu\lambda} + \partial^d_\nu F^d_{\lambda\mu} = 0, \tag{2.2.37b}$$

$$\partial^d_\mu F^{d\mu\nu} = -J^{d\nu}. \tag{2.2.37c}$$

As we shall see, the nontriviality of the isodual special relativity is illustrated by the fact that isodual electromagnetic waves experience gravitational repulsion when in the field of matter.

2.2.9 Inequivalence of Isodual and Spacetime Inversions

As it is well known (see, the fundamental spacetime symmetries of the 20-th century are the continuous (connected) component of the Poincaré symmetry plus discrete symmetries characterized by *space reversal* (also called *parity*) and *time reversal.*

As noted earlier, antiparticles are assumed in the above setting to exist in the same representation spacetime and to obey the same symmetries as those of particles. On the contrary, according to the isodual theory, antiparticles are represented in a spacetime and possess symmetries distinct from those of particles, although connected to the latter by the isodual transform.

The latter occurrence requires the introduction of the *isodual spacetime inversions,* that is, the isodual images of space and time inversions, first identified in Ref. [9], that can be formulated in unified coordinate form as follows

$$\begin{aligned} x^{d\mu\prime} \ &= \ \pi^d \times^d x^d \ = \ -\pi \times x = \\ &= (-r, x^4), \ \ \tau^d \times^d x^d = -\tau \times x \ = \ -(r, -x^4), \end{aligned} \tag{2.2.38}$$

with field theoretical extension (here expressed for simplicity for a scalar field)

$$\pi^d \times^d \phi^d(x^d) \times^d \pi^{d\dagger} = \phi^d(x'^d, x'^d = (-r^d, t^d) = (r, -t), \tag{2.2.39a}$$

$$\tau^d \times^d \phi^d(x^d) \times^d \tau^{d\dagger} = \bar{\phi}^d(x''^d, x''^d = (r^d, -t^d) = (-r, t), \tag{2.2.39b}$$

where $r^d(= -r)$ is the *isodual coordinate* on space $E^d(r^d,\, \delta^d,\, R^d)$, and t^d is the *isodual time* on $E^d(t^d,\, 1,\, R^d_t)$.

LEMMA 2.2.2 [9]: *Isodual inversions and spacetime inversions are inequivalent.*

Proof. Spacetime inversions are characterized by the change of sign $x \to -x$ *by always preserving the original metric measured with positive units*, while isodual inversions imply the map $x \to x^d = -x$ *but now measured with an isodual metric* $\eta^d = -\eta$ *with negative units* $I^d = -I$, thus being inequivalent. **q.e.d.**

Despite their simplicity, isodual inversions (or isodual discrete symmetries) are not trivial (Figure 2.6). In fact, all measurements are done in our spacetime, thus implying the need to consider the *projection* of the isodual discrete symmetries into our spacetime which are manifestly different than the conventional forms.

In particular, they imply a sort of interchange, in the sense that the conventional *space* inversion $(r, t) \to (-r, t)$ emerges as belonging to the projection in our spacetime of the isodual *time* inversion, and vice-versa.

Note that the above "interchange" of parity and time reversal of isodual particles projected in our spacetime could be used for experimental verifications, but this aspect is left to interested readers.

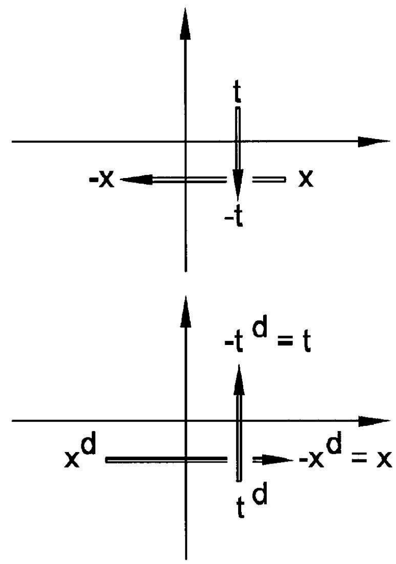

Figure 2.6. A schematic view of the additional peculiar property that the projection in our spacetime of the isodual space inversion appears as a time inversion and vice versa. In fact, a point in the isodual spacetime is given by $(x^d, t^d) = (-x, -t)$. The projection in our spacetime of the isodual space inversion $(x^d, t^d) \rightarrow (-x^d, t^d)$ is then given by $(x, -t)$, thus appearing as a time (rather than a space) inversion. Similarly, the projection in our spacetime of the isodual time inversion $(x^d, t^d) \rightarrow (x^d, -t^d)$ appears as $(-x, t)$, that is, as a space (rather than time) inversion. Despite its simplicity, the above occurrence has rather deep implications for all discrete symmetries in particle physics indicated later on.

In closing this subsection, we point out that the notion of isodual parity has intriguing connections with the parity of antiparticles in the $(j, 0) + (0, j)$ representation space more recently studied by Ahluwalia, Johnson and Goldman [10]. In fact, the latter parity results in being opposite that of particles which is fully in line with isodual space inversion (isodual parity).

2.2.10 Dunning-Davies Isodual Thermodynamics of Antimatter

An important contribution to the isodual theory has been made by J. Dunning-Davies [11] who introduced in 1999 the first, and only known consistent thermodynamics for antimatter, here called *Dunning-Davies antimatter thermodynamics* with intriguing results and implications.

As conventionally done in the field, let us represent heat with Q, internal energy with U, work with W, entropy with S, and absolute temperature with T. *Dunning-Davies isodual thermodynamics of antimatter* is evidently defined via the isodual quantities

$$Q^d = -Q,\ U^d = -U,\ W^d = -W,\ S^d = -S,\ T^d = -T \tag{2.2.40}$$

on isodual spaces over the isodual field of real numbers $R^d = R^d(n^d, +^d, \times^d)$ with isodual unit $I^d = -1$.

Recall from Section 2.1.3 that *differentials are isoselfdual* (that is, invariant under isoduality). Dunning-Davies then has the following:

THEOREM 2.2.1 [21]: *Thermodynamical laws are isoselfdual.*

Proof. For the *First Law of thermodynamics* we have

$$dQ = dU - dW \ \equiv \ d^dQ^d = d^dU^d - d^dW^d. \tag{2.2.41}$$

Similarly, for the *Second Law of thermodynamics* we have

$$dQ = T \times dS \ \equiv \ d^dQ^d = T^d \times^d S^d, \tag{2.2.42}$$

and the same occurs for the remaining laws. **q.e.d.**

Despite their simplicity, Dunning-Davies results [21] have rather deep implications. First, the identity of thermodynamical laws, by no means, implies the identity of the thermodynamics of matter and antimatter. In fact, *in Dunning-Davies isodual thermodynamics the entropy must always decrease in time*, since the isodual entropy is always negative and is defined in a space with evolution backward in time with respect to us. However, these features are fully equivalent to the conventional increase of the entropy tacitly referred to positive units.

Also, Dunning-Davies results indicate that *antimatter galaxies and quasars cannot be distinguished from matter galaxies and quasars via the use of thermodynamics,* evidently because their laws coincide, in a way much similar to the identity of the trajectories of particles and antiparticles of Lemma 2.2.1.

This result indicates that the only possibility known at this writing to determine whether far-away galaxies and quasars are made up of matter or of antimatter is that via the predicted gravitational repulsion of the light emitted by antimatter called *isodual light* (see next section and Chapter 5).

2.2.11 Isodual General Relativity

For completeness, we now introduce the *isodual general relativity* for the classical gravitational representation of antimatter. A primary motivation for its study is the incompatibility with antimatter of the positive-definite character of the energy-momentum tensor of the conventional general relativity studied in Chapter 1.

The resolution of this incompatibility evidently requires a structural revision of general relativity [33] for a consistent treatment of antimatter. The *only* solution known to the author is that offered by isoduality.[4]

It should be stressed that this study is here presented merely for completeness, since the achievement of a consistent treatment of negative-energies, by no means, resolves the serious inconsistencies of gravitation on a Riemannian space caused by curvature, as studied in Section 1.2, thus requiring new geometric vistas beyond those permitted by the Riemannian geometry (see Chapters 3 and 4).

As studied in Section 2.1.7, the *isodual Riemannian geometry* is defined on the isodual field $R^d(n^d, +^d, \times^d)$ for which *the norm is negative-definite*, Eq. (2.1.18). As a result, *all quantities that are positive in Riemannian geometry become negative under isoduality, thus including the energy-momentum tensor.*

In fact, the energy-momentum tensor of isodual electromagnetic waves (2.2.37) is negative-definite [8,9]

$$T^d_{\mu\nu} \ = \ (4 \times \pi)^{-1d} \times^d (F^d_{\mu\alpha} \times^d F^d_{\alpha_\nu} + (1/4)^{-1d} \times^d g^d_{\mu\nu} \times^d F^d_{\alpha\beta} \times^d F^{d\alpha\beta}). \quad (2.2.43)$$

The *Einstein-Hilbert isodual equations for antimatter in the exterior conditions in vacuum* are then given by [6,9]

$$G^d_{\mu\nu} \ = \ R^d_{\mu\nu} - \frac{1}{2}^d \times^d g^d_{\mu\nu} \times^d R^d \ = \ k^d \times^d T^d_{\mu\nu}. \quad (2.2.44)$$

The rest of the theory is then given by the use of the isodual Riemannian geometry of Section 2.1.7.

The explicit study of this gravitational theory of antimatter is left to the interested reader due to the indicated inconsistencies of gravitational theories on a Riemannian space for the *conventional case of matter* (Section 1.2). These inconsistencies multiply when treating antimatter, as we shall see.

2.3 OPERATOR ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.3.1 Basic Assumptions

In this section we study the operator image of the classical isodual theory of the preceding section; we prove that the operator image of isoduality is equivalent to charge conjugation; and we show that isodual mathematics resolves all known objections against negative energies.

A main result of this section is the identification of a simple, structurally new formulation of quantum mechanics known as *isodual quantum mechanics* or, more

[4]The author would be grateful to colleagues who care to bring to his attention other "classical" gravitational theories of antimatter compatible with the negative-energy solutions needed by antimatter.

properly, as the *isodual branch of hadronic mechanics* first proposed by Santilli in Refs. [5]. Another result of this section is the fact that all numerical predictions of operator isoduality coincide with those obtained via charge conjugation on a Hilbert space, thus providing the experimental verification of the isodual theory of antimatter at the operator level.

Despite that, the isodual image of quantum mechanics is not trivial because of a number of far reaching predictions we shall study in this section and in the next chapters, such as: the prediction that antimatter emits a new light distinct from that of matter; antiparticles in the gravitational field of matter experience antigravity; bound states of particles and their antiparticles can move backward in time without violating the principle of causality; and other predictions.

Other important results of this section are a new interpretation of the conventional Dirac equation that escaped detection for about one century, as well as the indication that the isodual theory of antimatter originated from the Dirac equation itself, not so much from the negative-energy solutions, but more properly from their two-dimensional unit that is indeed negative-definite, $I_{2\times 2} = Diag.(-1,-1)$.

As we shall see, Dirac's "hole theory", with the consequential restriction of the study of antimatter to the sole second quantization and resulting scientific imbalance indicated in Section 1.1, were due to Dirac's lack of knowledge of a *mathematics based on negative units.*

Intriguingly, had Dirac identified the quantity $I_{2\times 2} = Diag.(-1,-1)$ as the unit of the *mathematics* treating the negative energy solutions of his equation, the physics of the 20-th century would have followed a different path because, despite its simplicity, the unit is indeed the most fundamental notion of all mathematical and physical theories.

2.3.2 Isodual Quantization

The isodual Hamiltonian mechanics (and its underlying *isodual symplectic geometry* [5a] not treated in this chapter for brevity) permit the identification of a new quantization channel, known as the *naive isodual quantization* [6] that can be readily formulated via the use of the Hamilton-Jacobi-Santilli isodual equations (2.2.21) as follows

$$A^{\circ d} \rightarrow -i^d \times^d \hbar^d \times^d Ln^d \psi^d(t^d, r^d), \tag{2.3.1a}$$

$$\partial^d A^{\circ d}/^d \partial^d t^d + H^d \ = \ 0 \rightarrow i^d \times^d \partial^d \psi^d /^d \partial^d t^d \ = \ = \ H^d \times^d \psi^d \ = \ E^d \times^d \psi^d, \tag{2.3.1b}$$

$$\partial^d A^{\circ d}/^d \partial^d x^{dk} - \hat{p}_k \ = \ 0 \rightarrow p^d_k \times^d \psi^d \ = \ -i^d \times^d \partial^d_k \psi^d, \tag{2.3.1c}$$

$$\partial^d A^{\circ d}/^d \partial^d p^d_k = 0 \rightarrow \partial^d \psi^d /^d \partial^d p^d_k = 0. \tag{2.3.1d}$$

Recall that the fundamental unit of quantum mechanics is Planck's constant $\hbar = +1$. It then follows that the fundamental unit of the isodual operator theory is the new quantity

$$\hbar^d = -1. \tag{2.3.2}$$

It is evident that the above quantization channel identifies the new mechanics known as *isodual quantum mechanics,* or the *isodual branch of hadronic mechanics.*

2.3.3 Isodual Hilbert Spaces

Isodual quantum mechanics can be constructed via the anti-unitary transform

$$U \times U^\dagger = \hbar^d = I^d = -1, \tag{2.3.3}$$

applied, for consistency, to the *totality* of the mathematical and physical formulations of quantum mechanics. We recover in this way the isodual real and complex numbers

$$n \rightarrow n^d = U \times n \times U^\dagger = n \times (U \times U^\dagger) = n \times I^d, \tag{2.3.4}$$

isodual operators

$$A \rightarrow U \times A \times U^\dagger = A^d, \tag{2.3.5}$$

the isodual product among generic quantities A, B (numbers, operators, etc.)

$$A \times B \rightarrow U \times (A \times B) \times U^\dagger =$$

$$= (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) = A^d \times^d B^d, \tag{2.3.6}$$

and similar properties.

Evidently, isodual quantum mechanics is formulated in the *isodual Hilbert space* $\mathcal{H}^d$ with *isodual states* [6]

$$|\psi >^d = -|\psi >^\dagger = - < \psi|, \tag{2.3.7}$$

where $< \psi|$ is a conventional dual state on $\mathcal{H}$, and *isodual inner product*

$$< \psi|^d \times (-1) \times |\psi >^d \times I^d, \tag{2.3.8}$$

with *isodual expectation values* of an operator A^d

$$< A^d >^d = (< \psi|^d \times^d A^d \times^d |\psi >^d /^d < \psi|^d \times^d |\psi >^d), \tag{2.3.9}$$

and *isodual normalization*

$$< \psi|^d \times^d |\psi >^d = -1 \tag{2.3.10}$$

defined on the *isodual complex field* C^d with unit -1 (Section 2.1.1).

The isodual expectation values can also be reached via anti-unitary transform (2.3.3),

$$< \psi| \times A \times |\psi > \rightarrow U \times (< \psi| \times A \times |\psi >) \times U^\dagger =$$
$$= (< \psi| \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times$$
$$\times (U \times |\psi >) \times (U \times U^\dagger) = < \psi|^d \times^d A^d \times^d |\psi >^d \times I^d. \tag{2.3.11}$$

The proof of the following property is trivial.

LEMMA 2.3.1 [5b]: *The isodual image of an operator A that is Hermitian on $\mathcal{H}$ over C is also Hermitian on $\mathcal{H}^d$ over C^d (isodual Hermiticity).*

It then follows that *all quantities that are observables for particles are equally observables for antiparticles represented via isoduality.*

LEMMA 2.3.2 [5b]: *Let H be a Hermitian operator on a Hilbert space $\mathcal{H}$ over C with positive-definite eigenvalues E,*

$$H \times |\psi >= E \times |\psi >, H = H^\dagger, E => 0. \tag{2.3.12}$$

Then, the eigenvalues of the isodual operator H^d on the isodual Hilbert space $\mathcal{H}^d$ over C^d are negative-definite,

$$H^d \times^d |\psi >^d = E^d \times^d |\psi >^d, H^d = H^{d\dagger^d}, E^d < 0. \tag{2.3.13}$$

This important property establishes an evident compatibility between the classical and operator formulations of isoduality.

We also mention the *isodual unitary laws*

$$U^d \times^d U^{d^\dagger} = U^{d^\dagger} \times^d U^d = I^d, \tag{2.3.14}$$

the *isodual trace*

$$Tr^d A^d = (Tr A^d) \times I^d \in C^d, \tag{2.3.15a}$$
$$Tr^d (A^d \times^d B^d) = Tr^d A^d \times^d Tr^d B^d, \tag{2.3.15b}$$

the *isodual determinant*

$$Det^d A^d = (Det A^d) \times I^d \in C^d, \tag{2.3.16a}$$
$$Det^d (A^d \times^d B^d) = Det^d \times^d Det^d B^d, \tag{2.3.16b}$$

the *isodual logarithm* of a real number n

$$Log^d n^d = -(Log\, n^d) \times I^d, \tag{2.3.17}$$

and other isodual operations.

The interested reader can then work out the remaining properties of the isodual theory of linear operators on a Hilbert space.

2.3.4 Isoselfduality of Minkowski's Line Elements and Hilbert's Inner Products

A most fundamental new property of the isodual theory, with implications as vast as the formulation of a basically new cosmology, is expressed by the following lemma whose proof is a trivial application of transform (2.3.3).

LEMMA 2.3.3 [23]: *Minkowski's line elements and Hilbert's inner products are invariant under isoduality (or they are isoselfdual according to Definition 2.1.2),*

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \equiv$$

$$\equiv \ (x^{d\mu} \times^d \eta^d_{\mu\nu} \times^d x^{d\nu}) \times I^d = x^{d2^d}, \tag{2.3.18a}$$

$$< \psi| \times |\psi > \times I \equiv \ < \psi|^d \times^d |\psi >^d \times I^d. \tag{2.3.18b}$$

As a result, *all relativistic and quantum mechanical laws holding for matter also hold for antimatter under isoduality.* The equivalence of charge conjugation and isoduality then follows, as we shall see shortly.

Lemma 2.3.3 illustrates the reason why isodual special relativity and isodual Hilbert spaces have escaped detection for about one century. Note, however, that invariances (2.3.18) require the prior discovery of *new numbers,* those with negative unit.

2.3.5 Isodual Schrödinger and Heisenberg's Equations

The fundamental dynamical equations of isodual quantum mechanics are the isodual images of conventional dynamical equations. They are today known as the *Schrödinger-Santilli isodual equations* [4] (where we assume hereon $\hbar^d = -1$, thus having $\times^d \hbar^d = 1$)

$$i^d \times^d \partial|\psi >^d /^d \partial^d t^d = H^d \times^d |\psi >^d, \tag{2.3.19a}$$

$$p^d_k \times^d |\psi >^d = -i^d \times^d \partial^d|\psi >^d /^d \partial^d r^d, \tag{2.3.19b}$$

and the *Heisenberg-Santilli isodual equations*

$$i^d \times^d d^d A^d /^d d^d t^d = A^d \times^d H^d - H^d \times^d A^d = [A^d, H^d]^d, \tag{2.3.20a}$$

$$[r^d_i, p^d_j]^d = i^d \times^d \delta^{di}_j, [r^d, r^{dj}]^d = [p^d_i, p^d_j]^d = 0. \tag{2.3.20b}$$

Note that, when written explicitly, Eq. (2.3.19a) is based on an associative modular action *to the left,*

$$- < \psi| \times^d H^d = (\partial^d < \psi|\partial^d t^d) \times^d i^d. \tag{2.3.21}$$

It is an instructive exercise for readers interested in learning the new mechanics to prove the equivalence of the isodual Schrödinger and Heisenberg equations via the anti-unitary transform (2.3.3).

2.3.6 Isoselfdual Re-Interpretation of Dirac's Equation

Isoduality has permitted a novel interpretation of the conventional *Dirac equation* (we shall here used the notation of Ref. [12]) in which the negative-energy states are reinterpreted as belonging to the isodual images of positive energy states, resulting in the first known *consistent representation of antiparticles in first quantization.*

This result should be expected since the isodual theory of antimatter applies at the Newtonian level, let alone that of first quantization. Needless to say, the treatment via isodual first quantization does not exclude that via isodual second quantization. The point is that the treatment of antiparticles is no longer restricted to second quantization, as a condition to resolve the scientific imbalance between matter and antimatter indicated earlier.

Consider the conventional Dirac equation [2]

$$[\gamma^{\mu} \times (p_{\mu} - e \times A_{\mu}/c) + i \times m] \times \Psi(x) = 0, \tag{2.3.22}$$

with realization of Dirac's celebrated gamma matrices

$$\gamma_k = \begin{pmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad \gamma^4 = i \times \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{2.3.23a}$$

$$\{\gamma_{\mu}, \tilde{\gamma}_{\nu}\} \ = \ 2\times\eta_{\mu\nu}, \quad \Psi = i \times \begin{pmatrix} \Phi \\ -\Phi^{\dagger} \end{pmatrix}. \tag{2.3.23b}$$

At the level of first quantization here considered, the above equation is rather universally interpreted as representing an electron under an external electromagnetic field.

The above equations are generally defined in the 6-dimensional space given by the Kronecker product of the conventional Minkowski spacetime and an internal spin space

$$M_{Tot} = M(x, \eta, R) \times S_{spin}, \tag{2.3.24}$$

with total unit

$$I_{Tot} = I_{orb} \times I_{spin} = Diag.(1,\ 1,\ 1,\ 1) \times Diag.(1,\ 1), \tag{2.3.25}$$

and total symmetry

$$P(3.1) = SL(2.C) \times T(3.1). \tag{2.3.26}$$

The proof of the following property is recommended to interested readers.

THEOREM 2.3.1 [5b]: *Pauli's sigma matrices and Dirac's gamma matrices are isoselfdual,*

$$\sigma_k \equiv \sigma^d_k, \tag{2.3.27a}$$

$$\gamma_{\mu} \equiv \gamma^d_{\mu}. \tag{2.3.27b).}$$

The above properties imply an important re-interpretation of Eq. (2.3.22), first identified in Ref. [9] and today known as the *Dirac-Santilli isoselfdual equation,* that can be written

$$[\tilde{\gamma}^{\mu} \times (p_{\mu} - e \times A_{\mu}/c) + i \times m] \times \tilde{\Psi}(x) = 0, \tag{2.3.28}$$

with re-interpretation of the gamma matrices

$$\tilde{\gamma}_k = \begin{pmatrix} 0 & \sigma_k^d \\ \sigma_k & 0 \end{pmatrix}, \quad \tilde{\gamma}^4 = i \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & I_{2\times 2}^d \end{pmatrix}, \tag{2.3.29a}$$

$$\{\tilde{\gamma}_{\mu}, \tilde{\gamma}_{\nu}\} = 2^d \times^d \eta_{\mu\nu}^d, \quad \tilde{\Psi} = -\tilde{\gamma}_4 \times \Psi = i \times \begin{pmatrix} \Phi \\ \Phi^d \end{pmatrix}, \tag{2.3.29b}$$

By recalling that isodual spaces coexist with, but are different from conventional spaces, we have the following:

THEOREM 2.3.2 [9]: *The Dirac-Santilli isoselfdual equation is defined on the 12-dimensional isoselfdual representation space*

$$M_{Tot} = \{M(x, \eta, R) \times S_{spin}\} \times \{M^d(x^d, \eta^d, R^d) \times^d S^d_{spin}\}, \tag{2.3.30}$$

with isoselfdual total 12-dimensional unit

$$I_{Tot} = \{I_{orb} \times I_{spin}\} \times \{I^d_{orb} \times^d I^d_{spin}\}, \tag{2.3.31}$$

and its symmetry is given by the isoselfdual product of the Poincaré symmetry and its isodual

$$S_{Tot} = \mathcal{P}(3.1) \times \mathcal{P}^d(3.1) =$$
$$= \{SL(2.C) \times T(3.1)\} \times \{SL^d(2.C^d) \times^d T^d(3.1)\}. \tag{2.3.32}$$

A direct consequence of the isoselfdual structure can be expressed as follows.

COROLLARY 2.3.2a [9]: *The Dirac-Santilli isoselfdual equation provides a joint representation of an electron and its antiparticle (the positron) in first quantization,*

$$Dirac\ \ Equation = Electron \times Positron. \tag{2.3.33}$$

In fact, the two-dimensional component of the wave function with positive-energy solution represents the electron and that with negative-energy solutions represent the positron without any need for second quantization, due to the physical behavior of negative energies in isodual treatment established earlier.

Note the complete democracy and equivalence in treatment of the electron and the positron in equation (2.3.28), in the sense that the equation can be equally used to represent an electron or its antiparticle. By comparison, according to the original Dirac interpretation, the equation could only be used to represent the electron [12], since the representation of the positron required the "hole theory".

It has been popularly believed throughout the 20-th century that Dirac's gamma matrices provide a "four-dimensional representation of the SU(2)-spin symmetry". This belief is disproved by the isodual theory, as expressed by the following

THEOREM 2.3.3 [5b]: *Dirac's gamma matrices characterize the direct product of an irreducible two-dimensional (regular) representation of the SU(2)-spin symmetry and its isodual,*

$$Dirac's\ \ Spin\ \ Symmetry : SU(2) \times SU^d(2). \tag{2.3.34}$$

In fact, the gamma matrices are characterized by the conventional, 2-dimensional Pauli matrices σ_k and related identity $I_{2\times2}$ as well as other matrices that have resulted in being the exact isodual images σ_k^d with isodual unit $I^d_{2\times2}$.

It should be recalled that the isodual theory was born precisely out of these issues and, more particularly, from the incompatibility between the popular interpretation of gamma matrices as providing a "four-dimensional" representation of the SU(2)-spin symmetry and the *lack of existence of such a representation in Lie's theory.*

The sole possibility known to the author for the reconciliation of Lie's theory for the SU(2)-spin symmetry and Dirac's gamma matrices was to assume that $-I_{2\times2}$ is the unit of a dual-type representation. The entire theory studied in this chapter then followed.

It should also be noted that, as conventionally written, Dirac's equation *is not* isoselfdual because it is not sufficiently symmetric in the two-dimensional states and their isoduals.

In summary, Dirac's was forced to formulate the "hole theory" for antiparticles because he referred the *negative* energy states to the conventional *positive* unit, while their reformulation with respect to *negative* units yields fully physical results.

It is easy to see that the same isodual reinterpretation applies for Majorana's spinorial representations [13] (see also [14,15]) as well as Ahluwalia's broader spinorial representations (1/2, 0) + (0, 1/2) [16] (see also the subsequent paper [17]), that are reinterpreted in the isoselfdual form (1, 2, 0) + (1, 2, 0)^d, thus extending their physical applicability to first quantization.

In the latter reinterpretation the representation (1/2, 0) is evidently done conventional spaces over conventional fields with unit +1, while the isodual representation (1/2, 0)^d is done on the corresponding isodual spaces defined on isodual fields with unit −1. As a result, all quantities of the representation (1/2, 0) change sign under isoduality.

It should be finally indicated that Ahluwalia treatment of Majorana spinors has a deep connection with isoduality because the underlying Class II spinors have a *negative norm* [16] precisely as it is the case for isoduality. As a result, the isodual reinterpretation under consideration here is quite natural and actually warranted for mathematical consistency, e.g., to have the topology characterized by a negative norm be compatible with the underlying fields.

2.3.7 Equivalence of Isoduality and charge conjugation

We come now to another fundamental point of this chapter, the proof that isoduality is equivalent to charge conjugation. This property is crucial for the experimental verification of isoduality at the particle level too. This equivalence was first identified by Santilli in Ref. [6] and can be easily expressed today via the following:

LEMMA 2.3.4 [6,5b,18]: *The isodual transform is equivalent to charge conjugation.*

Proof. Charge conjugation is characterized by the following transform of wavefunctions (see, e.g., Ref. [12], pages 109 and 176)

$$\Psi(x) \rightarrow C\Psi(x) = c \times \Psi^{\dagger}(x), \tag{2.3.35}$$

where

$$|c| = 1, \tag{2.3.36}$$

thus being manifestly equivalent to the isodual transform

$$\Psi(x) \rightarrow \Psi^{d}(x^{d}) = -\Psi^{\dagger}(-x^{t}), \tag{2.3.37}$$

where t denotes transpose.

A reason why the two transforms are equivalent, rather than identical, is the fact that charge conjugation maps spacetime into itself, while isoduality maps spacetime into its isodual. **q.e.d.**

Let us illustrate Lemma 2.3.4 with a few examples. As well known, the Klein-Gordon equation for a free particle

$$\partial^{\mu}\partial_{\mu}\Psi - m^{2} \times \Psi = 0 \tag{2.3.38}$$

is invariant under charge conjugation, in the sense that it is turned into the form

$$c \times [\bar{\Psi}\partial^{\mu}\partial_{\mu} - \bar{\Psi} \times m^{2}] = 0, \ \ |c| = 1, \tag{2.3.39}$$

where the upper bar denotes complex conjugation (since $\bar{\Psi}$ is a scalar), while the Lagrangian density

$$L = -(\hbar \times \hbar/2 \times m) \times \{\partial^{\mu}\bar{\Psi} - i \times e \times A^{\mu}/\hbar \times c) \times \bar{\Psi}]\times$$

$$\times[\partial\Psi + (i \times e \times A_\mu/\hbar \times c) \times \Psi] + m \times m \times \bar{\Psi} \times \Psi \tag{2.3.40}$$

is left invariant, and the four-current

$$J_\mu = -(i \times \hbar/2 \times m) \times [\bar{\psi} \times \partial_\mu\Psi - (\partial_\mu\bar{\Psi}) \times \Psi] \tag{2.3.41}$$

changes sign

$$J_\mu \to CJ_\mu = -J_\mu. \tag{2.3.42}$$

By recalling the selfduality of ordinary derivatives, Eq. (2.1.30), under isoduality the Klein-Gordon Equation becomes

$$[\partial^\mu\partial_\mu\Psi - m^2 \times \Psi]^d = \Psi^d\partial^{d\mu}\partial^d_\mu - \Psi^d \times^d m^d \times^d m^d =$$

$$= -[\bar{\Psi}\partial^\mu\partial_\mu - \bar{\Psi} \times m^2] = 0, \tag{2.3.43}$$

thus being equivalent to Eq. (2.3.39), while the Lagrangian changes sign and the four-current changes sign too,

$$J^d_\mu = -(i \times \hbar/2 \times m) \times [\bar{\Psi} \times \partial_\mu\Psi - (\partial_\mu\bar{\Psi}) \times \Psi]^d =$$

$$= (i \times \hbar/2 \times m) \times [\bar{\Psi} \times \partial_\mu\Psi - (\partial_\mu\bar{\Psi}) \times \Psi], \tag{2.3.44}$$

(where we have used the isoselfduality of the imaginary number i).

The above results confirm Lemma 2.3.4 because of the equivalent behavior of the equations of motion and the four-current, while the change of sign of the Lagrangian does not affect the numerical results.

As it is also well known, the Klein-Gordon equation for a particle under an external electromagnetic field [12]

$$[(\partial_\mu + i \times e \times A_\mu/\hbar \times c)\times$$

$$\times(\partial^\mu + i \times e \times A^\mu/\hbar \times c) - m^2] \times \Psi = 0, \tag{2.3.45}$$

is equally invariant under charge conjugation in which *either* e *or* A_μ change sign, in view of the known invariance

$$C(i \times e \times A_\mu/\hbar \times c) = i \times e \times A_\mu/\hbar \times c, \tag{2.3.46}$$

while the four-current also changes sign. By noting that the preceding invariance persists under isoduality,

$$(i \times e \times A_\mu/\hbar \times c)^d = i \times e \times A_\mu/\hbar \times c, \tag{2.3.47}$$

Eq. (2.3.45) remains invariant under isoduality, while the Lagrangian density changes sign and the four-current, again, changes sign.

Similarly, consider Dirac equation (see also Ref. [12], pp. 176-177)

$$[\gamma^\mu \times (\partial_\mu\Psi - (i \times e \times A_\mu/\hbar \times c) \times \Psi + m \times \Psi = 0, \tag{2.3.48}$$

with Lagrangian density

$$L = (\hbar \times c/2) \times \{\tilde{\Psi} \times \gamma^\mu \times [\partial_\mu \Psi + (i \times e \times A_\mu/\hbar \times c) \times \Psi] -$$

$$-(\partial^\mu \tilde{\Psi} - (i \times e \times A^\mu/\hbar \times c) \times \tilde{\Psi}] \times \gamma_\mu - m \times \tilde{\Psi} \times \Psi, \qquad (2.3.49a)$$

$$\tilde{\Psi} = \Psi^\dagger \times \gamma_4, \qquad (2.3.49b)$$

and four-current

$$J_\mu = i \times c \times \tilde{\Psi} \times \gamma_\mu \times \Psi = i \times c \times \Psi^\dagger \times \gamma_4 \times \gamma_\mu \times \Psi. \qquad (2.3.50)$$

The charge conjugation for Dirac's equations is given by the transform [12]

$$\Psi \to C\Psi = c \times S_C^{-1} \times \tilde{\Psi}^t \qquad (2.3.51)$$

where S_C is a unitary matrix such that

$$\gamma_\mu \to -\gamma_\mu^t = S_C \times \gamma_\mu \times S_C^{-1}, \qquad (2.3.52)$$

and there is the change of sign *either* of e *or* of A_μ, under which the equation is transformed into the form

$$[\partial_\mu \tilde{\Psi} - (i \times e \times A_\mu/\hbar \times c) \times \tilde{\Psi}] \times \gamma^\mu - m \times \tilde{\Psi} = 0, \qquad (2.3.53)$$

while the Lagrangian density changes sign and the four-current remains the same,

$$L \to CL = -L, \quad J_\mu \to CJ_\mu = J_\mu. \qquad (2.3.54)$$

It is easy to see that isoduality provides equivalent results. In fact, we have for Eq. (2.3.48)

$$\{[\gamma^\mu \times (\partial_\mu \Psi - i \times e \times A_\mu/\hbar \times c) \times \Psi + m \times \Psi\}^d =$$

$$= [\partial_\mu \Psi^\dagger - (i \times e \times A_\mu/\hbar \times c) \times \Psi^\dagger] \times \gamma^\mu - m \times \Psi^\dagger = 0, \qquad (2.3.55)$$

that, when multiplied by γ_4 reproduces Eq. (2.3.53) identically. Similarly, by recalling that Dirac' s gamma matrices are isoselfdual (Theorem 2.3.1), and by noting that

$$\tilde{\Psi}^d = (\Psi^\dagger \times \gamma_4)^d = \gamma_4 \times \Psi, \qquad (2.3.56)$$

we have

$$L^d = L, \qquad (2.3.57)$$

while for the four-current we have

$$J_\mu^d = -i \times c \times \Psi^\dagger \times \gamma_\mu \times \gamma_4 \times \psi. \qquad (2.3.58)$$

But the γ_μ and γ_4 anticommutate. As a consequence, the four-current does not change sign under isoduality as in the conventional case.

Note that the lack of change of sign under isoduality of Dirac's four-current J_μ confirms reinterpretation (2.3.28) since, for the latter equation, the total charge is null.

The equivalence between isoduality and charge conjugation of other equations, such as those by Weyl, Majorana, etc., follows the same lines.

2.3.8 Experimental Verification of the Isodual Theory of Antimatter in Particle Physics

In Section 2.2.3. we have established the experimental verification of the isodual theory of antimatter in classical physics that, in particle physics, requires no detailed elaboration since it is established by the equivalence of charge conjugation and isoduality (Lemma 2.3.4), and we can write:

LEMMA 2.3.5 [6,5b,18], [7]: *All experimental data currently available for antiparticles represented via charge conjugation are equally verified by the isodual theory of antimatter.*

2.3.9 Elementary Particles and their Isoduals

We assume the reader is familiar with the conventional definition of *elementary particles* as irreducible unitary representations of the spinorial covering of the Galilei symmetry $G(3.1)$ for nonrelativistic treatments and those of the Poincaré symmetry $P(3.1)$ for relativistic treatments. We therefore introduce the following:

DEFINITION 2.3.1: Elementary isodual particles (antiparticles) are given by irreducible unitary representations of the spinorial covering of the Galilei-Santilli's isodual symmetry $G^d(3.1)$ *for nonrelativistic treatments and those of the Poincaré-Santilli isodual symmetry* $P^d(3.1)$ *for relativistic treatments.*

A few comments are now in order. Firstly, one should be aware that "isodual particles" and "antiparticles" do not represent the same notion, evidently because of the negative mass, energy and time of the former compared to positive mass, energy and time of the latter. In the rest of this chapter, unless otherwise stated, the word "antiparticle" will be referred to as the "isodual particle."

For instance the word "positron" e^+ is more appropriately intended to represent the "isodual electron" with symbol e^{-d}. Similarly the, "antiproton" p^- is intended to represent the "isodual proton" p^{+d}.

Secondly, the reader should note the insistence on the *elementary* character of the antiparticles here admitted. The reason is that the antigravity studied in Chapter 4 is specifically formulated for "elementary" isodual particles, such as the isodual electron, due to a number of unsettled aspects pertaining to composite particles.

Consider, as an illustration, the case of mesons. If the π° is a bound state of a particle and its isodual, the state is isoselfdual and, as such, it *cannot* experience antigravity, as illustrated in the next section. A number of ambiguities then follow for the study of the gravity of the charged mesons $\pi^\pm$, such as the problem of ascertaining which of the two mesons is a particle and which is its isodual or, whether the selected antiparticle is indeed the isodual image of the particle as a necessary condition for meaningful study of their gravity.

Note that essentially the same ambiguities prohibit the use of muons for a serious theoretical and experimental studies of the gravity of antiparticles, again, because of unsettled problems pertaining to the structure of the muons themselves. Since the muons are naturally unstable, they cannot be credibly believed to be elementary. Therefore, serious theoretical and experimental studies on the gravity of muons require the prior identification of their constituents with physical particles.

Finally, the reader should be aware that *Definition 2.3.1 excludes the use of quark conjectures for the gravitational studies of this monograph.* This is due to the well-known basic inconsistency of quark conjecture of not admitting any gravitation at all (see, e.g., the Appendix of Ref. [18]). In fact, gravity can only be defined in our spacetime while quarks can only be defined in their mathematical unitary internal space with no known connection with our spacetime due to the O'Rafearthaigh theorem.[5]

Also, the only "masses" that can be credibly claimed as possessing inertia are the eigenvalues of the second-order Casimir invariant of the Poincaré symmetry $p_\mu \times p^\mu = m^2$. Quarks cannot be characterized via such a fundamental symmetry, as well known. It then follows that "quark masses" are mere mathematical parameters defined in the mathematical internal complex-unitary space that cannot possibly be used as serious basis for gravitational tests.

2.3.10 Photons and their Isoduals

As it is well known, photons have no charge and, therefore, they are invariant under charge conjugation, as transparent from the simple plane-wave representation

$$\Psi(t, r) = N \times e^{i \times (k \times r - E \times t)}, \quad N \in R, \tag{2.3.59}$$

with familiar relativistic form

$$\Psi(x) = N \times e^{i \times k_\mu \times x^\mu}, \tag{2.3.60}$$

and familiar expression for the energy

$$E = h \times \nu. \tag{2.3.61}$$

As a result, matter and antimatter have been believed throughout the 20-th century to emit the same light. In turn, this belief has left fundamentally unsettled basic questions in astrophysics and cosmology, such as the lack of quantitative

[5] The possible connection between internal and spacetime symmetries offered by supersymmetric theories cannot be credibly used for gravitational tests due to their highly unsettled character and the prediction of a zoo of new particles none of which has been experimentally detected to the author's best knowledge.

studies as to whether far-away galaxies and quasars are made up of matter or of antimatter.

One of the most intriguing and far reaching implications of the isodual theory is that, while remaining evidently invariant under charge conjugation, *the photon is not invariant under isoduality,* thus admitting a conjugate particle first submitted by Santilli in Ref. [18] under the name of *isodual photon.* In particular, the isodual photon emerges as having physical characteristics that can be experimentally measured as being different from those of the photon.

Therefore, the isodual theory offers the first known possibilities of quantitative theoretical and experimental studies as to whether a far-away galaxy or quasar is made of matter or of antimatter due to detectable physical differences of their emitted light.

Note that the term "antiphoton" could be misleading because the prefix "anti" is generally assumed as referring to charge conjugation. For this reason the name of "isodual photon" appears to be preferable, also because it represents, more technically, the intended state.

In fact, the photon is mapped by isoduality into a new particle possessing all negative-definite physical characteristics, with the following simple isodual plane-wave representation

$$\Psi^d(t^d, r^d) = N^d \times^d e_d^{i^d \times^d (k^d \times^d r^d - E^d \times^d t^d)}, \;\; N^d \in R^d, \tag{2.3.62}$$

with relativistic expression on isodual Minkowski space

$$\Psi^d(x^d) = N^d \times^d e_d^{i^d \times^d k^d_\mu \times^d x^{d\mu}}, \tag{2.3.63}$$

and isodual expression for the energy

$$E^d \; = \; h^d \times^d \nu^d, \tag{2.3.64}$$

where e_d is the isodual exponentiation (2.1.26b).

Note that, since i is isoselfdual, Eq. (2.1.20), *the exponent of the plane-wave representation is invariant under both charge conjugation and isoduality,* as illustrated by the following expression

$$i^d \times^d (k^d \times^d r^d - E^d \times^d t^d) \; \equiv \; i \times (k \times r - E \times t), \tag{2.3.65}$$

or its relativistic counterpart

$$i^d \times^d k^d_\mu \times^d x^{d\mu} \; \equiv \; i \times k_\mu \times x^\mu, \tag{2.3.66}$$

thus confirming the lack of contradiction between charge conjugation and isoduality.

Moreover, both the photon and the isodual photon travel in vacuum with the same (absolute) speed $|c|$, for which we have the additional identity

$$k_\mu^d \times^d k^{d\mu} \equiv k_\mu \times k^\mu = 0. \tag{2.3.67}$$

Despite the above identities, energy and time are positive-definite for the photon, while they are negative-definite for the isodual photon. As we shall see, the latter property implies that photons are attracted by the gravitational field of matter while isodual photons are repelled, thus providing a physically detectable difference.

Additional differences between light emitted by matter and that emitted by antimatter, such as those pertaining to parity and other discrete symmetries, require additional study.

All in all, the isodual theory of antimatter permits the first possibilities known to the author for future experimental measurements as to whether far-away galaxies and quasars are made up of matter or of antimatter.

2.3.11 Electrons and their Isoduals

The next truly elementary particles and antiparticles are the electron e^- and its antiparticle, the positron e^+ or the isodual electron e^{-d}. The differences between the "positron" and the "isodual electron" should be kept in mind. In fact, the former has positive rest energy and moves forward in time, while the latter has negative rest energy and moves backward in time.

Also, the electron is known to experience gravitational attraction in the field of matter, as experimentally established. As conventionally defined, the positron too is predicted to experience gravitational attraction in the field of matter (because its energy is positive).

However, as we shall see in Chapter 4, the isodual electron is predicted to experience antigravity when immersed in the field of matter, and this illustrates again the rather profound physical differences between the "positron" and the "isodual electron".

Note that, in view of their truly elementary character, isodual electrons are the ideal candidates for the measurement of the gravitational field of antiparticles.

2.3.12 Protons and their Isoduals

The next particles demanding comments are the proton p^+, the antiproton p^- and the isodual proton p^{+d}. In this case the differences between the "antiproton" and the "isodual proton" should be kept in mind to avoid major inconsistencies with the isodual theory, such as the study of the possible antigravity for antiprotons in the field of matter which antigravity cannot exist for the isodual theory (due, again, to the positive mass of the antiproton).

Note that these particles are not elementary and, as such, they are not admitted by Definition 2.3.1. moreover, as stressed earlier [18], when represented in term of quark conjectures both the proton and the antiproton cannot admit any gravity at all, let alone antigravity. As a result, extreme scientific care should be exercised before extending to all antimatter any possible gravitational measurements for antiprotons.

2.3.13 The Hydrogen Atom and its Isodual

The understanding of this chapter requires the knowledge that studies conducted on the *antihydrogen atom* (see, e.g., the various contributions in Proceedings [19]), even though evidently interesting per se, have no connection with the *isodual hydrogen atom,* because the antihydrogen atom has positive mass, for which antigravity is prohibited, and emits conventional photons. Therefore, it is important to inspect the differences between these two formulations of the simplest possible atom of antimatter.

We assume as exactly valid the conventional quantum mechanical theory of *bound states of point-like particles at large mutual distances,*[6] as available in quantum mechanical books so numerous to discourage even a partial listing.

For the case of two particles denoted with the indices 1, 2, the total state in the Hilbert space is the familiar tensorial product of the two states

$$|\psi >= |\psi_1 > \times |\psi_2 > . \tag{2.3.68}$$

The total Hamiltonian H is the sum of the kinetic terms of each state plus the familiar interaction term $V(r)$ depending on the mutual distance r,

$$H = p_1 \times p_1/2 \times m_1 + p_2 \times p_2/2 \times m_2 + V(r). \tag{2.3.69}$$

The total angular momentum is computed via the familiar expressions for angular momenta and spins

$$J = J_1 \times I + I \times J_2, \quad S = S_1 \times I + I \times S_2, \tag{2.3.70}$$

where the I's are trivial units, with the usual rules for couplings, addition, etc. One should note that the unit for angular momenta is three-dimensional while that for spin has a generally different dimension.

A typical example of two-body bound states of particles is the *hydrogen atom* that experiences attraction in the gravitational field of matter with the well established emission of conventional photons.

[6] We are here referring to the large mutual distances as occurring in the atomic structure and exclude the short mutual distances as occurring in the structure of hadrons, nuclei and stars since a serious study of the latter is dramatically beyond the capabilities of quantum mechanics, as shown beyond scientific doubt in Chapter 3.

The study of *bound states of point-like isodual particles at large mutual distances* is an important part of isodual quantum mechanics. These bound states can be studied via an elementary isoduality of the corresponding bound states for particles, that is, via the use of the isodual Hilbert spaces $\mathcal{H}^d$ studied earlier.

The *total isodual state* is the tensorial product of the two isodual states

$$|\psi^d(r^d)>^d = |\psi_1^d(r^d)>^d \times^d |\psi_2^d(r^d)>^d = - < \psi_1(-r)| \times < \psi_2(-r)|. \quad (2.3.71)$$

The *total isodual Hamiltonian* is the sum of the isodual kinetic terms of each particle plus the isodual interaction term depending on the isodual mutual distance,

$$H^d = p_1^d \times^d p_1^d /^d 2^d \times^d m_1^d + p_2^d \times^d p_2^d /^d 2^d \times^d m_2^d + V^d(r^d). \quad (2.3.72)$$

The total *isodual angular momentum* is based on the expressions for isodual angular momenta and spin

$$J^d = J_1^d \times^d I^d + I^d \times^d J_2^d, \quad (2.3.73a)$$

$$S^d = S_1^d \times^d I^d + I^d \times^d S_2^d, \quad (2.3.73b)$$

The remaining aspects (couplings, addition theory of angular momenta, etc.) are then given by a simple isoduality of the conventional theory that is here omitted for brevity.

Note that all eigenvalues that are positive for the conventional case measured with positive units become negative under isoduality, yet measured with negative units, thus achieving full equivalence between particle and antiparticle bound states.

The simplest possible application of the above isodual theory is that for the *isodual hydrogen atom* (first worked out in Ref. [18]). The novel predictions of isoduality over that of the antihydrogen atom is that the isodual hydrogen atom is predicted to experience antigravity in the field of matter and emits isodual photons that are also repelled by the gravitational field of matter.

2.3.14 Isoselfdual Bound States

Some of the most interesting and novel bound states predicted by the isodual theory are the *isoselfdual bound states,* that is, bound states that coincide with their isodual image. The simplest case is the bound state of one elementary particle and its isodual, such as the *positronium.*

The condition of isoselfduality requires that the basic symmetry must be itself isoselfdual, e.g., for the nonrelativistic case the total symmetry must be

$$G_{Tot} = G(3.1) \times G^d(3.1), \quad (2.3.74)$$

where $\times$ is the Kronecker product (a composition of states thus being isoselfdual), with a simple relativistic extension here assumed as known from the preceding sections.

The total unit must also be isoselfdual,

$$I_{Tot} = I \times I^d, \tag{2.3.75}$$

where I represents the space, time and spin units.

The total Hilbert space and related states must also be isoselfdual,

$$\mathcal{H}_{Tot} = \mathcal{H} \times \mathcal{H}^d, \tag{2.3.76a}$$

$$|\psi >_{Tot} = |\psi > + |\psi >^d = |\psi > - < \psi|, \tag{2.3.76b}$$

and so on.

A main feature is that isoselfdual states exist in both the spacetime of particles and that of antiparticles. Therefore, the computation of the total energy must be done *either* in $\mathcal{H}$, in which case the total energy is positive, *or* in $\mathcal{H}^d$, in which case the total energy is negative.

Suppose that a system of one elementary particle and its isodual is studied in our laboratory of matter. In this case the eigenvalues for both particle and its isodual must be computed in $\mathcal{H}$, in which case we have the equation

$$\begin{aligned} i \times \partial_t |\psi > &= (p \times p/2 \times m) \times |\psi > + \\ &+ (p^d \times^d p^d /^d 2^d \times^d m^d) \times^d |\psi > + V(r) \times |\psi > = \\ &= [p \times p/2 \times m + V(r)] \times |\psi > = E \times |\psi >, \end{aligned} \tag{2.3.77}$$

under which the total energy E is evidently positive.

When the same isoselfdual state is detected in the spacetime of antimatter, it must be computed with respect to $\mathcal{H}^d$, in which case the total energy is negative, as the reader is encouraged to verify.

The total angular momentum and other physical characteristics are computed along similar lines and they also result in having positive values when computed in $\mathcal{H}$, as occurring for the conventional charge conjugation.

As we shall see shortly, the positive character of the total energy of bound states of particles and their antiparticles is crucial for the removal of the inconsistencies of theories with negative energy.

The above properties of the isoselfdual bound states have the following implications:

1) Isoselfdual bound states of elementary particles and their isoduals are predicted to be attracted in both, the gravitational field of matter and that of antimatter because their total energy is positive in our world and negative in the isodual world. This renders necessary an experimental verification of the gravitational behavior of isoselfdual bound states, independently from that of individual

antiparticles. Note that the prediction holds only for bound states of truly elementary particles and their isoduals, such as the positronium. No theoretical prediction for the muonium and the pionium is today feasible because the unsettled nature of their constituents.

2) Isoselfdual bound states are predicted to have a null internal total time $t + t^d = 0$ and therefore acquires the time of the matter or antimatter in which they are immersed, although the physical time t of the observer (i.e., of the bound state equation) is not null. This is readily understood by noting that the quantity t of Eq. (2.3.77) is our own time, i.e., we merely study the behavior of the state with respect to our own time. A clear understanding illustrated previously with the "isodual cube" of Section 2.1 is that the description of a state with our own time, by no means, implies that its intrinsic time necessarily coincides with our own. Note that a similar situation occurs for the energy because the intrinsic total energy of the positronium is identically null, $E + E^d = 0$. Yet the energy measured by us is $E_{part.} - E^d_{antipart.} = 2E > 0$. A similar situation occurs for all other physical quantities.

3) Isoselfdual bound states may result in being the microscopic image of the main characteristics of the entire universe. Isoselfduality has in fact stimulated a new cosmology, the *isoselfdual cosmology* [21] studied in Chapter 5, that is patterned precisely along the structure of the positronium or of Dirac's equation in our isoselfdual re-interpretation. In this case the universe results in having null total physical characteristics, such as null total energy, null total time, etc., thus implying no discontinuity at its creation.

2.3.15 Resolution of the Inconsistencies of Negative Energies

The treatment of antiparticles with negative energies was rejected by Dirac because of incompatibility with their physical behavior. Despite several attempts made during the 20-th century, the inconsistencies either directly or indirectly connected to negative energies have remained unresolved.

The isodual theory of antimatter resolves these inconsistencies for the reason now familiar, namely, that the inconsistencies emerge when one refers negative energies to conventional numbers with positive units, while the same inconsistencies cannot be evenly formulated when negative energies are referred to isodual numbers and their negative units.

A good illustration is given by the known objection according to which the creation of a photon from the annihilation of an electron-positron pair, with the electron having a positive energy and the positron having a negative energy, would violate the principle of conservation of the energy.

In fact, such a pair could be moved upward in our gravitational field without work and then annihilated in their new upward position. The resulting photon would then have a blueshift in our gravitational field of Earth, thus having more energy than that of the original photon.

Presumed inconsistencies of the above type cannot be even formulated within the context of the isodual theory of antimatter because, as shown in the preceding section, the electron-positron state is isoselfdual, thus having a non-null *positive* energy when observed in our spacetime. Consequently, the lifting upward of the pair does indeed require work and no violation of the principle of conservation of the energy can be expected.

A considerable search has established that all other presumed inconsistencies of negative energy known to the author cannot even be formulated within the context of the isodual theory of antimatter. Nevertheless, the author would be particularly grateful to any colleague who brings to his attention inconsistencies of negative energies that are really applicable under negative units.

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Chapter 3

LIE-ISOTOPIC BRANCH OF HADRONIC MECHANICS AND ITS ISODUAL

3.1 INTRODUCTION

3.1.1 Conceptual Foundations

As recalled in Chapter 1, the systems generally considered in the 20-th century are the conventional *exterior dynamical systems,* consisting of closed-isolated and reversible systems of constituents approximated as being point-like while moving in vacuum under sole action-at-a-distance potential interactions, as typically represented by planetary and atomic systems.

More technically, we can say that *exterior dynamical systems are characterized by the exact invariance of the Galilean symmetry for the nonrelativistic case and Poincaré symmetry for relativistic treatments,* with the consequential verification of the well known ten total conservation laws.

In this chapter we study the more general *interior dynamical systems of extended particles* and, separately, of *extended antiparticles,* consisting of systems that are also closed-isolated, thus verifying the same ten total conservation laws of the exterior systems, yet admit additional internal force of nonlocal-integral and nonpotential type due to actual contact and/or mutual penetration of particles, as it is the case for the structure of planets at the classical level (see Figure 3.1), and the structure of hadrons, nuclei, stars, and other systems at the operator level (see Figure 3.2).

To avoid excessive complexity, the systems considered in this chapter will be assumed to be *reversible,* that is, invariant under time reversal. The open-irreversible extension of the systems will be studied in the next chapter.

The most important methodological differences between exterior and interior systems are the following:

1) Exterior systems are completely represented with the knowledge of only *one* quantity, the Hamiltonian, while the representation of interior systems requires

the knowledge of the Hamiltonian for the potential forces, plus additional quantities for the representation of nonpotential forces, as done in the *true Lagrange and Hamilton equations,* those with external terms,

$$\frac{d}{dt}\frac{\partial L(t,r,v)}{\partial v_a^k} - \frac{\partial L(t,r,v)}{\partial r_a^k} = F_{ak}(t,r,v), \tag{3.1.1a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t,r,p)}{\partial r_a^k} + F_{ak}(t,r,p), \tag{3.1.1b}$$

$$L = \Sigma_a \frac{1}{2} \times m_a \times v_{ak} \times v_a^k - V(t,r,v), \tag{3.1.1c}$$

$$H = \Sigma_a \frac{p_{ak} \times p_{ak}}{2 \times m_a} + V(t,r,p), \tag{3.1.1d}$$

$$V = \Sigma_a U(t,r)_{ak} \times v_a^k + U_o(t,r), \tag{3.1.1e}$$

$$F(t,r,v) = F(t,r,p/m), \tag{3.1.1f}$$

$$a = 1,2,3,\ldots,N; \quad k = 1,2,3.$$

Consequently, by their very conception, interior systems are structurally beyond the representational capability of classical and quantum Hamiltonian mechanics, in favor of covering disciplines.

2) Exterior systems are of *Keplerian type*, while interior systems are not, since they do not admit a Keplerian center (see, again, Figures 3.1 and 3.2). Consequently, also by their very conception, interior systems cannot be characterized by the Galilean and Poincaré symmetries in favor of covering symmetries.

3) Exterior systems are local-differential, that is, they describe a finite set of isolated points, thus being fully treatable with the mathematics of the 20-th century, beginning with conventional local-differential topologies. By contrast, interior systems are nonlocal-integral, that is, they admit internal interactions over finite surfaces or volumes that cannot be consistently reduced to a finite set of isolated points. Consequently, interior systems cannot be consistently treated via the mathematics of classical and quantum Hamiltonian mechanics in favor of a basically new mathematics.

4) The time evolution of the Hamiltonian treatment of exterior systems characterizes a *canonical transformation* at the classical level, and a *unitary transformation* at the operator level, that we shall write in the unified form

$$U \times U^\dagger = U^\dagger \times U = I, \tag{3.1.2}$$

where $\times$ represents the usual (associative) multiplication.[1] By contrast, the time evolution of interior systems, being non-Hamiltonian, characterizes *noncanoni-*

[1]Since we shall use several types of multiplications, to avoid confusions, it is essential to identify the assumed multiplication in any mathematical treatment.

Figure 3.1. A view of Jupiter, a most representative interior dynamical system, where one can see with a telescope the dramatic differences with exterior systems, such as internal exchanges of linear and angular momentum always in such a way to verify total conservation laws. As repeatedly stated in the literature on hadronic mechanics, the structure of Jupiter has been assumed as fundamental for the construction of new structure models of hadrons, nuclei and stars, and the development of their new clean energies and fuels.

cal transformations at the classical level and *nonunitary transformations* at the operator level, that we shall jointly write

$$U \times U^{\dagger} \neq I. \tag{3.1.3}$$

In particular, the noncanonical-nonunitary character is necessary to exit from the class of equivalence of classical and quantum Hamiltonian theories.

5) The *invariance* (rather than "covariance") of exterior systems under the Galilean or Poincaré symmetry has the fundamental implication of preserving the basic units, predicting the same numerical values under the same conditions at different times, and admitting all conditions needed for consistent applications of the theory to experimental measurements. By comparison, the loss of the Galilean and Poincaré invariance, combined with the necessary noncanonical-nonunitary structure of interior systems, activates the *theorems of catastrophic mathematical and physical inconsistencies* studied in Chapter 1 whenever treated with the mathematics of canonical-unitary theories.

In this chapter we report the rather long scientific journey that lead to a mathematically and physically consistent, classical and operator treatment of interior dynamical systems via the *isotopic branch of hadronic mechanics for*

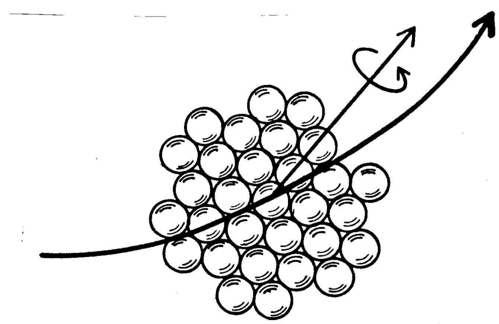

Figure 3.2. A schematic view of nuclei as they are in the physical reality, bound states of extended particles without a Keplerian center, under which conditions quantum mechanics cannot possibly be exact due to the breaking of the fundamental Galilean and Poincaré symmetries in favor of covering theories. As we shall see in this chapter, even though these breakings are small (because nucleons are in conditions of mutual penetration in nuclei of about 10^{-3} parts of their volumes), said breakings permit the prediction and industrial development of new clean energies and fuels that are prohibited by the exact validity of quantum mechanics.

matter, and the *isodual isotopic branch for antimatter* including the resolution of all the above problems.

Besides a number of experimental verifications reviewed in this chapter, the achievement of a consistent treatment of interior systems offers basically new structure models of hadrons, nuclei, stars, Cooper pairs, molecules and other interior structures. In turn, these new models permit quantitative studies of new clean energies and fuels already under industrial, let alone scientific development.

Stated in a nutshell, a primary aim of this chapter is to show that the assumption of a final character of quantum mechanics and special relativity beyond the conditions of their original conception (isolated point particles in vacuum) is the primary origin of the current alarming environmental problems.

The reader should be aware that, nowadays, the literature on hadronic mechanics is rather vast, having surpassed the mark of 15,000 pages of published research. As such, to avoid a prohibitive length, *the presentation in this chapter is restricted to the outline of the origination* of each topic and of the *most important developments.* Scholars interested in a comprehensive list of literature are suggested to consult the quoted references as well as those of Chapter 1.

Also to avoid a prohibitive length, the presentation of this chapter is restricted to studies of direct relevance for hadronic mechanics, namely, research fundamentally dependent on a generalization of the basic unit. The quotation of related studies not fundamentally dependent on the generalization of the basic unit cannot be reviewed for brevity.

3.1.2 Closed Non-Hamiltonian Systems

The first step in the study of hadronic mechanics is the dispelling of the belief that nonpotential forces, being nonconservative, do not permit total conservation laws, namely, that the external terms in the analytic equations (3.1.1) solely applies for open-nonconservative systems, such as an extended object moving within a resistive medium considered as external.

This belief was disproved, apparently for the first time, by Santilli in monographs [1,2]. Ref. [1] presented a comprehensive treatment of the integrability conditions for the existence of a potential or a Hamiltonian, *Helmholtz's conditions of variational selfadjointness,* according to which the total force is divided into the following two components

$$F(t, r, p, \ldots) = F^{SA}(t, r, p) + F^{NSA}(t, r, p, \ldots), \tag{3.1.4}$$

where the selfadjoint(SA) component F^{SA} admits a potential and the nonselfadjoint (NSA) component F^{NSA} does not.

We should also recall for clarity that, to be Newtonian as currently understood, a force should solely depend on time t, coordinates r and velocity $v = dr/dt$ or momenta $p = m \times v$, $F = F(t, r, v)$. Consequently, forces depending on derivatives of the coordinates of order bigger than the first, such as forces depending on the acceleration $F = F(t, r, v, a)$, $a = dv/dt$, are not generally considered Newtonian forces.

Ref. [2] then presented the broadest possible realization of the conditions of variational selfadjointness via analytic equations derivable from a variational principle, and included the first known identification of *closed non-Hamiltonian systems* (Ref. [2], pages 233–236), namely, systems that violate the integrability conditions for the existence of a Hamiltonian, yet verify all ten total conservation laws of conventional Hamiltonian systems.

Let us begin by recalling the following well known property:

THEOREM 3.1.1: Necessary and sufficient conditions for a system of N particles to be closed, that is, isolated from the rest of the universe, are that the following ten conservation laws are verified along an actual path

$$\frac{dX_i(t, r, p)}{dt} = \frac{\partial X_i}{\partial b^\mu} \times \frac{db^\mu}{dt} + \frac{\partial X_i}{\partial t} = 0, \tag{3.1.5a}$$

$$X_1 = E_{tot} = H = T + V, \tag{3.1.5b}$$

$$(X_2, X_3, X_4) = \mathbf{P}_{tot} = \Sigma_a \mathbf{p}_a, \tag{3.1.5c}$$

$$(X_5, X_6, X_7) = \mathbf{J}_{tot} = \Sigma_a \mathbf{r_a} \wedge \mathbf{p}_a, \tag{3.1.5d}$$

$$(X_8, X_9, X_{10}) = \mathbf{G}_{Tot} = \Sigma_a (m_a \times \mathbf{r}_a - t \times \mathbf{p}_a), \tag{3.1.5e}$$

$$i = 1, 2, 3, \ldots, 10; \quad k = 1, 2, 3; \quad a = 1, 2, 3, \ldots, N.$$

It is also well known that Galilean or Poincaré invariant systems do verify the above conservation laws since the X_i quantities are the generators of the indicated symmetries. However, in this case all acting forces are derivable from a potential and the systems are Hamiltonian.

Assume now the most general possible dynamical systems, those according to the true Lagrange's and Hamilton equations (3.1.1) where the selfadjoint forces are represented with the Lagrangian or the Hamiltonian and the nonselfadjoint forces are external.

DEFINITION 3.1.1 [2]: Closed-isolated non-Hamiltonian systems of particles are systems of $N \geq 2$ particles with potential and nonpotential forces characterized by the following equations of motion

$$\frac{db_a^\mu}{dt} = \begin{pmatrix} dr_a^k/dt \\ dp_{ka}/dt \end{pmatrix} = \begin{pmatrix} p_{ak}/m_a \\ F_{ka}^{SA} + F_{ka}^{NSA} \end{pmatrix}, \tag{3.1.6}$$

verifying all conditions (3.1.5), where the term "non-Hamiltonian" denotes the fact that the systems cannot be entirely represented with the Hamiltonian, thus requiring additional quantities, such as the external terms.

The case $n = 2$ is exceptional, yet it admits solutions, and closed non-Hamiltonian systems with $N = 1$ evidently cannot exist (because a single free particle is always Hamiltonian).

Closed non-Hamiltonian systems can be classified into:

CLASS α: systems for which Eqs. (3.1.5) are first integrals;

CLASS β: systems for which Eqs. (3.1.5) are invariant relations;

CLASS γ: systems for which Eqs. (3.1.5) are subsidiary constraints.

The case of closed non-Hamiltonian systems of antiparticles are defined accordingly.

The study of closed non-Hamiltonian systems of Classes β and γ is rather complex. For the limited scope of this presentation it is sufficient to see that interior systems of Class α exist.

THEOREM 3.1.2 [2]: Necessary and sufficient conditions for the existence of a closed non-Hamiltonian systems of Class α are that the nonselfadjoint forces *verify the following conditions:*

$$\sum_a \mathbf{F}_a^{NSA} \equiv 0, \tag{3.1.7a}$$

$$\sum_a \mathbf{p}_a \otimes \mathbf{F}_a^{NSA} \equiv 0, \tag{3.1.7b}$$

$$\sum_a \mathbf{r}_a \wedge \mathbf{F}_a^{NSA} \equiv 0. \tag{3.1.7c}$$

Proof. Consider first the case $N > 2$ and assume first for simplicity that $\mathbf{F}_a^{SA} = 0$. Then, the first nine conservation laws are verified when

$$\frac{\partial X_i}{\partial p_{ka}} \times F_{ka}^{NSA} \equiv 0, \tag{3.1.8}$$

in which case the 10-th conservation law, Eq. (3.1.5e), is automatically verified, and this proves the *necessity* of conditions (3.1.7) for $N > 2$.

The sufficiency of the conditions is established by the fact that Eqs. (3.1.7) consist of seven conditions on $3N$ unknown functions F_{ka}^{NSA}. Therefore, a solution always exists for $N \geq 3$.

The case $N = 2$ is special inasmuch as motion occurs in a plane, in which case Eqs. (3.1.7) reduce to *five* conditions on *four* functions $\mathbf{F}_{ka}^{NSA}$, and the system appears to be overdetermined. Nevertheless, solutions always exist because the verification of the first four conditions (3.1.5) automatically implies the verification of the last one, Eqs. (3.1.5e). As shown in Ref. [2], Example 6.3, pages 272–273, a first solution is given by the *non-Newtonian force*

$$\mathbf{F}_1^{NSA} = -\mathbf{F}_2^{NSA} = K \times a = K \times \frac{dv}{dt}, \tag{3.1.9}$$

where K is a constant. Another solution is given by

$$\mathbf{F}_1^{NSA} = -\mathbf{F}_2^{NSA} = M \times \frac{dr}{dt} \times \phi(M \times \dot{r} + V), \quad M = \frac{m_1 \times m_2}{m_1 + m_2}. \tag{3.1.10}$$

Other solutions can be found by the interested reader. The addition of a non-null selfadjoint force leaves the above proof unchanged. **q.e.d.**

The search for other solutions is recommended to readers interested in acquiring a technical knowledge of hadronic mechanics because such solutions are indeed useful for applications. A general solution of Eqs. (3.1.7), as well as of their operator counterpart and of their isodual images for antimatter will be identified later on in this chapter after the identification of the applicable mathematics.

It should be noted that the proof of Theorem 3.1.2 is not necessary because the existence of closed non-Hamiltonian systems is established by visual observations (Figure 3.1). At any rate, the representation of Jupiter's structure via one single function, the Lagrangian or the Hamiltonian, necessarily implies the belief in the perpetual motion within physical media, due to the necessary condition that constituents move inside Jupiter with conserved energy, linear momentum and angular momentum.

As recalled in Chapter 1, whenever exposed to departures from closed Hamiltonian systems, a widespread posture is the claim that the non-Hamiltonian character of the systems is "illusory" (*sic*) because, when the systems are reduced to their elementary constituents, all nonpotential forces "disappear" (*sic*) and conventional Hamiltonian disciplines are recovered in full.

The political-nonscientific character of the above posture is established by the following property of easy proof by any graduate student in physics:

THEOREM 3.1.3 [3]: A classical non-Hamiltonian system cannot be consistently reduced to a finite number of quantum mechanical point-like particles and, vice-versa, a finite ensemble of quantum mechanical point-like particles cannot consistently characterize a classical non-Hamiltonian system.

The above property establishes that, rather than being "illusory," *nonpotential effect originate at the deepest and most elementary level of nature.* The property also establishes the need for the identification of methods suitable for the invariant treatment of classical and operator non-Hamiltonian systems in such a way to constitute a covering of conventional Hamiltonian treatments.

This chapter is devoted to the mathematical theoretical and experimental study of classical and operator interior system of particles and antiparticles, their experimental verifications and their novel applications.

3.1.3 Need for New Mathematics

By following the main guidelines of hadronic mechanics, we adapt the mathematics to nature, rather than adapting nature to preferred mathematics. For this purpose, we shall seek a mathematics capable of representing the following main features of interior dynamical systems:

1) Points have no dimension and, consequently can only have action-at-a-distance potential interactions. Therefore, the first need for the new mathematics is the representation of the *actual, extended, generally nonspherical shape of the wavepackets and/or of the charge distribution of the particles considered,* that we shall assume in this monograph for simplicity to have the shape of spheroidal ellipsoids with diagonal form

$$Shape_a = Diag.(n_{a1}^2, n_{a2}^2, n_{a3}^2), \quad a = 1, 2, 3, \ldots, N, \tag{3.1.11}$$

with more general non-diagonal expressions not considered for simplicity, where $n_{a1}^2, n_{a2}^2, n_{a3}^2$ represent the semiaxes of the spheroidal ellipsoids assumed as *deviation* from, or normalized with respect to the perfect sphericity

$$n_{a1}^2 = n_{a2}^2 = n_{a3}^2 = 1. \tag{3.1.12}$$

The n's are called *characteristic quantities* of the particles considered. It should be stressed that, contrary to a rather popular belief, *the n-quantities are not parameters because they represent the actual shape as derived from experimental measurements.*

To clarify this important point, by definition a "parameter" can assume any value as derived form the fit of experimental data, while this is not the case for the characteristic quantities here considered. As an example, the use for the n's of value of the order of 10^{-16} cm to represent a proton would have no physical value because the proton charge distribution is a spheroidal ellipsoid of the order of 10^{-13} cm.

2) Once particles are assumed as being extended, there is the consequential need to represent their *density.* This task can be achieved via a fourth set of quantities

$$Density_a = n_{a4}^2, \tag{3.1.13}$$

representing the *deviation* of the density of the particle considered from the density of the vacuum here assumed to be one,

$$n_{Vacuum,4}^2 = 1. \tag{3.1.14}$$

Again, n_4 is not a free parameter because its numerical value is fixed by experimental data. As an example for the case of a hadron of mass m and radius r we have the density

$$n_4^2 = \frac{m \times c^2}{\frac{4}{3} \times \pi \times r^3}, \tag{3.1.15}$$

thus establishing that n_{a4} is not a free parameter capable of assuming.

Predictably, most nonrelativistic studies can be conducted with the sole use of the space components characterizing the shape. Relativistic treatments require the additional use of the density as the forth component, resulting in the general form

$$(Shape - Density)_a = Diag.(n_{a1}^2, n_{a2}^2, n_{a3}^2, n_{a4}^2), \quad a = 1, 2, 3, \ldots, N. \tag{3.1.16}$$

3) Perfectly rigid bodies exist in academic abstractions, but not in the physical reality. Therefore, the next need is for a meaningful representation of the *deformation of shape* as well as *variation of density* that are possible under interior conditions. This is achieved via the appropriate functional dependence of the

characteristic quantities on the energy E_a, linear momentum p_a, pressure P and other characteristics, and we shall write

$$n_{ak} = n_{ak}(E, p, P, \ldots), \quad k = 1, 2, 3, 4. \tag{3.1.17}$$

The reader is suggested to meditate a moment on the fact that Lagrangian or Hamiltonian theories simply cannot represent the actual shape and density of particles. The impossibility of representing deformations of shapes and variations of density are well known, since the pillar of contemporary relativities, the rotational symmetry, is notoriously incompatible with the theory of elasticity.

4) Once particles are represented as they are in the physical reality (extended, nonspherical and deformable), there is the emergence of the following new class of interactions nonexistent for point-particles (for which reason these interactions have been generally ignored throughout the 20-th century), namely, interactions of:

I) *contact type,* that is, due to the actual physical contact of extended particle; consequently, of

II) *zero range type,* since all contacts are dimensionless; consequently of

III) *nonpotential type,* that is, not representable with any possible action-at-a-distance potential; consequently, of

IV) *non-Hamiltonian type,* that is, not representable with any Hamiltonian; consequently, of

V) *noncanonical type* at the classical level and *nonunitary type at the operator level;* as well as of

VI) *nonlinear type,* that is, represented via nonlinear differential equations, such as depending on power of the wavefunction greater than one; and, finally, of

VII) *nonlocal-integral type.* Interactions among point-particles are local-differential, that is, reducible to a finite set of isolated points, while contact interactions among extended particles and/or their wavepackets are, by conception, nonlocal-integral in the sense of being dependent on a finite surface or volume that, as such, cannot be reduced to a finite set of isolated points (see Figure 3.3).

5) Once the above new features of interior systems have been identified, there is the need not only of their mathematical representation, but above all of their *invariant representation* in order to avoid the theorem of catastrophic inconsistencies of Chapter 1.

As an illustration, Coulomb interactions have reached their towering position in the physics of the 20-th century because *the Coulomb potential is invariant under the basic symmetries of physics,* thus predicting the same numerical values under the same conditions at different times with consequentially consistent physical applications. The same occurs for other interactions derivable from a potential (except gravitation represented with curvature as shown in Section 1.4).

Along the same lines, any representation of the extended, nonspherical and deformable character of particles, their densities and their novel nonlinear, non-

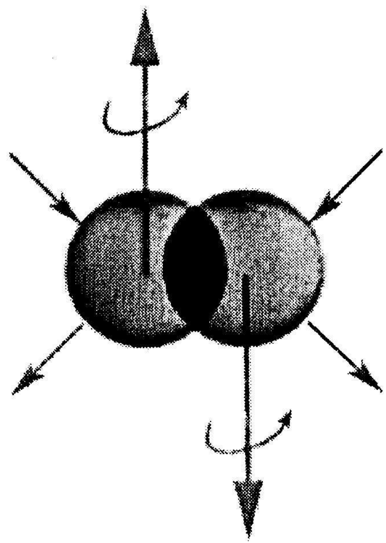

Figure 3.3. A schematic view of the fundamental interactions studied in this monograph, those originating from deep wave-overlappings of the wavepackets of particles also for the case with point-like charge as occurring in electron valence bonds, Cooper pairs in superconductivity, Pauli's exclusion principle, and other basic structures. These interactions have been ignored throughout the 20-th century, resulting in the problematic aspects or sheer inconsistencies identified in Chapter 1. As we shall see in this chapter, the representation of the new interactions here depicted with generalized units of type (3.1.19) permits the achievement of the first known, exact and invariant representation of molecular data and other data that have escaped an exact and invariant representation via quantum mechanics for about one century.

local and nonpotential interactions cannot possibly have physical value unless it is also *invariant,* and not "covariant," again, because the latter would activate the theorems of catastrophic inconsistencies of Chapter 1.

It should be indicated that an extensive search conducted by the author in 1978–1983 in the advanced libraries of Cambridge, Massachusetts, identified numerous integral geometries and other nonlocal mathematics. However, none of them verifies all the following conditions necessary for physical consistency:

CONDITION 1: *The new nonlocal-integral mathematics must admit the conventional local-differential mathematics as a particular case under a well identified limit procedure,* because new physical advances must be a covering of preceding results. This condition alone is not verified by any integral mathematics the author could identify.

CONDITION 2: *The new nonlocal-integral mathematics must permit the clear separation of the contributions of the new nonlocal-integral interactions from those* *of local-differential interactions.* This second condition too was not met by any of the integral mathematics the author could identify.

CONDITION 3: *The new nonlocal-integral mathematics must permit the invariant formulation of the new interactions.* This latter condition was also violated by all integral mathematics the author could identify, thus ruling them out in a final form for consistent physical applications.

After clarifying that the mathematics needed for the correct treatment of interior systems was absent, the author was left with no other choice than that of constructing the needed mathematics. After extensive search, Santilli [4,5] suggested as the *only* possible or otherwise known solution, the invariant representation of nonlinear, nonlocal and nonpotential interactions via a generalization of the trivial unit of conventional theories. The selection was based on the fact that, whether conventional or generalized, the unit is the basic invariant of any theories. We reach in this way the following:

FUNDAMENTAL ASSUMPTION OF HADRONIC MECHANICS [4-10]: The actual, extended, nonspherical and deformable shape of particles, their variable densities and their nonlinear, nonlocal and nonpotential interactions can be invariantly represented with a generalization of the basic spacetime unit of conventional Hamiltonian theories

$$I = Diag.(1,1,1,1), \tag{3.1.18}$$

into nowhere singular, sufficiently smooth, most general possible integro-differential forms, today called "Santilli isounit", of the type here expressed for simplicity for the case of two particles:

$$\begin{gathered}\hat{I} = \hat{I}^\dagger = \hat{I}_{1-2} = Diag.(n_{11}^2, n_{12}^2, n_{13}^2, n_{14}^2)\times \\ \times Diag.(n_{21}^2, n_{22}^2, n_{23}^2, n_{24}^2)\times \\ \times e^{\Gamma(t,r,\psi,\psi^\dagger,\dots)\times\int dr^3\times\psi^\dagger(r)\times\psi(r)} = 1/\hat{T} > 0,\end{gathered} \tag{3.1.19}$$

with trivial generalizations to multiparticle and nondiagonal forms, where the n_{ak}^2 represents the semiaxes of the spheroidal shape of particle a, n_{a4}^2 represents its density, the expression $\Gamma(t,r,\psi,\psi,\dots)$ represents the nonlinearity of the interaction and $\int dr^3 \times \psi^\dagger(r) \times \psi(r)$ provides a simple representation of its nonlocality. The corresponding features of antiparticles are represented by Santilli's isodual isounit

$$\hat{I}^d = -\hat{I}^\dagger = -\hat{I} < 0, \tag{3.1.20}$$

and mixed states of particles and antiparticles are represented by the tensorial product of the corresponding units and their isoduals.

Explicit examples of classical (operator) systems with nonpotential forces represented via generalized units will be given in Section 2.3 (Section 2.4).

As we shall see, the entire structure of hadronic mechanics follows uniquely and unambiguously from the assumption of the above basic unit. As a matter of fact, some of the main features of hadronic mechanics can already be derived from the above basic assumption.

First, the maps, called in the literature *Santilli liftings*

$$I \rightarrow \hat{I}, \quad I^d \rightarrow \hat{I}^d; \tag{3.1.21}$$

(where $I^d = -I$ is the isodual unit of Chapter 2 [8]) require two corresponding generalizations of the totality of the mathematical and physical formulations of conventional classical and quantum Hamiltonian theories without any exception known to this author (to avoid catastrophic inconsistencies).

As we shall see in this chapter, even basic notions such as trigonometric functions, Fourier transforms, differentials, etc. have to be lifted into two forms admitting the new quantity $\hat{I}$ and $\hat{I}^d$ as the correct left and right units.

In view of the assumed Hermiticity and positive-definiteness of $\hat{I}$, the resulting new mathematics is called in the literature *Santilli's isotopic mathematics* or *isomathematics* for short, with the corresponding *isodual isomathematics* for antimatter in interior conditions. The resulting new physical formulations are known as *Santilli isotopic mechanics* or *isomechanics* for short for the case of particles, with the *isodual isomechanics* for antiparticles.

Again in view of the fact that $\hat{I}$ is Hermitian and positive-definite, at the abstract, realization-free level there is no topological difference between I and $\hat{I}$ and, for this reason $\hat{I}$ is called *Santilli isotopic unit* or *isounit* for short.

Consequently, the new mathematical and physical formulations are expected to be *new realizations of the same axioms of conventional Hamiltonian mechanics,* and they should not be intended as characterizing "new theories" since they do not admit new abstract axioms. This illustrates the name of *isotopic mathematics* from the Greek meaning of preserving the topology.[2]

Finally, Santilli isounit $\hat{I}$ identifies in full the *covering* nature of isomechanics over conventional mechanics, as well as the type of resulting covering. This covering character is illustrated by the fact that at sufficiently large mutual distances of particles the integral in the exponent of Eq. (3.1.19) is null

$$\lim_{r \gg 1\,\mathrm{Fm}} \int dr^3 \times \psi^\dagger(r) \times \psi(r) = 0, \tag{3.1.22}$$

[2]When $\hat{I}$ is no longer Hermitian, we have the more general *genotopic mathematics* studied in Chapter 4.

in which case the actual shape of particles has no impact in the interactions and the generalized unit recovers the conventional unit[3]

$$\lim_{r \gg 1\,\mathrm{Fm}} \hat{I} = I = Diag.(1,\, 1,\, 1,\, 1), \tag{3.1.23}$$

under which limit hadronic mechanics recovers conventional quantum mechanics identically and uniquely.

The above limits also identify the important feature according to which *hadronic mechanics coincides with quantum mechanics for all mutual distances of particles sufficiently bigger than their wavepackets, while at mutual distances below that value hadronic mechanics provides a generally small corrections to quantum mechanics* (see Figure 3.3).

In this chapter we review the long and laborious scientific journey by mathematicians, theoreticians and experimentalists (see the bibliography of Chapter 1) for the achievement of maturity of formulation of the isotopic branch of hadronic mechanics, its experimental verification, its novel industrial applications, and its isodual for antimatter.

We shall begin with a review of recent developments in the construction of isomathematics that have occurred following the publication of the second edition of Vol. I of this series in 1995 [6] since these developments have important implications. We shall then identify the recent developments in physical theories occurred since the second edition of Vol. II of this series [7]. We shall then review the novel industrial applications developed since the appearance of Volumes I and II.

It should be noted that in this chapter we shall merely present recent developments. As a consequence, Volumes I and II of this series [6,7] remain useful for all detailed aspects that will not be repeated in this final volume.

A primary motivation of this volume is to present *industrial applications.* Consequently, we have selected the simplest possible mathematical treatment accessible to any experimentalists. Readers interested in utmost mathematical rigor are suggested to consult the specialized mathematical literature in the field.

Finally, the literature on the mathematics, physics and chemistry of classical and quantum Hamiltonian theories is so vast to discourage discriminatory quotations. For this reason, unless there is a contrary need, we shall abstain from quotations of works on pre-existing methods since their knowledge is a pre-requisite for the understanding of this monograph in any case.

[3]When the exponent of Eq. (3.1.19) is null, that is, when the mutual distances of particles are large, the characteristic quantities are constant and, consequently, terms such as $Diag.(n_{11}^{-2},\, n_{12}^{-2},\, n_{13}^{-2}, n_{14}^{-2})$ factor out of all equations, resulting in reduction (3.1.23).

3.2 ELEMENTS OF SANTILLI'S ISOMATHEMATICS AND ITS ISODUAL

3.2.1 Isounits, Isoproducts and their Isoduals

As indicated earlier, *Santilli isotopic mathematics*, [4–10] or *isomathematics* for short, is characterized by the map, called *lifting,* of the trivial unit $I = +1$ into a generalized unit $\hat{I}$

N-dimensional unit

$$I = +1 \rightarrow \hat{I}(t, r, p, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \dots), \tag{3.2.1}$$

or, more generally, by the lifting of N-dimensional units

$$I = (I^i_j) = Diag.(1, 1, 1, \dots),\ i, j = 1, 2, \dots, N$$

of conventional Hamiltonian theories[4] into a nowhere singular, Hermitian and positive-definite, matrix $\hat{I}$ of the same dimension N whose elements $\hat{I}^i_j$ have an arbitrary, nonlinear and integral dependence on time t, space coordinates r, momenta p, wavefunctions ψ, their derivatives $\partial\psi$, and any other needed quantity [*loc. cit.*]

$$\begin{aligned} I \ &= \ (I^i_j) = Diag.(1, 1, \dots) > 0 \rightarrow \\ \rightarrow \hat{I} \ &= \ (\hat{I}^j_i) \ = \ \hat{I}(t, r, p, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \dots) \ = \ 1/\hat{T} \ > \ 0. \end{aligned} \tag{3.2.2}$$

Isomathematics can then be defined as the lifting of all possible branches of mathematics with left and right unit I into forms admitting $\hat{I}$ as the new left and right unit.

Recall that I is the right and left unit under the conventional *associative product* $A \times B = AB$, where A, B are generic quantities (e.g., numbers, vector-fields, operators, *etc.*) for which $I \times A = A \times I = A$ for all element A of the considered set.

It is easy to see that $\hat{I}$ cannot be a unit under the same product because $\hat{I} \times A \neq A$. Therefore, for consistency, the conventional associative product $A \times B$ must be lifted into the new form first proposed by Santilli in Ref. [5] of 1978,

$$A \times B \rightarrow A \hat{\times} B = A \times \hat{T} \times B = A \times (1/\hat{I}) \times B, \tag{3.2.3}$$

where $\hat{T}$ is fixed for the set considered, under which product $\hat{I}$ is indeed the correct left and right new unit,

$$I \times A = A \times I = A \rightarrow \hat{I} \hat{\times} A = A \hat{\times} \hat{I} = A, \tag{3.2.4}$$

---

[4]For instance, Hamiltonian theories in 3-dimensional Euclidean space are based on the unit $I = Diag.(1, 1, 1)$ of the rotational and Euclidean symmetries, while Hamiltonian theories in Minkowski space are based on the unit $I = Diag.(1, 1, 1, 1)$ that is at the foundation of Lie's theory of the Lorentz and Poincaré symmetries.

for all elements A of the considered set. In this case (only) $\hat{I}$ is called *Santilli's isotopic unit*, or *isounit* for short, and $\hat{T}$ is called *Santilli's isotopic element,* or *isoelement* for short.

Isomathematics was first submitted by Santilli in memoirs [*loc. cit.*] of 1978 and then worked out in various additional contributions by the same author, as well as by numerous mathematicians and theoreticians (see the references of Chapter 1 as well as of this section).

The most salient feature of Santilli's liftings (3.2.2) and (3.2.3) is that they are *axiom preserving,* from which feature they derived their name "isotopic" [*loc. cit.*], recently contracted to the prefix "iso."

In fact, $\hat{I}$ preserves the basic topological characteristics of I. Therefore, isomathematics is expected to provide *new realizations* of the abstract axioms of the mathematics admitting I as left and right unit. In particular, the preservation of the original abstract axioms is an important guiding principle in the consistent construction of isomodels and their applications.

At this introductory stage the axiom-preserving character of generalized product (3.2.3) is easily verified by the fact that it preserves all basic axioms of the original product. In fact, the isoproduct verifies the *right and left isoscalar laws*

$$n\hat{\times}(A\hat{\times}B) = (n\hat{\times}A)\hat{\times}B, \tag{3.2.5a}$$

$$(A\hat{\times}B)\hat{\times}n = A\hat{\times}(B\hat{\times}n), \tag{3.2.5b}$$

the *right and left isodistributive laws*[5]

$$A\hat{\times}(B + C) = A\hat{\times}B + A\hat{\times}C, \tag{3.2.6a}$$

$$(A + B)\hat{\times}C = A\hat{\times}C + B\hat{\times}C, \tag{3.2.6b}$$

and the *isoassociative law*

$$A\hat{\times}(B\hat{\times}C) = (A\hat{\times}B)\hat{\times}C. \tag{3.2.7}$$

A verification of the preservation of the axioms of all subsequent constructions is crucial for a serious study and application of hadronic mechanics.

The simplest method for the construction of isomathematics as needed for various applications is given by the use of a positive-definite N-dimensional *noncanonical transform* at the classical level or a *nonunitary transform* at the operator level, here written in the unified form

$$U \times U^{\dagger} \neq I, \tag{3.2.8}$$

[5]The reader should keep in mind that the verification of the right and left scalar and distributive laws are necessary for any product to characterize an *algebra* as commonly understood in contemporary mathematics.

and its identification with the basic isounit of the theory

$$\hat{I} = U \times U^\dagger = 1/\hat{T} > 0, \tag{3.2.9}$$

realization first introduced by Santilli in Ref. [6,7] of 1993.

In this case, the Hermiticity of $\hat{I}$ is guaranteed because of the property,

$$(U \times U^\dagger)^\dagger = U \times U^\dagger. \tag{3.2.10}$$

Therefore, realization (3.2.9) of the isounit only requires that $U \times U^\dagger$ be a positive-definite N-dimensional matrix other than the unit matrix, from which the nowhere singularity follows, e.g., via condition

$$\mathrm{Det}(U \times U^\dagger) > 0, \neq I. \tag{3.2.11}$$

Once the fundamental realization (3.2.9) is assumed, the construction of isomathematics follows in a simple, unique and unambiguous way. In fact, *isomathematics can be constructed by submitting conventional mathematics with left and right unit I to said noncanonical-nonunitary transform,* with very few exception, such as the isodifferential calculus that escapes construction via noncanonical-nonunitary transforms.

To begin, the isounit itself is simply given by said noncanonical-nonunitary transform of the conventional unit,

$$I \rightarrow U \times I \times U^\dagger = \hat{I}, \tag{3.2.12}$$

the isoproduct too is simply given by said noncanonical-nonunitary transform of the conventional product

$$\begin{aligned} A \times B \rightarrow U \times (A \times B) \times U^\dagger = \\ = (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) = \\ = \hat{A} \times \hat{T} \times \hat{B} = \hat{A} \hat{\times} \hat{B}, \end{aligned} \tag{3.2.13}$$

and the same simple transform holds for the construction of other aspects of isomathematics, as illustrated in this section.

As a matter of fact, the use of the above transform provides a method for the construction of isomathematics that is more rigorous than empirical liftings. For instance, by comparing Eqs. (3.2.3) and (3.2.13), we see that the lifting of the unit $I \rightarrow \hat{I} = U \times I \times U^\dagger$ implies not only the lifting of the associative product $\times \rightarrow \hat{\times} = \times (U \times U^\dagger)^{-1} \times$, but also the lifting of all elements of the set considered, $A \rightarrow \hat{A} = U \times A \times U^\dagger$.

In view of the above, the claim often expressed in the nontechnical physics literature that "the mathematics of hadronic mechanics is too difficult to comprehend" is just a case of venturing judgment without any serious knowledge of the topic.

The reader should be aware that other generalizations of the associative product, such as

$$A \bigotimes B = \hat{T} \times A \times B, \tag{3.2.14a}$$

$$A \bigodot B = A \times B \times \hat{T}, \tag{3.2.14b}$$

are unacceptable because they violate either the right or the left distributive and scalar laws, thus being unable to characterize an algebra. As such, liftings (3.2.14) are not isotopic in Santilli's sense [*loc. cit.*].

Examples of isounits have been given in Section 3.1.3. Additional examples will be provided in Sections 3.3 and 3.4. Note that, since they are Hermitian by assumption, isounits can always be diagonalized into the form of type (3.1.19).

Santilli isodual isomathematics [6–10] is the image of isomathematics under the anti-isomorphic *isodual map* of an arbitrary quantity

$$\begin{aligned} A(t, r, p, \psi, \psi^\dagger, \ldots) &\rightarrow A^d(t^d, r^d, p^d, \psi^d, \psi^{\dagger d}, \ldots) \\ &\rightarrow -A^\dagger(-t, -r^t, -p^t, -\psi^\dagger, -\psi^\dagger, \ldots), \end{aligned} \tag{3.2.15}$$

(where t denotes transposed) first submitted by Santilli in Ref. [8] of 1985 (see also Chapter 2).

The basic quantity of isodual isomathematics is then the *isodual isounit*

$$\hat{I}^d = -\hat{I}^\dagger(-t, -r^\dagger, -p^\dagger, -\psi^\dagger, -\partial\psi^\dagger, \ldots) = 1/\hat{T}^d. \tag{3.2.16}$$

Similarly, we have the *isodual isoproduct*

$$B^\dagger \times \hat{T}^d \times A^\dagger = B^\dagger \hat{\times}^d A^\dagger, \tag{3.2.17}$$

under which $\hat{I}^d$ is indeed the right and left unit,

$$\hat{I}^d \hat{\times}^d A = A \hat{\times}^d \hat{I}^d = A, \tag{3.2.18}$$

for all A of the considered set.

Note that, *isodual map (3.2.15) must be applied for consistency to the totality of quantities of isomathematics as well as of their operations.* As an illustration, the application of the isodual map only to the quantities A, B of a product $A \times B$ and not to the product itself $\times$, leads to a host of inconsistencies.

For this and other reasons the conventional associative product is written in this monograph with the explicit notation $A \times B$ rather than the conventional notation AB. In fact, the latter would lead to gross misunderstandings and inconsistencies under the various liftings of hadronic mechanics.

Also, the construction of isomathematics is indeed recommended for physicists to be done via a noncanonical-nonunitary transform (3.2.9), while the construction of isodual isomathematics is recommended via the isodual map (3.2.15) and not via the use of an anti-isomorphic transform.

In fact, the use of anti-isomorphic transforms causes ambiguities in the very central issue, the achievement of equivalence of the isodual operator theory with charge conjugation due to ambiguities and other technical aspects. In turn, this occurrence illustrates the significance and uniqueness of Santilli isodual map (3.2.15).

Note also that isodual isomathematics preserves the axioms, not of conventional mathematics, but of the isodual mathematics of Chapter 2, that with the simplest possible isounit unit $I^d = -I$.

Needless to say, mathematicians do not need the above elementary construction of isomathematics and its isodual since they can be formulated on abstract realization-free grounds from basic axioms.

3.2.2 Isonumbers, Isofields and their Isoduals

The first necessary isotopic lifting following that of the basic unit and product, is that of ordinary numbers. The resulting new numbers were first presented by Santilli at the 1980 meeting in Clausthal, Germany, on *Differential Geometric Methods in Mathematical Physics* and then published in a variety of papers, such as Ref. [8] of 1985, Vols. [15,16] of 1991, memoir [9] of 1993 and other works. A comprehensive presentation is available in Vol. I [6] of 1995 that also presents industrial applications of the new numbers for cryptograms and other fields. As a result of these contributions the new numbers are today known as *Santilli's isonumbers.*

The new numbers have also been studied by various authors. An important contribution has been made by E. Trell [11] in 1998 consisting in a proof of Fermat's celebrated theorem that is the simplest on record and, therefore, credibly conceivable by Fermat (as compared to other proof requiring mathematics basically unknown during Fermat's time). Unfortunately, Fermat left no record of the proof of his celebrated theorem and, therefore, there is no evidence that Fermat first studied numbers with arbitrary units. Nevertheless, Trell's proofs of Fermat's theorems remains the most plausible known to this author for being conceived during Fermat's time.

Numerous additional studies on isonumbers have been conducted by other authors. For a complete bibliography we refer interested readers to the monograph on *Santilli isonumber theory* by C.-X. Jiang [12] of 2002. Additional studies on isonumbers have occurred for their use as basis of other isostructures. Related references will be quoted in the appropriate subsequent sections.

Santilli's isonumbers have also been subjected to a generalization called *pseudo-isonumbers* identified in Ref. [9] and studies by various authors, including N. Kamiya [13] and others. However, the latter generalization violates the axioms of a field and, as such, it cannot be used for hadronic mechanics.

The reader should be aware that in this section we merely present the minimal possible properties of isonumbers sufficient for industrial applications.

Let us consider: the field $R(n,+,\times)$ of *real numbers* n with ordinary sum $+$ and product $\times$; the field $C(c,+,\times)$ of *complex numbers* $c = n_1 + i \times n_2$ where i is the imaginary unit and $n_1, n_2 \in R$; and the field $Q(q,+,\times)$ of *quaternions* $q = i_o + i_1 \times n_1 + i_2 \times n_2 + i_3 \times n_3$, where i_o is the 2-dimensional unit matrix, $i_k, k = 1,2,3$ are Pauli's matrices and $n_1, n_2, n_3 \in R$. These fields are hereon represented with the unified notation[6]

$$F(a,+,\times) : a = n,\ c,\ q, \tag{3.2.19}$$

In this section we present first the simplest possible method for the lifting of numbers via the use of a positive-definite (thus invertible) noncanonical-nonunitary transform identified with Santilli's isounit

$$I \to \hat{I} = U \times I \times U^{\dagger} = 1/\hat{T} > 0, \quad U \times U^{\dagger} \neq I. \tag{3.2.20}$$

We shall then pass to a mathematical presentation.

The isotopic lifting of ordinary numbers is easily achieved via the above map resulting in *Santilli isonumbers* for the characterization of *matter*

$$a \to \hat{a} = U \times a \times U^{\dagger} = a \times (U \times U^{\dagger}) = a \times \hat{I}, \tag{3.2.21}$$

and related *isoproduct*

$$a \times b \to U \times (a \times b) \times U^{\dagger} = \hat{a} \times \hat{T} \times \hat{b} = \hat{a} \hat{\times} \hat{b}, \tag{3.2.22}$$

under which $\hat{I}$ is the correct right and left isounit, Eq. (3.2.4), with the element *isozero* coinciding with the ordinary zero

$$0 \to \hat{0} = U \times 0 \times U^{\dagger} \equiv 0, \tag{3.2.23}$$

and, consequently, the *isosum* coinciding with the ordinary sum,

$$a + b \to U \times (a + b) \times U^{\dagger} = \hat{a} \hat{+} \hat{b} \equiv \hat{a} + \hat{b}. \tag{3.2.24}$$

The above liftings result in: *Santilli isofield* $\hat{R}(\hat{n}, \hat{+}, \hat{\times})$ of *isoreal isonumbers*; the isofield $\hat{C}(\hat{c}, \hat{+}, \hat{\times})$ of *isocomplex isonumbers*; and the isofield $\hat{Q}(\hat{q}, \hat{+}, \hat{\times})$ of *isoquaternionic isonumbers*; hereon represented with the unified notation

$$\hat{F}(\hat{a}, \hat{+}, \hat{\times}), \quad \hat{a} = \hat{n}, \hat{c}, \hat{q}. \tag{3.2.25}$$

[6]Octonions are not considered "numbers" because they violate the associativity property of the axioms of a field.

Needless to say, the liftings of the unit and of the product require a corresponding lifting of all conventional operations of numbers depending on the multiplication. By using the above noncanonical-nonunitary map, one can easily prove the *isopowers*

$$\hat{a}^{\hat{n}} = \hat{a} \hat{\times} \hat{a} \hat{\times} \ \ldots \ \hat{\times} \hat{a} \ \ (n \ \text{ times}) = a^n \times \hat{I}. \tag{3.2.26}$$

An important particular case is the property that *isopowers of the isounits reproduce the isounit identically,*

$$\hat{I}^{\hat{n}} = \hat{I} \hat{\times} \hat{I} \hat{\times} \ldots \hat{\times} \hat{I} \equiv \hat{I}. \tag{3.2.27}$$

Similarly we have the *isosquare isoroot*

$$\hat{a}^{\widehat{1/2}} = a^{1/2} \times \hat{I}^{1/2}; \tag{3.2.28}$$

the *isoquotient*

$$\hat{a} \hat{/} \hat{b} = (\hat{a}/\hat{b}) \times \hat{I} = (a/b) \times \hat{I}; \tag{3.2.29}$$

and the *isonorm*

$$\widehat{|}\hat{a}\widehat{|} = |a| \times \hat{I}, \tag{3.2.30}$$

where $|a|$ is the conventional norm. All these properties were first introduced by Santilli in Refs. [6–9]. The reader can now easily construct the desired isotopic image of any other operation on numbers.

Despite their simplicity, isonumbers are nontrivial. As an illustration, the assumption of the isounit $\hat{I} = 3$ implies that "2 multiplied by 3" = 18, while 4 becomes a prime number.

The best way to illustrate the nontriviality of the new numbers is to indicate the **industrial applications of Santilli's isonumbers,** that are a primary objective of this monograph as indicated earlier.

To begin, *all* applications of hadronic mechanics are based on isonumbers, and they will be presented later on in this chapter. In addition to that, *Santilli's isonumbers have already found a direct industrial application consisting of the isotopic lifting of cryptograms used by the industry to protect secrecy, including banks, credit cards. etc.* This industrial application was first presented by Santilli in Appendix 2.C of the second edition of Vol. I [6] of 1995, and will be reviewed later on in this chapter.

At this moment we merely mention that all cryptograms based on the multiplication depend on only one value of the unit, the quantity +1 dating back to biblical times. A mathematical theorem establishes that a solution of any cryptogram can be identified in a finite period of time. As a result of this occurrence, banks and other industries are forced to change continuously their cryptograms to properly protect their secrecy.

By comparison, *Santilli's isocryptograms* are based on the isoproduct and, as such, they admit an *infinite number of possible isounits,* such as, for instance, the values

$$\hat{I} = 7.2; \quad 0.98364; \quad 236; \quad 1,293' \ 576; \quad \text{etc.} \tag{3.2.31}$$

Consequently, it remains to be seen whether Santilli isocryptograms can be broken in a finite period of time under the availability of an infinite number of possible isounits.

Independently from that, with the use of isocryptograms banks and other industries do not have to change the entire cryptogram for security, but can merely change the value of the isounit to keep ahead of possible hackers, and even that process can be computerized for frequent automatic changes of the isounit, with clearly added safety.

Finally, another application of Santilli isocryptograms permitted by their simplicity is their use to protect the access to personal computers.

It is hoped this illustrates the industrial significance of Santilli isonumbers *per se,* that is, independently from their basic character for hadronic mechanics.

We now pass to a mathematical presentation of the new numbers.

DEFINITION 3.2.1 [9]: Let $F = F(a, +, \times)$ be a field of characteristic zero as per Definition 2.1.1. Santilli's isofields are rings $\hat{F} = \hat{F}(\hat{a}, \hat{+}, \hat{\times})$ with: elements

$$\hat{a} = a \times \hat{I}, \tag{3.2.32}$$

where $a \in F$, $\hat{I} = 1/\hat{T}$ is a positive-definite quantity generally outside F and $\times$ is the ordinary product of F; the isosum $\hat{+}$ coincides with the ordinary sum $+$,

$$\hat{a} \hat{+} \hat{b} \equiv \hat{a} + \hat{b}, \ \ \forall \ \hat{a}, \hat{b} \in \hat{F}, \tag{3.2.33}$$

consequently, the element $\hat{0} \in \hat{F}$ coincides with the ordinary $0 \in F$; and the isoproduct $\hat{\times}$ is such that $\hat{I}$ is the right and left isounit of $\hat{F}$,

$$\hat{I} \hat{\times} \hat{a} = \hat{a} \hat{\times} \hat{I} \equiv \hat{a}, \ \ \forall \ \hat{a} \in \hat{F}. \tag{3.2.34}$$

Santilli's isofields verify the following properties:

1) For each element $\hat{a} \in \hat{F}$ there is an element $\hat{a}^{-\hat{1}}$, called isoinverse, for which

$$\hat{a} \hat{\times} \hat{a}^{-\hat{1}} = \hat{I}, \ \ \forall \ \hat{a} \in \hat{F}; \tag{3.2.35}$$

2) The isosum is isocommutative

$$\hat{a} \hat{+} \hat{b} = \hat{b} \hat{+} \hat{a}, \tag{3.2.36}$$

and isoassociative

$$(\hat{a} \hat{+} \hat{b}) + \hat{c} = \hat{a} \hat{+} (\hat{b} \hat{+} \hat{c}), \ \ \forall \hat{a}, \hat{b}, \hat{c} \in \hat{F}; \tag{3.2.37}$$

3) The isoproduct is not necessarily isocommutative

$$\hat{a}\hat{\times}b \neq \hat{b}\hat{\times}\hat{a}, \tag{3.2.38}$$

but isoassociative

$$\hat{a}\hat{\times}(\hat{b}\hat{\times}\hat{c}) = (\hat{a}\hat{\times}\hat{b})\hat{\times}\hat{c}, \quad \forall \hat{a}, \hat{b}, \hat{c} \in \hat{F}; \tag{3.2.39}$$

4) The set $\hat{F}$ is closed under the isosum,

$$\hat{a}\hat{+}\hat{b} = \hat{c} \in \hat{F}, \tag{3.2.40}$$

the isoproduct,

$$\hat{a}\hat{\times}\hat{b} = \hat{c} \in \hat{F}, \tag{3.2.41}$$

and right and left isodistributive compositions,

$$\hat{a}\hat{\times}(\hat{b}\hat{+}\hat{c}) = \hat{d} \in \hat{F}, \tag{3.2.42a}$$

$$(\hat{a}\hat{+}\hat{b})\hat{\times}c = \hat{d} \in \hat{F}, \quad \forall \hat{a}, \hat{b}, \hat{c}, \hat{d} \in \hat{F}; \tag{3.2.42b}$$

5) The set $\hat{F}$ verifies the right and left isodistributive law

$$\hat{a}\hat{\times}(\hat{b}\hat{+}\hat{c}) = (\hat{a}\hat{+}\hat{b})\hat{\times}\hat{c} = \hat{d}, \ \forall \hat{a}, \hat{b}, \hat{c}, \hat{d} \in \hat{F}. \tag{3.2.43}$$

Santilli's isofields are called of the first (second) kind when $\hat{I}$ is (is not) an element of F.

The basic axiom-preserving character of the isotopies of numbers is illustrated by the following:

LEMMA 3.2.1 [9]: Isofields of first and second kind are fields (namely, they verify all axioms of a field).

Note that the isotopic lifting does indeed change the *operation* of the multiplication but not that of the sum because the isotopies here considered do change the multiplicative unit I, but not the additive unit 0, Eq. (3.2.23). This is a crucial property of hadronic mechanics best illustrated by the following property:

LEMMA 3.2. [9]: Nontrivial liftings of the additive unit 0 *and related sum violates the axioms of a field (for which reason, they are called "pseudoisofields")*

In fact, suppose that one wants to change the value of the element 0, e.g.,

$$0 \to \hat{0} = K \neq 0, \ \ K \in F. \tag{3.2.44}$$

Then, for $\hat{0}$ to remain the new additive unit, one must alter the sum into a new form admitting $\hat{0}$ as left and right additive unit, e.g.,

$$a\hat{+}b = a + (-\hat{0}) + b, \tag{3.2.45}$$

under which

$$a\hat{+}\hat{0} = \hat{0}\hat{+}a \equiv a, \ \ \forall a \in F. \tag{3.2.46}$$

However, there is no single lifting of the product such that

$$\hat{0}\hat{\times}a \neq \hat{0}, \ \ \forall a \in F, \tag{3.2.47}$$

under which there is the loss of the distributive axiom of a field, i.e.,

$$(a\hat{+}b) \times c \neq a \times c \ \ \hat{+} \ \ b \times c. \tag{3.2.48}$$

In turn, the loss of the distributive law causes very serious physical inconsistencies, such as preventing experimental applications of the theory. Therefore, *being axiom-preserving, hadronic mechanics is solely based on the isotopic lifting of the multiplicative unit and related product, but not on any lifting of the additive unit and related sum.*

Santilli's isodual isonumbers for the characterization of *antimatter* can be uniquely and unambiguously characterized via the isodual map (3.2.15). They are characterized by the *additive and multiplicative isodual isounit*

$$\hat{0} \rightarrow \hat{0}^d \equiv 0, \tag{3.2.49a}$$

$$\hat{I}^d = -\hat{I} < 0, \tag{3.2.49b}$$

where one should recall that $\hat{I}$ is real valued and positive-definite, thus Hermitian. Isodual isonumbers are then explicitly given by

$$\hat{a}^d = -\hat{a}^\dagger = -\hat{I} \times \hat{a}^\dagger. \tag{3.2.50}$$

The isodual isonumbers were first introduced by Santilli in Ref. [8] of 1985, treated mathematically in Ref. [9] of 1993 and studied extensively in Vol. I of this series [6].

The use of the same isodual map then identifies the *isodual isosum*

$$\hat{a}^d \hat{+}^d \hat{b}^d = \hat{a}^d + \hat{b}^d, \tag{3.2.51}$$

the *isodual isoproduct*

$$(\hat{a}\hat{\times}\hat{b})^d = \hat{b}^d \times^d \hat{T}^d \times^d \hat{A}^d = -\hat{b}^d \hat{\times} \hat{a}^d = -\hat{b}^\dagger \hat{\times} \hat{a}^\dagger, \tag{3.2.52}$$

and the *isodual isonorm*

$$\hat{|}\hat{a}\hat{|}^d = -\hat{|}\hat{a}\hat{|} = -|a| \times \hat{I}. \tag{3.2.53}$$

that is always *negative-definite.*

The above liftings result in: *Santilli's isodual isofield* $\hat{R}^d(\hat{n}^d, \hat{+}^d, \hat{\times}^d)$ of *isodual isoreal isonumbers*; the isodual isofield $\hat{C}^d(\hat{c}^d, \hat{+}^d, \hat{\times}^d)$ of *isodual isocomplex isonumbers*; and the isodual isofield $\hat{Q}^d(\hat{q}^d, \hat{+}^d, \hat{\times}^d)$ of *isodual isoquaternionic isonumbers*; hereon represented with the unified notation

$$\hat{F}^d(\hat{a}^d, \hat{+}^d, \hat{\times}^d), \hat{a}^d = \hat{n}^d, \hat{c}^d, \hat{q}^d. \tag{3.2.54}$$

DEFINITION 3.2.3 [9]: Let $\hat{F}(\hat{a}, \hat{+}, \hat{\times})$ *be an isofield as per Definition 3.2.1. Then Santilli isodual isofields* $\hat{F}^d(\hat{a}^d, \hat{+}^d, \hat{\times}^d)$ *are the image of* $\hat{F}$ *under the isodual map (3.2.15).*

LEMMA 3.2.3 [9]: Isodual isofields are fields (that is, they verify all axioms of a field).

LEMMA 3.2.4 [9]: Isodual isofields are anti-isomorphic to isofields.

As we shall see in this chapter, the latter property, jointly with the anti-isomorphic character of the isodual map, will result to be crucial for a consistent treatment of antimatter composed of extended particles with potential and non-potential internal forces.

The above properties establish the fact (first identified in Ref. [8]) that, by no means, the axioms of a field require that the multiplicative unit to be the trivial unit $+1$, because the basic unit can be a negative-definite quantity -1 as it occurs for the isodual mathematics of Chapter 2, an arbitrary positive-definite quantity $\hat{I} > 0$ as occurring in isomathematics, or an arbitrary negative-definite quantity $\hat{I}^d < 0$ as it occurs for the isodual isomathematics.

The reader should be aware that an in depth knowledge of Santilli's isonumbers and their isoduals requires an in depth study of memoir [9] or of Chapter 2 of Vol. I of this series, Ref. [6], and that an in depth knowledge of Santilli's isonumbers theory requires a study of Jiang's monograph [12].

Finally, the reader should meditate a moment on the viewpoint expressed several times in this writing to the effect that *there cannot be really new physical theories without new mathematics, and there cannot be really new mathematics without new numbers.* The basic novelty of hadronic mechanics can, therefore, be reduced to the novelty of Santilli's isonumbers.

By remembering that all "numbers" have been fully identified centuries ago, the novelty of hadronic mechanics can be reduced to the discovery that the axioms of conventional fields admit new realizations with nonsingular, but otherwise arbitrary multiplicative units.

3.2.3 Isospaces and Their Isoduals

Following the lifting of units, products and fields, the next necessary lifting is that of N-dimensional *metric or pseudo-metric spaces* with local coordinates r and Hermitian, thus diagonalized metric m over a field F, here written in the unified notation

$$S(r,m,F): \ r=(r^k), \ m=[m_{ij}(r,\ldots)]=Diag.(m_{11},m_{22},\ldots,m_{NN}), \quad i,j,k=1,2,\ldots,N, \tag{3.2.55}$$

basic invariant

$$r^2=(r^i\times m_{ij}\times r^j)\times I=(r^t\times m\times r)\times I \ \in \ F(a,+,\times), \tag{3.2.56}$$

(where t stands for transposed) and fundamental N-dimensional unit[7]

$$I=Diag.(1,1,\ldots,1). \tag{3.2.57}$$

As now familiar, isotopies are based on the lifting of the above N-dimensional unit via a positive-definite noncanonical-nonunitary transform in the same dimension with an otherwise unrestricted functional dependence

$$I=Diag.(1,1,\ldots,1)\rightarrow\hat{I}(t,r,p,\psi,\psi^\dagger,\ldots)=U\times I\times U^\dagger=1/\hat{T}>0, \tag{3.2.58}$$

The above liftings requires that of spaces $S(r,m,R)$ into *isotopic spaces,* or *isospaces* for short, for the treatment of *matter,* hereon denoted $\hat{S}(\hat{r},\hat{M},\hat{F})$, where $\hat{r}$ denotes the *isocoordinates,* and $\hat{M}$ denotes the *isometric* defined on the isofields $\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times})$ of Section 3.2.2.

Isospaces were first proposal by Santilli in Ref. [14] of 1983 for the axiom-preserving isotopies of the Minkowskian spacetime and special relativity that are at the foundations of hadronic mechanics. Isospaces were then used by Santilli for the liftings of the various spacetime and internal symmetries (such as $SU(2)$, $SO(3)$, $SO(3.1)$, $SL(2.C)$, $G(3.1)$, $P(3.1)$, $SU(3)$, etc.) as studied later on in this chapter.

A comprehensive presentation of isospaces first appeared in monographs [15,16] of 1991 and in the first edition of Volumes I and II of this series, Ref. [6,7] of 1993 (see the second edition of 1995 for various upgradings). A mathematical study of isospaces by Santilli was presented in memoir [10] of 1996. In view of all these contributions, the new spaces are today known as *Santilli's isospaces.*

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[7] The basic character of the unit should be recalled here. For the case of the three-dimensional Euclidean space, $I=Diag.(1,1,1)$ is not only the basic geometric unit, but also the unit of the entire Lie theory of the rotational and Euclidean symmetries. Similarly, for the case of the Minkowski spacetime, the unit $I=Diag.(1,1,1,1)$ is at the foundations of the entire Lie theory for the Lorentz and Poincaré symmetries. We begin to see in this way the far reaching implications of isotopic generalization of the basic unit.

Following the appearances of these contributions, isospaces have been also studied by a number of authors for both mathematical and physical applications to be studied in subsequent sections, including the definition of isocontinuity, isotopology, isomanifolds, etc. The related literature will be presented in the appropriate subsequent sections.

In this section we identify the basic notions of Santilli isospaces. Specific types of isospaces needed for applications will be studied in subsequent sections.

The coordinates r of ordinary spaces $S(r, m, F)$ are defined on the base field $F = F(a, +, \times)$, thus being real numbers for $F = R$, complex numbers for $F = C$ and quaternionic numbers for $F = Q$.

Consequently, the *isocoordinates* $\hat{r}$ on isospaces $\hat{S}(\hat{r}, \hat{m}, \hat{F})$ must be defined on the isofields $\hat{F} = \hat{F}(\hat{a}, \hat{+}, \hat{\times})$, namely, must be *isonumbers* and, more particularly, be isoreal isonumbers for $\hat{F} = \hat{R}$, isocomplex isonumbers for $\hat{F} = \hat{C}$, and isoquaternionic isonumbers for $\hat{F} = \hat{Q}$.

Since isocoordinates are isonumbers, they can be easily constructed via the same lifting used for isonumbers, resulting in the simple definition

$$r \rightarrow \hat{r} = U \times r \times U^{\dagger} = r \times (U \times U^{\dagger}) = r \times \hat{I}. \tag{3.2.59}$$

Similarly, the metric m on $S(r, m, F)$ is an ordinary matrix in N-dimension whose elements m_{ij} are functions defined on the base field F, thus being real, complex or quaternionic functions depending on the corresponding character of F.

As we shall see shortly, a necessary condition for $\hat{S}(\hat{r}, \hat{M}, \hat{F})$ to preserve the geometric axioms of $S(r, m, F)$ (that is, for $\hat{S}$ to be an isotope of S), is that, when the unit is lifted in the amount $I \rightarrow \hat{I} = 1/\hat{T}$, the metric is lifted by the *inverse* amount $m \rightarrow \hat{m} = \hat{T} \times m$, thus yielding the transform (where the diagonal character of m is taken into account)

$$\begin{aligned} m \rightarrow U^{\dagger -1} \times m \times U^{-1} = (U \times U^{\dagger})^{-1} \times m = \\ = \hat{T} \times m = (\hat{m}_{ij}) = (\hat{T}_i^k \times m_{kj}), \end{aligned} \tag{3.2.60}$$

However, in this case the elements $\hat{m}_{ij}$ are not properly defined on $\hat{S}$ because they are not isonumbers on $\hat{F}$. For this purpose, the correct definition of the *isometric* is given by

$$\hat{M} = \hat{m} \times \hat{I} = (\hat{m}_{ij} \times \hat{I}) = (\hat{m}_{ij}) \times \hat{I}. \tag{3.2.61}$$

As we shall see in the next section, the above definition is independently confirmed by the isotopies of matrices. We, therefore, have the following

DEFINITION 3.2.3 [14]: Let $S(r, m, F)$ be an N-dimensional metric or pseudo-metric space with contravariant coordinates $r = (r^k)$, metric $m = (m_{ij})$ and invariant $r^2 = (r_k \times r^k) \times I = (r^i \times m_{ij} \times r^j) \times I$ over a field F with trivial unit I.

Then, Santilli's isospaces are the N-dimensional isovector spaces

$$\hat{S}(\hat{r}, \hat{M}, \hat{F}) : \quad \hat{r} = (\hat{r}^k) = (r^k) \times \hat{I} \ \in \hat{F}, \tag{3.2.62a}$$

$$\hat{M} = (\hat{T} \times m) \times \hat{I} = (T_i^k \times m_{ki}) \times \hat{I} \ \in \ \hat{F}, \quad \hat{M}^{ij} = [(\hat{M}_{pq})^{-1}]^{ij} \ \in \hat{F}, \tag{3.2.62b}$$

$$\hat{r}^k = \hat{M}^{ki} \hat{\times} \hat{r}_i = \hat{m}^{ki} \times r_i \times \hat{I}, \quad \hat{r}_k = \hat{M}_{ki} \hat{\times} \hat{r}^i = \hat{m}_{ki} \times r^i \times \hat{I}, \tag{3.2.62c}$$

$$\hat{r}^{\hat{2}} = \hat{r}^k \hat{\times} \hat{r}_k = \hat{r}^i \hat{\times} \hat{M}_{ij} \hat{\times} \hat{r}^j = (r^i \times \hat{m}_{ij} \times r^j) \times \hat{I} \ \in \ \hat{F}, \tag{3.2.62d}$$

$$i, j, k, p, q = 1, 2, \ldots, N,$$

and its projection on the original space $S(r, m, F)$, is characterized by

$$\hat{S}(r, \hat{m}, F) : \quad r = (r^k) = (r^k) \times I \ \in \ F; \tag{3.2.63a}$$

$$\hat{m} = \hat{T} \times m = (\hat{T}_i^k \times m_{kj}) \ \in F, \quad \hat{m}^{ij} = [(\hat{m}_{ps})^{-1}]^{ij} \ \in \ F, \tag{3.2.63b}$$

$$r^k = \hat{m}^{ki} \times r_i \ \in \ R, \quad r_k = \hat{m}_{ki} \times r^1 \ \in \ F, \tag{3.2.63c}$$

$$r^2 = r^i \times \hat{m}_{ij} \times r^j) \times I = r^i \times (\hat{T}_i^k \times m_{kj}) \times r^j) \times I \ \in \ F. \tag{3.2.63d}$$

As one can see, expression (3.2.62) is the proper formulation of the isoinvariant on isospaces over the base isofield, and we shall write $\hat{S}(\hat{r}, \hat{M}, \hat{F})$, while expression (3.2.63) is the "projection" of the preceding space in the original space S, and we shall write $\hat{S}(r, \hat{m}, F)$, because the latter space is defined with conventional coordinates, units and products over the conventional field F by construction.

It should be stressed that *isospaces are mathematical spaces and, therefore, all physical calculations and applications will be done in the projection of isospaces over conventional spaces.* In fact, experimental measurements and events can only occur in our space time. Therefore, all physical applications of isospaces can only occur in their projection in our spacetime.

A simple visual inspection of invariants (3.2.56) and (3.2.62) establish the following

THEOREM 3.2.1 [10]: All line elements of metrics or pseudo-metric spaces with metric m and unit I, and all their isotopes possess the following invariance property

$$I \to \hat{I} = n^2 \times I, \quad m \to \hat{m} = n^{-2} \times m, \tag{3.2.64}$$

where n is a non-null parameter.

This property too will soon acquire fundamental character, since it permits the identification, for the first time, of the property that *the Galilean and Poincaré* *symmetries are "eleven" dimensional,* and not ten-dimensional as believed throughout the 20-th century.

In particular, the 11-th invariance is "hidden" in conventional line elements and will permit the first and perhaps only known, axiomatically consistent grand unification of electroweak and gravitational interactions, as studied later on in this chapter.

The nontriviality of isospaces is then expressed by the following

THEOREM 3.2.2 [14]: Even though preserving all topological properties of m (from the positive-definiteness of $\hat{I}$), the projection $\hat{m}$ of the isometric $\hat{M}$ on $\hat{S}$ over $\hat{F}$ into the original space S over F acquires an unrestricted functional dependence on any needed local variables or quantities,

$$\hat{M} \rightarrow \hat{m} = \hat{m}(t, r, p, \psi, \psi^{\dagger}, \ldots). \tag{3.2.65}$$

As we shall see, the above property has truly fundamental implications, since it will permit the first and only known *geometric unification of the Minkowskian and Riemannian geometries with the consequential unification of special and general relativities,* and other applications of manifestly fundamental nature.

By recalling that the basic invariant r^2 represents the square of the "distance" in S, from Eqs. (3.2.56) and (3.2.62) we derive the following additional property

THEOREM 3.2.3 [6,7,10]: The basic invariant of a metric or pseudometric space has the structure:

$$Invariant \quad = \quad [Length]^2 \times [Unit]^2 \tag{3.2.66}$$

The above property will soon have deep geometric implications, such as permitting different shapes, sizes and dimension for the same object under inspection by different observers, all in a way compatible with our sensory perception.

Note that invariant structure (3.2.66) is indeed new because identified for the first time by the isotopies, since the multiplication of the invariant by the unit is trivial for conventional studies and, as such, it was ignored.

It is now important to indicate the *differences between Santilli isospaces $\hat{S}(\hat{r}, \hat{M}, \hat{F})$ or $\hat{S}(r, \hat{m}, F)$ and deformed spaces* that, as well known, are given by the sole deformations of the metric, for which we use the notation $S(r, \hat{m}, F)$.

It is easy to see that *deformed spaces $S(r, \hat{m}, F)$ have a conventional noncanonical or nonunitary structure,* thus activating the theorems of catastrophic inconsistencies of Section 3.4. By comparison, Santilli isospaces have been constructed precisely to resolve these catastrophic inconsistencies via the reconstruction of canonicity or unitarity on isospaces over isofields.

Moreover, *deformed metric spaces $S(r, \hat{m}, F)$ necessarily break the symmetries of the original spaces $S(r, m, F)$, while, as we shall soon see, isospaces $\hat{S}(\hat{r}, \hat{M}, \hat{F})$ reconstruct the exact symmetries of $S(r, g, F)$.*

The implications of the latter property alone are far reaching because *all symmetries believed to be broken in the 20-th century can be proved to remain exact on suitable isospaces over isofields.* In different terms, the "breakings of space-time and internal symmetries" studies through the 20-th century are a direct manifestation of the adaptation of new physical events to a rather limited, pre-existent mathematics because, if the underlying mathematics is suitably lifted, all believed breakings cease to exist, as already proved in Vol. II of this series [7] and updated in this volume.

Santilli's isodual isospaces for the treatment of *antimatter* are the anti-isomorphic image of isospaces under the isodual map (3.2.15) and can be written

$$\hat{S}^d(\hat{r}^d, \hat{M}^d, \hat{F}^d): \quad \hat{r}^d = -\hat{r}^\dagger, \quad \hat{M}^d = -\hat{M}, \tag{3.2.67a}$$

$$\hat{r}^{\hat{2d}} = \hat{r}^d \hat{\times}^d \hat{M}^d \hat{\times}^d \hat{r}^{t,d}. \tag{3.2.67b}$$

Isodual isospaces were introduced in Vol. I of this series [6] and then treated in various other works (see, e.g., [10,17,18]). As we shall see, they play a crucial role for the treatment of antimatter in interior conditions. The tensorial product of isospaces and their isoduals appears to be significant for basic advances in biology, e.g., to achieve a quantitative mathematical representation of bifurcations and other biological behavior.

As we shall see, all **industrial applications** of hadronic mechanics are based on isospaces to such an extent that the new isogeometries have acquired evident relevance for new patents assuredly without prior art, evidently in view of their novelty.

3.2.4 Isofunctional Analysis and its Isodual

The lifting of fields evidently requires a corresponding lifting of functional analysis into a form known as *Kadeisvili isofunctional analysis* since it was first studied by J. V. Kadeisvili [19,20] in 1992. Additional studies were done by A. K. Aringazin *et al.* [21] in 1995 and other authors.

A detailed study of isofunctional analysis was also provided in monographs [6,7] of 1995. A knowledge of these studies is necessary for any application of hadronic mechanics because all conventional functions and transforms have to be properly lifted for consistent applications, while the use of conventional (or improperly lifted) functions and transforms leads to catastrophic inconsistencies.

In essence, the consistent formulation of isofunctional analysis requires not only the preservation of the original axioms, but also the preservation of the original numerical values when formulated on isospaces over isofields, under which conditions the broadening of conventional formulations emerge in the projection of the isotopic treatment in the original space.

The latter mathematical requirement has deep physical implications, such as the preservation of the speed of light *in vacuum* as the universal invariant on *isospaces over isofield,* with consequential preservation under isotopies of all axioms of special relativity, while locally varying speeds of light within physical media emerge in the *projection* of the isospace in our spacetime, as we shall see in subsequent sections.

The scope of this section is essentially that of providing the guidelines for the updating of Refs. [19,20,16,6,7] along the above requirements to achieve compatibility with the main lines of this presentation.

DEFINITION 3.2.4 [19,20,21, 6,7] Let $f(x)$ be an ordinary (sufficiently smooth) function on a vector space S with local variable x (such as a coordinate) over the reals R. The isotopic image of $f(x)$, called isofunctions, can be constructed via the use of a noncanonical-nonunitary transform

$$U \times f(x) \times U^{\dagger} = f(x) \times \hat{I} \quad \in \quad \hat{F}, \tag{3.2.68}$$

reformulated on isospace $\hat{S}(\hat{x}, \hat{F})$ over the isofield $\hat{F}$

$$f(x) \times \hat{I} = f(\hat{T} \times \hat{x}) \times \hat{I} = \ \hat{f}(\hat{x}) \in \hat{F}, \tag{3.2.69}$$

with projection in the original space $S(x, F)$

$$f(\hat{T} \times x) \in F. \tag{3.2.70}$$

As one can see, expression (3.2.68) coincides with the definition of isofunction in the quoted references. A feature identified since that time is the re-interpretation in such a way that the function $f(x)$ preserves its numerical value when formulated as $\hat{f}(\hat{x})$ on the isospace $\hat{S}$ over the isofield $\hat{F}$ because the variable $\hat{x}$ is multiplied by $\hat{T}$ while the unit to which such a variable is referred to is multiplied by the *inverse* amount $\hat{I} = 1/\hat{T}$. All numerical differences emerge in the *projection* of $\hat{f}(\hat{x})$ in the original space.

This is essentially the definition of isofunctions that will allow us to preserve the basic axioms of special relativity on isospaces over isofields and actually expand their applicability from motion in empty space to motion within physical media.

For the case of the simple function $f(x) = x$ we have the lifting

$$\hat{x} = U \times x \times U^{\dagger} = x \times (U \times U^{\dagger}) = x \times \hat{I} = \hat{T} \times \hat{x} \times \hat{I} \ \in \ \hat{F}, \tag{3.2.71}$$

with the projection in the original space S being simply given in this case by $\hat{T} \times x$.

More instructive is the lifting of the exponentiation into the *isoexponentiation* given by

$$e^x \rightarrow U \times e^x \times U^{\dagger} =$$

$$= U \times (I + x/1! + x \times x/2! + \ldots) \times U^\dagger =$$
$$= \hat{I} + \hat{x}\hat{/}\hat{1}! + \hat{x}\hat{\times}\hat{x}\hat{/}\hat{2}! + \ldots) =$$
$$= \hat{e}^{\hat{x}} = (e^{\hat{x}\times\hat{T}}) \times \hat{I} = \hat{I} \times (e^{\hat{T}\times\hat{x}}) \ \in \ \hat{F}, \tag{3.2.72}$$

with projection in the original space S given by

$$\hat{e}^x = (e^x \times \hat{T}) \times I = I \times (e^{\hat{T}\times x}) \ \in F, \tag{3.2.73}$$

where one should note that the function in isospace is computed over $\hat{F}$ while its projection in the original space is computed in the original field F.

The above lifting is nontrivial because of the appearance of the nonlinear integro-differential quantity $\hat{T}(t, x, \psi, \partial\psi, \ldots)$ in the exponent. As we shall see shortly, this feature permits the first known extension of the linear and local Lie theory to nonlinear and nonlocal formulations.

Let $M(x) = (M_{ij}(x))$ be an N-dimensional matrix with elements $M_{ij}(x)$ on a conventional space $S(x, F)$ with local coordinates x over a conventional field F with unit I. Then, the isotopic image of $M(x)$ or it isomatrix, is defined by

$$\hat{M}(\hat{x}) = (\hat{M}_{ij}(\hat{x})) = M(\hat{T} \times \hat{x}) \times \hat{I}, \ \ \hat{M}_{ij} \in \hat{F}, \tag{3.2.74}$$

Similarly, the *isodeterminant* of $\hat{M}$ is defined by

$$\hat{\mathrm{Det}}\hat{M} = [\mathrm{Det}(\hat{T} \times M)] \times \hat{I} \tag{3.2.75}$$

where Det represents the conventional determinant, with the preservation of the conventional axioms, e.g.,

$$\hat{\mathrm{Det}}(\hat{M}_1 \hat{\times} \hat{M}_2) = \hat{\mathrm{Det}}(\hat{M}_1) \hat{\times} \hat{\mathrm{Det}}(\hat{M}_2); \tag{3.2.76a}$$

$$\hat{\mathrm{Det}}(\hat{M}^{-\hat{I}}) = (\hat{\mathrm{Det}}\hat{M})^{-\hat{I}}, \tag{3.2.76b}$$

Note that, by construction, isomatrices and isodeterminant preserve the original values on isospaces over isofields, although show deviations when the same quantities are observed from the original space, that is, referred to the original unit.

Similarly, the *isotrace* of $\hat{M}$ is defined by[8]

$$\hat{Tr}\hat{M} = [Tr(\hat{T} \times M)] \times \hat{I}, \tag{3.2.77}$$

[8]The isodeterminant introduced in Ref. [6], Eq. (6.3.19) is the correct form as in Eq. (3.2.77) above. However, the isotrace introduced in Eq. (6.3.20a) of Ref. [6] preserves the axioms of a trace, but not its value, as a consequence of which it is not fully invariant, the correct definition of isotrace being given by Eq. (3.2.77) above.

where Tr is the conventional trace, and it also verifies the conventional axioms, such as

$$\hat{T}r(\hat{M}_1\hat{\times}\hat{M}_2) = \hat{T}r\hat{M}_1\hat{\times}\hat{T}r\hat{M}_2, \tag{3.2.78a}$$

$$\hat{T}r(\hat{M}^{-\hat{I}}) = (\hat{T}r\hat{M})^{-\hat{I}}. \tag{3.2.78b}$$

The *isologarithm* is hereon defined by[9]

$$\hat{\log}_{\hat{e}}\hat{a} = \log_e a \times \hat{I}, \tag{3.2.79}$$

and admit the unique solution

$$\hat{\log}_{\hat{e}}\hat{a} = \log_e(\hat{T} \times a) \times \hat{I}, \tag{3.2.80}$$

under which the conventional axioms are preserved,

$$\hat{e}^{\hat{\log}_{\hat{e}}\hat{a}} = \hat{a}, \tag{3.2.81a}$$

$$\hat{\log}_{\hat{e}}\hat{e} = \hat{I}, \ \ \hat{\log}_{\hat{e}}\hat{I} = 0, \tag{3.2.81b}$$

$$\hat{\log}_{\hat{e}}(\hat{a}\hat{\times}\hat{b}) = \hat{\log}_{\hat{e}}\hat{a} + \hat{\log}_{\hat{e}}\hat{b}, \tag{3.2.81c}$$

$$\hat{\log}_{\hat{e}}(\hat{a}\hat{/}\hat{b}) = \hat{\log}_{\hat{e}}\hat{a} - \hat{\log}_{\hat{e}}\hat{b}, \tag{3.2.81d}$$

$$\hat{\log}_{\hat{e}}(\hat{a}^{-\hat{I}}) = -\hat{\log}_{\hat{e}}\hat{a}, \tag{3.2.81e}$$

$$\hat{b}\hat{\times}\hat{\log}_{\hat{e}}\hat{a} = \hat{\log}_{\hat{e}}(\hat{a}^{\hat{b}}). \tag{3.2.81f}$$

The lifting of trigonometric functions is intriguing and instructive (see Chapter 6 of Ref. [6] and Chapter 5 of Ref. [7] whose results in this case require no upgrading). Let $E(r, \delta, R)$ be a conventional two-dimensional Euclidean space with coordinates $r = (x, y)$ on the reals R and polar representation $x = r \times \cos\theta$ and $y = r \times \sin\theta$, $x^2 + y^2 = r^2 \times (\cos^2\theta + \sin^2\theta) = r^2$. Consider now the *iso-Euclidean space* in two dimension

$$\hat{E}(\hat{r}, \hat{\delta}, \hat{R}): \quad \hat{\delta} = Diag.(n_1^{-2}, n_2^{-2}), \ \ \hat{I} = Diag.(n_1^2, n_2^2), \tag{3.2.82a}$$

$$\hat{r}^{\hat{2}} = (x^2/n_1^2 + y^2/n_2^2) \times \hat{I} \ \ \in \ \ \hat{R}. \tag{3.2.82b}$$

Then, the *isopolar coordinates* and related *isotrigonometric functions* on $\hat{E}$ are defined by

$$\hat{x} = \hat{r}\hat{\times}\hat{\cos}\hat{\phi}, \tag{3.2.83a}$$

$$\hat{\cos}\hat{\phi} = n_1 \times \cos(\phi/n_1 \times n_2), \tag{3.2.83b}$$

[9]Note, again, that a different definition of isologarithm was assumed in Eq. (6.7.5) of Ref. [6].

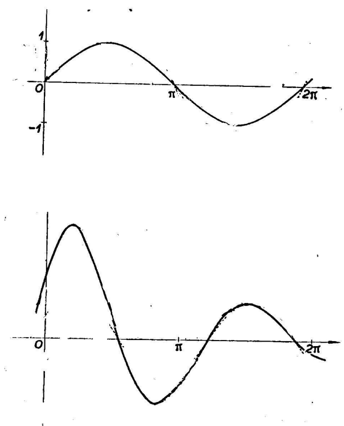

Figure 3.4. A schematic view of the conventional sinus function in Euclidean and iso-Euclidean spaces (top view) and of the projection of a possible example of the isosinus function in the conventional space.

$$\hat{y} = \hat{r}\hat{\times}\hat{\sin}\hat{\phi}, \tag{3.2.83c}$$

$$\hat{\sin}\hat{\phi} = n_2 \times \sin(\phi/n_1 \times n_2), \tag{3.2.83d}$$

and they preserve the axioms of conventional trigonometric functions, such as,

$$\hat{r}^{\hat{2}} = (x^2/n_1^2 + y^2/n_2^2) \times \hat{I} \ = r^2 \times \hat{I} \in \ \hat{R}. \tag{3.2.84}$$

The isotopy of spherical coordinates are treated in detail in Section 5.5 of Ref. [7]. For self-sufficiency of this volume we recall that their definition requires a three-dimensional iso-Euclidean space

$$\hat{E}(\hat{r}, \hat{\delta}, \hat{R}) : \ \ \hat{\delta} = Diag.(n_1^{-2}, n_2^{-2}, n_3^{-2}), \ \ \hat{I} = Diag.(n_1^2, n_2^2, n_3^2), \tag{3.2.85a}$$

$$\hat{r}^{\hat{2}} = (x^2/n_1^2 + y^2/n_2^2 + z^2/n_3^2) \times \hat{I} \ \in \ \hat{R}. \tag{3.2.85b}$$

The isotopies of the conventional spherical coordinates in $E(r, \delta, R)$ then yields the following *isospherical coordinates* here presented in the projected form on $\hat{E}(r, \hat{\delta}, R)$

$$x = r \times n_1 \times \sin(\theta/n_3) \times \sin(\phi/n_1 \times n_2), \tag{3.2.86a}$$

$$y = r \times n_2 \times \sin(\theta/n_3) \times \cos(\phi/n_1 \times n_2), \tag{3.2.86b}$$

$$z = r \times n_3 \times \cos(\theta/n_3). \tag{3.2.86c}$$

Via the use of the above general rules, the reader can now construct all needed isofunctions.

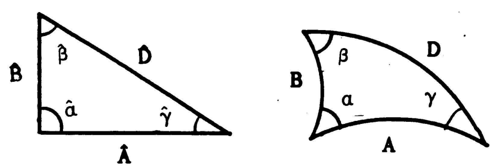

Figure 3.5. An intriguing application of isotrigonometric functions, the generalization of the conventional Pythagorean Theorem (left view) to triangles with curvilinear sides (right view). This is due to the fact that conventional triangles and the Pythagorean theorem are preserved identically on isospaces over isofields, but the projection on conventional Euclidean spaces of straight lines in isospaces over isofields are curves. Therefore in isospace we have expressions such as $\hat{A} = \hat{D}\hat{\times}\text{isosinus}(\hat{\gamma})$ with projections in the conventional space for curvilinear sides $A = D \times \text{isosinus}(\gamma)$, where A and D are now the lengths of the curvilinear sides.

The reader should meditate a moment on the isotrigonometric functions. In fact, they provide a *generalization of the Pythagorean theorem to curvilinear triangles.* This is due to the fact that the projection of $\hat{E}(\hat{r}, \hat{\delta}, \hat{R})$ into the original space $E(r, \delta, R)$ characterizes indeed curvilinear triangles, trivially, because the n-characteristic quantities are functions.

However, the reader is suggested to verify that the *isotriangle*, that is, the image on $\hat{E}$ of an ordinary triangle on E coincides with the latter because the changes caused by the lifting are compensated by the inverse changes of the unit.

By noting that their value must be isonumbers, the *isointegral* can be defined by (here expressed for the simple case of isounits independent form the integration variable)

$$\hat{\int} d\hat{r} = \hat{I} \times \int \hat{T} \times d(r \times \hat{I}) = \hat{I} \times \int dr, \qquad (3.2.87)$$

whose extension to the case of isounits with an explicit functional dependence on the integration variables has a complexity that goes beyond the elementary level of this presentation.

Isointegrals and isoexponentiations then permit the introduction of the following *Fourier-Kadeisvili isotransforms*, first studied in Ref. [19,20] (also represented here to avoid excessive mathematical complexities for the simpler case of isounits without an explicit dependence on the integration variables)[10]

$$\hat{f}(\hat{x}) = (\hat{1}\hat{/}\hat{2\pi})\hat{\times}\hat{\int}_{-\infty}^{+\infty} \hat{g}(\hat{k})\hat{\times}\hat{e}^{\hat{i}\hat{\times}\hat{x}}\hat{d}\hat{k}, \qquad (3.2.88a)$$

[10]The reader should be aware that in most applications of hadronic mechanics the isounits can be effectively approximated into constants, thus avoiding the complex mathematics needed for isointegrals and isotransforms with an explicit functional dependence on the integration variables.

$$\hat{g}(\hat{x}) = (\hat{1}\hat{/}2\hat{\pi})\hat{\times}\hat{\int}_{-\infty}^{+\infty} \hat{f}(\hat{k})\hat{\times}\hat{e}^{\hat{i}\hat{\times}\hat{x}}\hat{d}\hat{x}, \tag{3.2.88b}$$

with similar liftings for Laplace transforms, etc. Other transforms can be defined accordingly [6].

We confirm in this way a major feature of isomathematics, the fact that *Hamiltonian quantities preserve not only their axioms, but also their numerical value under isotopic lifting when defined on isospaces over isofields, and all deviations occur in the projection of the lifting into the original space.*

The explicit construction of the *isodual isofunctional analysis* is also instructive and intriguing because they reveal properties that have essentially remained unknown until recently, such as the fact that *the isofourier transforms are isoselfdual* (see also Refs. [6,7]).

3.2.5 Isodifferential Calculus and its Isodual

As indicated in Chapter 1, the delay to complete the construction of hadronic mechanics since its proposal in 1978 [5] was due to difficulties in identifying the origin of the non-invariance of its initial formulation, that is, the lack of prediction of the same numerical values for the same quantities under the same conditions, but at different times, a fundamental invariance property fully verified by quantum mechanics.

These difficulties were related to the lack of a consistent isotopic lifting of the familiar quantum mechanical momentum. More particular, *all* aspects of quantum mechanics could be consistently and easily lifted via a nonunitary transforms, except the eigenvalue equation for the linear momentum, as shown by the following lifting

$$\begin{aligned} p \times \psi(t,r) = -i \times \hbar \times \frac{\partial}{\partial r}\psi(t,r) = K \times \psi(t,r) \rightarrow \\ \rightarrow U \times [p \times \psi(t,r)] = (U \times p \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times [U \times \psi(t,r)] = \\ = \hat{p} \times \hat{T} \times \hat{\psi}(\hat{t},\hat{r}) = \hat{p}\hat{\times}\hat{\psi}(\hat{t},\hat{r}) = \\ = -i \times \hbar \times U[\frac{\partial}{\partial r}\psi(t,r)] = K \times U \times \psi(t,r) = \hat{K}\hat{\times}\hat{\psi}(\hat{t},\hat{r}), \end{aligned} \tag{3.2.89}$$

where $\hat{K} = K \times \hat{I}$ is an isonumber.

As one can see, the initial and final parts of the lifting are elementary. The problem rested in the impossibility of achieving a consisting lifting of the intermediate step, that based on the partial derivative.

In the absence of a consistent isotopy of the linear momentum, the early studies of hadronic mechanics lacked consistent formulations of physical quantities depending on the isomomentum, such as the isotopies of angular momentum, kinetic energy, etc.

The origin of the above problem resulted in being where expected the least, in the *ordinary differential calculus,* and this explains the delay in the resolution of the impasse.

The above problem was finally resolved by Santilli in the second edition of Refs. [6,7] of 1995 (see Section 5.4.B of Vol. I and Section 8.4.A of Vol. II) with a mathematical presentation in memoir [10] of 1996. The resulting generalization of the ordinary differential calculus, today known as *Santilli's isodifferential calculus,* plays a fundamental role for these studies beginning with the first known structural generalization of Newton's equations in Newtonian mechanics, and then passing to the correct invariant formulation of all dynamical equations of hadronic mechanics.

For centuries, since its discovery by Newton and Leibnitz in the mid 1600, the ordinary differential calculus had been assumed to be independent from the basic unit and field, and the same assumption was kept in the earlier studies on hadronic mechanics, resulting in the lack of full invariance, inability to formulate physical models and other insufficiencies.

After exhausting all other possibilities, an inspection of the differential calculus soon revealed that, contrary to an erroneous belief kept in mathematics for about four centuries, the ordinary differential calculus is indeed dependent on the basic unit and related field.

In this section we review Santilli's isodifferential calculus in its version needed for applications and verifications of hadronic mechanics. This update is recommendable because of various presentations in which the role of $\hat{I}$ and $\hat{T}$ were interchanged, resulting in possible ambiguities that could cause loss of invariance even under the lifting of the differential calculus.

A main feature is that, *unlike all other aspects of hadronic mechanics, the isotopies of the differential calculus cannot be reached via the use of a noncanonical or nonunitary transform, and have to be built via different, yet compatible methods.*

Let $S(r, m, R)$ an N-dimensional metric or pseudo-metric space with *contravariant* coordinates $R = (r^k)$, metric $m = (m_{ij})$, $i, j, k = 1, 2, \ldots, N$, and conventional unit $I = Diag.(1, 1, \ldots, 1)$ on the reals R. Let $f(r)$ be an ordinary (sufficiently smooth) function on S, let dr^k be the differential in the local coordinates, and let $\partial f(r)/\partial r^k$ be its partial derivative.

As it is well known, the connection between covariant and contravariant coordinates is characterized by the familiar rules

$$r^k = m^{kj} \times r_j, \quad r_i = m_{ik} \times r^k, \tag{3.2.90a}$$

$$m^{ij} = [(m_{qw})^{-1}]^{ij}. \tag{3.2.90b}$$

Let $\hat{S}(\hat{r}, \hat{M}, \hat{R})$ be an isotope of S with N-dimensional isounit $\hat{I} = (\hat{I}^i_j)$, contravariant isocoordinates $\hat{r} = (r^k) \times \hat{I}$ and isometric $\hat{M} = (\hat{M}_{ij}) = (\hat{T}^s_i \times m_{sj}) \times \hat{I}$ on the isoreals $\hat{R}$.

The connection between covariant and contravariant isocoordinates is then given by

$$\hat{r}^k = \hat{M}^{kj} \hat{\times} \hat{r}_j, \quad \hat{r}_i = \hat{M}_{ik} \hat{\times} \hat{r}^k, \tag{3.2.91a}$$

$$\hat{M}^{ij} = [(\hat{M}_{qw})^{-1}]^{ij}. \tag{3.2.91b}$$

Therefore, on grounds of compatibility with the metric and subject to verifications later on geometric grounds, we have the following:

LEMMA 3.2.5 [10]: Whenever the isounit of contravariant coordinates $\hat{r}^k$ on an isospace $\hat{S}(\hat{r}, \hat{M}, \hat{R})$ is given by

$$\hat{I} = (\hat{I}^i_j(t, r, \ldots)) = 1/\hat{T} = (\hat{T}^j_i)^{-1}, \tag{3.2.92}$$

the isounit for the related covariant coordinates $\hat{r}_k$ is given by its inverse

$$\hat{T} = (\hat{T}^i_j(t, r, \ldots)) = 1/\hat{I} = (\hat{I}^i_j)^{-1}, \tag{3.2.93}$$

and viceversa.

The *ordinary differential* of the contravariant isocoordinates is given by $d\hat{r}^k$ with covariant counterpart $d\hat{r}_k$ and they clearly do not constitute an isotopy. The condition for the preservation of the original axioms and value for constant isounits then leads to the following

DEFINITION 3.2.5 [6,7,10]: The isodifferentials of contravariant and covariant coordinates are given respectively by[11]

$$\hat{d}\hat{r}^k = \hat{d}(r^k \times \hat{I}) = \hat{T}^k_i \times d(r^i \times \hat{I}), \tag{3.2.94a}$$

$$\hat{d}\hat{r}_k = \hat{d}(r_k \times \hat{T}) = \hat{I}^i_k \times d(r_i \times \hat{T}). \tag{3.2.94b}$$

LEMMA 3.2.6 [loc. cit.]: For one-dimensional isounits independent from the local variables, isodifferentials coincide with conventional differentials,

$$\hat{d}\hat{r}^k \equiv dr^k, \quad \hat{d}\hat{r}_k \equiv dr_k. \tag{3.2.95}$$

[11] It should be noted that the role of $\hat{I}$ and $\hat{T}$ in this definition and that of Ref. [10] are inverted. Also, the reader should keep in mind that, since they are assumed to be Hermitian, isounits can always be diagonalized. In fact, diagonal isounits are sufficient for the verifications and applications of hadronic mechanics, while leaving to the interested reader the formulation of hadronic mechanics according to the broader isodifferential calculus of Refs. [6,7,10].

Note that the above property constitutes a *new invariance of the differential calculus.* Its trivial character explains the reason isodifferential calculus escaped detection for centuries. Needless to say, the above triviality is lost for isounit with nontrivial functional dependence from the local variables as it is generally the case for hadronic mechanics.

The *ordinary derivative* of an isofunction of contravariant coordinates is evidently given by

$$\frac{\partial \hat{f}(\hat{r}^k)}{\partial \hat{r}^k} = \lim_{\hat{d}\hat{r}^k \to 0} \frac{\hat{f}(\hat{r}^k + \hat{d}\hat{r}^k) - \hat{f}(\hat{r}^k)}{\hat{d}\hat{r}^k} \tag{3.2.96}$$

with covariant version

$$\frac{\partial \hat{f}(\hat{r}_k)}{\partial \hat{r}_k} = \lim_{\hat{d}\hat{r}_k \to 0} \frac{\hat{f}(\hat{r}_k + \hat{d}\hat{r}_k) - \hat{f}(\hat{r}_k)}{\hat{d}\hat{r}_k}. \tag{3.2.97}$$

It is then simple to reach the following

DEFINITION 3.2.4 [loc. cit.]: The isoderivative of isofunctions on contravariant and covariant isocoordinates are given respectively by

$$\frac{\hat{\partial} \hat{f}(\hat{r}^k)}{\hat{\partial} \hat{r}^k} = \hat{I}^i_k \times \frac{\partial \hat{f}(\hat{r}^i)}{\partial \hat{r}^k}, \tag{3.2.98a}$$

$$\frac{\hat{\partial} \hat{f}(\hat{r}_k)}{\hat{\partial} \hat{r}_k} = \hat{T}^k_i \times \frac{\partial \hat{f}(\hat{r}_i)}{\partial \hat{r}^k}, \tag{3.2.98b}$$

where the isoquotient is tacitly assumed.[12]

A few examples are now in order to illustrate the axiom-preserving character of the isodifferential calculus. Assume that the isounit is not dependent on r. Then, for $\hat{f}(\hat{r}^k) = \hat{r}^k$ we have

$$\frac{\hat{d}\hat{r}^i}{\hat{d}\hat{r}^j} = \hat{\delta}^i_j = \delta^i_j \times \hat{I}. \tag{3.2.99}$$

Similarly we have

$$\frac{\hat{d}(\hat{r}^i)^{\hat{n}}}{\hat{d}\hat{r}^j} = \hat{\delta}^i_j \times \hat{(}r^i)^{\hat{n}-\hat{1}}. \tag{3.2.100}$$

[12]Note that the isofunction in the numerator contains an additional isounit, $\hat{f} = f \times \hat{I}$, that, however, cancels out with the isounit of the isoquotient, $\hat{/} = / \times \hat{I}$, resulting in expressions (3.2.98). Note also the lack of presence of a *factorized* isounit in the definition of the isodifferentials and isoderivatives, and this explains why the isodifferential calculus cannot be derived via noncanonical or nonunitary transforms.

It is instructive for the reader interested in learning Santilli isodifferential calculus to prove that *isoderivatives in different variables "isocommute" on isospace over isofields,*

$$\frac{\hat{\partial}}{\hat{\partial}\hat{r}^i}\frac{\hat{\partial}}{\hat{\partial}\hat{r}^j} = \frac{\hat{\partial}}{\hat{\partial}\hat{r}^j}\frac{\hat{\partial}}{\hat{\partial}\hat{r}^i}, \tag{3.2.101}$$

but their projections on ordinary spaces over ordinary fields do not necessarily "commute".

We are now sufficiently equipped to point out the completion of the construction of hadronic mechanics. First, let us verify the axiom-preserving character of the isoderivative of the isoexponent in a contravariant coordinate for the simple case in which the isounit does not depend on the local variables. In fact, we have the expression

$$\frac{\hat{\partial}}{\hat{\partial}\hat{r}}\hat{e}^{\hat{r}} = \hat{I} \times \frac{\partial}{\partial\hat{r}}[\hat{I} \times e^{\hat{T}\times\hat{r}}] = \hat{I} \times \hat{T} \times [\hat{I} \times e^{\hat{T}\times\hat{r}}] = \hat{e}^{\hat{r}}. \tag{3.2.102}$$

Consider now the *isoplanewave* as a simply isotopy of the conventional planewave solution (again for the case in which the isounit does not depend explicitly on the local coordinates),

$$\hat{e}^{\hat{i}\hat{\times}\hat{r}\hat{\times}\hat{K}} = \hat{I} \times e^{i\times\hat{T}\times K\times\hat{r}}, \tag{3.2.103}$$

for which we have the isoderivatives

$$\frac{\hat{\partial}}{\hat{\partial}\hat{r}}\hat{e}^{\hat{i}\hat{\times}\hat{r}\hat{\times}\hat{K}} = \hat{I} \times \frac{\partial}{\partial\hat{r}}[\hat{I} \times e^{i\times\hat{T}\times K\times\hat{r}}] =$$

$$= -i \times K \times \hat{I} \times e^{i\times\hat{T}\times K\times\hat{r}} = \hat{i}\hat{\times}\hat{K}\hat{\times}\hat{e}^{\hat{i}\hat{\times}\hat{r}\hat{\times}\hat{K}}. \tag{3.2.104}$$

We reach in this way the following fundamental definition of *isomomentum*, first achieved by Santilli in Refs. [6,7] of 1995, that completed the construction of hadronic mechanics (its invariance will be proved later on in Section 3.5).

DEFINITION 3.2.7 [6,7,10]: The isolinear momentum on an iso-Hilbert space over the isofield of isocomplex numbers $\hat{C}$ (see Section 3.5 for details) is characterized by

$$\hat{p}_k\hat{\times}\hat{\psi}(\hat{t},\hat{r}) = -\hat{i}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{r}^k}\hat{\psi}(\hat{t},\hat{r}) = -i\hat{\times}\hat{I}_k^i \times \frac{\partial}{\partial\hat{r}^i}\hat{\psi}(\hat{t},\hat{r}) = \hat{K}\hat{\times}\hat{\psi}(\hat{t},\hat{r}). \tag{3.2.105}$$

Comparing the above formulation with Eq. (3.2.89), and in view of invariance (3.2.95),we reach the following

THEOREM 3.2.4 [6,7,10]: Planck's constant $\hbar$ is the fundamental unit of the differential calculus underlying quantum mechanics, i.e., quantum mechanical eigenvalue equations can be identically reformulated in terms of the isodifferential calculus with basic isounit $\hbar$,

$$p \times \psi(t,r) = -i \times \hbar \times \frac{\partial}{\partial r}\psi(t,r) \equiv -i \times \frac{\hat{\partial}}{\hat{\partial} r}\psi(t,r). \tag{3.2.106}$$

In conclusion, *Santilli's isodifferential calculus establishes that the isounit not only is the algebraic unit of hadronic mechanics, but also replaces Planck's constant with an integro-differential operator $\hat{I}$, as needed to represent contact, nonlinear, nonlocal and nonpotential effects.*

More specifically, Santilli's isodifferential calculus establishes that, while in exterior dynamical systems such as atomic structures, we have the conventional quantization of energy, in interior dynamical systems such as in the structure of hadrons, nuclei and stars, we have a superposition of quantized energy level at atomic distances plus continuous energy exchanges at hadronic distances.

Needless to say, all models of hadronic mechanics will be restricted by the condition

$$\lim_{r \to \infty} \hat{I} \equiv \hbar, \tag{3.2.107}$$

under which hadronic mechanics recovers quantum mechanics uniquely and identically.

DEFINITION 3.2.8 [6,7,17]: The isodual isodifferentials are defined by

$$\hat{d}^d \hat{r}^d = (-\hat{d}^\dagger)(-\hat{r}^\dagger) = \hat{d}\hat{r}, \tag{3.2.108}$$

while isodual isoderivatives are given by

$$\hat{\partial}^d \hat{f}^d(\hat{r}^d)\hat{/}^d \hat{d}^d \hat{r}^d = -\hat{\partial}\hat{f}(\hat{r})\hat{/}\hat{d}\hat{r}. \tag{3.2.109}$$

THEOREM 3.2.5 [6,7,17]: Isodifferentials are isoselfduals.

The latter new invariance constitutes an additional, reason why the isodual theory of antimatter escaped attention during the 20-th century.

3.2.6 Kadeisvili's Isocontinuity and its Isodual

The notion of continuity on an isospace was first studied by Kadeisvili [19] in 1992 and it is today known as *Kadeisvili's isocontinuity.* A review up to 1995 was presented in monographs [6,7]. Rigorous mathematical study of isocontinuity has been done by Tsagas and Sourlas [22–23], R. M. Falcón Ganfornina and J. Núñez Valdés [24–26] and others. For mathematical studies we refer the interested reader to the latter papers. For the limited scope of this volume we shall present the notion of isocontinuity in its most elementary possible form.

Let $\hat{f}(\hat{r}) = f(\hat{T} \times \hat{r}) \times \hat{I}$ be an isofunction on an isospace $\hat{S}$ over the isofield $\hat{R}$. The *isomodulus* of said isofunction is defined by [19]

$$\hat{|}\hat{f}(\hat{r})\hat{|} = |f(\hat{T} \times \hat{r})| \times \hat{I}. \tag{3.2.110}$$

DEFINITION 3.2.9 [19,20]: An infinite sequence of isofunctions $\hat{f}_1(\hat{r})$, $\hat{f}_2(\hat{r})$, ... is said to be "strongly isoconvergent" to the isofunction $\hat{f}(\hat{r})$ when

$$\lim_{k \to \infty} \hat{|}\hat{f}_k(\hat{r}) - \hat{f}(\hat{r})\hat{|} \hat{=} \hat{0}. \tag{3.2.111}$$

while the "iso-Cauchy condition" can be defined by

$$\hat{|}\hat{f}_m(\hat{r}) - \hat{f}_n(\hat{r})\hat{|} \ < \ \hat{\delta} = \delta \times \hat{I}, \tag{3.2.112}$$

where δ is a sufficiently small real number, and m and n are integers greater than a suitably chosen neighborhood of δ.

The isotopies of other notions of continuity, limits, series, etc. can be easily constructed (see Refs. [6,7] for physical treatments and Refs. [22–26] for mathematical treatments).

Note that *functions that are conventionally continuous are also isocontinuous.* Similarly, *a series that is strongly convergent is also strongly isoconvergent.* However, a series that is strongly isoconvergent is not necessarily strongly convergent. We reach in this way the following important

THEOREM 3.2.6 [6,7]: Under the necessary continuity and regularity conditions, a series that is conventionally divergent can always be turned into a convergent isoform under a suitable selection of the isounit.

This mathematically trivial property has far reaching implications, e.g., the achievement, for the first time in physics, of convergent perturbative series for strong interactions, which perturbative treatments are conventionally divergent (see Section 3.4).

Similarly, the reader may be interested in knowing that, given a function which is not square-integrable in a given interval, there always exists an isotopy which turns the function into a square-integrable form [6,7]. The novelty is due to the fact that the underlying mechanism is not that of a weight function, but that of altering the underlying field.

The *isodual isocontinuity* is a simple isodual image of the preceding notions of continuity and will be hereon assumed.

3.2.7 TSSFN Isotopology and its Isodual

Topology is the ultimate foundation of quantitative sciences because it identifies on rigorous mathematical grounds the limitations of the ensuing description.

Throughout the 20-th century, all quantitative sciences, including particle physics, nuclear physics, astrophysics, superconductivity, chemistry, biology, etc., have been restricted to the use of mathematics based on the conventional *local-differential topology,* with the consequence that the sole admitted representations are those dealing with a finite number of isolated point-like particles.

Since points are dimensionless, they cannot have contact interactions. Therefore, an additional consequence is that the sole possible interactions are those of action-at-a-distance type representable with a potential.

In conclusion, the very assumption of the conventional local-differential topology, such as the conventional topology for the Euclidean space, or the Zeeman topology for the Minkowski space, uniquely and unambiguously restrict the admitted systems to be local, differential and Hamiltonian.

This provided an approximation of systems that proved to be excellent whenever the mutual distances of particles are much greater than their size as it is the case for planetary and atomic systems.

However, the above conditions are the exception and not the rule in nature, because all particles have a well defined extended wavepacket and/or charge distribution of the order of 10^{-13} cm. It is well known in pure and applied mathematics that the representation of the actual shape of particles is impossible with a local-differential topology.

Moreover, once particles are admitted as being extended, there is the emergence of the additional contact, zero-range nonpotential interactions that are nonlocal in the sense of occurring in a finite surface or volume that cannot be consistently reduced to a finite number of isolated points.

Consequently, it is equally know by experts that conventional local-differential topologies cannot represent extended particles at short distances and their nonlocal-nonpotential interactions, as expected in the structure of planets, strongly interacting particles, nuclei, molecules, stars and other interior dynamical systems.

The need to build a new topology, specifically conceived and constructed for hadronic mechanics was suggested since the original proposal [5] of 1978. It was not only until 1995 that the Greek mathematicians Gr. Tsagas and D. S. Sourlas [22,23] proposed the first *isotopology* on scientific record formulated on isospaces over ordinary fields. In 1996, the Italian-American physicist R. M. Santilli [10] extended the formulation to isospaces over isofields. Finally, comprehensive studies on isotopology were conducted by the Spanish Mathematicians R. M. Falcón Ganfornina and J. Núñez Valdés [24,25]. As a result, the new topology is hereon called the *Tsagas-Sourlas-Santilli-Falcón-Núñez isotopology* (or TSSFN Isotopology for short).

The author has no words to emphasize the far reaching implications of the new TSSFN isotopology because, for the first time in the history of science, mathematics can consistently represent the actual extended, generally nonspherical and deformable shape particles, their densities as well as their nonpotential and nonlocal interactions.

As an example, Newton's equations have remained unchanged in Newtonian mechanics since the time of their conception to represent point-particles. No consistent generalization was possible due to the underlying local-differential topology and related differential calculus. As we shall see in the next section, the isodifferential calculus and underlying isotopology will permit the first known structural generalization of Newton's equations in Newtonian mechanics for the representation of extended particles.

New coverings of quantum mechanics, quantum chemistry, special relativity, and other quantitative sciences are then a mere consequence. Perhaps more importantly, the new clean energies and fuels permitted by hadronic mechanics can see their origin precisely in the TSSFN isotopology, as we shall see later on in this chapter.

In their most elementary possible form accessible to experimental physicists, the main lines of the new isotopology can be summarized as follows. Being nowhere singular, Hermitian and positive-definite, N-dimensional isounits can always be diagonalized into the form

$$\hat{I} = Diag.(n_1^2, n_2^2, \ldots, n_N^2), \quad n_k = n_k(t, r, v, \ldots) > 0, \quad k = 1, 2, \ldots, N. \quad (3.2.113)$$

Consider N isoreal isofields $\hat{R}_k(\hat{n}, \hat{+}, \hat{\times})$ each characterized by the isounit $\hat{I}_k = n_k^2$ with (ordered) Cartesian product

$$\hat{R}^N = \hat{R}_1 \times \hat{R}_2 \times \ldots \times \hat{R}_N. \quad (3.2.114)$$

Since each isofield $\hat{R}_k$ is isomorphic to the conventional field of real numbers $R(n, +, \times)$, it is evident that $\hat{R}^N$ is isomorphic to the Cartesian product of N ordinary fields

$$R^N = R \times R \times \ldots \times R. \quad (3.2.115)$$

Let

$$\tau = \{R^N, K_i\} \quad (3.2.116)$$

be the conventional *topology* on R^N (whose knowledge is here assumed for brevity), where K_i represents the subset of R^N defined by

$$K_i = \{P = (a_1, a_2, \ldots, a_N)/n_i < a_1, a_2, \ldots, a_N < m; \quad n_i, m_i, a_i \in R\}. \quad (3.2.117)$$

We therefore have the following:

DEFINITION 3.2.8 [10,22-25]: The isotopology can be defined as the simple lifting on $\hat{R}^N$ of the conventional topology on R^N, and we shall simply write

$$\hat{\tau} = \{\hat{R}^N, \hat{K}_i\}, \tag{3.2.118a}$$

$$\hat{K}_i = \{\hat{P} = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_N)\hat{/}\hat{n}_i < \hat{a}_1, \hat{a}_2, \ldots, \hat{a}_N < \hat{m};\ \ \hat{n}_i, \hat{m}_i, \hat{a}_i \in \hat{R}\}. \tag{3.2.118b}$$

As one can see, the above isotopology coincides everywhere with the conventional topology of R^N except at the isounit $\hat{I}$. In particular, $\hat{\tau}$ is everywhere local-differential, except at $\hat{I}$ which can incorporate nonlocal integral terms.

It is evident that isotopology can characterize for the first time in scientific history, extended, nonspherical and deformable particles. In fact, for the case of three-dimensions in diagonal representation (3.2.113), we have the characterization of deformable spheroidal ellipsoids with variable semiaxes n_1^2, n_2^2, n_3^2 depending on local quantities, such as energy, density, pressure, etc. For the case of four-dimension the quantity n_4^2 represents, for the first time in scientific record, the density of the particle considered[13].

The reader should be aware that the above formulation of the isotopology is the simplest possible one, being restricted to the description of *one* isolated *isoparticle,* that is, an extended and nonspherical particle on isospace over isofields that, as such, has no interactions.

Consequently, numerous generalizations of the above formulations are possible and actually needed for hadronic mechanics. The first broadening is given by the case of *two* or more isoparticles in which case the basic isounit is given by the Cartesian product of two isounits of type (3.2.113). The second broadening is given by exponential factors incorporating nonlinear integral terms as in the general isounit (3.1.19). In the preceding formulation, these exponential factors have been incorporated in the n's since they are common factors.

A lesser trivial broadening of the above formulation of isotopology is given by *nondiagonal isounits* that are capable of representing nonspheroidal shapes and other complex geometric occurrences (see in Ref. [6], page 213 the case of a nondiagonal isotopy contracting the dimensions from three to one, also reviewed in the next section). The study of the latter more general formulations of isotopology is left to the interested reader.

DEFINITION 3.2.11 [22-25]: An isotopological isospace $\hat{\tau}(\hat{R}^N)$ is the isospace $\hat{R}^N$ equipped with the isotopology $\hat{\tau}$. An isocartesian isomanifold $\hat{M}(\hat{R}^N)$ is the

[13]The reader is encouraged to inspect any desired textbook in particle physics and verify the complete lack of representation of the density of the particle considered.

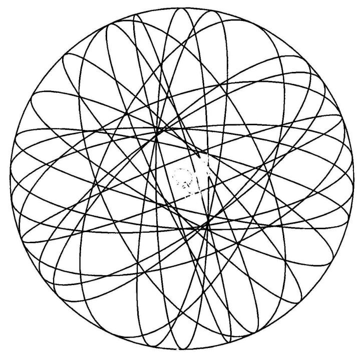

Figure 3.6. A schematic view of the "isosphere", namely, the perfect sphere on isospace over isofield represented by isoinvariant (3.2.121), that is assumed as the geometric representation of hadrons used in this monograph. The actual nonspherical and deformable shape of hadrons is obtained by projecting the isosphere in our Euclidean space, as illustrated in the last identify of Eq. (3.2.122).

isotopological isospace $\hat{M}(\hat{R}^N)$ equipped with a isovector structure, an isoaffine structure and the mapping

$$\hat{F} : \hat{R}^N \rightarrow \hat{R}^N; \quad \hat{a} \rightarrow \hat{f}(\hat{a}), \ \ \forall \hat{a} \in \hat{R}^N. \tag{3.2.119}$$

An iso-Euclidean isomanifold $\hat{M}(\hat{E}(\hat{r}, \hat{\delta}, \hat{R}))$ occurs when the N-dimensional isospace $\hat{E}$ is realized as the Cartesian product (3.2.106) and equipped with isotopology (3.2.118) with basic isounit (3.2.113).

The *isodual isotopology* and related notions can be easily constructed with the isodual map (3.2.15) and its explicit study is left as an instructive exercise for the interested reader.

3.2.8 Iso-Euclidean Geometry and its Isodual

The isotopies of the Euclidean space and geometry were introduced for the first time by Santilli in Ref. [14] of 1983 as a particular case of the broader isotopies of the Minkowski space and geometry treated in the next section.

The same isotopies were then studied in various works by the same author and a comprehensive treatment was presented in Chapter 5 of Vol. I [6]. These isotopies are today known as the *Euclid-Santilli isospace and isogeometry.* The presentation of Vol. I will not be repeated here for brevity. We merely limit ourselves to outline the main aspects for minimal self-sufficiency of this monograph.

Consider the fundamental isospace for nonrelativistic hadronic mechanics, the three-dimensional *Euclid-Santilli isospace* with contravariant isocoordinates $\hat{r}$, isometric $\hat{\delta}$ over the isoreals $\hat{R} = \hat{R}(\hat{n}, \hat{+}, \hat{\times})$ (see Section 3.3)

$$\hat{E}(\hat{r}, \hat{\delta}, \hat{R}): \quad \hat{r} = (\hat{r}^k) = (\hat{x}, \hat{y}, \hat{z}) = (r^k) \times \hat{I} = (x, y, z) \times \hat{I}, \quad k = 1, 2, 3; \tag{3.2.120a}$$

$$\hat{I} = Diag.(n_1^2, n_2^2, n_3^2) = 1/\hat{T} > 0, \quad n_k = n_k(t, r, v, a, \mu, \tau, \ldots) > 0, \tag{3.2.120b}$$

$$\hat{\Delta} = \hat{\delta} \times \hat{I}; \quad \hat{\delta} = \hat{T} \times \delta = Diag.(n_1^{-2}, n_2^{-2}, n_3^{-2}), \tag{3.2.120c}$$

with basic isoinvariant on $\hat{E}$

$$\hat{r}^{\hat{2}} = \hat{r}^i \hat{\times} \hat{\Delta}_{ij} \hat{\times} \hat{r}^j = \hat{r}^i \times \hat{\delta}_{ij} \times \hat{r}^j = \hat{r}^i \times (\hat{T}_i^k \times \delta_{kj}) \times \hat{r}^j =$$

$$= \hat{x}^{\hat{2}} + \hat{y}^{\hat{2}} + \hat{z}^{\hat{2}} = \frac{\hat{x}^2}{n_1^2} + \frac{\hat{y}^2}{n_2^2} + \frac{\hat{z}^2}{n_3^2} \in \hat{R}. \tag{3.2.121}$$

and projection on the conventional Euclidean space

$$r^2 = \frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} \in R. \tag{3.2.122}$$

where the scalar functions n_k, besides being sufficiently smooth and positive-definite, have an unrestricted functional dependence on time t, coordinates r, velocities v, acceleration a, density μ, temperature τ, and any needed local variable.

The *Euclid-Santilli isogeometry* is the geometry of the above isospaces. A knowledge of the following main features is essential for an understanding of nonrelativistic hadronic mechanics.

Since the isospaces $\hat{E}$ are all locally isomorphic to the conventional Euclidean space $E(r, \delta, R)$, it is evident that *the Euclid-Santilli isogeometry verifies all axioms of the conventional geometry,* as proved in detail in Section 5.2 of Vol. I [6]. In fact, the conventional and isotopic geometries coincide at the abstract, realization free level to such an extent that they can be expressed with the same abstract symbols, the differences between the conventional and the isotopic geometries emerging only in the selected realizations of said abstract axioms.

Note that, while the Euclidean space and geometry are unique, there exist an infinite family of different yet isomorphic Euclid-Santilli isospaces and isogeometries, evidently characterized by different isometrics in three dimension and signature $(+, +, +)$.

Recall from Section 3.2.3 that the structure of the basic invariant is given by Eq. (3.2.66). Therefore, the *isosphere*, namely, the image on $\hat{E}$ of the perfect sphere on E remains a perfect sphere. However, the projection of the isosphere on the original space E is a spheroidal ellipsoid, as clearly indicated by invariant (3.2.121). Therefore, *the isosphere on isospace over isofields unifies all possible spheroidal ellipsoids on ordinary spaces over ordinary fields.* These features are

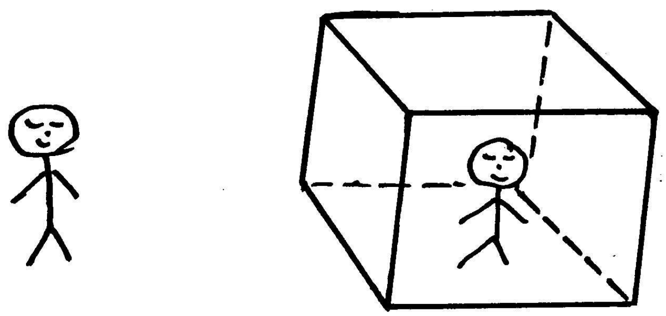

Figure 3.7. A schematic view of the "space isocube", namely, an ordinary cube inspected by two observers, an exterior observer in Euclidean space with basic units of measurements $I = Diag.(1\ \mathrm{cm}, 1\ \mathrm{cm}, 1\ \mathrm{cm})$ and an interior observer on isospace with basic isounits $\hat{I} = Diag.(n_x^2\ \mathrm{cm}, n_y^2\ \mathrm{cm}, n_z^2\ \mathrm{cm})$. It it then evident that, if the exterior observer measures, for instance, the sides of the cube to be $3m$, the interior observers measures different length that can be bigger or smaller than $3m$ depending on whether the isounit is smaller or bigger, respectively, than the original unit. Also, for the case of the Euclidean observer, the units in the three space directions are the same, while the corresponding isounits have different values for different directions. Therefore, the same object appears as a cube of a given size to the external observer, while having a completely different shape and size for the internal observer.

crucial to understand later on the reconstruction of the *exact* rotational symmetry for *deformed* spheres (see Fig. 3.6).

Since the functional dependence of the isometric is unrestricted except verifying the condition of positive-definiteness, it is easy to see that *the Euclid-Santilli isogeometry unifies all possible three-dimensional geometries with the signature)* $(+, +, +)$*,* thus including as particular cases the Riemannian, Finslerian, non-Desarguesian and other geometries. As an example, the Riemannian metric $g_{ij}(r) = g^t$ is a trivial particular case of Santilli's isometric $\hat{\delta}_{ij}(t, r, \ldots)$. This occurrence has profound physical implications that will be pointed out in Section 3.5.

Yet another structural difference between conventional and isotopic geometries is that the former has the same unit for all three reference axes. In fact, the geometric unit $I = Diag.(1, 1, 1)$ is a dimensionless representation of the selected units, for instance, $I = diag.(1\,\mathrm{cm}, 1\,\mathrm{cm}, 1\,\mathrm{cm})$. In the transition to the isospace, the units are different for different axes and we have, for instance, $\hat{I} = Diag.(n_1^2\,\mathrm{cm}, n_2^2\,\mathrm{cm}, n_3^2\,\mathrm{cm})$. It then follows that *shapes detected by our sensory perception are not necessarily absolute, in the sense that they may appear basically different for an isotopic observer* (see Fig. 3.7).

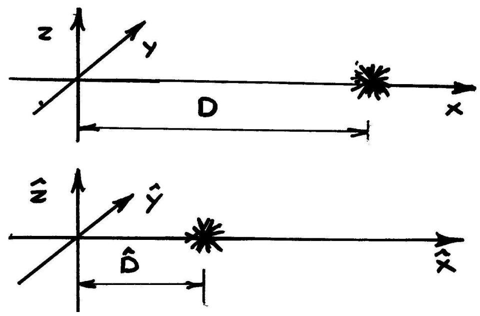

Figure 3.8. A schematic view of the geometric propulsion studied in greater details in Chapter 12, here illustrated via the contraction of distances in the transition from our coordinmates to the isotopiuc ones.

Note that in the conventional space $E(r, \delta, R)$ there are two trivially different trivial units, namely, the unit $I = +1$ of the base field R and the unit $I = Diag.(1, 1, 1)$ of the space, related geometry and symmetries. The isotopies have identified for the first time the fact that *the unit of the space must coincide with the unit of the base field.*

In fact, the isounit of isospace $\hat{E}(\hat{r}, \hat{\delta}, \hat{R})$ must coincide with the isounit of the isofield $\hat{R}$. It is then evident that, at the limit $\hat{I} \to I = Diag.(1, 1, 1)$ the unit matrix $I = Diag.(1, 1, 1)$ must be the unit of both the Euclidean space and of the basic field. This implies a trivial reformulation of R that is ignored hereon.

Another important notion is that of *isodistance* between two points P_1 and P_2 on $\hat{E}$ that can be defined by the expression

$$\hat{D}^2_{1-2} = (\hat{x}_1 - \hat{x}_2)^2/n_1^2 + (\hat{y}_1 - \hat{y}_2)^2/n_2^2 + (\hat{z}_1 - \hat{z}_2)^2/n_3^2. \tag{3.2.123}$$

It then follows that *local alterations of the space geometry cause a change in the distance,* an occurrence first identified in Ref. [6] as originating from a lifting of the units, and today known as *isogeometric locomotion* studied in Chapter 13. We are here referring to a new form of non-Newtonian locomotion in which objects can move without the application of a force or, equivalently, without any application of the principle of action and reaction (see Figure 3.8).

Finally, it is important to point out that *the dimensionality of the original Euclidean space is not necessarily preserved under isotopies.* This occurrence constitutes another intriguing epistemological feature because isotopies are axiom-preserving. Therefore, our senses based on the three Eustachian lobes perceive no difference in dimension between a conventional and an isotopic shape.

The epistemological question raised by the isotopies is then whether our perception of space as three-dimensional is real, in the sense of being intrinsic, or it is a mere consequence of our particular sensory perception, with different dimensions occurring for other observers.[14]

The occurrence was discovered by Santilli in Ref. [6], page 213, via the following isotopic element

$$\hat{\mathbf{T}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \tag{3.2.124}$$

that is positive definite since $\operatorname{Det} \hat{T} = 1$, thus being a fully acceptable isotopic element.

It is easy to see that the isoinvariant of the Euclid-Santilli isospace characterized by the above non-diagonal isotopy is given by

$$\hat{r}^{\hat{2}} = \hat{r}^i \times \hat{T}_i^k \times \delta_{kj} \times \hat{r}^j =$$
$$= \hat{x} \times \hat{z} + \hat{y} \times \hat{z} - \hat{z} \times \hat{y} = \hat{x} \times \hat{x}, \tag{3.2.125}$$

namely, in this case the *isotopic image of the three-dimensional Euclidean space is one dimensional.*

This occurrence provides another illustration of the fact that, despite their simplicity, the geometric implications of the isotopies are rather deep indeed.

The *isodual Euclid-Santilli isospace* in three dimension can be represented by the expressions

$$\hat{E}^d(\hat{r}^d, \hat{\Delta}^d, \hat{R}^d): \quad \hat{r}^d = (-\hat{x}, -\hat{y}, -\hat{z}); \tag{3.2.126a}$$
$$\hat{I}^d = Diag.(-n_1^2, -n_2^2, -n_3^2) = -1/\hat{T} > 0, \quad n_k = n_k(t, r, \ldots) > 0,$$
$$\hat{\Delta}^d = \hat{\delta}^d \times \hat{I}, \quad \hat{\delta}^d = \hat{T}^d \times^d \delta^d = Diag.(-n_1^{-2}, -n_2^{-2}, -n_3^{-2}), \tag{3.2.126b}$$

with isodual isoinvariant on $\hat{R}^d$

$$\hat{r}^{d\hat{2}^d} = \hat{r}^{di} \hat{\times}^d \hat{\Delta}^d_{ij} \hat{\times}^d \hat{r}^{dj} =$$
$$= -\hat{x}^{d\hat{2}^d} - \hat{y}^{d\hat{2}^d} - \hat{z}^{d\hat{2}^d} \quad \in \quad \hat{R}^d. \tag{3.2.127}$$

and projection on the isodual Euclidean space

$$r^{d^2} = (-x^2/n_1^2 - y^2/n_2^2 - z^2/n_3^2) \times \hat{I} \quad \in \quad R^d. \tag{3.2.128}$$

A study of the *isodual Euclid-Santilli isogeometry* from Vol. I [6] is essential for a study of antimatter in interior conditions.

[14] As we shall see in Chapter 4, an even deeper epistemological issue emerges from our hyper-isotopies in which the unit is characterized by a *set* of values. In this case, space can be "three-dimensional" yet be "hyper-dimensional", in the sense that each dimension can be multi-valued.

3.2.9 Minkowski-Santilli Isogeometry and its Isodual

3.2.9A. Conceptual Foundations. The isotopies of the Minkowski space and geometry are the main mathematical methods of relativistic hadronic mechanics, because they are at the foundations of the Poincaré-Santilli isosymmetry, and related broadening of special relativity for relativistic interior dynamical systems.

The isotopies of the Minkowski space and geometry were first proposed by Santilli in Ref. [14] of 1983 and then studied in numerous papers (see monographs [6,7,14,15] and papers quoted therein) and are today known as *Minkowski-Santilli isospace and isogeometry.*

Due to their fundamental character, the new spaces and geometry were treated in great details in Refs. [6,7], particularly in the second edition of 1995, and that presentation is here assumed as known for brevity.

The primary purpose of this section is to identify the most salient advances occurred since the second edition of Refs. [6,7] with particular reference to the geometric treatment of gravitation.

In essence, the original efforts in the construction of relativistic hadronic mechanics were based on *two different isotopies,* the isotopies of the Minkowskian geometry for nongravitational profiles, and the isotopies of the Riemannian geometry for gravitational aspects. The presentation of Refs. [6,7] was based on this dual approach.

Subsequently, it became known that *the isotopies of the Riemannian geometry could not resolve the catastrophic inconsistencies of gravitation identified in Chapter 1 because they are inherent in the background Riemannian treatment itself, thus persisting under isotopies.*

The resolution of these catastrophic inconsistencies was finally reached by Santilli in Ref. [26] of 1998 via the *unification of the Minkowskian and Riemannian geometries into Minkowski-Santilli isogeometry.* In fact, the isometric of the latter geometry admits, as a particular cases, all possible Riemannian metrics.

Consequently, it became clear that the various methods used for the Riemannian geometry (such as covariant derivative, Christoffel symbols, etc.) are inapplicable to the conventional Minkowski space evidently because flat, but the same methods are fully applicable to the Minkowski-Santilli isogeometry.

The achievement of a geometric unification of the Minkowskian and Riemannian geometries reached in memoir [26] permitted truly momentous advances, such as the geometric unification of the special and general relativities, an axiomatically consistent grand unification of electroweak and gravitational interactions, the first known axiomatically consistent operator form of gravity, and other basic advances reviewed in Section 3.5.

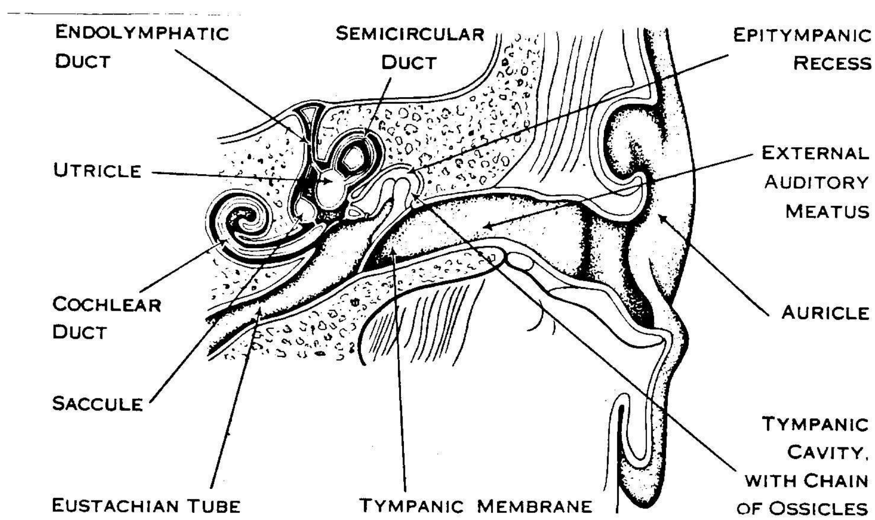

Figure 3.9. A view of the three Eustachian lobes allowing us to perceive three-dimensional shapes. The intriguing epistemological issue raised by the Euclid-Santilli isogeometry is whether living organisms with different senses perceive the same object with different shape and size than ours. As illustrated with the isobox of Figure 3.7, the same object can appear with dramatically different shapes and sizes to a conventional and an isotopic observer, as well as in dimension different than the original ones, as illustrated in the text. Another illustration of the meaning and importance of isotopies is that being axiom-preserving, different shapes, sizes and dimensions on isospaces are rendered compatible with our sensory perception.

3.2.9B. Minkowski-Santilli Isospaces. We now review in this subsection the foundations of the Minkowski-Santilli isospaces by referring interested readers to volumes [6,7] for details.

DEFINITION 3.2.12 [26]: Consider the conventional Minkowski space

$$M = M(x, \eta, R) : x = (x^\mu) = (r, c_o t), \tag{3.2.129a}$$

$$x^\mu = \eta^{\mu\nu} \times x_\nu, \;\; x_\mu = \eta_{\mu\nu} \times x^\nu, \tag{3.2.129b}$$

where c_o is the speed of light in vacuum, metric

$$\eta = (\eta_{\mu\nu}) Diag.(+1, +1, +1, -1), \;\; \eta^{\mu\nu} = [(\eta_{\alpha,\beta})^{-1}]^{\mu\nu}, \tag{3.2.130}$$

basic unit

$$I = Diag.(+1, +1, +1, +1), \tag{3.2.131}$$

and invariant on the reals

$$x^2 = x^\mu \times x_\mu = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \;\; \in \;\; R = R(n, +, \times), \tag{3.2.132}$$

$$\mu, \nu, \alpha, \beta = 1, 2, 3, 4.$$

Then, the Minkowski-Santilli isospaces can be defined by isotopies

$$\hat{M} = \hat{M}(\hat{x}, \hat{G}, \hat{R}) : \hat{x} = (\hat{x}^\mu) = (r, c_o t) \times \hat{I}, \tag{3.2.133a}$$

$$\hat{x}^\mu = \hat{G}^{\mu\nu} \hat{\times} \hat{x}_\nu, \quad \hat{x}_\mu = \hat{G}_{\mu\nu} \hat{\times} \hat{x}^\nu, \tag{3.2.133b}$$

with isometric on isospaces over isofields

$$\hat{G} = \hat{\eta} \times \hat{I} = (\hat{T}_\mu^\rho \times \eta_{\rho\nu}) \times \hat{I} =$$

$$= Diag.(\hat{T}_{11}, \hat{T}_{22}, \hat{T}_{33}, \hat{T}_{44}) \times \hat{I} \quad \in \quad \hat{R} = \hat{R}(\hat{n}, \hat{+}, \hat{\times}), \tag{3.2.134a}$$

$$\hat{G}^{\mu\nu} = [(\hat{G}_{\alpha,\beta})^{-1}]^{\mu\nu}, \tag{3.2.134b}$$

and isounit

$$\hat{I} = Diag.(\hat{T}_{11}^{-1}, \hat{T}_{22}^{-1}, \hat{T}_{33}^{-1}, \hat{T}_{44}^{-1}), \tag{3.2.135}$$

where $\hat{T}_{\mu\nu}$ are positive-definite functions of spacetime coordinates x, velocities v, accelerations a, densities μ, temperature τ, wavefunctions, their derivatives and their conjugates and any other needed quantity

$$\hat{T}_{\mu\nu} = \hat{T}_{\mu\nu}(x, v, a, \mu, \tau, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \ldots) \quad > \quad 0 \tag{3.2.136}$$

isoinvariant on isospaces over the isofield of isoreal numbers

$$\hat{x}^{\hat{2}} = \hat{x}^\mu \hat{\times} \hat{x}_\mu = (\hat{x}^\mu \hat{\times} \hat{G}_{\mu\nu} \hat{\times} \hat{x}^\nu) \times I \quad \in \quad \hat{R} = R(\hat{n}, \hat{+}, \hat{\times}) \tag{3.2.137}$$

with projection in our spacetime

$$\hat{M}(x, \hat{\eta}, R) : x = (x^\mu) \times I, \tag{3.2.138a}$$

$$x^\mu = \hat{\eta}^{\mu\nu} \times x_\nu, \quad x_\mu = \hat{\eta}_{\mu\nu} \times x^\nu, \tag{3.2.138b}$$

metric over the field of real numbers

$$\hat{\eta} = (\hat{\eta}_{\mu\nu}) = (\hat{T}_\mu^\rho \times \eta_{\rho\nu}) = Diag.(\hat{T}_{11}, \hat{T}_{22}, \hat{T}_{33}, \hat{T}_{44}) \in R = R(n, +, \times), \tag{3.2.139a}$$

$$\hat{\eta}^{\mu\nu} = [(\hat{\eta}_{\alpha,\beta})^{-1}]^{\mu\nu}, \tag{3.2.139b}$$

and invariant in our spacetime over the reals

$$x^2 = x^\mu \times x_\nu = x^\mu \times \hat{\eta}_{\mu\nu}(x, v, a, \mu, \tau, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \ldots) \times x^\nu =$$

$$= T_{11} \times x_1^2 + \hat{T}_{22} \times x_2^2 + \hat{T}_{33} \times x_3^2 - \hat{T}_{44} \times x_4^2 \ \in \ R. \tag{3.2.140}$$

Note that all scalars on M must be lifted into *isoscalars* to have meaning for $\hat{M}$, i.e., they must have the structure of the isonumbers $\hat{n} = n \times \hat{I}$. This condition requires the re-definition $x \ \rightarrow \ \hat{x} = x \times \hat{I}$, $\eta_{\mu\nu} \ \rightarrow \ \hat{G}_{\mu\nu} = \hat{\eta}_{\mu\nu} \times \hat{I}$, $x^2 \rightarrow \hat{x}^{\hat{2}}$, etc.

The reader interested in learning in depth the new isogeometry should also study from the preceding sections the different realizations of the isometry whether realized in the original Minkowskian coordinates or in the isocoordinates, since the functional dependence is different in these two cases.

Note however the redundancy in practice for using the forms $\hat{x} = x \times \hat{I}$ and $\hat{G} = \hat{\eta} \times \hat{I}$ because of the identity $\hat{x}^{\hat{2}} = \hat{x}^\mu \hat{\times} \hat{G}_{\mu\nu} \hat{\times} \hat{x}^\nu \equiv (x^\mu \times \hat{\eta}_\nu \times x^\nu) \times \hat{I}$. For simplicity we shall often use the conventional coordinates x and the isometric will be referred to $\hat{\eta} = \hat{T} \times \eta$. The understanding is that the full isotopic formulations are needed for mathematical consistency.

A fundamental property of the infinite family of generalized spaces (3.2.133) is the lifting of the basic unit $I \rightarrow \hat{I}$ while the metric is lifted of the *inverse* amount, $\eta \rightarrow \hat{\eta} = \hat{T} \times \eta$, $\hat{I} = \hat{T}^{-1}$. This implies the preservation of all original axioms, and we have the following:

THEOREM 3.2.7 [26]: All infinitely possible isominkowski spaces $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ over the isofields $\hat{R}(\hat{n}, \hat{+}, \hat{\times})$ with a common positive-definite isounit $\hat{I}$ preserve all original axioms of the Minkowski space $M(x, \eta, R)$ over the reals $R(n, +, \times)$.

The nontriviality of the lifting is that *the Minkowskian axioms are preserved under an arbitrary functional dependence of the metric $\hat{\eta} = \hat{\eta}(x, v, a, \mu, \tau, \ldots)$* for which the sole x-dependence of the Riemannian metric $g(x)$ is only a simple *particular case*. As a matter of fact, we have the following

THEOREM 3.2.8 [26]: Minkowski-Santilli isospaces are "directly universal" in spacetime, that is, they represent all infinitely possible spacetimes with signature $(+, +, +, -)$ ("universality"), directly with the isometric and without any use of the transformation theory ("direct universality").

Note that all possible "deformations" of the Minkowski space are also particular cases of the above isospaces. However, the former are still referred to the old unit I, thus losing the isomorphic between deformed and Minkowski spaces, while the isotopies preserve the original axioms by construction.

A fundamental physical characteristic of the Minkowski-Santilli isospaces is that *it alters the units of space and time.* Recall that the unit

$$I = Diag.(\{1, 1, 1\}, 1)$$

of the Minkowski space represents in a dimensionless form the units of the three Cartesian axes and time, e.g., $I = (+1\,\text{cm}, +1\,\text{cm}, +1\,\text{cm}, +1\,\text{sec})$. Recall also that the Cartesian space-units are *equal for all axes.*

Consider now the isospaces, and recall that $\hat{I}$ is positive-definite. Consequently, we have the following lifting of the units in which the $\hat{T}_{\mu\mu}$ quantities are reinter-

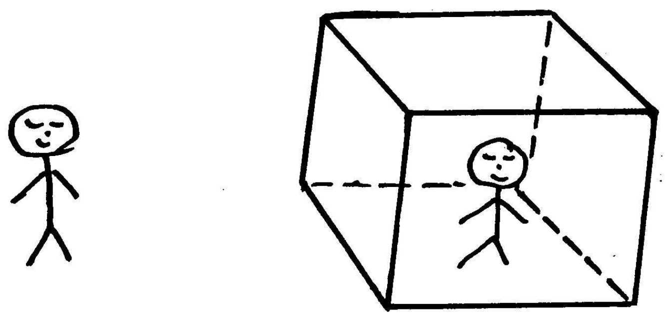

Figure 3.10. A view of the "spacetime isocube" characterized by the "space isocube" of Figure 3.7 now inspected in two spacetimes, the conventional Minkowski spacetime in the exterior and Santilli isospacetime in the interior. In addition to the variations of shape, size and dimensions indicated in Figure 3.7, the same object can be in different times for the two observers, all in a way fully compatible with our sensory perception. Consequently, seeing in a telescope a far away quasar or galaxy it does not mean that astrophysical structure is necessarily in our time, since it could be evolving far away in the future or in the past.

preted as constants

$$I = (+1\,\mathrm{cm}, +1\,\mathrm{cm}, +1\,\mathrm{cm}, +1\,\mathrm{sec}) \rightarrow$$
$$\rightarrow \hat{I} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) = 1/\hat{T},\ \hat{I}_\mu^\mu = n_\mu^2, n_\mu > 0. \tag{3.2.141}$$

This means that, not only the original units are now lifted into arbitrary positive values, but the *units of different space axes generally have different values.* Jointly, the components of the metric are lifted by the *inverse* amounts n_μ^{-2}. This implies the preservation on $\hat{M}$ over $\hat{R}$ of the original *numerical* values on M over R, including the crucial preservation of the maximal causal speed c_o, as we shall see in Section 3.5.

Note also the necessary condition that *the isospace and isofield have the same isounit $\hat{I}$.* This condition is absent in the conventional Minkowski space where the unit of the space is the unit *matrix* $I = Diag.(1, 1, 1, 1)$, while that of the underlying field is the *number* $I = +1$. Nevertheless, the latter can be trivially reformulated with the common unit matrix I, by achieving in this way the form admitted as a particular case by the covering isospaces

$$M(x, \eta, R):\ x = \{x^\mu \times I\},\ x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \in R. \tag{3.2.142}$$

The structure of both the conventional and isotopic invariants is therefore given by Theorem 3.2.66, namely

$$\text{Basic Invariant} = (\text{Length})^2 \times (\text{Unit})^2, \tag{3.2.143}$$

which illustrates more clearly the preservation under the dual lifting $\eta \rightarrow \hat{\eta} = \hat{T} \times \eta$ and $I \rightarrow \hat{I} = 1/\hat{T}$ of the original axioms as well as numerical values.

THEOREM 3.2.9 [6,7,26]: Conventional and isotopic symmetries of spacetime are 11-dimensional.

Proof. In addition to the 10-dimensionality of the Poincaré symmetry, there is an additional 11-th dimensionality characterized by the isotransform

$$\eta \rightarrow \hat{\eta} = \eta/n^2, \quad I \rightarrow \hat{I} = n^2 \times I, \tag{3.2.144}$$

where n is a non-null constant. **q.e.d.**

Note the crucial role of Santilli's isonumbers in the above property. This explains why the 11-th dimensionality remained undiscovered throughout the 20-th century.

A significant difference between the conventional space M and its isotopes $\hat{M}$ is that the former admit only *one* formulation, the conventional one, while the latter admit *two* formulations: that on isospace itself (i.e., expressed with respect to the isounit $\hat{I}$) and its *projection* in the original space M (i.e., expressed with respect to the conventional unit I).

Note that the projection of $\hat{M}(\hat{x}, \hat{M}, \hat{R})$ into $M(x, \eta, R)$ is not a conformal map, but an *inverse isotopic map* because it implies the transition from generalized units and fields to conventional units and fields.

The axiomatic motivation for constructing the isotopies of the Minkowskian geometry is that any modification of the Minkowski metric requires the use of *noncanonical transforms* $x \rightarrow x'(x)$,

$$\eta_{\mu\nu} \rightarrow \hat{\eta}_{\mu\nu} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} \eta_{\alpha\beta} \frac{\partial x'^{\beta}}{\partial x^{\nu}} \neq \eta_{\mu\nu}, \tag{3.2.145}$$

and this includes the case of the transition from the Minkowskian metric η to the Riemannian metric $g(x)$.

In turn, all noncanonical theories, thus including the Riemannian geometry, do not possess invariant units of space and time, thus having the catastrophic inconsistencies studied in Chapter 1. A primary axiomatic function of the isospace is that of restoring the invariance of the basic units, as established by the Poincaré-Santilli isosymmetry.

This is achieved by embedding all noncanonical content in the generalization of the unit. Invariance for noncanonical structures such as Riemannian metrics is then assured by the fact indicated earlier that, whether conventional or generalized, the unit is the basic invariant of any theory.

Stated in different terms, a primary axiomatic difference between the special and general relativities is that the time evolution of the former is a *canonical* *transform*, thus implying the majestic mathematical and physical consistency of special relativity recalled in Chapter 1, while the time evolution of the latter is a *noncanonical transform*, thus implying a number of unresolved problematic aspects that have been lingering throughout this century.

The reformulation of the Riemannian geometry in terms of the Minkowskian axioms is the sole possibility known to this author for achieving axiomatic consistency under a nontrivial functional dependence of the metric.

In summary, Minkowski-Santilli isospaces have the following primary applications. First, they are used for a re-interpretation of the Riemannian metrics $g(x)$ for the particular case

$$\hat{\eta} = \hat{\eta}(x) = g(x) \tag{3.2.146}$$

characterizing *exterior gravitational problems in vacuum.* Second, the same isospaces are used for the characterization of *interior gravitational problems* with isometrics of unrestricted functional dependence

$$\hat{\eta} = \hat{\eta}(x, v, a, \mu, \tau, \ldots) = g(x, v, a, \mu, \tau, \ldots) \tag{3.2.147}$$

while preserving the original Minkowskian axioms.

Since the explicit functional dependence is inessential under isotopies, our studies will be generally referred to the interior gravitational problem. Unless otherwise stated, only diagonal realizations of the isounits will be used hereon for simplicity. An example of nondiagonal isounits inherent in a structure proposed by Dirac is indicated in Section 3.5. More general liftings of the Minkowski space of the so-called *genotopic and multivalued-hyperstructural type* will be indicated in Chapter 4.

3.2.9C. Isoderivative, Isoconnection, and Isoflatness. In the preceding subsections we have presented the *Minkowskian* aspects of the new isogeometry. We are now sufficiently equipped to present the novel part of the Minkowski-Santilli isogeometry, its *Riemannian* character as first derived in Ref. [26].

Our study is strictly in local coordinates representing the *fixed* frame of the observer without any un-necessary use of the transformation theory or abstract treatments. Our presentation will be as elementary as possible without reference to advanced topological requirements, such as Kadeisvili's isocontinuity (Section 3.2.6), isomanifolds and related TSSFN isotopology (Section 3.2.7) .

Also, our presentation is made, specifically, for the (3+1)-dimensional isospacetime, with the understanding that the extension to arbitrary dimensions and signatures or signatures different than the conventional one $(+,+,+,-)$ is elementary, and will be left to interested readers.

Let $\hat{M}(\hat{x}, \hat{G}, \hat{R})$ be a Minkowski-Santilli isospace and let $\hat{M}(x, \hat{\eta}, R)$ be its projection in our spacetime as per Definition 3.2.12. To illustrate the transition from isocoordinates $\hat{x}$ to conventional spacetime coordinates x, we shall denote the projection $\hat{M} = \hat{M}(\hat{x}, \hat{\eta}, R)$. This notation emphasizes that the referral of the isospace to the conventional units and field causes the reduction of the isometric from the general form $\hat{G} = \hat{\eta} \times \hat{I}$ to $\hat{\eta} = \hat{T} \times \eta$, where, as now familiar, $\hat{I} = 1/\hat{T}$ and $\eta = Diag.(1, 1, 1, -1)$ is the familiar Minkowskian metric.

According to this notation the Riemannian content of the Minkowski-Santilli isogeometry can be unified in both its isospace formulation properly speaking and its projection in our spacetime. All differences in the interpretations whether occurring in isospace or in our spacetime are then deferred to the selection of the basic unit.

Consider now the infinitesimal version of isoinvariant (3.2.137) permitted by the isodifferential calculus

$$\hat{d}\hat{s}^{\hat{2}} = \hat{d}\hat{x}_\mu \hat{\times} \hat{d}\hat{x}^\mu \in \hat{R}. \tag{3.2.148}$$

The *isonormal coordinates* occur when the isometric $\hat{\eta}$ is reduced to the Minkowski metric η as in conventional Riemannian geometry. Consequently, isonormal coordinates coincide with the conventional normal coordinates, and the Minkowski-Santilli isogeometry verifies the *principle of equivalence* as for the conventional Riemannian geometry.

By using the isodifferential calculus, we now introduce the *isodifferential of a contravariant isovector field* on $\hat{M}$ over $\hat{R}$ [15]

$$\hat{d}\hat{X}^\beta = (\hat{\partial}_\mu \hat{X}^\beta) \hat{\times} \hat{d}\hat{x}^\mu = \hat{I}^\rho_\mu \times (\partial_\rho \hat{X}^\beta) \hat{\times} \hat{T}^\mu_\sigma \times d\hat{x}^\sigma \equiv$$
$$\equiv (\partial_\mu X^\beta) \times d\hat{x}^\mu = (\partial^\rho X^\beta) \times \hat{\eta}_{\rho\sigma} \times d\hat{x}^\sigma, \tag{3.2.149}$$

where the last expression is introduce to recall that the contractions are in isospace. The preceding expression then shows that *isodifferentials of isovector fields coincide at the abstract level with conventional differentials for all isotopies of the class here admitted* (that with $\hat{I} > 0$).

DEFINITION 3.2.13 [26]: The isocovariant isodifferential are defined by

$$\hat{D}\hat{X}^\beta = \hat{d}\hat{X}^\beta + \hat{\Gamma}^\beta_{\alpha\gamma} \hat{\times} \hat{X}^\alpha \hat{\times} \hat{d}\hat{x}^\gamma, \tag{3.2.150}$$

with corresponding isocovariant derivative

$$\hat{X}^\beta_{\hat{|}\mu} = \hat{\partial}_\mu \hat{X}^\beta + \hat{\Gamma}^\beta_{\alpha\mu} \hat{\times} \hat{X}^{\hat{\alpha}}, \tag{3.2.151}$$

where the iso-Christoffel's symbols are given by

$$\hat{\Gamma}^\beta_{\alpha\gamma}(x, v, a, \mu, \tau, \ldots) = \frac{\hat{1}}{\hat{2}} \hat{\times} (\hat{\partial}_\alpha \hat{\eta}_{\beta\gamma} + \hat{\partial}_\gamma \hat{\eta}_{\alpha\beta} - \hat{\partial}_\beta \hat{\eta}_{\alpha\gamma}) \times \hat{I} = \hat{\Gamma}_{\gamma\beta\alpha}, \tag{3.2.152a}$$

[15] We should note that the role of the isounit and of the isoelement in this presentation and in that of Ref. [26] are interchanged for general compatibility with the various applications and developments.

$$\hat{\Gamma}^{\beta}_{\alpha\gamma} = \hat{\eta}^{\beta\rho} \times \hat{\Gamma}_{\alpha\rho\gamma} = \hat{\Gamma}^{\beta}_{\gamma\alpha}. \quad (3.2.152b)$$

Note the unrestricted functional dependence of the connection which is notoriously absent in conventional treatments. Note also the abstract identity of the conventional and isotopic connections. Note finally that *local numerical values of the conventional and isotopic connections coincide when computed in their respective spaces.* This is due to the fact that in Eq.s (3.2.152) $\hat{\eta} \equiv g(x)$ for exterior problems, while the value of derivatives ∂_μ and isoderivatives $\hat{\partial}_\mu$ coincide when computed in their respective spaces.

Note however that, when projected in the conventional spacetime, the conventional and isotopic connections are different even in the exterior problem in which $\hat{\eta} = g(x)$,

$$\hat{\Gamma}_{\alpha\beta\gamma} = \frac{1}{2} \times (\hat{I}^{\mu}_{\alpha} \times \partial_\mu g_{\beta\gamma} + \hat{I}^{\rho}_{\gamma} \times \partial_\rho \hat{\eta}_{\alpha\beta} - \hat{I}^{\sigma}_{\beta} \times \partial_\sigma g_{\alpha\gamma}) \times \hat{I} \neq \Gamma_{\alpha\beta\gamma} \times \hat{I}. \quad (3.2.153)$$

The extension to covariant isovector fields and covariant or contravariant isotensor fields is consequential.

Without proof we quote the following important result from Ref. [26]:

LEMMA 3.2.7 (Iso-Ricci Lemma) [26]: Under the assumed conditions, the isocovariant derivatives of all isometrics on Minkowski-Santilli isospaces spaces are identically null,

$$\hat{\eta}_{\alpha\beta\hat{|}\gamma} \equiv 0, \quad \alpha, \beta, \gamma = 1, 2, 3, 4. \quad (3.2.154)$$

The novelty of the isogeometry is then illustrated by the fact that *the Ricci property persists under an arbitrary dependence of the metric, as well as under Minkowskian, rather than Riemannian axioms.*

The *isotorsion* on $\hat{M}$ is defined by

$$\hat{\tau}^{\beta}_{\alpha\gamma} = \hat{\Gamma}^{\beta}_{\alpha\gamma} - \hat{\Gamma}^{\beta}_{\gamma\alpha}, \quad (3.2.155)$$

and coincides again with the conventional torsion at the abstract level, although the two torsions have significant differences in their explicit forms when both projected in our space-time.

DEFINITION 3.2.14 [26]: The Minkowski-Santilli isogeometry is characterized by the following isotensor: the isoflatness isotensor

$$\hat{R}^{\beta}_{\alpha\gamma\delta} = \hat{\partial}_\delta \hat{\Gamma}^{\beta}_{\alpha\gamma} - \hat{\partial}_\gamma \hat{\Gamma}^{\beta}_{\alpha\delta} + \hat{\Gamma}^{\beta}_{\rho\delta} \hat{\times} \hat{\Gamma}^{\rho}_{\alpha\gamma} - \hat{\Gamma}^{\beta}_{\rho\gamma} \hat{\times} \hat{\Gamma}^{\rho}_{\alpha\delta}; \quad (3.2.156)$$

the iso-Ricci isotensor

$$\hat{R}_{\mu\nu} = \hat{R}^{\beta}_{\mu\nu\beta}; \quad (3.2.157)$$

the isoflatness isoscalar

$$\hat{R} = \hat{\eta}^{\alpha\beta} \times \hat{R}_{\alpha\beta}; \tag{3.2.158}$$

the iso-Einstein isotensor

$$\hat{G}_{\mu\nu} = \hat{R}_{\mu\nu} - \frac{\hat{1}}{2} \hat{\times} \hat{N}_{\mu\nu} \hat{\times} \hat{R},\ \hat{N}_{\mu\nu} = \hat{\eta}_{\mu\nu} \times \hat{I}; \tag{3.2.159}$$

and the isotopic isoscalar

$$\begin{aligned}\hat{\Theta} = \hat{N}^{\alpha\beta} \hat{\times} \hat{N}^{\gamma\delta} \hat{\times} (\hat{\Gamma}_{\rho\alpha\delta} \hat{\times} \hat{\Gamma}^{\rho}_{\gamma\beta} - \Gamma_{\rho\alpha\beta} \hat{\times} \hat{\Gamma}^{\rho}_{\gamma\delta}) = \\ = \hat{\Gamma}_{\rho\alpha\beta} \hat{\times} \hat{\Gamma}^{\rho}_{\gamma\delta} \hat{\times} (\hat{N}^{\alpha\delta} \hat{\times} \hat{N}^{\gamma\beta} - \hat{N}^{\alpha\beta} \hat{\times} \hat{N}^{\gamma\delta});\end{aligned} \tag{3.2.160}$$

the latter being new for the Minkowski-Santilli isogeometry.

Note the lack of use of the term "isocurvature" and the use instead of the term "isoflatness". This is due to the fact that the prefix "iso-" represents the preservation of the original axioms. The term "isocurvature" would then be inappropriate because the basic axioms of the geometry are flat.

In any case, the main problem underlying the studies herein reported is, as indicated in Chapter 1, that *curvature is the ultimate origin of the catastrophic inconsistencies of general relativity.* Consequently, all geometric efforts are here aimed at the replacement of the notion of curvature with a covering notion resolving the indicated catastrophic inconsistencies.

As we shall see better in Section 3.5, the notion of "isoflatness" does indeed achieve the desired objectives because flatness and its related invariance of gravitation under the Poincaré-Santilli isosymmetry is reconstructed on isospaces over isofields, while the ordinary curvature emerge as a mere projection in our spacetime.

3.2.9D. The Five Identities of the Minkowski-Santilli Isogeometry. By continuing our review of memoir [26], tedious but simple calculations yield the following *five basic identities of the Minkowski-Santilli isogeometry:*

Identity 1: *Antisymmetry of the last two indices of the isoflatness isotensor*

$$\hat{R}^{\beta}_{\alpha\gamma\delta} = -\hat{R}^{\beta}_{\alpha\delta\gamma}; \tag{3.2.161}$$

Identity 2: *Symmetry of the first two indices of the isoflatness isotensor*

$$\hat{R}_{\alpha\beta\gamma\delta} \equiv \hat{R}_{\beta\alpha\gamma\delta}; \tag{3.2.162}$$

Identity 3: *Vanishing of the totally antisymmetric part of the isoflatness isotensor*

$$\hat{R}^{\beta}_{\alpha\gamma\delta} + \hat{R}^{\beta}_{\gamma\delta\alpha} + \hat{R}^{\beta}_{\delta\,\alpha\gamma} \equiv 0; \tag{3.2.163}$$

Figure 3.11. Primary objectives of the Minkowski-Santilli isogeometry are the resolution of the catastrophic inconsistencies of the Riemannian formulation of exterior gravitation (Section 1.4) and a representation of interior gravitation as occurring for the Sun depicted in this figure and any other massive object. These objectives are achieved via the isotopies of the Minkowskian geometry since they are flat in isospace, thus admitting a well defined invariance for all possible gravitation, by adding sources requested by the Freud identity and other reasons, and by unifying exterior and interior gravitational problem in a single formulation in isospace that formally coincides with that for the exterior problem, the interior effects being incorporated in the isounit (see Section 3.5).

Identity 4: *Iso-Bianchi identity*

$$\hat{R}^{\beta}_{\alpha\gamma\delta\hat{|}\rho} + \hat{R}^{\beta}_{\alpha\rho\gamma\hat{|}\delta} + \hat{R}^{\beta}_{\alpha\delta\rho\hat{|}\gamma} \equiv 0; \tag{3.2.164}$$

Identity 5: *Iso-Freud identity*

$$\hat{R}^{\alpha}_{\beta} - \frac{\hat{1}}{2}\hat{\times}\hat{\delta}^{\alpha}_{\beta}\hat{\times}\hat{R} - \frac{\hat{1}}{2}\hat{\times}\hat{\delta}^{\alpha}_{\beta}\hat{\times}\hat{\Theta} = \hat{U}^{\alpha}_{\beta} + \hat{\partial}_{\rho}\hat{V}^{\alpha\rho}_{\beta}, \tag{3.2.165}$$

where $\hat{\Theta}$ is the isotopic isoscalar and

$$\hat{U}^{\alpha}_{\beta} = -\frac{1}{2}\frac{\hat{\partial}\hat{\Theta}}{\hat{\partial}\hat{\eta}^{\alpha\beta}_{\hat{|}\alpha}}\hat{\eta}^{\alpha\beta}_{\hat{|}\beta}, \tag{3.2.166a}$$

$$\hat{V}^{\alpha\rho}_{\beta} = \frac{1}{2}[\hat{\eta}^{\gamma\delta}(\delta^{\alpha}_{\beta}\hat{\Gamma}^{\rho}_{\alpha\delta} - \delta^{\rho}_{\beta}\hat{\Gamma}^{\rho}_{\gamma\delta})+ \tag{3.2.166b}$$

$$+(\delta^{\rho}_{\beta}\hat{\eta}^{\alpha\gamma} - \delta^{\alpha}_{\beta}\hat{\eta}^{\rho\gamma})\hat{\Gamma}^{\delta}_{\gamma\delta} + \hat{\eta}^{\rho\gamma}\hat{\Gamma}^{\alpha}_{\beta\gamma} - \hat{\eta}^{\alpha\gamma}\hat{\Gamma}^{\rho}_{\beta\gamma}], \tag{3.2.166c}$$

Note that the conventional Riemannian geometry is generally thought to possess only *four* identities. In fact, the *fifth* identity (3.2.165) is generally unknown in the contemporary literature in gravitation as the reader is encouraged to verify in the specialized literature in the Riemannian geometry (that is so vast to discourage discriminatory listings).

The latter identity was introduced by Freud [27] in 1939, treated in detail by Pauli in his celebrated book [28] of 1958 and then generally forgotten for a half a century, apparently because of its evident incompatibility between Einstein's conception of exterior gravitation in vacuum as pure curvature without source (see Section 3.4)

$$G^{\alpha}_{\beta} = R^{\alpha}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}R = 0, \tag{3.2.167}$$

and the need for a source term also in exterior gravitation in vacuum mandated by the Freud identity and other reasons

$$R^{\alpha}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}R - \frac{1}{2}\delta^{\alpha}_{\beta}\Theta = U^{\alpha}_{\beta} + \hat{\partial}_{\rho}V^{\alpha\rho}_{\beta}. \tag{3.2.168}$$

Freud's identity was rediscovered by the author during his accurate study of Pauli's historical book and studied in detail in Refs. [6,7] of 1992. Additional studies of the Freud identity were done by Yilmaz [30]. Following a suggestion by the author, the late mathematician Hanno Rund [29] studied the identity in one of his last papers and proved that:

LEMMA 3.2.8 (Rund's Lemma) [29]: Freud's identity is a bona fide identity for all Riemannian spaces irrespective of dimension and signature.

In this way, Rund confirmed the general need of a source also in vacuum (see Sections 1.4 and 3.5).

Following Ref. [26], in this paper we have presented the isotopies of the Freud identity on Minkowski-Santilli isospaces, as characterized by the isodifferential calculus. Its primary functions for this monograph is to identify the geometric structure of the *interior* gravitational problem. The persistence of the source in vacuum as per the Freud identity, electrodynamics and other needs will then be consequential, thus confirming the inconsistency of Einstein's conception of gravity in vacuum as pure curvature without source.

Note that *all conventional and isotopic identities coincide at the abstract level.*

3.2.9E. Isoparallel Transport and Isogeodesics. An isovector field $\hat{X}^{\beta}$ on $\hat{M} = \hat{M}(\hat{x}, \hat{M}, \hat{R})$ is said to be transported by *isoparallel* displacement from a point $\hat{m}(\hat{x})$ on a curve $\hat{C}$ on $\hat{M}$ to a neighboring point $\hat{m}'(\hat{x} + \hat{d}\hat{x})$ on $\hat{C}$ if

$$\hat{D}\hat{X}^{\beta} = \hat{d}\hat{X}^{\beta} + \hat{\Gamma}^{\beta}_{\alpha\gamma}\hat{\times}\hat{X}^{\alpha}\hat{\times}\hat{d}\hat{x}^{\gamma} \equiv 0, \tag{3.2.169}$$

or in integrated form

$$\hat{X}^{\beta}(\hat{m}') - \hat{X}^{\beta}(m) = \hat{\int}_{\hat{m}}^{\hat{m}'} \frac{\hat{\partial}\hat{X}^{\beta}}{\hat{\partial}\hat{x}^{\alpha}} \frac{\hat{d}\hat{x}^{\alpha}}{\hat{d}\hat{s}} \hat{\times} \hat{d}\hat{s}, \tag{3.2.170}$$

where one should note the isotopic character of the integration. The isotopy of the conventional case then yields the following:

LEMMA 3.2.9 [26]: Necessary and sufficient condition for the existence of an isoparallel transport along a curve $\hat{C}$ on a $(3+1)$-dimensional Minkowski-Santilli isospace is that all the following equations are identically verified along $\hat{C}$

$$\hat{R}^{\beta}_{\alpha\gamma\delta} \hat{\times} \hat{X}^{\alpha} = 0, \quad \alpha, \beta, \gamma, \delta = 1, 2, 3, 4. \tag{3.2.171}$$

Note, again, the abstract identity of the conventional and isotopic parallel transport. However, it is easy to see that the projection of the isoparallel transport in ordinary spacetime is structurally different than the conventional parallel transport.

Consider, as an example, an extended object in gravitational fall in atmosphere (see Figure 3.12). Its trajectory is evidently irregular and depends on the actual shape of the object, as well as its weight. The understanding of the new Minkowski-Santilli isogeometry requires the knowledge of the fact that said trajectory is represented on isospace over isofields as a *straight line,* that is, via the trajectory in the absence of the resistive medium. The actual, irregular trajectory appears only in the projection of said isotrajectory in our spacetime.

If the latter treatment is represented by a rocket, one would note a twisting action as occurring in the reality of motion within physical media, which is evidently absent in the exterior case.

Along similar lines, we say that a smooth isopath $\hat{x}_{\alpha}$ on $\hat{M}$ with isotangent $\hat{v}_{\alpha} = \hat{d}\hat{x}_{\alpha}/\hat{d}\hat{s}$ is an *isogeodesic* when it is solution of the isodifferential equations

$$\frac{\hat{D}\hat{v}^{\beta}}{\hat{D}\hat{s}} = \frac{\hat{d}\hat{v}}{\hat{d}\hat{s}} + \hat{\Gamma}_{\alpha\beta\gamma} \hat{\times} \frac{\hat{d}\hat{x}^{\alpha}}{\hat{d}\hat{s}} \hat{\times} \frac{\hat{d}\hat{x}^{\gamma}}{\hat{d}\hat{s}} = 0. \tag{3.2.172}$$

It is easy to prove the following:

LEMMA 3.2.10 [26]: The isogeodesics of a Minkowski-Santilli isospace $\hat{M}$ are the isocurves verifying the isovariational principle

$$\hat{\delta} \hat{\int} [\hat{G}_{\alpha\beta}(\hat{x}, \hat{v}, \hat{a}, \mu, \tau, \ldots) \hat{\times} \hat{d}\hat{x}^{\alpha} \hat{\times} \hat{d}\hat{x}^{\beta}]^{\hat{1/2}} = 0, \tag{3.2.173}$$

where again isointegration is understood.

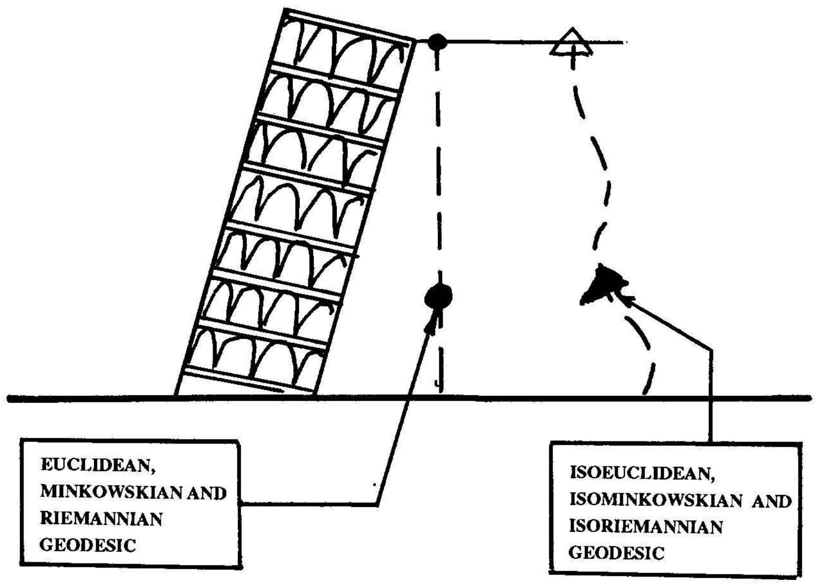

Figure 3.12. A schematic view of two objects released from the Pisa tower. The vertical trajectory represents the approximate geodesic considered by Galileo, used by Einstein and adopted until the end of the 20-th century, namely, the approximation under the lack of resistance due to our atmosphere. The Minkowski-Santilli isogeometry has been built to represent as isogeodesics actual trajectories within physical media.

Finally, we point out the property inherent in the notion of isotopies according to which

COROLLARY 3.2.10A: [26]: Trajectories in an ordinary Riemannian space coincide with the corresponding isogeodesic trajectories in Minkowski-Santilli isospace, but not with the projection of the latter in the original space.

For instance, if a circle is originally a geodesic, its image under isotopy in isospace remains the perfect circle, the *isocircle* (Section 3.2.9), even though its projection in the original space can be an ellipse. The same preservation in isospace occurs for all other curves.

The differences between a geodesic and an isogeodesic therefore emerge only when projecting the latter in the space of the former.

An empirical but conceptually effective rule is that *interior physical media "disappear" under their isogeometrization*, in the sense that actual trajectories under resistive forces due to physical media (which are not geodesics of a Minkowski space) are turned into isogeodesics in isospace having the shape of the geodesics in the absence of resistive forces.

The simplest possible example is given by the iso-Euclidean representation of a straight stick partially immersed in water. In conventional representations the stick penetrating in water with an angle α appears as bended at the point of immersion in water with an angle $\gamma = \alpha + \beta$, where β is the angle of refraction. In iso-Euclidean representation the stick remains straight also in its immersion because the isoangle $\hat{\gamma} = \gamma \times \hat{I}_\gamma$ recovers the original angle α with $\hat{I}_\gamma = \alpha/(\alpha+\beta)$.

The situation is essentially the same for our representation of interior gravitation because the latter is represented in isospace over isofield via field equations (this time necessarily with sources) that formally coincide with conventional equations on a conventional Riemannian spacetime. Being noncanonical, all interior features are invariantly represented via generalized units.

3.2.9F. Isodual Minkowski-Santilli isospaces and isogeometry. The *isodual Minkowski-Santilli isospaces* were introduced for the first time by Santilli in Ref. [8] of 1985 and then studied in various works (see the references of Chapter 1), and can be written

$$\hat{M}^d = \hat{M}^d(\hat{x}^d, \hat{\eta}^d, \hat{R}^d):$$

$$\hat{x}^d = \{x^{\mu d}\} \times^d \hat{I}^d = \{x^\mu\} \times (-\hat{I}) = \{r^d, c_o^d \times^d t^d\} \times^d \hat{I}^d, \tag{3.2.174a}$$

$$\hat{\eta}^d = -\hat{\eta}. \tag{3.2.174b}$$

The *isodual Minkowski-Santilli isogeometry* is the geometry of isodual isospaces M^d over R^d and was studied for the first time by Santilli in Ref. [26] of 1998.

The physically and mathematically most salient property of the latter geometry is that it is *characterized by negative units of space, time, etc., and negative norms.* Therefore, in addition to a change in the sign of the charge, we also have change of sign of masses, energies, and other quantities normally positive for matter. Similarly, we have the *isodual isospace and isotime coordinates*

$$\hat{x}^d = \hat{x}^d \times^d \hat{I} = -\hat{x}, \;\; \hat{t}^d = t^d \times^d \hat{I}^d = -\hat{t}. \tag{3.2.175}$$

Thus, motion under isoduality is in a time direction *opposite* to the conventional motion. These features are necessary so as to have a classical representation of antimatter in interior conditions whose operator image yields indeed antiparticles (rather than particles with the wrong sign of the charge).

We also have the following important

LEMMA 3.2.12 [17]: Isodualities are independent from spacetime inversions

$$r' = \pi \times r = -r, \;\; t' = \tau \times t = -t. \tag{3.2.176}$$

Proof. Inversions occur within the same original space and keep the unit fixed, while isodualities require a map to a different space, and change the sign of the unit. Therefore, in addition to maps in different spaces, isodualities have numerical value different than the inversions. **q.e.d.**

These are the conceptual roots for the isodual theory of antimatter to predict a *new photon*, the *isodual photon* emitted by antimatter [17]. When applied to the photon, charge conjugation and, more generally, the PCT theorem, do not yield a new photon, as well known. This is not the case under isoduality because all physical characteristics change in sign and numerical value. As a result, *the isodual photon is indistinguishable from the ordinary photon under all interactions except gravitation*. In fact, as indicated in Chapter 1, the isodual photon is predicted to experience antigravity in the field of matter, thus offering, apparently for the first time, a possibility for the future study whether far away galaxies and quasars are made up of matter or of antimatter.

Another important property of isoduality is expressed by the following:

LEMMA 3.2.13 [26]: The intervals of conventional and isotopic Minkowskian spaces are invariant under the joint isodual maps $\hat{I}^d \to \hat{I}^d$ and $\hat{\eta} \to \hat{\eta}^d$,

$$\hat{x}^2 = (x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I} \equiv [x^\mu \times (-\hat{\eta}_{\mu\nu}) \times x^\nu] \times (-\hat{I}). \tag{3.2.177}$$

As a result, *all physical laws applying in conventional Minkowskian geometry for the characterization of matter also apply to its isodual image for the characterization of antimatter.*

Note that, strictly speaking, the intervals are not isoselfdual because

$$\hat{x}^{\hat{2}} = \hat{x}^\mu \hat{\times} \hat{M}_{\mu\nu} \hat{\times} \hat{x}^\mu \ \to \ \hat{x}^{d\hat{2}d} = \hat{x}^{\mu d} \times^d \hat{M}^d_{\mu\nu} \times^d \hat{x}^{\mu d} = \hat{x}^{d\hat{2}d} = -\hat{x}^{\hat{2}}. \tag{3.2.178}$$

To outline the *Riemannian* characteristics of the isodual Minkowski-Santilli isogeometry, we consider an *isodual isovector isofield* $\hat{X}^d(\hat{x}^d)$ on $\hat{M}^d$ which is explicitly given by $\hat{X}^d(\hat{x}^d) = -X^t(-x^t \times \hat{I}) \times \hat{I}$. The *isodual exterior isodifferential* of $\hat{X}^d(\hat{x}^d)$ is given by

$$\hat{D}^d \hat{X}^{\mu d}(\hat{x}^d) = \hat{d}^d \hat{X}^{\mu d}(\hat{x}^d) \hat{+}^d \hat{\Gamma}^{d\,\mu}_{\alpha\beta} \hat{\times}^d \hat{X}^{\alpha d} \hat{\times}^d \hat{d}^d \hat{x}^{\beta d} = \hat{D} \hat{X}^{t\mu}(-\hat{x}^t), \tag{3.2.179}$$

where the $\hat{\Gamma}^d$'s are the components of the *isodual isoconnection*. The *isodual isocovariant isoderivative* is then given by

$$\hat{X}^{\mu d}(\hat{x}^d)_{\hat{|}d\nu} = \hat{\partial}^d \hat{X}^{\mu d}(\hat{x}^d) \hat{/}^d \hat{\partial}^d \hat{x}^{\nu d} \hat{+} \hat{\Gamma}^{d\,\mu}_{\alpha\nu} \hat{\times}^d \hat{X}^{\alpha d}(\hat{x}^d) = -\hat{X}^{t\mu}(-\hat{x}^t)_{\hat{|}k}. \tag{3.2.180}$$

The interested reader can then easily derive the remaining notions of the new geometry. It is an instructive exercise for the interested reader to prove the following isodualities:

$$
\begin{array}{cc}
\text{Isodual isounit} & \hat{I} \rightarrow \hat{I}^d = -\hat{I}, \\
\text{Isodual isometric} & \hat{\eta} \rightarrow \hat{\eta}^d = -\eta, \\
\text{Isodual isoconnection coefficients} & \hat{\Gamma}_{\alpha\beta\gamma} \rightarrow \hat{\Gamma}^d_{\alpha\beta\gamma} = \hat{\Gamma}_{\alpha\beta\gamma}, \\
\text{Isoflatness isotensor} & R_{\alpha\beta\gamma\delta} \rightarrow R^d_{\alpha\beta\gamma\delta} = -R_{\alpha\beta\gamma\delta}, \\
\text{Isodual iso-Ricci isotensor} & \hat{R}_{\mu\nu} \rightarrow \hat{R}^d_{\mu\nu} = \hat{R}_{\mu\nu}, \\
\text{Isodual iso-Ricci isoscalar} & \hat{R} \rightarrow \hat{R}^d = \hat{R}, \\
\text{Isodual iso-Freud isoscalar} & \hat{\Theta} \rightarrow \hat{\Theta}^d = -\hat{\Theta}, \\
\text{Isodual Iso-Einstein isotensor} & \hat{G}_{\mu\nu} \rightarrow \hat{G}^d_{\mu\nu} = -\hat{G}_{\mu\nu}, \\
\text{Isodual electromagnetic potentials} & A_\mu \rightarrow A^d_\mu = -A_\mu, \\
\text{Isodual electromagnetic field} & F_{\mu\nu} \rightarrow F^d_{\mu\nu} = -F_{\mu\nu}, \\
\text{Isodual elm energy-mom. isotensor} & T_{\mu\nu} \rightarrow T^d_{\mu\nu} = -T_{\mu\nu}.
\end{array}
\tag{3.2.181}
$$

More detailed isogeometric studies are left to interested readers. Specific applications to gravitational treatments of matter and antimatter are presented in Section 3.5.

3.2.10 Isosymplectic Geometry and its Isodual

As it is well known, the *symplectic geometry* had an important role in the construction of quantum mechanics because it permitted the mathematically rigorous verification, known as *symplectic quantization,* that original quantization procedures, known also as *naive quantization,* were correct.

No broadening of quantum mechanics can be considered mature unless it admits fully equivalent procedures in the map from classical to operator forms known as *isoquantization* also called *hadronization* (rather than quantization).

For this purpose. Santilli [31] presented in 1988 the first known *isotopies of the symplectic geometry,* subsequently studied in various works, with a general presentation available in Vols. I, II of this series (see in particular Chapter 5 of Vol. I [6]). The new geometry is today known as *Santilli's isosymplectic geometry.*

We cannot possibly review here the isosymplectic geometry in detail and have to suggest interested readers to study Refs. [6,7]. Nevertheless, an indication of the basic lines is important for the self-sufficiency of this monograph.

Let us ignore the global (also called abstract) formulation of the symplectic geometry and consider for clarity and simplicity only its realization in a local chart (or coordinates).[16] A *topological manifold* $M(R)$ on the reals R admits the local realization as an Euclidean space $E(r, \delta R)$ with local contravariant coordinates

[16] Again, the literature on the conventional symplectic geometry is so vast to discourage discriminatory quotations.

$r = (r^i),\ i = i, 2, \ldots, N$. The *cotangent bundle* T^*M then becomes the ordinary phase space with local coordinates $(r, p) = (r^i, p_i)$, where p_i represents the tangent vectors (physically the linear momentum). The *canonical one-form* then admits the local realization

$$\theta = p_i \times dr^i. \tag{3.2.182}$$

The *fundamental (canonical) symplectic form* is then given by the exterior derivative of the preceding one form

$$\omega = d\theta = p_i \wedge dr^i, \tag{3.2.183}$$

and one can easily prove that it is closed, namely, that $d\omega \equiv 0$.

Consider now the *isotopological isomanifold* (introduced earlier) $\hat{M}(\hat{R})$ on the isoreals $\hat{R}$ with basic isounit $\hat{I}$. Its realization on local coordinates is given by the Euclid-Santilli isospace $\hat{E}(\hat{r}, \hat{\Delta}, \hat{R})$ with local contravariant isocoordinates $\hat{r} = (r^i) \times \hat{I}$. Then, the *isocotangent isobundle* $\hat{T}^*\hat{M}$ admits as local realization the *isophase isospace* with local coordinates $(\hat{r}^i, \hat{p}_i)$, where $\hat{p}$ is again a tangent isovector. The novelty is given by the fact that the unit of $\hat{p}$ is the *inverse* of that of $\hat{r}$ and we shall write

$$\hat{r} = r \times \hat{I},\ \ \hat{p} = p \times \hat{T},\ \ \hat{I} = 1/\hat{T}. \tag{3.2.184}$$

This property was identified for the first time by Santilli [31] (for a mathematical treatment see also Ref. [10]) because not identifiable in the conventional symplectic geometry due to the use of the trivial unit for which $I^{-1} \equiv I = +1$.

Consequently, we have the isodifferentials

$$\hat{d}\hat{r} = \hat{T} \times d(r \times I),\ \ \hat{d}\hat{p} = \hat{I} \times d(p \times \hat{T}). \tag{3.2.185}$$

The *isocanonical one-isoform* is then given by

$$\hat{\theta} = \hat{p} \hat{\times} \hat{d}\hat{r} = (p \times \hat{T}) \times \hat{I} \times \hat{d}(\hat{r}) = p \times \hat{T} \times d(r \times \hat{I}). \tag{3.2.186}$$

The *fundamental isocanonical two-isoform* is then given by

$$\hat{\omega} = \hat{d}\hat{\theta} = \hat{p} \hat{\wedge} \hat{d}\hat{r} = dp_i \wedge dr^i \equiv \omega, \tag{3.2.187}$$

from which the preservation of closure under isotopy, $\hat{d}\hat{\omega} \equiv \hat{0} = 0$ trivially follows.

LEMMA 3.2.14 [31,10]: The fundamental symplectic and isosymplectic two-forms coincide.

The identity of the fundamental isocanonical and canonical two-forms explains why isosymplectic geometry escaped detection by mathematicians for centuries.

It is evident that, in view of the positive-definiteness of the isounit, *the symplectic and isosymplectic geometries coincide at the global (abstract) realization-free level* to such an extent that there is not even the need of changing formulae in the literature of the symplectic geometry because the isosymplectic geometry can be expressed with the pre-existing formalism and merely subject it to a broader realization.

Despite this simplicity, the physical implications are by far non-trivial. In fact, unlike the conventional two-form, and thanks to the background TSSFN isotopology, the fundamental isocanonical two-form is universal for all possible (sufficiently smooth and regular but otherwise arbitrary) nonlocal and non-Hamiltonian systems. To illustrate this feature, let us consider a *vector field* of the cotangent bundle that must be strictly *local-differential* to avoid catastrophic inconsistencies with the underlying local-differential Euclidean topology, T^*M

$$X(r,p) = A_i(r,p) \times \frac{\partial}{\partial r^i} + B^i(r,p) \times \frac{\partial}{\partial p_i}, \tag{3.2.188}$$

or in unified notations

$$b = (b^\mu) = (r^i, p_j), \quad \mu = 1, 2, \ldots, 2N, \tag{3.2.189}$$

$$X(b) = X_\mu(b) \times \frac{\partial}{\partial b^\mu}, \tag{3.2.190}$$

is said to be a *Hamiltonian vector field* when there exists a function $H(r,p) = H(b)$ on T^*M, called the *Hamiltonian,* verifying the identity

$$A_i \times dr^i + B^i \times dp_i = -dH(r,p) \tag{3.2.191}$$

or in unified notation

$$X\rfloor\omega = dH, \tag{3.2.192}$$

that is

$$\omega_{\mu\nu} \times X^\mu \times db^\nu = -dH, \tag{3.2.193}$$

where the fundamental symplectic form has the components

$$\omega = dp_i \wedge dr^i = \frac{1}{2} \times \omega_{\mu\nu} \times db^\mu \wedge db^\nu, \tag{3.2.194}$$

$$(\omega_{\mu\nu}) = \begin{pmatrix} O_{N\times N} & -I_{N\times N} \\ I_{N\times N} & O_{N\times N} \end{pmatrix}. \tag{3.2.195}$$

Eq. (3.2.192) can hold if and only if

$$\omega_{\mu\nu} \times \frac{db^\nu}{dt} = \frac{\partial H}{\partial b^\mu}, \tag{3.2.196}$$

from which one recovers the familiar truncated Hamilton's equations

$$\frac{dr^i}{dt} = \frac{\partial H}{\partial p_i}, \quad \frac{dp_i}{dt} = -\frac{\partial H}{\partial r^i}. \tag{3.2.197}$$

The main physical limitations is that *the condition for a vector field to be Hamiltonian constitutes a major restrictions because vector fields in the physical reality are generally non-Hamiltonian, besides existing from the limitations of the topology underlying the symplectic geometry.*

As we shall see in Section 3.3, the above restrictions is removed for Santilli isosymplectic geometry that acquire the character of *direct universality,* that is, the capability of representing all sufficiently smooth and regular but otherwise arbitrary vector fields (universality) in the local chart of the experimenter (direct universality).

In fact, expression (3.2.192) is lifted into the form

$$\hat{\omega}_{\mu\nu} \hat{\times} \frac{\hat{d}\hat{b}^\nu}{\hat{d}\hat{t}} = \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{b}^\mu}, \tag{3.2.198}$$

that, under the assumption for simplicity that $\hat{t} = t$, and by removing common factors, reduces to

$$\frac{dr^i}{dt} = \frac{\hat{\partial} H}{\hat{\partial} p_i} = \hat{T}^i_j(r,p) \times \frac{\partial H}{\partial p_j}; \tag{3.2.199}$$

$$\frac{dp_i}{dt} = -\frac{\hat{\partial} H}{\hat{\partial} r^i} = -\hat{I}^j_i \times \frac{\partial H}{\partial r^j}. \tag{3.2.200}$$

As we shall see better in Section 3.3, direct universality then follows from the number of free functions $\hat{T}^j_i$ as well as the arbitrariness of their functional dependence.

We shall also show that the achievement of a direct isogeometric representation of nonlocal and non-Hamiltonian vector fields representing interior dynamical problems permits their consistent map into an operator form, by therefore reaching hadronic mechanics in a mathematically rigorous, unique and unambiguous way.[17]

The construction of the *isodual isosymplectic geometry* [6] is an instructive exercise for readers interested in serious studies of antimatter in interior dynamical conditions.

[17]Note the crucial role of the isodifferential calculus for the isosymplectic geometry and its implications.

3.2.11 Isolinearity, Isolocality, Isocanonicity and Their Isodualities

In Section 3.1 we pointed out that the primary physical characteristics of particles and antiparticles in interior conditions (such as a neutron in the core of a neutron star) are nonlinear, nonlocality and noncanonicity due to the mutual penetration-overlapping of their wavepackets with those of the surrounding medium.

In the preceding subsections we have identified isotopic means for mapping linear, local and canonical systems into their most general possible nonlinear, nonlocal and noncanonical form. In this section we show how the isotopies permit the reconstruction of linearity, locality and canonicity on isospaces over isofields, called *isolinearity, isolocality and isocanonicity* for the case of particles, with their isodual counterpart for antiparticles.

The understanding of this seemingly impossible task requires the knowledge that conventional methods have only one formulation. By contrast, all isotopic methods have a dual formulation, the first in isospace over isofields, and the second when projected in ordinary spaces over ordinary fields. Deviations from conventional properties can only occur in the latter formulation because in the former all original axiomatic properties are preserved by construction.

Let $S(r, R)$ be a conventional real vector space with local coordinates r over the reals $R = R(n, +, \times)$, and let

$$r' = A(w) \times r, \quad r'^t = r^t \times A^t(w), \quad w \in R. \tag{3.2.201}$$

be a conventional right and left linear, local and canonical transformation on S, where t denotes transpose.

The isotopic lifting $S(r, R) \rightarrow \hat{S}(\hat{r}, \hat{R})$ requires a corresponding necessary isotopy of the transformation theory. In fact, it is instructive for the interested reader to verify that the application of conventional linear transformations to the isospace $\hat{S}(\hat{r}, \hat{R})$ causes the loss of linearity, transitivity and other basic properties.

For these and other reasons, Santilli submitted in the original proposals [4,5] of 1978 (see monographs [6,7] for comprehensive treatments and applications) the isotopy of the transformation theory, called *isotransformation theory,* which is characterized by isotransforms (where we make use of the notion of isofunction of Section 3.2.4)

$$\hat{r}' = \hat{A}(\hat{w}) \hat{\times} \hat{r} = \hat{A}(\hat{w}) \times \hat{T} \times \hat{r} = [A(\hat{T} \times w) \times \hat{I}] \times \hat{T} \times (r \times \hat{I}) =$$

$$= A[\hat{T}(r, \ldots) \times w] \times \hat{r}, \tag{3.2.202a}$$

$$\hat{r}'^{\hat{t}} = \hat{r}^{\hat{t}} \hat{\times} \hat{A}^{\hat{t}} \hat{w} = \hat{r}^{\hat{t}} \times A^t[\hat{T}(r) \times w]. \tag{3.2.202b}$$

The most dominant aspect in the transition from the conventional to the isotopic transforms is that, while the former are linear, local and canonical, the latter are nonlinear in the coordinates as well as other quantities and their derivatives of arbitrary order, nonlocal-integral in all needed quantities, and noncanonical when projected in the original spaces $S(r, R)$. This is due to the unrestricted nature of the functional dependence of the isotopic element $\hat{T} = \hat{T}(r, \ldots)$.

But the conventional and isotopic transforms coincide at the abstract level where we have no distinction between the modular action $A(w) \times r$ and $\hat{A}(\hat{w})\hat{\times}\hat{r}$. Therefore, isotransforms (3.2.202) are *isolinear* when formulated on isospace $\hat{S}$ over the isofield $\hat{R}$, because they verify the conditions

$$\hat{A}\hat{\times}(\hat{n}\hat{\times}\hat{r} + \hat{m}\hat{\times}\hat{p}) = \hat{n}\hat{\times}\hat{A}\hat{\times}\hat{r} + \hat{m}\hat{\times}\hat{A}\hat{\times}\hat{p}, \quad \hat{r}, \hat{p} \in \hat{S}, \quad \hat{n}, \hat{m} \in \hat{R}. \tag{3.2.203}$$

Note that conventional transforms are characterized by a *right modular associative action* $A \times r$. Isotransforms are then characterized by the *right isomodular isoassociative action* $\hat{A}\hat{\times}\hat{r}$. Therefore, we do have the preservation of the original axiomatic structure and isotransforms are indeed an isotopy of conventional transforms.

The situation for locality and canonicity follows the same lines [4,5,6,7]. Conventional methods are local in the sense that they are defined at a finite set of isolated points. The isotopic methods are *isolocal* in the sense that they verify the condition of locality in isospaces over isofields. However, their projection on conventional space is nonlocal-integral, because that is the general characteristic of the isotopic element $\hat{T}$, as illustrated, e.g., in Eq. (3.1.202).

Similarly, conventional methods are canonical in the sense that they can be characterized via a first-order canonical action in phase space (or cotangent bundle). The isotopic methods are *isocanonical* in the sense that, as we shall see in Section 3.3, they are derivable from an *isoaction* that is first-order and canonical on isospaces over isofields, although, when projected on ordinary spaces over ordinary fields, such an isoaction is of arbitrary order.

LEMMA 3.2.15 [6,7]: All possible nonlinear, nonlocal and noncanonical transforms on a vector space $S(r, R)$

$$r' = B(w, r, \ldots) \times r, \quad r \in S, \quad w \in R, \tag{3.2.204}$$

can always be rewritten in an identical isolinear, isolocal and isocanonical form, that is, there always exists at least one isotopy of the base field, $R \to \hat{R}$, and a corresponding isotopy of the space $S(r, R) \to \hat{S}(\hat{r}, \hat{R})$, such as

$$B(w, r, \ldots) \equiv A(\hat{T} \times w), \tag{3.2.205}$$

under which

$$r' = B(w, r, \ldots) \times r \equiv A(\hat{T} \times w) \times r \equiv \hat{A}(\hat{w})\hat{\times}r, \tag{3.2.206}$$

from which the isolinear form (3.2.202) follows.

COROLLARY 3.2.15A [6,7]: Under sufficient continuity and regularity conditions, all possible ordinary differential equations that are nonlinear in ordinary spaces over ordinary fields can always be turned into an identical form that is isolinear on isospaces over isofields,[18]

$$\dot{r} - E(\dot{r}, w, \ldots) \rightarrow \hat{\dot{r}} - A[\hat{T}(\dot{r}, w, \ldots) \times \dot{r} - B[\hat{T}(\dot{r}, w, \ldots] \equiv$$
$$\equiv \hat{\dot{r}} - \hat{A}(\hat{w}) \hat{\times} \hat{\dot{r}} - \hat{B}(w) = 0. \tag{3.2.207}$$

The above properties are at the foundation of the *direct universality* of isotopic methods, that is, their applicability to all possible (sufficiently smooth and regular) nonlinear, nonlocal and noncanonical systems (universality) in the frame of the experimenter (direct universality).

In order to apply isotopic methods to a nonlinear, nonlocal and noncanonical system, one has merely to identity one of its possible isolinear, isolocal and isocanonical identical reformulations in the same system of coordinates. The applicability of the methods studied in this monograph then follows.

The *isodual isotransforms* are given by the image of isotransforms (3.2.202) under isoduality, and, as such, are defined on the isodual isospace $\hat{S}^d(\hat{r}^d, \hat{R}^d)$ over the isodual isofield $\hat{R}^d$ with isodual isounit $\hat{I}^d = 1/\hat{T}^d = -\hat{I}^\dagger$. [6,7] with evident properties

$$\hat{A}^d \hat{\times}^d (\hat{n}^d \hat{\times}^d \hat{r}^d + \hat{m}^d \hat{\times}^d \hat{p}^d) =$$
$$= \hat{n}^d \hat{\times}^d \hat{A}^d \hat{\times}^d \hat{r}^d + \hat{m}^d \hat{\times}^d \hat{A}^d \hat{\times}^d \hat{p}^d, \ \ \hat{r}^d, \hat{p}^d \in \hat{S}^d, \ \ \hat{n}^d, \hat{m}^d \in \hat{R}^d. \tag{3.2.208}$$

The definition of *isodual isolinearity, isolocality and isocanonicity* then follows.

From now on, we shall use isotransforms for the study of interior dynamical systems of particles and their isodual for interior systems of antiparticles.

3.2.12 Lie-Santilli Isotheory and its Isodual

3.2.12A. Statement of the Problem. As it is well known, Lie's theory has permitted outstanding achievements in various disciplines throughout the 20-th century. Nevertheless, in its current conception and realization, Lie's theory is linear, local-differential and canonical-Hamiltonian.[19]

[18] The author has proposed for over a decade that mathematicians use the property of this Corollary 3.2.15A to identify simpler methods for the solution of nonlinear differential equations, but the request has not been met as yet, to our best knowledge.

[19] The literature on Lie's theory is also vast to discourage discriminatory listings. In any case, its knowledge is a necessary pre-requisite for the understanding of this section.

As such, Lie's theory is exactly valid for exterior dynamical systems, but possesses clear limitations for interior dynamical systems since the latter are nonlinear, nonlocal and noncanonical. This occurrence mandates a suitable revision of Lie's theory such to be exactly valid for interior dynamical systems without approximations.

Independently from that, Lie's theory in its current formulation is solely applicable to matter, evidently because there exists no antiautomorphic version of the conventional Lie's theory as necessary for the correct treatment of antimatter beginning at the classical level, as shown in Chapters 1 and 2.

Another central problem addressed in these studies is the construction of the universal *symmetry* (and not "covariance") of gravitation for matter and, independently, for antimatter, that is, a symmetry for all possible exterior and interior gravitational line elements of matter and, under antiautomorphic image, of antimatter.

Yet another need in physics is the identification of the exact symmetry that can effectively replace broken Lie symmetries, which exact symmetry cannot possibly be a conventional Lie symmetry due to the need of preserving the original dimensions so as to avoid the prediction on nonphysical effects and/or hypothetical new particles.

It is evident that Lie's theory in its current formulation is unable to solve the above identified problems. In a memoir of 1978, Santilli [4] proposed a step-by-step generalization of the conventional Lie theory specifically conceived for nonlinear, nonlocal-integral and nonpotential-noncanonical systems.

The generalized theory was subsequently studied by Santilli in a variety of papers (see monographs [1,2,6,7,14,15] and references quoted therein). The theory was also studied by a number of mathematicians and theoreticians, and it is today called the *Lie-Santilli isotheory* (see, e.g., monographs [32–37] and references quoted therein, as well as specialized papers [38–43]).

A main characteristic of the Lie-Santilli isotheory, that distinguishes it from other possible generalizations, is its isotopic character, that is, the preservation of the original Lie axioms when formulated on isospaces over isofields, despite its nonlinear, nonlocal and noncanonical structure when projected in ordinary spaces. This basic feature is evidently permitted by the reconstruction of linearity, locality and canonicity on isospaces over isofields studied in the preceding section.

To begin, let us recall that Lie's theory is centrally dependent on the basic N-dimensional unit $I = Diag.(1, 1, \ldots, 1)$ of the enveloping algebra. The main idea of the Lie-Santilli isotheory [4] is the reformulation of the entire conventional theory with respect to the most general possible isounit $\hat{I}(x, \dot{x}, \ddot{x}, \ldots)$.

One can therefore see from the very outset the richness and novelty of the isotopic theory since isounits with different topological features (such as Hermiticity, non-Hermiticity, positive-definiteness, negative-definiteness, etc.) characterize different generalized theories.

In this section we outline the rudiments of the Lie-Santilli isotheory properly speaking, that with positive-definite isounits and its isodual with negative-definite isounits. A knowledge of Lie's theory is assumed as a pre-requisite. A true technical knowledge of the Lie-Santilli isotheory can only be acquired from the study of mathematical works such as monographs [2,6,14,36,37].

In inspecting the literature, the reader should be aware that Santilli [4] constructed the isotopies of Lie's theory as a particular case of the broader Lie-admissible theory studied in Chapter 4 occurring for non-Hermitian generalized units, and known as *Lie-Santilli genotheory.* As a matter of fact, a number of aspects of the isotheory can be better identified within the context of the broader genotheory.

The extension to non-Hermitian isounits (that was the main object of the original proposal [4]) requires the exiting of Lie's theory in favor of the covering Lie-admissible theory, and will be studied in Chapter 4.

The isotopies of Lie's theory were proposed by Santilli from first axiomatic principles without the use of any map or transform. It is today known that the isotheory cannot be entirely derived via the use of noncanonical-nonunitary transforms since some of the basic structures (such as the isodifferential calculus) are not entirely derivable via noncanonical-nonunitary transforms.

3.2.12B. Universal Enveloping Isoassociative Algebras. Let ξ be an *associative algebra* over a field $F = F(a, +, \times)$ of characteristic zero with generic elements $A, B, C, \ldots$, trivial associative product $A \times B$ and unit I. The infinitely possible isotopes $\hat{\xi}$ of ξ were first introduced in Ref. [4] under the name of *isoassociative algebras*. In the original proposal $\hat{\xi}$ coincides with ξ as vector spaces but is equipped with Santilli's isoproduct so as to admit the isounit as the correct left and right unit

$$\hat{I}(x, \dot{x}, \ddot{x}, \ldots) = 1/\hat{T} > 0, \tag{3.2.209a}$$

$$\hat{A}\hat{\times}\hat{B} = \hat{A} \times \hat{T} \times \hat{B}, \;\; \hat{A}\hat{\times}(\hat{B}\hat{\times}\hat{C}) = (\hat{A}\hat{\times}\hat{B})\hat{\times}\hat{C}, \tag{3.2.209b}$$

$$\hat{I}\hat{\times}\hat{A} = \hat{A}\hat{\times}\hat{I} \equiv \hat{A}, \;\; \forall \hat{A} \in \hat{\xi}, \tag{3.2.209c}$$

where $\hat{A}, \hat{B}, \ldots$ denote the original elements $A, B, \ldots$ formulated on isospace over isofields.

Let $\xi = \xi(L)$ be the *universal enveloping associative algebra* of an N-dimensional Lie algebra L with ordered basis X_k, $k = 1, 2, \ldots, N$, and attached antisymmetric algebra isomorphic to the Lie algebras, $[\xi(L)]^- \approx L$ over F, and let the infinite-dimensional basis $I, X_k, X_i \times X_j, i \leq j, \ldots$ of $\xi(L)$ be characterized by the *Poincaré-Birkhoff-Witt theorem.*

A fundamental property submitted in the original proposal [4] (see also [2], pp. 154–163) is the following

THEOREM 3.2.11 (Poincaré-Birkhoff-Witt-Santilli isotheorem): Isocosets of the isounit and the standard, isomonomials

$$\hat{I},\ \ X_k,\ \ \hat{X}_i\hat{\times}\hat{X}_j,\ i\leq j,\ \ \hat{X}_i\hat{\times}\hat{X}_j\hat{\times}\hat{X}_k,\ i\leq j\leq k,\ \ldots, \tag{3.2.210}$$

form a basis of universal enveloping isoassociative algebra $\hat{\xi}(L)$ of a Lie algebra L (also called isoenvelope for short).

The first application of the above infinite-dimensional basis is a rigorous characterization of the isoexponentiation, Eq. (3.2.72), i.e.,

$$\hat{e}^{\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}} = \hat{e}^{i\times w\times\hat{X}} =$$

$$= \hat{I} + \hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}\hat{/}\hat{1}! + (\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X})\hat{\times}(\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X})\hat{/}\hat{2}! + \ldots =$$

$$= \hat{I}\times(e^{i\times w\times\hat{T}\times\hat{X}}) = (e^{i\times w\times\hat{X}\times\hat{T}})\times\hat{I},\ \ \hat{i} = i\times\hat{I}, \hat{w} = w\times\hat{I}\in\hat{F}. \tag{3.2.211}$$

The nontriviality of the Lie-Santilli isotheory is illustrated by the emergence of the nonlinear, nonlocal and noncanonical isotopic element $\hat{T}$ directly in the exponent, thus ensuring the desired generalization.

The implications of Theorem 3.2.11 also emerge at the level of isofunctional analysis because all structures defined via the conventional exponentiation must be suitably lifted into a form compatible with Theorem 3.2.11, as illustrated by the *iso-Fourier transforms,* Eq. (3.2.88).

It is today known that the main lines of isoenvelopes can indeed be derived via the use of noncanonical-nonunitary transforms, such as

$$U\times U^{\dagger}\neq I, \tag{3.2.212a}$$

$$I\rightarrow\hat{I} = U\times I\times U^{\dagger}, \tag{3.2.212b}$$

$$X_i\times X_j\rightarrow U\times(X_i\times X_j)\times U^{\dagger} = \hat{X}_i\hat{\times}\hat{X}_j, \tag{3.2.212c}$$

$$X_i\times X_j\times X_k\rightarrow U\times(X_i\times X_j\times X_k)\times U^{\dagger} = \hat{X}_i\hat{\times}\hat{X}_j\hat{\times}\hat{X}_k,\ \ \text{etc.} \tag{3.2.212d}$$

Nevertheless, the uncontrolled use of the above transforms may lead to misrepresentations. In fact, a primary objective of the Lie-Santilli isotheory is that of preserving the original generators and parameters and change instead the associative and Lie products in an axiom-preserving way to accommodate the treatment of nonlinear, nonlocal and noncanonical interactions.

The preservation of the generators is, in particular, necessary for physical consistency because they represent conserved total quantities (such as the total energy, total angular momentum, etc.). These total quantities remain unchanged in the transition from closed Hamiltonian and non-Hamiltonian systems (see Section 3.1.2). Equivalently, the generators of Lie's theory cannot be altered by non-Hamiltonian effects.

This physical requirement can only be achieved by preserving conventional generators X_k and lifting instead their product $X_i \times X_j \rightarrow X_i \hat{\times} X_j = X_i \times \hat{T} \times X_j$, which is the original formulation of the Lie-Santilli isotheory [4] and remain the formulation needed for applications to this day. It is essentially given by the projection of the isotopic formulation on conventional spaces over conventional fields.

3.2.12C. Lie-Santilli Isoalgebras. As it is well known, Lie algebras are the antisymmetric algebras $L \approx [\xi(L)]^-$ attached to the universal enveloping algebras $\xi(L)$. This main characteristic is preserved although enlarged under isotopies (see [4,2] for details). We therefore have the following

DEFINITION 3.2.15 [4]: A finite-dimensional isospace $\hat{L}$ with generic elements $\hat{A}, \hat{B}, \ldots$, over the isofield $\hat{F}$ with isounit $\hat{I} = 1/\hat{T} > 0$ is called a "Lie-Santilli isoalgebra" over $\hat{F}$ when there is a composition $[\hat{A}\hat{,}\hat{B}]$ in $\hat{L}$, called "isocommutator", that is isolinear as an isovector space and such that all the following axioms are satisfied

$$[\hat{A}\hat{,}\hat{B}] = -[\hat{B}\hat{,}\hat{A}], \tag{3.2.213a}$$

$$[\hat{A}\hat{,}[\hat{B}\hat{,}\hat{C}]] + [\hat{B}\hat{,}[\hat{C}\hat{,}\hat{A}]] + [\hat{C}\hat{,}[\hat{A}\hat{,}\hat{B}]] \equiv 0, \tag{3.2.213b}$$

$$[\hat{A}\hat{\times}\hat{B}\hat{,}\hat{C}] = \hat{A}\hat{\times}[\hat{B}\hat{,}\hat{C}] + [\hat{A}\hat{,}\hat{C}]\hat{\times}\hat{B}, \ \ \forall \hat{A}, \hat{B}, \hat{C} \in \hat{L}. \tag{3.2.213c}$$

The isoalgebras are said to be: *isoreal, isocomplex or isoquaternionic* depending on the assumed isofield and *isoabelian* when $[\hat{A}\hat{,}\hat{B}] \equiv \forall \hat{A}, \hat{B} \in \hat{L}$. A subset $\hat{L}^o$ of $\hat{L}$ is said to be an *isosubalgebra* of $\hat{L}$ when $[\hat{L}^o\hat{,}\hat{L}^o] \subseteq \hat{L}^o$. $\hat{L}^o$ is called an *isoideal* of $\hat{L}$ when $[\hat{L}^o\hat{,}\hat{L}] \subseteq \hat{L}^o$. A *maximal isoideal* verifying the property $[\hat{L}^o\hat{,}\hat{L}^o] = 0$ is called the *isocenter* of $\hat{L}$.

For the isotopies of additional conventional notions, theorems and properties of Lie algebras, one may see monograph [2,6,36,37].

We merely recall the *isotopic generalizations of the celebrated Lie's First, Second and Third Theorems* introduced in the original proposal [4], but which we do not review here for brevity. For instance, the *Lie-Santilli Second Isotheorem* reads

$$[\hat{X}_i\hat{,}\hat{X}_j] = \hat{X}_i\hat{\times}\hat{X}_j - \hat{X}_j\hat{\times}\hat{X}_i = \tag{3.2.214a}$$

$$= \hat{X}_i \times \hat{T}(x, \dot{x}, \ddot{x}, \ldots) \times \hat{X}_j - \hat{X}_j \times \hat{T}(x, \dot{x}, \ddot{x}, \ldots) \times \hat{X}_i = \hat{C}^k_{ij}(x, \dot{x}, \ddot{x}, \ldots)\hat{\times}\hat{X}_k, \tag{3.2.214b}$$

where the C's, called the *structure isofunctions*, generally have an explicit dependence on the underlying isovariable (see the examples later on), and verify certain restrictions from the Isotopic Third Theorem.

It is today known that Lie-Santilli isoalgebras can be reached via a noncanonical-nonunitary transform of conventional Lie algebras. In fact, we have

$$\begin{gathered}[X_i, X_j] = C_{ij}^k \times X_k \rightarrow \\ U \times [X_i, X_j] \times U^\dagger = [\hat{X}_i \hat{,} \hat{X}_j] = \\ U \times (C_{ij}^k \times X_k) \times U^\dagger = \hat{C}_{ij}^k(x, \dot{x}, \ddot{x}, \ldots) \hat{\times} \hat{X}_k. \end{gathered} \tag{3.2.215}$$

However, again, this type of derivation of the isotheory may be misleading in physical applications due to the need to preserve the original generators unchanged, in accordance with the original formulation [4] of 1978. In this case we shall use the following projection of the isoalgebras on the original space over the original field

$$[X_i \hat{,} X_j] = X_i \times \hat{T} \times X_j - X_j \times \hat{T} \times X_i = C_{ij}^k(x, \dot{x}, \ldots) \times X_k. \tag{3.2.216}$$

It has been proved (see, e.g., [2,4,6] for details) that *Lie-Santilli isoalgebras $\hat{L}$ are isomorphic to the original algebra L*. In other words, the isotopies with $\hat{I} > 0$ cannot characterize any new algebra because all possible Lie algebras are known from Cartan classification. Therefore, Lie-Santilli isoalgebras merely provide new nonlinear, nonlocal and noncanonical realizations of existing algebras. It should be stresses that the above isomorphism is lost for more general liftings as shown in the next chapter.

3.2.12D. Lie-Santilli Isogroups. Under certain integrability conditions hereon assumed, Lie algebras L can be "exponentiated" to their corresponding *Lie transformation groups* G and, vice-versa, Lie transformation groups G admit their corresponding Lie algebra L when computed in the neighborhood of the identity I.

These basic properties are preserved under isotopies although broadened to the most general possible nonlinear, nonlocal and noncanonical transformations groups.

DEFINITION 3.2.16 [4]: A right isomodular Lie-Santilli isotransformation group $\hat{G}$ on an isospace $\hat{S}(\hat{x}, \hat{F})$ over an isofield $\hat{F}$ with common isounit $\hat{I} = 1/\hat{T} > 0$ is a group mapping each element $\hat{x} \in \hat{S}$ into a new element $\hat{x}' \in \hat{S}$ via the isotransformations

$$\hat{x}' = \hat{g}(\hat{w}) \hat{\times} \hat{x}, \;\; \hat{x}, \hat{x}' \in \hat{S}, \;\; \hat{w} \in \hat{F}, \tag{3.2.217}$$

such that:

1) The map $\hat{g}\hat{\times}\hat{S}$ *into* $\hat{S}$ *is isodifferentiable* $\forall \hat{g} \in \hat{G}$*;*
2) $\hat{I}$ *is the left and right unit*

$$\hat{I}\hat{\times}\hat{g} = \hat{g}\hat{\times}\hat{I} \equiv \hat{g}, \ \ \forall \hat{g} \in \hat{G}; \tag{3.2.218}$$

3) the isomodular action is isoassociative, i.e.,

$$\hat{g}_1\hat{\times}(\hat{g}_2\hat{\times}\hat{x}) = (\hat{g}_1\hat{\times}\hat{g}_2)\hat{\times}\hat{x}, \ \ \forall \hat{g}_1, \hat{g}_2 \in \hat{G}; \tag{3.2.219}$$

4) in correspondence with every element $\hat{g}(\hat{w}) \in \hat{G}$ *there is the inverse element* $\hat{g}^{-\hat{I}} = \hat{g}(-\hat{w})$ *such that*

$$\hat{g}(\hat{0}) = \hat{g}(\hat{w})\hat{\times}\hat{g}(-\hat{w}) = \hat{I}; \tag{3.2.220}$$

5) following composition laws are verified

$$\hat{g}(\hat{w})\hat{\times}\hat{g}(\hat{w}') = \hat{g}(\hat{w}')\hat{\times}\hat{g}(\hat{w}) = \hat{g}(\hat{w} + \hat{w}'), \forall \hat{g} \in \hat{G}, \ \hat{w} \in \hat{F}. \tag{3.2.221}$$

The I left isotransformation group is defined accordingly.

The notions of *connected or simply connected transformation groups* carry over to the isogroups in their entirety.

The most direct realization of the (connected) isotransformation groups is that via isoexponentiation,

$$\hat{g}(w) = \prod_k \hat{e}^{\hat{i}\hat{\times}\hat{w}_k\hat{X}_k} = \Big(\prod_k e^{i\times w_k\times X_k\times\hat{T}(x,\dot{x},\ddot{x},\ldots)}\Big) \times \hat{I}, \tag{3.2.222}$$

where the X's and w's are the infinitesimal generators and parameters, respectively, of the original algebra L, with corresponding connected isotransformations

$$\hat{x}' = \hat{g}(\hat{w})\hat{\times}\hat{x} = \Big(\prod_k \hat{e}^{\hat{i}\hat{\times}\hat{w}_k\hat{X}_k}\Big) \times \hat{I} \times \hat{T} \times x \times \hat{I} =$$

$$= \Big(\prod_k e^{i\times w_k\times X_k\times\hat{T}(x,\dot{x},\ddot{x},\ldots)}\Big) \times x \times \hat{I}. \tag{3.2.223}$$

Equations (3.2.223) hold in some open neighborhood N of the isoorigin of $\hat{L}$ and, in this way, characterize some open neighborhood of the isounit of $\hat{G}$. Consequently, under the assumed continuity and connectivity properties, Lie-Santilli isoalgebras can be obtained as infinitesimal versions of finite Lie-Santilli isogroups, as illustrated by the following finite isotransform

$$\hat{A}(\hat{w}) = (\hat{e}^{\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}})\hat{\times}\hat{A}(\hat{0})\hat{\times}(\hat{e}^{-\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}}) =$$

$$= (e^{i\times w\times\hat{X}\times\hat{T}}) \times \hat{A}(\hat{0}) \times (e^{-i\times w\times\hat{T}\times\hat{X}}) \tag{3.2.224}$$

with infinitesimal version in the neighborhood of $\hat{I}$

$$\hat{A}(\hat{d}\hat{w}) = (\hat{I} + \hat{i}\hat{\times}\hat{d}\hat{w}\hat{\times}\hat{X} + \ldots)\hat{\times}\hat{A}(0)\hat{\times}(\hat{I} - \hat{i}\hat{\times}\hat{d}\hat{w}\hat{\times}\hat{X} + \ldots) =$$
$$= \hat{A}(\hat{0}) + \hat{i}\hat{\times}\hat{d}\hat{w}\hat{\times}\hat{X}\hat{\times}\hat{A}(\hat{0}) - \hat{i}\hat{\times}\hat{d}\hat{w}\hat{\times}\hat{A}(\hat{0})\hat{\times}\hat{X}, \qquad (3.2.225)$$

that can be written

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}(\hat{w})}{\hat{d}\hat{w}} = \hat{A}\hat{\times}\hat{X} - \hat{X}\hat{\times}\hat{A} = [\hat{A}\hat{,}\hat{X}]. \qquad (3.2.226)$$

Note the crucial appearance of the isotopic element $\hat{T}(x, \dot{x}, \ddot{x}, \ldots)$ in the exponent of the isogroup. This ensures a structural generalization of Lie's theory of the desired nonlinear, nonlocal and noncanonical form.

Still another important property is that conventional group composition laws admit a consistent isotopic lifting, resulting in the following *Baker-Campbell-Hausdorff-Santilli Isotheorem* [4]

$$(\hat{e}^{\hat{X}_1})\hat{\times}(\hat{e}^{X_2}) = \hat{e}^{\hat{X}_3}, \qquad (3.2.227a)$$

$$\hat{X}_3 = \hat{X}_1 + \hat{X}_2 + [\hat{X}_1\hat{,}\hat{X}_2]\hat{/}\hat{2} + [(\hat{X}_1 - \hat{X}_2)\hat{,}[\hat{X}_1\hat{,}\hat{X}_2]]\hat{/}\hat{1}\hat{2} + \ldots \,. \qquad (3.2.227b)$$

Let $\hat{G}_1$ and $\hat{G}_2$ be two isogroups with respective isounits $\hat{I}_1$ and $\hat{I}_2$. The *direct isoproduct* $\hat{G}_1\hat{\times}\hat{G}_2$ is the isogroup of all ordered pairs

$$(\hat{g}_1, \hat{g}_2), \quad \hat{g}_1 \in \hat{G}_1, \hat{g}_2 \in \hat{G}_2, \qquad (3.2.228)$$

with isomultiplication

$$(\hat{g}_1, \hat{g}_2)\hat{\times}(\hat{g}_1', \hat{g}_2') = (\hat{g}_1\hat{\times}\hat{g}_1', \hat{g}_2\hat{\times}\hat{g}_2'), \qquad (3.2.229)$$

total isounit $(\hat{I}_1, \hat{I}_2)$ and inverse $(\hat{g}_1^{-\hat{I}_1}, \hat{g}_2^{-\hat{I}_2})$.

The following particular case is important for the isotopies of inhomogeneous groups. Let $\hat{G}$ be an isogroup and $\hat{G}_{\hat{a}}$ the isogroup of all its inner isoautomorphisms. Let $\hat{G}^o_{\hat{a}}$ be a subgroup of $\hat{G}_{\hat{a}}$, and let $\Lambda(\hat{g})$ be the image of $\hat{g} \in \hat{G}$ under $\hat{G}_{\hat{a}}$. The *semidirect isoproduct* $\hat{G}\hat{\times}\hat{G}^o_{\hat{a}}$ is the isogroup of all ordered pairs

$$(\hat{g}, \hat{\Lambda})\hat{\times}(\hat{g}^o, \hat{\Lambda}^o) = (\hat{g}, \hat{\Lambda}(\hat{g}^o), (\hat{\Lambda}, \hat{\Lambda}^o), \qquad (3.2.230)$$

with total isounit given by $\hat{I}_{tot} = \hat{I} \times \hat{I}^o$.

The studies of the isotopies of the remaining aspects of the structure theory of Lie groups is then consequential.

It is hoped that the reader can see from the above elements that the entire conventional Lie theory does indeed admit a consistent and nontrivial lifting into the covering Lie-Santilli formulation.

3.2.12E. Isorepresentations of Lie-Santilli Isoalgebras. Despite considerable research on the Lie-Santilli isotheory over the past 26 years, the study of the *isorepresentations* of the Lie-Santilli isoalgebras remains vastly unknown at this writing (summer 2004), with the sole exception of the *fundamental (or regular) isorepresentations* that were also identified by Santilli in the original proposal [4].

In this monograph we shall primarily use in the applications of hadronic mechanics the fundamental isorepresentations or other isorepresentations reducible to the latter.

Let L be an N-dimensional Lie algebra with N-dimensional unit $I = Diag.(1, 1, \ldots, 1)$. Let R be the fundamental, N-dimensional matrix representation of L. Let $\hat{L}$ be the isotope of L characterized by the N-dimensional isounit $\hat{I} = U \times U^\dagger > 0$. It is then evident that the *fundamental isorepresentation* of $\hat{L}$ is given by

$$\hat{R} = U \times R \times U^\dagger, \quad U \times U^\dagger = \hat{I} \neq I, \hat{I} > 0. \tag{3.2.231}$$

Interested colleagues are encouraged to study the isorepresentation theory because, as we shall see in the next sections, the fundamental notion of hadronic mechanics, that of *isoparticles,* is characterized by an irreducible isorepresentation of the Poincaré-Santilli isosymmetry.

3.2.12F. Isodual Lie-Santilli Isotheory. As indicated Chapters 1 and 2, the contemporary formulation of Lie's theory is one of the most serious obstacles for a consistent *classical* representation of antimatter, because it lacks an appropriate conjugate formulation that, after quantization, is compatible with charge conjugation.[20]

It is easy to verify that the isotheory presented above admits a consistent antiautomorphic image under isoduality, thus permitting the treatment of antimatter under nonlinearity, nonlocality and noncanonicity as occurring in interior conditions, such as for the structure of an antimatter star.

In fact, we have the *isodual universal enveloping isoassociative isoalgebra* $\hat{\xi}^d$ characterized by the *isodual Poincaré-Birkhoff-Witt-Santilli isotheorem* with infinite dimensional basis

$$\hat{I}^d, \ \ X^d_k, \ \ \hat{X}^d_i \hat{\times}^d \hat{X}^d_j, \ i \leq j, \ \ \hat{X}^d_i \hat{\times}^d \hat{X}^d_j \hat{\times}^d \hat{X}^d_k, \ i \leq j \leq k, \ \ldots. \tag{3.2.232}$$

The *isodual Lie-Santilli isoalgebra* $\hat{L}^d \approx (\hat{\xi}^d)^-$ attached to $\hat{\xi}^d$ is characterized by the *isodual Lie-Santilli Second Isotheorem*

$$[\hat{X}^d_i \hat{,} \hat{X}^d_j] = \hat{X}^d_i \hat{\times}^d \hat{X}^d_j - \hat{X}^d_j \hat{\times}^d \hat{X}^d_i = \hat{C^d_{ij}}^k \hat{\times}^d \hat{X}^d_k. \tag{3.2.233}$$

[20]The reader is urged to verify that the classical treatment of antimatter via the so-called *dual Lie algebras* does not achieve antiparticles under quantization, trivially, because of the uniqueness of the quantization channel for both particles and antiparticles.

Under the needed continuity and connectivity property, the *isodual exponentiation* of $\hat{L}^d$ characterizes the *connected isodual Lie-Santilli transformation isogroup*

$$\hat{x}'^d = (\hat{g}^d(\hat{w}^d) = \hat{\prod}_k \hat{e}^{d^{\hat{i}^d \hat{\times}^d \hat{w}_k^d \hat{X}_k^d}}) \hat{\times}^d \hat{x}^d. \tag{3.2.234}$$

Interested readers can then easily derive any additional needed isodual property.

3.2.13 Unification of All Simple Lie Algebras into Lie-Santilli Isoalgebras

The original proposal [4] of 1978 included the *conjecture that all simple Lie algebras of dimension N can be unified into a single Lie-Santilli isoalgebra of the same dimension,* and gave an explicit example. The conjecture was subsequently proved by the late mathematicians Gr. Tsagas [42] in 1996 for all simple Lie algebras of type A, B, C and D. The premature departure of Prof. Tsagas while working at the problem prevented him to complete the proof of the conjecture for the case of all exceptional Lie algebras. As a result, the proof of the indicated conjecture remain incomplete at this writing.

For the unification here considered it is important to eliminate the restriction that the isounits are necessarily positive definite, while preserving all other characteristics, such as nowhere singularity and Hermiticity. As a result, in its simple possible form, the isounit can be diagonalized into the form whose elements can be either positive or negative,

$$\hat{I} = Diag.(\pm n_1^2, \pm n_2^2, \ldots, \pm n_N^2) = 1/\hat{T},\ n_k \in R,\ n_k \neq 0,\ k = 1, 2, \ldots, N. \tag{3.2.235}$$

The example provided in the original proposal [4], subsequently studied in detail in Refs. [8], consisted in the *classification of all possible simple Lie algebra of dimension 3.* In this case, Cartan's classification produces two non-isomorphic Lie algebras, the compact rotational algebra in three dimension $SO(3)$ and the noncompact algebra $SO(2.1)$.

The distinction between compact and noncompact algebras is lost under the class of isotopies here considered. In fact, the *classification of all possible, simple, three-dimensional Lie-Santilli isoalgebras $\hat{L}_3$ for the case of diagonal isounits* is characterized by the isounit itself and can be written

$$\hat{I} = Diag.(+1, +1, +1),\ \ \hat{L}_3 \approx SO(3), \tag{3.2.236a}$$

$$\hat{I} = Diag.(+1, +1, -1),\ \ \hat{L}_3 \approx SO(2.1), \tag{3.2.236b}$$

$$\hat{I} = Diag.(+1, -1, +1),\ \ \hat{L}_3 \approx SO(2.1), \tag{3.2.236c}$$

$$\hat{I} = Diag.(-1, +1, +1),\ \ \hat{L}_3 \approx SOI(2.1), \tag{3.2.236d}$$

$$\hat{I} = Diag.(-1,-1,-1), \quad \hat{L}_3 \approx SO(3)^d, \tag{3.2.236e}$$

$$\hat{I} = Diag.(-1,-1,+1), \quad \hat{L}_3 \approx SO(2.1)^d, \tag{3.2.236f}$$

$$\hat{I} = Diag.(-1,+1,-1), \quad \hat{L}_3 \approx SO(2.1), \tag{3.2.236g}$$

$$\hat{I} = Diag.(+1,-1,-1), \quad \hat{L}_3 \approx SO(2.1)^d, \tag{3.2.236h}$$

$$\hat{I} = Diag.(+n_1^2,+n_2^2,+n_3^2), \quad \hat{L}_3 \approx SO(3), \tag{3.2.236i}$$

$$\hat{I} = Diag.(+n_1^2,+n_2^2,-n_3^2), \quad \hat{L}_3 \approx SO(2.1), \tag{3.2.236j}$$

$$\hat{I} = Diag.(+n_1^2,-n_2^2,+n_3^2), \quad \hat{L}_3 \approx SO(2.1), \tag{3.2.236k}$$

$$\hat{I} = Diag.(-n_1^2,+n_2^2,+n_3^2), \quad \hat{L}_3 \approx SOI(2.1), \tag{3.2.236l}$$

$$\hat{I} = Diag.(-n_1^2,-n_2^2,-n_3^2), \quad \hat{L}_3 \approx SO(3)^d, \tag{3.2.236m}$$

$$\hat{I} = Diag.(-n_1^2,-n_2^2,+n_3^2), \quad \hat{L}_3 \approx SO(2.1)^d, \tag{3.2.236n}$$

$$\hat{I} = Diag.(-n_1^2,+n_2^2,-n_3^2), \quad \hat{L}_3 \approx SO(2.1), \tag{3.2.236o}$$

$$\hat{I} = Diag.(+n_1^2,-n_2^2,-n_3^2), \quad \hat{L}_3 \approx SO(2.1)^d, \tag{3.2.236p}$$

In conclusion, when studying simple algebras from the viewpoint of the covering Lie-Santilli isoalgebras, there exist *only one single isoalgebra in three dimensions,* $\hat{L}_3$ without any distinction between compact and noncompact algebras.

The *realization* of the simple isoalgebra $\hat{L}_3$ with diagonal isounits consists of 21 different Lie-Santilli isoalgebras in three dimension that can be reduced to 4 topologically different Lie algebras, namely $SO(3)$, $SO(2.1)$, $SO(3)^d$ and $SO(2.1)^d$.

All distinctions between these 21 different realizations are lost at the level of abstract Lie-Santilli isoalgebra $\hat{L}_3$.

It should be stressed that, by no means, the 21 realizations (3.2.236) exhaust all possible forms of Lie-Santilli simple isoalgebras in three dimensions because in realizations (3.2.236) we have excluded nondiagonal realizations of the isounit, as well as imposed additional restrictions on the isounit, such as single valuedness and Hermiticity.

Essentially the same results hold for the unification of the Lie Algebras of type A, B, C, and D studied by Tsagas [42].

It is hoped that interested mathematicians can complete the proof of Santilli's conjecture for the remaining exceptional algebras. In considering the problem, mathematicians are suggested to keep in mind that Hermitian and diagonal realizations of the isounit (3.2.135) are expected to be insufficient, thus implying the possible use of *nowhere singular, Hermitian, nondiagonal isounits,* or *nowhere singular, Hermitian, nondiagonal and multivalued isounits,* or *nowhere singular, non-Hermitian, nondiagonal and multivalued isounits.*

3.2.14 The Fundamental Theorem for Isosymmetries and Their Isoduals

The fundamental symmetries of the 20-th century physics deal with point-like abstractions of particles in vacuum under linear, local and potential interactions, and are the *Galilei symmetry* $G(3.1)$ for nonrelativistic treatment or the *Poincaré symmetry* for relativistic formulations.

A central objective of hadronic mechanics is the broadening of these fundamental spacetime symmetries to represent extended, nonspherical and deformable particles under linear and nonlinear, local and nonlocal and potential as well as nonpotential interactions.

In fact, as we shall see, all novel industrial applications of hadronic mechanics are crucially dependent on the admission of the extended character of particles or of their wavepackets in conditions of deep mutual penetration. In turn, the latter conditions imply new effects permitting basically new energies and fuels that are completely absent for conventional spacetime and other symmetries.

Alternatively and equivalently a central problem of hadronic mechanics is the *construction in an explicit form of the symmetries of all possible nonsingular, but otherwise arbitrary deformations of conventional spacetime and internal invariants.*

All these problems and others are resolved by the following important:

THEOREM 3.2.12 [6]: Let G be an N-dimensional Lie symmetry group of a K-dimensional metric or pseudo-metric space $S(x, m, F)$ over a field F,

$$G: \ \ x' = \Lambda(w) \times x, \ \ y' = \Lambda(w) \times y, \ \ x, y \in \hat{S}, \tag{3.2.237a}$$

$$(x' - y')^\dagger \times \Lambda^\dagger \times m \times \Lambda \times (x - y) \equiv (x - y)^\dagger \times m \times (x - y), \tag{3.2.237b}$$

$$\Lambda^\dagger(w) \times m \times \Lambda(w) \equiv m. \tag{3.2.237c}$$

Then, all infinitely possible isotopies $\hat{G}$ of G acting on the isospace $\hat{S}(\hat{x}, \hat{M}, \hat{F})$, $\hat{M} = \hat{m} \times \hat{I} = (\hat{T}_i^k \times m_{kj}) \times \hat{I}$ characterized by the same generators and parameters of G and new isounits $\hat{I} = 1/\hat{T} > 0$ leave invariant the isocomposition on the projection $\hat{S}(x, \hat{m}, F)$ of $\hat{S}(\hat{x}, \hat{M}, \hat{F})$ on the original space $S(x, m, F)$

$$\hat{G}: \ \ x' = \hat{\Lambda}(w) \times x, \ \ y' = \hat{\Lambda}(w) \times y, \ \ x, y \in \hat{S}, \tag{3.2.238a}$$

$$(x' - y')^\dagger \times \hat{\Lambda}^\dagger \times \hat{m} \times \hat{\Lambda} \times (x - y) \equiv (x - y)^\dagger \times \hat{m} \times (x - y), \tag{3.2.238b}$$

$$\hat{\Lambda}^\dagger(\hat{w}) \times \hat{m} \times \hat{\Lambda}(\hat{w}) \equiv \hat{m}. \tag{3.2.238c}$$

Similarly,all infinitely possible isodual isotopies $\hat{G}^d$ of $\hat{G}$ acting on the isodual isospace $\hat{S}^d(\hat{x}^d, \hat{M}^d, \hat{F}^d)$, $\hat{M}^d = (\hat{T}^d \times m^d) \times \hat{I}^d$ characterized by the isodual generators $\hat{X}_k^d$ parameters $\hat{w}^d$ and isodual isounit $\hat{I}^d = 1/\hat{T}^d < 0$ leave invariant the *isodual isocomposition on the projection* $\hat{S}^d(x^d, \hat{m}^d, F^d)$

$$\hat{G}^d: \quad x'^d = \hat{\Lambda}^d \times^d x^d, \;\; y'^d = \hat{\Lambda}^d \times^d y^d, \;\; x^d, y^d \in \hat{S}^d, \tag{3.2.239a}$$

$$(x'-y')^{\dagger^d} \times^d \hat{\Lambda}^{\dagger^d} \times^d \hat{m}^d \times^d \hat{\Lambda}^d \times^d (x-y)^d \equiv (x-y)^{\dagger^d} \times^d \hat{m}^d \times^d (x-y)^d, \tag{3.2.239b}$$

$$\hat{\Lambda}^{\dagger^d} \times^d \hat{m}^d \times^d \hat{\Lambda}^d \equiv \hat{m}^d. \tag{3.2.239c}$$

Proof. Assume that $N = K$ and the representation Λ is the fundamental one. Recall that metrics, isometrics and isounits are diagonal. Then on $\hat{S}(x, \hat{m}, F)$ we have the identities

$$\hat{I} = U \times U^\dagger \neq I, \;\; \hat{T} = (U \times U^\dagger)^{-1}, \tag{3.2.240a}$$

$$\begin{aligned} & U \times (\Lambda \times m \times \Lambda) \times U^\dagger = \\ & = (U \times \Lambda \times U^\dagger) \times (U^{\dagger^{-1}} \times m \times U^{-1}) \times (U \times \Lambda \times U^\dagger) = \\ & = \hat{\Lambda} \times (\hat{T} \times m) \times \hat{\Lambda} = \hat{\Lambda} \times \hat{m} \times \hat{\Lambda} = \hat{m}. \end{aligned} \tag{3.2.240b}$$

The proof of the remaining cases are equally trivial. **q.e.d.**

Note that the isotopic symmetries and their isoduals can be uniquely and explicitly constructed with the methods summarized in this section via the sole use of the original symmetry and the isounit characterizing the deformation of the original metric m.

Under our assumptions, the isosymmetries can be constructed in the needed, explicit, nonlinear, nonlocal and noncanonical forms. In fact, the existence of the original symmetry transformations plus the condition $\hat{I} > 0$ ensure the convergence of the infinite isoseries of the isoexponentiation, resulting in the needed explicit form, as we shall see in various examples in the next sections.

3.3 CLASSICAL LIE-ISOTOPIC MECHANICS FOR MATTER AND ITS ISODUAL FOR ANTIMATTER

3.3.1 Introduction

One of the reasons for the majestic consistency of quantum mechanics is the existence of axiomatically consistent and invariant classical foundations, given by *classical Lagrangian and Hamiltonian mechanics,* namely, the discipline based on the *truncated analytic equations*

$$\frac{d}{dt}\frac{\partial L(t,r,v)}{\partial v_a^k} - \frac{\partial L(t,r,v)}{\partial r_a^k} = 0, \tag{3.3.1a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t,r,p)}{\partial r_a^k}, \tag{3.3.1b}$$
$$k = 1,2,3; \quad a = 1,2,3,\ldots,N,$$

with a unique and unambiguous map into operator forms.

Following the original proposal [5] of 1978 to build hadronic mechanics, this author did not consider the new discipline sufficiently mature for experimental verifications and industrial applications until the new discipline had equally consistent and invariant classical foundations with an equally unique and unambiguous map into operator formulations.

Intriguingly, the *operator* foundations of hadronic mechanics were sufficiently identified in the original proposal [5], as we shall see in the next section. However, the identification of the *classical* counterpart turned out to be a rather complex task that required decades of research.

The objective, fully identified in 1978, was the construction of a covering of classical Lagrangian and Hamiltonian mechanics, namely, a covering of Eqs. (3.3.1), admitting a unique and unambiguous map into the already known Lie-isotopic equations of hadronic mechanics.

The mandatory starting point was the consideration of the *true Lagrange and Hamilton equations,* those with external terms

$$\frac{d}{dt}\frac{\partial L(t,r,v)}{\partial v_a^k} - \frac{\partial L(t,r,v)}{\partial r_a^k} = F_{ak}(t,r,v), \tag{3.3.2a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t,r,p)}{\partial r_a^k} + F_{ak}(t,r,p), \tag{3.3.2b}$$

since they were conceived, specifically, for the interior dynamical systems treated by hadronic mechanics.

In fact, the legacy of Lagrange and Hamilton is that classical systems *cannot* be entirely represented with one single function today called a Lagrangian or a Hamiltonian used for the representation of forces derivable from a potential, but require additional quantities for the representation of contact nonpotential forced represented precisely by the external terms.

As such, the true Lagrange and Hamilton equations constitute excellent candidates for the classical origin of hadronic mechanics.

3.3.2 Insufficiencies of Analytic Equations with External Terms

It was indicated by Santilli [4] also in 1978 (see the review in Chapter 1 for more details) that the true analytic equations cannot be used for the construction of a consistent covering of conventional analytic equations because the new algebraic brackets of the time evolution of a generic quantity $A(r,p)$ in phase space

$$\frac{dA}{dt} = (A,H,F) = [A,H] + \frac{\partial A}{\partial r^k} \times F_k =$$
$$= \frac{\partial A}{\partial r^k} \times \frac{\partial H}{\partial p_k} - \frac{\partial H}{\partial r^k} \times \frac{\partial A}{\partial p_k} + \frac{\partial A}{\partial r^k} \times F_k, \tag{3.3.3}$$

violate the right distributive and scalar laws, Eqs. (3.2.5) and (3.2.6). Consequently, *the true analytic equations in their original formulation lose "all" possible algebras, let alone all possible Lie algebras.* No axiomatically consistent covering can then be build under these premises.[21]

The above insufficiency essentially established the need of rewriting the true analytic equations into a form admitting a consistent algebra in the brackets of the time evolution laws and, in addition, achieves the same invariance possessed by the truncated analytic equations.

Even though its main lines were fully identified in 1978, the achievement of the new covering mechanics resulted to require a rather long and laborious scientific journey.

This section is intended to outline the final formulation of the classical mechanics underlying hadronic mechanics in order to distinguish it from the numerous attempts that were published with the passing of time.

As a brief guide to the literature, the reader should be aware that the true analytic equations (3.3.2) are generally set for *open nonconservative systems.* These systems require the broader *Lie-admissible branch of hadronic mechanics* that will be studied in the next chapter.

Therefore, the reader should be aware that several advances in Lie-isotopies have been obtained and can be originally identified as particular cases of the broader Lie-admissible theories.

This chapter is dedicated to the study of classical and operator closed-isolated systems verifying conventional total conservation laws while having linear and nonlinear, local and nonlocal as well as potential and nonpotential internal forces.

The verification of conventional total conservation law requires classical brackets that, firstly, verify the right and left distributive and scalar laws (as a condition to characterize an algebra), and, secondly, the brackets are necessarily antisymmetric.

The brackets of conventional Hamiltonian mechanics are Lie. Therefore, a necessary condition to build a true covering of Hamiltonian mechanics is the search of brackets that are of the broader Lie-isotopic type. As a matter of

[21] For additional problematic aspects of the true analytic equations, one may consult Ref. [4] or the review in Chapter 1.

fact, this feature, fully identified in 1978 [4,5], was the very motivation for the construction of the isotopies of the Lie theory reviewed in Section 3.2.12.

In summary, the construction of a covering of the conventional Hamiltonian mechanics as the classical foundations of the Lie-isotopic branch of hadronic mechanics must be restricted to a reformulation of the true analytic equations (3.3.2) in such a way that the underlying brackets are Lie-isotopic, and the resulting mechanics is invariant.

3.3.3 Insufficiencies of Birkhoffian Mechanics

Santilli dedicated the second volume of *Foundations of Theoretical Mechanics* published by Springer-Verlag [2] in 1982 to the construction of a covering of classical Hamiltonian mechanics along the above indicated requirement. The resulting new mechanics was released under the name of *Birkhoffian mechanics* to honor G. D. Birkhoff who first discovered the underlying analytic equations in 1927.[22]

Conventional Hamiltonian mechanics is based on the *canonical action principle*

$$\delta A^o = \delta \int (p_k \times dr^k - H \times dt) = 0, \tag{3.3.4}$$

and, via the use of the unified notation

$$b = (b^\mu) = (r^i, p_j), \tag{3.3.5a}$$

$$R^o = (R^o_\mu) = (p_k, 0), \quad \mu = 1, 2, \ldots, 6, \tag{3.3.5b}$$

can be written

$$\begin{aligned} \delta A^o = \delta \int (R^o_\mu \times db^\mu - H \times dt) \equiv \\ \equiv \delta \int (p_k \times dr^k - H \times dt) = 0. \end{aligned} \tag{3.3.6}$$

from which the conventional Hamilton's equations (3.3.1b) acquire the unified form

$$\omega_{\mu\nu} \times \frac{db^\nu}{dt} = \frac{\partial H}{\partial b^\mu}, \tag{3.3.7}$$

where

$$\omega_{\mu\nu} = \frac{\partial R^o_\nu}{\partial b^\mu} - \frac{\partial R^o_\mu}{\partial b^\nu} \tag{3.3.8}$$

is the fundamental (canonical) symplectic tensor (3.2.187).

[22] Interested readers should consult, for brevity, the historical notes of Ref. [2].

The fundamental (conventional Poisson) brackets of the time evolution then acquire the unified form

$$\frac{dA}{dt} = [A, H] = \frac{\partial A}{\partial b^\mu} \times \omega^{\mu\nu} \times \frac{\partial H}{\partial b^\nu}, \tag{3.3.9}$$

where

$$\omega^{\mu\nu} = [(\omega_{\alpha\beta})^{-1}]^{\mu\nu} \tag{3.3.10}$$

is the fundamental (canonical) Lie tensor.

Santilli [2] based the construction of a covering isotopic (that is, axiom-preserving) mechanics on the most general possible *Pfaffian action principle*

$$\delta A = \delta \int (R_\mu \times db^\mu - B \times dt) = 0, \tag{3.3.11}$$

where the $R_\mu(b)$ functions are now arbitrary functions in phase space, e.g., of the type

$$R(b) = (R_\mu) = (E_i(r,p), D^j(r,p)), \tag{3.3.12}$$

verifying certain regularity conditions [2].

It is easy to see that principle (3.3.11) characterizes the following analytic equations[23]

$$\Omega_{\mu\nu} \times \frac{db^\nu}{dt} = \frac{\partial B}{\partial b^\mu}, \tag{3.3.13a}$$

$$\Omega_{\mu\nu} = \frac{\partial R_\nu}{\partial b^\mu} - \frac{\partial R_\mu}{\partial b^\nu} \tag{3.3.13b}$$

is the most general possible symplectic tensor in local coordinates. Eqs. (3.3.12) were called *Birkhoff's equations* because, following a considerable research, they resulted to have been first identified by D. G. Birkhoff in 1927. The function B was called the *Birkhoffian* in order to distinguish it from the conventional Hamiltonian, since the latter represent the total energy, while the former does not.

The fundamental brackets of the time evolution then acquire the unified form

$$\frac{dA}{dt} = \frac{\partial A}{\partial b^\mu} \times \Omega^{\mu\nu} \times \frac{\partial B}{\partial b^\nu}, \tag{3.3.14a}$$

$$\Omega^{\mu\nu} = [(\Omega_{\alpha\beta})^{-1}]^{\mu\nu}. \tag{3.3.14b}$$

The covering nature of Eqs. (3.3.11)–(3.3.14) over the conventional Eqs. (3.3.4)–(3.3.10) is evident. In particular, brackets (3.3.14) are antisymmetric and verify the Lie axioms, although in the generalized Lie-Santilli isotopic form.

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[23] The equations are called "analytic" in the sense of being derivable from a variational principle.

Moreover, Birkhoffian mechanics was proved in Ref. [2] to be "directly universal", that is, capable of representing "all" possible (sufficiently smooth and regular) Newtonian systems directly in the "frame of the observer" without any need for the transformation theory.

Therefore, at the time of releasing monograph [2] in 1982, the Birkhoffian mechanics appeared to have all the necessary pre-requisites to be the classical foundation of hadronic mechanics.

Unfortunately, subsequent studies established that *Birkhoffian mechanics cannot be used for consistent physical applications* because it is afflicted by the catastrophic inconsistencies studied in Section 1.4.1, with particular reference to the lack of invariance, namely, the inability to predict the same numbers for the same physical conditions at different times owing to the noncanonical character of the time evolution.

Moreover, canonical action (3.3.4) is independent from the momenta, $A^o = A^o(r)$, while this is not the case for the Pfaffian action (3.3.11) for which we have $A = A(r,p)$. Consequently, any map into an operator form implies "wavefunctions" dependent on both coordinates and momenta, $\psi(r,p)$. Therefore, the operator image of Birkhoffian mechanics is beyond our current knowledge, and its study is deferred to future generations.

The above problems requested the resumption of the search for the consistent classical counterpart of hadronic mechanics from its beginning.

Numerous additional generalized classical mechanics were identified but they still missed the achievement of the crucial invariance (for brevity, see monographs [15,16] of 1991 and the first edition of monograph [6,7] of 1993).

By looking in retrospect, the origin of all the above difficulties resulted to be where one would expect them the least, in the use of the ordinary differential calculus.

Following the discovery in 1995 (see the second edition of monographs [6,7] and Ref. [10]) of the isodifferential calculus, the identification of the final, axiomatically consistent and invariant form of the classical foundations of hadronic mechanics emerged quite rapidly.

3.3.4 Newton-Santilli Isomechanics for Matter and its Isodual for Antimatter

The fundamental character of *Newtonian Mechanics* for all scientific inquiries is due to the preservation at all subsequent levels of treatment (such as Hamiltonian mechanics, Galileo's relativity, special relativity, quantum mechanics, quantum chemistry, quantum field theory, etc.) of its main structural features, such as:

1) The underlying local-differential Euclidean topology;
2) The ordinary differential calculus; and
3) The consequential point-like approximation of particles.

Nevertheless, Newton's equations have well known notable limitations to maintain such a fundamental character for the entirely of scientific knowledge without due generalization for so many centuries.

As indicated in Chapter 1, the point-like approximation is indeed valid for very large mutual distances among particles compared to their size, as occurring for planetary and atomic systems (*exterior dynamical systems*). However, the same approximation is excessive for systems of particles at short mutual distances, as occurring for the structure of planets, hadrons, nuclei and stars (*interior dynamical systems*).

Also, dimensionless particles cannot experience any contact or resistive interactions. Consequently, dissipative or, more generally, nonconservative forces used for centuries in Newtonian mechanics are a mere *approximation* of contact nonpotential nonlocal-integral interactions among extended constituents, the approximation being generally achieved via power series expansion in the velocities.

It should be finally recalled on historical grounds that *Newton had to construct the differential calculus as a pre-requisite for the formulation of his celebrated equations.*

No genuine structural broadening of the disciplines of the 20-th century is possible without a consistent structural generalization of their foundations, Newton's equations in Newtonian mechanics.

Santilli's isomathematics has been constructed to permit *the first axiomatically consistent structural generalization of Newton's equations in Newtonian mechanics since Newton's time, for the representation of extended, nonspherical and deformable particles under linear and nonlinear, local and nonlocal and potential as well as nonpotential interactions as occurring in the physical reality of interior dynamical systems.*

By following Newton's teaching, the author has dedicated primary efforts to the isotopic lifting of the conventional differential calculus, topology and geometries [6,10] as a pre-requisite for the indicated structural generalization of Newton's equations.

To outline the needed isotopies, let us recall that Newtonian mechanics is formulated on a 7-dimensional representation space characterized by the following Kronecker products of Euclidean spaces

$$S_{tot} = E(t, R_t) \times E(r, \delta, R_r) \times E(v, \delta, R_v), \tag{3.3.15}$$

of the one dimensional space $E(t, R_t)$ representing time t, the tree dimensional Euclidean space $E(r, \delta, R_r)$ of the coordinates $r = (r^k_a)$ (where $k = 1, 2, 3$ are the Euclidean axes and $a = 1, 2, \dots, N$ represents the number of particles), and the velocity space $E(v, \delta, R_v), v = dr/dt$.

It is generally assumed that all variables t, r, and v are defined on the same field of real numbers R. However, the unit of time is the *scalar* $I = +1$, while the unit of the Euclidean space is the *matrix*, and the same happens for the velocities, $I_r = I_v = Diag.(1,1,1)$.

Therefore, on rigorous grounds, the representation space of Newtonian mechanics must be defined on the Kronecker product of the corresponding fields

$$R_{tot} = R_t \times R_r \times R_v \tag{3.3.16}$$

with total unit

$$I_{Tot} = 1_t \times Diag.(1,1,1)_r \times Diag.(1,1,1)_v. \tag{3.3.17}$$

The above total unit can be factorized into the production of seven individual units for time and the two sets of individual Euclidean axes a, y, a with corresponding factorization of the fields

$$I_{tot} = 1_t \times 1_{rx} \times 1_{ry} \times 1_{rz} \times 1_{vx} \times 1_{vy} \times 1_{vz}, \tag{3.3.18a}$$

$$R_{tot} = R_t \times R_{rx} \times R_{ry} \times R_{rz} \times R_{vx} \times R_{vy} \times R_{vz}, \tag{3.3.18b}$$

that constitute the foundations of the conventional *Euclidean topology* here assumed as known.

Via the use of Eqs. (3.1.5), *Newton's equations for closed-non-Hamiltonian systems* can then be written

$$m_a \times a_{ka} = m_a \times \frac{dv_{ka}}{dt} = F_{ka}(t, r, v) = F_{ka}^{SA} + F_{ka}^{NSA}, \tag{3.3.19a}$$

$$\sum_a \mathbf{F}_a^{NSA} = 0, \tag{3.3.19b}$$

$$\sum_a \mathbf{r}_a \bigodot \mathbf{F}_a^{NSA} = 0, \tag{3.3.19c}$$

$$\sum_a \mathbf{r}_a \bigwedge \mathbf{F}_a^{NSA} = 0, \tag{3.3.19d}$$

where SA (NSA) stands for *variational selfadjointness (variational nonselfadjointness),* namely, the verification (violation) of the integrability conditions for the existence of a potential [1], and conditions (3.3.xx), (3.3.xx) and (3.3.xx) assure the verification of conventional total conservation laws.

The isotopies of Newtonian mechanics, today known *Newton-Santilli isomechanics*, were first submitted in the second edition of monograph [5] and in the mathematical treatment [10].

They require the use of: the *isotime* $\hat{t} = t \times \hat{I}_t$ with isounit $\hat{I}_t = 1/\hat{T}_t > 0$ and related isofield $\hat{R}_t$; the *isocoordinates* $\hat{r} = (\hat{r}_a^k) = r \times \hat{I}_r$, with isounit $\hat{I}_r = 1/\hat{T}_r > 0$ and related isofield $\hat{R}_r$; and the *isovelocities* $\hat{v} = (v_{ka}) = v \times \hat{I}_v$ with isounit $\hat{I}_v = 1/\hat{T}_v > 0$ and related isofield $\hat{R}_v$.

The Newton-Santilli isomechanics is then formulated on the 7-dimensional isospace

$$\hat{S}_{tot} = \hat{E}(\hat{t}, \hat{R}_{\hat{t}}) \times \hat{E}(\hat{r}, \hat{\delta}_r, \hat{R}_{\hat{r}}) \times \hat{E}(\hat{v}, \hat{\delta}_v, \hat{R}_{\hat{v}}), \tag{3.3.20}$$

with isometrics

$$\hat{\delta}_r = \hat{T}_r \times \delta = (\hat{T}^k_{ir} \times \delta_{kj}), \quad \hat{\delta}_v = \hat{T}_v \times \delta = (\hat{T}^k_{iv} \times \delta_{kj}), \tag{3.3.21}$$

over the Kronecker product of isofields

$$\hat{R}_{tot} = \hat{R}_t \times \hat{R}_r \times \hat{R}_v, \tag{3.3.22}$$

with total isounit

$$\begin{aligned}\hat{I}_{tot} = \hat{I}_t \times \hat{I}_r \times \hat{I}_v = \\ = n_t^2 \times Diag.(n^2_{rx}, n^2_{ry}, n^2_{rz}) \times Diag.(n^2_{vx}, n^2_{vy}, n^2_{vz}).\end{aligned} \tag{3.3.23}$$

Consequently, the isounit can also be factorized into the product of the following seven distinct isounits, with related product of seven distinct isofields

$$\hat{I}_{tot} = n_t^2 \times n^2_{rx} \times n^2_{ry} \times n^2_{rz} \times n^2_{vx} \times n^2_{vy} \times n^2_{vz}, \tag{3.3.24a}$$

$$\hat{R}_{tot} = \hat{R}_t \times \hat{R}_{rx} \times \hat{R}_{ry} \times \hat{R}_{rz} \times \hat{R}_{vx} \times \hat{R}_{vy} \times \hat{R}_{vz}, \tag{3.3.24b}$$

and consequential applicability of the fundamental *Tsagas-Sourlas-Santilli-Falcón-Núñez isotopology* (or TSSFN Isotopology) that allows, for the first time to the author's best knowledge, a consistent representation of extended, nonspherical and deformable shapes of particles in newtonian mechanics, here represented via the semiaxes $n^2_\alpha = n^2_\alpha(t, r, v, \ldots)$, $\alpha = t, r, v$.

Note that the isospeed is the given by

$$\hat{v} = \frac{\hat{d}\hat{r}}{\hat{d}\hat{t}} = \hat{I}_t \times \frac{d(r \times \hat{I}_r)}{dt} = v \times \hat{I}_t \times \hat{I}_r + r \times \hat{I}_t \times \frac{d\hat{I}_r}{dt} = v \times \hat{I}_v, \tag{3.3.25}$$

thus illustrating that the isounit of the isospeed cannot be the same as that for the isocoordinates, having in particular the value

$$\hat{I}_v = \hat{I}_t \times \hat{I}_r \times \left(1 + \frac{r}{v} \times \frac{1}{\hat{I}_r} \times \frac{d\hat{I}_r}{dt}\right). \tag{3.3.26}$$

The *Newton-Santilli isoequation* [6,10] can be written

$$\hat{m}_a \hat{\times} \frac{\hat{d}\hat{v}_{ka}}{\hat{d}\hat{t}} = -\frac{\hat{\partial}\hat{V}(\hat{r})}{\hat{\partial}\hat{r}^k_a}, \tag{3.3.27}$$

namely, *the equations are conceived in such a way to formally coincide with the conventional equations for selfadjoint forces when formulated on isospace over* *isofields, while all nonpotential forces are represented by the isounits or, equivalently, by the isodifferential calculus.*

Such a conception is the only one known permitting the representation of extended particles with contact interactions that is invariant, thus avoiding the catastrophic inconsistencies of Section 1.4.1 and, in addition, achieves closure, namely, the verification of all conventional total conservation laws.

An inspection of Eqs. (3.3.27) is sufficient to see that *the Newton-Santilli isomechanics reconstructs linearity, locality and canonicity on isospaces over isofields,* as studied in Section 3.2.11. Note that this would not be the case if nonselfadjoint forces appear in the right hand side of Eqs. (3.3.27) as in Eqs. (3.3.2).

Note the truly crucial role of the isodifferential calculus for the above structural generalization of Newtonian mechanics (as well as of the subsequent mechanics), that justifies *a posteriori* its construction.

The verification of conventional total conservation laws is established by a visual inspection of Eqs. (3.3.27) since their symmetry is the *Galileo-Santilli isosymmetry* [14,15] that is isomorphic to the conventional Galilean symmetry, only formulated on isospace over isofields. By recalling that conservation laws are represented by the generators of the underlying symmetry, conventional total conservation laws then follow from the indicated invariance.

When projected in the conventional representation space S_{tot}, Eqs. (3.3.27) can be explicitly written

$$\hat{m}\hat{\times}\frac{\hat{d}\hat{v}}{\hat{d}\hat{t}} = m \times \hat{I}_t \times \frac{d(v \times \hat{I}_v)}{dt} =$$

$$= m \times \frac{dv}{dt} \times \hat{I}_t \times \hat{I}_v + m \times v \times \hat{I}_t \times \frac{d\hat{I}_v}{dt} = -\frac{\hat{\partial}\hat{V}(\hat{r})}{\hat{\partial}\hat{r}} = -\hat{I}_r \times \frac{\partial V}{\partial r}, \qquad (3.3.28)$$

that is

$$m \times \frac{dv}{dt} = -\hat{T}_t \times \hat{T}_v \times \hat{I}_r \times \frac{\partial V}{\partial r} - m \times v \times \hat{T}_v \times \frac{d\hat{I}_v}{dt}. \qquad (3.3.29)$$

The necessary and sufficient conditions for the representation of all possible SA and NSA forces are given by

$$\hat{I}_r = \hat{T}_t \times \hat{T}_r, \qquad (3.3.30a)$$

$$m \times v \times \hat{T}_v \times \frac{d\hat{I}_v}{dt} = F^{NSA}, \qquad (3.3.30b)$$

and they always admit a solution, since they constitute a system of $6n$ *algebraic* (rather than differential) equations in the $6N + 1$ unknowns given by $\hat{I}_t$, and the diagonal $3N$-dimensional matrices $\hat{I}_r$ and $\hat{I}_v$.

Note that for $\hat{T}_t = 1$ we recover from a dynamical viewpoint the condition $\hat{I}_r = 1/\hat{I}_v$ obtained in Section 3.2.4 and 3.2.10 on geometric grounds.

As a simple illustration among unlimited possibilities, we have the following equations of motion of an *extended* particle with the ellipsoidal shape experiencing a resistive force $F^{NSA} = -\gamma \times v$ because moving within a physical medium

$$m \times \frac{dv}{dt} = \int d\sigma\, \Gamma(\sigma, \mathbf{r}, p, \dots) \approx -\gamma \times v, \tag{3.3.31a}$$

$$\hat{I}_v = Diag.(n_1^2, n_2^2, n_3^2) \times e^{\gamma \times t/m}, \tag{3.3.31b}$$

where the nonlocal-integral character with respect to a kernel Γ is emphasized. Interested readers can then construct the representation of *any* desired non-Hamiltonian Newtonian system (see also memoir [10] for other examples).

Note the natural appearance in the NSA forces of the velocity dependence, as typical of resistive forces. Note also that the representation of the extended character of particles occurs only in isospace because, when Eqs. (3.3.xx) are projected in the conventional Newtonian space, factorized isounits cancel out and the point characterization of particles is recovered.

Note finally the *direct universality* of the Newton-Santilli isoequations, namely, their capability of representing all infinitely possible Newton's equations in the frame of the observer.

As now familiar earlier, Eqs. (3.3.27) can only describe a system of *particles.* The *isodual Newton-Santilli isoequations* for the treatment of a system of *antiparticles* are given by [6,10]

$$\hat{m}_a^d \hat{\times}^d \frac{\hat{d}^d \hat{v}_{ka}^d}{\hat{d}^d \hat{t}^d} = -\frac{\hat{\partial}^d \hat{V}^d(\hat{r}^d)}{\hat{\partial}^d \hat{r}_a^{kd}}. \tag{3.3.32}$$

The explicit construction of the remaining isodualities of the above isomechanics are instructive for the reader seriously interested in a classical study of antimatter under interior dynamical conditions.

3.3.5 Hamilton-Santilli Isomechanics for Matter and its Isodual for Antimatter

3.3.5A. Isoaction Principle and its Isodual. The isotopies of classical Hamiltonian mechanics were first introduced by Santilli in various works (see monographs [6,7] and references quoted therein), and are today known as the *Hamilton-Santilli isomechanics.*

To identify its representation space, recall that the conventional Hamiltonian mechanics is represented in a 7-dimensional space of time, coordinates and momenta (rather than velocity), the latter characterizing phase space (or cotangent bundle of the symplectic geometry).

Correspondingly, the new isomechanics is formulated in the 7-dimensional isospace of isotime $\hat{t}$, isocoordinates $\hat{r}$ and isomomenta $\hat{p}$

$$\hat{S}_{tot} = \hat{E}(\hat{t}, \hat{R}_{\hat{t}}) \times \hat{E}(\hat{r}, \hat{\delta}_r, \hat{R}_{\hat{r}}) \times \hat{E}(\hat{p}, \hat{\delta}_p, \hat{R}_{\hat{p}}), \tag{3.3.33}$$

with isometrics

$$\hat{\delta}_{\hat{r}} = \hat{T}_{\hat{r}} \times \delta = (\hat{T}^k_{ir} \times \delta_{kj}), \hat{\delta}_{\hat{p}} = \hat{T}_{\hat{p}} \times \delta = (\hat{T}^k_{ip} \times \delta_{kj}), \tag{3.3.34}$$

over the Kronecker product of isofields and related isounits

$$\hat{R}_{tot} = \hat{R}_t \times \hat{R}_r \times \hat{R}_p, \tag{3.3.35a}$$

$$\hat{I}_{tot} = \hat{I}_{\hat{t}} \times \hat{I}_{\hat{r}} \times \hat{I}_{\hat{p}} =$$
$$= n_t^2 \times Diag.(n_{rx}^2, n_{ry}^2, n_{rz}^2) \times Diag.(n_{px}^2, n_{py}^2, n_{pz}^2). \tag{3.3.35b}$$

The following new feature now appears. The *isophasespace,* or, more technically, the *isocotangent bundle* of the isosymplectic geometry in local isochart $(\hat{r}, \hat{p})$ requires that the isounits of the variables $\hat{r}$ and $\hat{p}$ are *inverse* of each others (Section 3.2.3 and 3.2.10)

$$\hat{I}_{\hat{r}} = 1/\hat{T}_{\hat{r}} = \hat{I}_{\hat{p}}^{-1} = \hat{T}_{\hat{p}} > 0. \tag{3.3.36}$$

Consequently, by ignoring hereon for notational simplicity the indices for the N particles, the total isounit of the isophase space can be written

$$\hat{I}_{tot} = \hat{I}_{\hat{t}} \times \hat{I}_{\hat{r}} \times \hat{T}_{\hat{r}} = \hat{I}_{\hat{t}} \times \hat{I}_6, \tag{3.3.37a}$$

$$\hat{I}_6 = (\hat{I}_\mu^\nu) = \hat{I}_{\hat{r}} \times \hat{T}_{\hat{r}}. \tag{3.3.37b}$$

The fundamental *isoaction principle* for the classical treatment of matter in interior conditions can be written in the explicit form in the $\hat{r}$ and $\hat{p}$ isovariables

$$\hat{\delta}\hat{A}^o = \hat{\delta} \int_{t_1}^{t_2} (\hat{p}_k \hat{\times} \hat{d}\hat{r}^k - \hat{H} \hat{\times} \hat{d}\hat{t}) = \hat{\delta} \int_{t_1}^{t_2} [p_k \times \hat{T}_{\hat{r}}^{k_i(t,r,p,\dots)} \times \hat{d}\hat{r}^i - \hat{H} \times \hat{T}_{\hat{t}} \times d\hat{t}] = 0, \tag{3.3.38}$$

where

$$\hat{H} = \hat{p}^{\hat{2}} \hat{/} \hat{2} \hat{\times} \hat{m} - \hat{V}(\hat{r}), \tag{3.3.39}$$

is the *isohamiltonian* or simple the Hamiltonian because its projection on conventional spaces represents the orginary total energy except an inessential multiplicative factor.

By using the unified notation

$$\hat{b} = (\hat{b}^\mu) = (\hat{r}^i, \hat{p}_j) = (r^i, p_j) \times \hat{I}_6 = b \times \hat{I}_6, \tag{3.3.40}$$

and the isotopic image of the canonical R^o functions, Eqs. (3.3.xx),

$$\hat{R}^o = (\hat{R}^o_\mu) = (\hat{r}, \hat{0}), \tag{3.3.41}$$

the fundamental isoaction principle can be written in unified notation

$$\hat{\delta}\hat{A}^o = \hat{\delta}\int_{t_1}^{t_2}(\hat{p}_k \hat{\times} \hat{d}\hat{r}^k - \hat{H}\hat{\times}\hat{d}\hat{t}) \equiv \hat{\delta}\int_{t_1}^{t_2}(\hat{R}^o_\mu \hat{\times} \hat{d}\hat{b}^\mu - \hat{H}\hat{\times}\hat{d}\hat{t}) =$$

$$= \hat{\delta}\int_{t_1}^{t_2}(R^o_\mu \times \hat{T}^\mu_{6\nu} \times d\hat{b}^\nu - H \times \hat{T}_{\hat{t}} \times d\hat{t}) = 0. \tag{3.3.42}$$

A visual inspection of principle (3.3.38) establishes the *isocanonicity* of Hamilton-Santilli isomechanics (Section 3.2.11), namely, the reconstruction of canonicity on isospaces over isofield that is crucial for the consistency of hadronic mechanics.

In fact, the conventional action principle (3.3.4) and isoprinciple (3.3.38) coincide at the abstract, realization-free level by conception and construction.

The direct universality of classical isomechanics can be seen from the arbitrariness of the integrand of isoaction functional (3.3.38) once projected on conventional spaces over conventional fields.

An important property of the isoaction is that its functional dependence on isospaces over isofields is restricted to that on isocoordinates only, i.e., $\hat{A} = \hat{A}(\hat{r})$. However, when projected on conventional spaces, the functional dependence is arbitrary, i.e., $\hat{A}(\hat{r}) = \hat{A}(r \times \hat{I}) = \hat{A}(t, r, p, \ldots)$. This feature will soon have a crucial role for the operator image of the classical isomechanics.

It should finally be noted that isoprinciple (3.3.38) essentially eliminates the entire field of *Lagrangian and action principles of orders higher than the first,* e.g., $L = L(t, r, \dot{r}, \ddot{r}, \ldots)$ because of these higher order formulations can be easily reduced to the isotopic first-order form (3.3.38).

Recall that the action principle has the important application via the use of the *optimal control theory* of optimizing dynamical systems, However, the latter can have only been Hamiltonian until now due to the lack of a universal action functional for non-Hamiltonian systems (that constitute, by far, the system most significant for optimization). Recall also that the optimal control theory can only be applied for local-differential systems due to the underlying Euclidean topology, thus secluding from the optimization process the most important systems, those of extended, and, therefore, of nonlocal type.

Note that isoaction principle (3.3.38) occurs for all possible non-Hamiltonian as well as nonlocal-integral systems, thanks also to the underlying TSSFN isotopology (Section 3.2.7). We, therefore, have the following important:

THEOREM 3.3.1 [6,10]: Isoaction principle (3.3.38) permits the (first known) optimization of all possible nonpotential/non-Hamiltonian and nonlocal-integral systems.

The *isodual isoaction principle* [10] for the classical treatment of antimatter in interior conditions is given by

$$\hat{\delta}^d \hat{A}^d = \hat{\delta}^d \int_{t_1}^{t_2}{}^{d} (\hat{p}_k^d \hat{\times}^d \hat{d}^d \hat{r}^{kd} - \hat{H}^d \hat{\times}^d \hat{d}^d \hat{t}^d) =$$

$$= \hat{\delta}^d \int_{t_1}^{t_2}{}^{d} (\hat{R}_\mu^{od} \hat{\times}^d \hat{d}^d \hat{b}^\mu - \hat{H}^d \hat{\times} \hat{d}^d \hat{t}^d) = 0. \qquad (3.3.43)$$

Additional isodual treatments are left to the interested reader.

3.3.5B. Hamilton-Santilli Isoequations and their Isoduals. The discovery of the isodifferential calculus in 1995 permitted Santilli [6,10] the identification of the following classical dynamical equations for the treatment of matter at the foundations of hadronic mechanics, today known as the *Hamilton-Santilli isoequations*. They are easily derived via the isovariational principle and can be written from isoprinciple (3.3.38) in disjoint notation

$$\frac{\hat{d}\hat{r}^k}{\hat{d}\hat{t}} = \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_k}, \quad \frac{\hat{\partial}\hat{p}_k}{\hat{d}\hat{t}} = -\frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{r}^k}. \qquad (3.3.44)$$

The same equations can be written in unified notation from principle (3.3.40)

$$\hat{\omega}_{\mu\nu} \hat{\times} \frac{\hat{d}\hat{b}^\mu}{\hat{d}\hat{t}} = \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{b}^\mu}, \qquad (3.3.45)$$

where

$$\hat{\omega}_{\mu\nu} = \omega_{\mu\nu} \times \hat{I}_6 \qquad (3.3.46)$$

is the *isocanonical isosymplectic tensor* that coincides with the conventional canonical symplectic tensor $\omega_{\mu\nu}$ except for the factorization of the isounit (Section 3.2.10).

To verify the latter property from an analytic viewpoint, it is instructive for the reader to verify the following identify under isounits (3.3.37)

$$\hat{\omega}_{\mu\nu} = \frac{\hat{\partial}\hat{R}_\nu^o}{\hat{\partial}\hat{b}^\mu} - \frac{\hat{\partial}\hat{R}_\mu^o}{\hat{\partial}\hat{b}^\nu} = \omega_{\mu\nu} \times \hat{I}_6. \qquad (3.3.47)$$

A simple comparison of the above isoanalytic equations with the isotopic and conventional Newton's equations established the following:

THEOREM 3.3.2: Hamilton-Santilli isoequations (3.3.5) are "directly universal" in Newtonian mechanics, that is, capable of representing all possible, conventional or isotopic, hamiltonian and non-Hamiltonian Newtonian systems directly in the fixed coordinates of the experimenter.

It is now important to show that Eqs. (3.3.45) provide an identical reformulation of the true analytic equations (3.3.2). For this purpose, we assume the simple case in which isotime coincide with the conventional time, that is, $\hat{t} = t$, $\hat{I}_t = +1$ and we write isoequations (3.3.45) in the explicit form

$$\begin{gathered}(\omega) \times \begin{pmatrix} dr^k/dt \\ dp_k/dt \end{pmatrix} = \begin{pmatrix} 0_{3\times3} & -I_{3\times3} \\ I_{3\times3} & 0_{3\times3} \end{pmatrix} \times \begin{pmatrix} dr^k/dt \\ dp_k/dt \end{pmatrix} = \\ \begin{pmatrix} -dp_k/dt \\ dr^k/dt \end{pmatrix} = \begin{pmatrix} \hat{\partial}\hat{H}/\hat{\partial}r^k \\ \hat{\partial}\hat{H}/\hat{\partial}p_k \end{pmatrix} = \begin{pmatrix} \hat{I}^i_k \times \partial\hat{H}/\partial r^i \\ \hat{T}^k_i \times \partial\hat{H}/\partial p_i \end{pmatrix}. \end{gathered} \tag{3.3.48}$$

It is easy to see that Eqs. (3.3.xx) *coincide* with the true analytic equations (3.3.2) under the trivial algebraic identification

$$\hat{I}_{\hat{r}} = Diag.[I - F/(\partial H/\partial r)]. \tag{3.3.49}$$

As one can see, the main mechanism of Eqs. (3.3.45) is that of *transforming the external terms $F = F^{NSA}$ into an explicit realization of the isounit $\hat{I}_3$*. As a consequence, *reformulation (3.3.45) constitutes direct evidence on the capability to represent non-Hamiltonian forces and effects with a generalization of the unit of the theory.*

Note in particular that *the external terms are embedded in the isoderivatives.* However, when written down explicitly, Eqs. (3.3.2) and (3.3.45) coincide. Note that $\hat{I}_3$ as in rule (3.3.49) is fully symmetric, thus acceptable as the isounit of isomathematics. Note also that all nonlocal and nonhamiltonian effects are embedded in $\hat{I}$.

The reader should note the extreme simplicity in the construction of a representation of given non-Hamiltonian equations of motion, due to the algebraic character of identifications (3.3.49).

Recall that *Hamilton's equations with external terms are not derivable from a variational principle.* In turn, such an occurrence has precluded the identification of the operator counterpart of Eqs. (3.3.2) throughout the 20-th century.

We now learn that *the identical* reformulation (3.3.45) of Eqs. (3.3.2) becomes fully derivable from a variational principle. In turn, this will soon permit the identification of the unique and unambiguous operator counterpart.

It should be noted that *the Hamilton-Santilli isoequations are generally irreversible* due to the general irreversibility of the external forces,

$$F(t,\ldots) \neq F(-t,\ldots), \quad \text{or} \tag{3.3.50a}$$

$$\hat{I}(t,\ldots) = Diag.[I - F(t,\ldots)/(\partial H/\partial t)] \neq \hat{I}(-t,\ldots). \tag{3.3.50b}$$

In particular, we have irreversibility under the conservation of the total energy (see next chapter for full treatment). This feature is important to achieve compatibility with thermodynamics, e.g., to have credible analytic methods for the representation of the internal increase of the entropy for closed-isolated systems such as Jupiter.

The study of these thermodynamical aspects is left to the interested reader. In this chapter we shall solely consider *reversible closed-isolated systems* that occur for external forces not explicitly dependent on time and verify other restrictions.

An important aspect is that *the Hamilton-Santilli isoequations coincide with the Hamilton equations without external terms at the abstract level.* In fact, all differences between I and $\hat{I}$, $\times$ and $\hat{\times}$, ∂ and $\hat{\partial}$, *etc.*, disappear at the abstract level. This proves the achievement of a central objective of isomechanics, the property that the analytic equations with external terms can indeed be *identically* rewritten in a form equivalent to the analytic equations without external terms, provided, however, that the reformulation occurs via the broader isomathematics.

The *isodual Hamilton-Santilli isoequations* for the classical treatment of antimatter, also identified soon after the discovery of the isodifferential calculus, are given by

$$\hat{\omega}^d_{\mu\nu} \hat{\times}^d \frac{\hat{d}^d \hat{b}^{d\mu}}{\hat{d}^d \hat{t}^d} = \frac{\hat{\partial}^d \hat{H}^d}{\hat{\partial}^d \hat{b}^{d\mu}}, \tag{3.3.51}$$

where

$$\hat{\omega}^d_{\mu\nu} = \omega^d_{\mu\nu} \times \hat{I}_6 \tag{3.3.52}$$

is the *isodual isocanonical isosymplectic tensor.* The derivation of other isodual properties is instructed for the interested reader.

3.3.5C. Classical Lie-Santilli Brackets and their Isoduals. It is important to verify that Eqs. (3.3.44) or (3.3.45) resolve the problematic aspects of external terms indicated in Section 3.3.2 [4]. In fact, the isobrackets of the time evolution of matter are given by

$$\frac{d\hat{A}}{\hat{d}\hat{t}} = [\hat{A} \hat{,} \hat{H}] = \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{r}^k} \hat{\times} \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_k} - \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{r}^k} \hat{\times} \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{p}_k}, \tag{3.3.53}$$

and they verify the left and right distributive and scalar laws, thus characterizing a consistent algebra. Moreover, that algebra results to be Lie-isotopic, for which reasons the above brackets are known as the *Lie-Santilli isobrackets.*

When explicitly written in our spacetime, brackets (3.3.53) recover the brackets (3.3.3) of the true analytic equations (3.3.2)

$$\frac{dH}{dt} = \frac{\partial H}{\partial r^k} \times \frac{\partial H}{\partial p_k} - \frac{\partial H}{\partial p_k} \times \frac{\partial H}{\partial r^k} + \frac{\partial H}{\partial p_k} \times F^k = \frac{\partial H}{\partial p_k} \times F^k \equiv 0, \tag{3.3.54}$$

where the last identity holds in view of Eqs. (3.3.49). Therefore, the Hamilton-Jacobi isoequations do indeed constitute a reformulation of the true analytic equations with a consistent Lie-isotopic algebraic brackets, as needed (Section 3.3.3).

Note that, in which of their anti-isomorphic character, isobrackets (3.3.53) represent the conservation of the Hamiltonian,

$$\frac{d\hat{H}}{dt} = [\hat{H}\hat{,}\hat{H}] = \frac{\hat{\partial}\hat{H}}{\hat{\partial}r^k} \hat{\times} \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_k} - \frac{\hat{\partial}\hat{H}}{\hat{\partial}r^k} \hat{\times} \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_k} \equiv 0. \tag{3.3.55}$$

This illustrates the reason for assuming closed-isolated Newtonian systems (3.3.19) at the foundations of this chapter.

Basic isobrackets (3.3.53) can be written in unified notation

$$[\hat{A}\hat{,}\hat{B}] = \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{b}^\mu} \hat{\times} \hat{\omega}^{\mu\nu} \hat{\times} \frac{\hat{\partial}\hat{B}}{\hat{\partial}\hat{b}^\nu}, \tag{3.3.56}$$

where $\hat{\omega}_{\mu\nu}$ is the Lie-Santilli isotensor. By using the notation $\hat{\partial}^\mu = \hat{\partial}/\hat{\partial}\hat{b}^\mu$, the isobrackets can be written

$$[\hat{A}\hat{,}\hat{B}] = \hat{\partial}_\mu \hat{A} \times \hat{T}^\mu_\rho \times \omega^{\rho\nu} \hat{\partial}_\nu \hat{\partial}_\nu \hat{B}, \tag{3.3.57}$$

and, when projected in our spacetime, the isobrackets can be written

$$[A\hat{,}B] = \partial_\mu A \times \omega^{\mu\rho} \times \hat{I}^\nu_\rho \times \partial_\nu B, \tag{3.3.58}$$

where $\omega^{\mu\nu}$ is the canonical Lie tensor.

The *isodual Lie-Santilli isobrackets* for the characterization of antimatter can be written

$$[\hat{A}^d\hat{,}\hat{B}^d] = \hat{\partial}^d_\mu \hat{A}^d \hat{\times}^d \hat{\omega}^{d\rho\nu} \hat{\partial}^d_\nu \hat{\partial}^d_\nu \hat{B}^{,} \tag{3.3.59}$$

where $\hat{\omega}^{d\mu\nu}$ is the *isodual Lie-Santilli isotensor*. Other algebraic properties can be easily derived by the interested reader.

3.3.5D. Hamilton-Jacobi-Santilli Isoequations and their isoduals. Another important consequence of isoaction principle (3.3.38) is the characterization of the following *Hamilton-Jacobi-Santilli isoequations* for matter [6,10]

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{t}} + \hat{H} = 0, \tag{3.3.60a}$$

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{r}^k} - \hat{p}_k = 0, \tag{3.3.60b}$$

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{p}_k} \equiv 0, \tag{3.3.60c}$$

which will soon have basic relevance for isoquantization.

Note the *independence of the isoaction $\hat{A}^o$ from the isomomenta* that will soon be crucial for consistent isoquantization.

The isodual equations for antimatter are then given by

$$\frac{\hat{\partial}^d\hat{A}^{od}}{\hat{\partial}^d\hat{t}^d} + \hat{H}^d = 0, \tag{3.3.61a}$$

$$\frac{\hat{\partial}^d\hat{A}^{od}}{\hat{\partial}^d\hat{r}^{kd}} - \hat{p}^d_k = 0, \tag{3.3.61b}$$

$$\frac{\hat{\partial}^d\hat{A}^{od}}{\hat{\partial}^d\hat{p}^d_k} \equiv 0. \tag{3.3.61c}$$

The latter equations will soon result to be essential for the achievement of a consistent operator image of the classical treatment of antimatter in interior conditions.

3.3.5E. Connection Between Isotopic and Birkhoffian Mechanics. Since the Hamilton-Santilli isoequations are directly universal, they can also represent Birkhoff's equations (3.3.13) in the fixed b-coordinates. In fact, by assuming for simplicity that the isotime is the ordinary time, we can write the identities

$$\frac{db^\mu}{dt} = \Omega^{\mu\mu}(b) \times \frac{\partial H(b)}{\partial b^\nu} \equiv \omega^{\mu\nu} \times \frac{\hat{\partial} H(b)}{\hat{\partial} b^\nu} = \omega^{\mu\rho} \times \hat{I}^\nu_{6\rho} \times \frac{\partial H}{\partial b^\nu}. \tag{3.3.62}$$

Consequently, we reach the following *decomposition of the Birkhoffian tensor*

$$\Omega^{\mu\nu}(b) = \omega^{\mu\rho} \times \hat{I}^\nu_{6\rho}(b). \tag{3.3.63}$$

Consequently, Birkhoff's equations can indeed be identically rewritten in the isotopic form, as expected. In the process, the reformulation provides additional insight in the isounit.

The reformulation also carries intriguing geometric implications since it confirms the *direct universality in symplectic geometry of the canonical two-form,* since a general symplectic two-form can always be identically rewritten in the isocanonical form via decomposition of type (3.3.xx) and then the embedding of the isounit in the isodifferential of the exterior calculus.

As an incidental note, the reader should be aware that the construction of an analytic representation via Birkhoff's equations is rather complex, inasmuch as it requires *the solution of nonlinear partial differential equations* or integral equations [2].

By comparison, the construction of the same analytic equations via Hamilton-Santilli isoequations (3.3.44) or (3.3.45) is truly elementary, and merely requires the identification of the isounit according to *algebraic* rule (3.3.49) for arbitrarily given external forces $F_k(t, r, p)$.

3.3.6 Simple Construction of Classical Isomechanics

The above classical isomechanics can be constructed via a simple method which does not need any advanced mathematics, yet it is sufficient and effective for practical applications.

In fact, *the Hamilton-Santilli isomechanics can be constructed via the systematic application of the following noncanonical transform to all quantities and operations of the conventional Hamiltonian mechanics*

$$U = \begin{pmatrix} \hat{I}_3^{1/2} & 0 \\ 0 & \hat{T}_3^{1/2} \end{pmatrix}, \tag{3.3.64a}$$

$$U \times U^t = \hat{I}_6 \neq I, \tag{3.3.64b}$$

$$\hat{I}_3 = I - \frac{F}{\partial H/\partial p} = I - \frac{F}{p/m}. \tag{3.3.64c}$$

The success of the construction depends on the application of the above noncanonical transform to the *totality* of Hamiltonian mechanics, with no exceptions. We have in this way the lifting of: the 6-dimensional unit of the conventional phase space into the isounit

$$I_6 \rightarrow \hat{I}_6 = U \times I_6 \times U^t; \tag{3.3.65}$$

numbers into the isonumbers,

$$n \rightarrow \hat{n} = U \times n \times U^t = n \times (U \times U^t) = n \times \hat{I}_6; \tag{3.3.66}$$

associative product $A \times B$ among generic quantities A, B into the isoassociative product with the correct expression and property for the isotopic element,

$$A \times B \rightarrow A \hat{\times} B = U \times (A \times B) \times U^t = A' \times \hat{T} \times B', \tag{3.3.67a}$$

$$A' = U \times A \times U^t, \quad B' = U \times B \times U^t, \quad \hat{T} = (U \times U^t)^{-1} = T^t; \tag{3.3.67b}$$

Euclidean into iso-Euclidean spaces (where we use only the space component of the transform)

$$\begin{gathered}x^2 = x^t \times \delta \times x \rightarrow \hat{x}^{\hat{2}} = U \times x^2 \times U^t = \\ = (x^t \times U^t) \times (U^{t-1} \times \delta \times U^{-1}) \times (U \times x) \times (U \times U^t) = \\ = [x'^t \times (\hat{T} \times \delta) \times x'] \times \hat{I};\end{gathered} \tag{3.3.68}$$

and, finally, we have the following isotopic lifting of Hamilton's into Hamilton-Santilli isoequations (here derived for simplicity for the case in which the transform does not depend explicitly on the local coordinates),

$$db/dt - \omega \times \partial H/\partial b = 0 \rightarrow$$

$$\rightarrow U \times db/dt \times U^t - U \times \omega \times \partial H/\partial b \times U^t =$$

$$= db/dt \times (U \times U^t) - (U \times \omega \times U^t) \times (U^{t\ -1} \times U^{-1}) \times$$

$$\times (U \times \partial H/\partial b \times U^t) \times (U \times U^t) =$$

$$= db/dt \times \hat{I} - \omega \times (\hat{\partial} H/\hat{\partial}\hat{b}) \times \hat{I} = 0, \tag{3.3.69}$$

where we have used the important property the reader is urged to verify

$$U \times \omega \times U^t \equiv \omega. \tag{3.3.70}$$

As one can see, the seemingly complex isomathematics and isomechanics are reduced to a truly elementary construction. e its universality.

3.3.7 Invariance of Classical Isomechanics

A final requirement is necessary for a physical consistency, and that is, *the invariance of isomechanics under its own time evolution*, as it occurs for conventional Hamiltonian mechanics.

Recall that a transformation $b \rightarrow b'(b)$ is called a *canonical transformation* when all the following identities hold

$$\frac{\partial b^\mu}{\partial b'^\alpha} \times \omega_{\mu\nu} \times \frac{\partial b^\nu}{\partial b'^\beta} = \omega_{\alpha\beta}. \tag{3.3.71}$$

The invariance of Hamiltonian mechanics follows from the property that its time evolution constitutes a canonical transformation, as well known.

The proof of the invariant of isomechanics is elementary. In fact, an isotransformation $\hat{b} \rightarrow \hat{b}'(\hat{b})$ constituted an *isocanonical isotransform* when all the following identities old

$$\frac{\hat{\partial}\hat{b}^\mu}{\hat{\partial}\hat{b}'^\alpha} \hat{\times} \hat{\omega}_{\mu\nu} \hat{\times} \frac{\hat{\partial}\hat{b}^\nu}{\hat{\partial}\hat{b}'^\beta} = \hat{\omega}_{\alpha\beta} = \omega_{\alpha\beta} \times \hat{I}_6. \tag{3.3.72}$$

But the above expression can be written

$$(\hat{I}_6^{\mu_\rho} \times \frac{\partial \hat{b}^\rho}{\partial \hat{b}^\alpha} \times \omega_{\mu\nu} \times \hat{I}_6^{\xi_\nu} \times \frac{\partial \hat{b}^\xi}{\partial \hat{b}'^\beta}) \times \hat{I}_6 = \omega_{\mu\nu} \times \hat{I}_6, \tag{3.3.73}$$

and they coincide with conditions (3.3.xx) in view of the identities

$$\hat{I}_6^{\mu_\rho} \times \omega_{\mu\nu} \times \hat{I}_6^{\nu_\xi} = \omega_{\rho\xi}. \tag{3.3.74}$$

Consequently, we have the following important

THEOREM 3.3.3 [6,10]: Following factorization of the isounit, isocanonical transformations are canonical.

The desired invariance of the Hamilton-Santilli isomechanics then follows.

It is an instructive exercise for the reader interested in learning isomechanics to verify that *all* catastrophic mathematical and physical inconsistencies of non-canonical theories pointed out in Chapter 1 (see Section 1.4.1 in particular) are indeed resolved by isomechanics as presented in this section.

3.4 OPERATOR LIE-ISOTOPIC MECHANICS FOR MATTER AND ITS ISODUAL FOR ANTIMATTER

3.4.1 Introduction

We are finally equipped to present the foundations of *the Lie-isotopic branch of nonrelativistic hadronic mechanics* for matter and its isodual for antimatter, more simply referred to as *operator isomechanics,* and its isodual for antimatter referred to as *isodual operator isomechanics.* The new mechanics will then be used in subsequent sections for various developments, experimental verifications and industrial applications.

The extension of the results of this section to *relativistic operator isomechanics* is elementary and will be done in the following sections whenever needed for specific applications. the case of *operator genomechanics* with a Lie-admissible, rather than the Lie-isotopic structure, will be studied in the next chapter.

A knowledge of Section 3.2 is necessary for a technical understanding of operator isomechanics. For the mathematically non-inclined readers, we present in Section 3.4.8 a very elementary construction of operator isomechanics via nonunitary transforms.

Unless otherwise specified, all quantities and operations represented with conventional symbols A, H, $\times$, *etc.*, denote quantities and operations on conventional Hilbert spaces over conventional fields. All quantities and symbols of the type $\hat{A}$, $\hat{H}$, $\hat{\times}$, *etc.*, are instead defined on isohilbert spaces over isofields.

Note the use of the terms "operator" isomechanics, rather than "quantum" isomechanics, because, as indicated in Chapter 1, the notion of quantum is fully established within the arena of its conception, the transition of electrons between different stable orbits of atomic structure (exterior problem), while the assumption of the same quantum structure for the same electrons when in the core of a star (interior problems) is a scientific religion at this writing deprived of solid experimental evidence.

3.4.2 Naive Isoquantization and its Isodual

An effective way to derive the basic dynamical equations of operator isomechanics is that via the isotopies of the conventional map of the classical Hamilton-Jacobi equations into their operator counterpart, known as *naive quantization.*. More rigorous methods, such as the isotopies of symplectic quantization, essentially yields the same operator equations and will not be treated in this section for brevity (see monograph [7] for a presentation).

Recall that the *naive quantization* can be expressed via the following map of the canonical action functional

$$A^o = \int_{t_1}^{t_2} (p_k \times dr^k - H \times dt) \to -i \times \hbar \times \ln |\psi\rangle, \tag{3.4.1}$$

under which the conventional Hamilton-Jacobi equations are mapped into the Schrödinger equations,

$$-\partial_t A^o = H \to i \times \hbar \times \partial_t |\psi\rangle = H \times |\psi\rangle, \tag{3.4.2a}$$

$$p_k = \partial_k A^o \to p_k \times |\psi\rangle = -i \times \hbar \times \partial_k |\psi\rangle, \tag{3.4.2b}$$

where $|\psi\rangle$ is the *wavefunction,* or, more technically, a state in a Hilbert space $\mathcal{H}$.

Isocanonical action (3.3.38) is evidently different than the conventional canonical action, e.g., because it is of higher order derivatives. As such, the above naive quantization does not apply.

In its place we have the following *naive isoquantization* first introduced by Animalu and Santilli [44] of 1990, and here extended to the use of the isodifferential calculus

$$\hat{A}^o = \int_{t_1}^{t_2} (\hat{p}_k \hat{\times} \hat{d}\hat{x}^k - \hat{H} \hat{\times} \hat{d}\hat{t}) \to -i \times \hat{I} \times \ln |\hat{\psi}\rangle, \tag{3.4.3}$$

where $\hat{i} = i \times \hat{I}$, $|\hat{\psi}\rangle$ is the *ospwavefunction,* or, more precisely, a state of the iso-Hilbert space $\hat{\mathcal{H}}$ outlined in the next section, and we should note that $\hat{i} \hat{\times} \hat{I} \times \ln |\hat{\psi}\rangle = i \times \mathrm{isoln} |\hat{\psi}\rangle$.

The use of Hamilton-Jacobi-Santilli isoequations (3.3.60) yields the following operator equations (here written for the simpler case in which $\hat{T}$ has no dependence on r, but admits a dependence on velocities and higher derivatives)

$$-\hat{\partial}_t \hat{A}^o = \hat{H} \to i \times \hat{\partial}_t |\hat{\psi}\rangle = \hat{H} \times \hat{T} \times |\hat{\psi}\rangle = \hat{H} \hat{\times} |\hat{\psi}\rangle, \tag{3.4.4a}$$

$$\hat{p}_k = \hat{\partial}_k \hat{A}^o \to \hat{p}_k \times \hat{T} \times |\hat{\psi}\rangle = \hat{p}_k \hat{\times} |\hat{\psi}\rangle = -\hat{i} \hat{\times} \hat{\partial}_k |\hat{\psi}\rangle, \tag{3.4.4b}$$

that constitutes the fundamental equations of operator isomechanics, as we shall see in the next section.

As it is well known, *Planck's constant $\hbar$ is the basic unit of quantum mechanics.* By comparing Eqs. (3.4.xx) and (3.4.xx) it is easy to see that $\hat{I}$ is the basic unit of operator isomechanics. Recall also that the isounits are defined at short distances as in Eqs. (3.1.xxx). We therefore have the following important

POSTULATE 3.4.1 [5]: In the transition from quantum mechanics to operator isomechanics Planck's unit $\hbar$ is replaced by the integrodifferential unit $\hat{I}$ under the condition of recovering the former at sufficiently large mutual distances,

$$\lim_{r \to \infty} \hat{I} = \hbar = 1. \tag{3.4.5}$$

Consequently, in the conditions of deep mutual penetration of the wavepackets and/or charge distributions of particles as studied by operator isomechanics there is the superposition of quantized and continuous exchanges of energy.

3.4.3 Isohilbert Spaces and their Isoduals

As it is well known, the Hilbert space $\mathcal{H}$ used in quantum mechanics is expressed in terms of states $|\psi\rangle,\ |\phi\rangle, \ldots$, with normalization

$$\langle\psi| \times |\psi\rangle = 1, \tag{3.4.6}$$

and inner product

$$\langle\phi| \times |\psi\rangle = \int dr^3 \phi^\dagger(r) \times \psi(r), \tag{3.4.7}$$

defined over the field of complex numbers $\mathcal{C} = C(c, +, \times)$.

The lifting $C(c, +, \times) \to \hat{C}(\hat{c}, \hat{+}, \hat{\times})$, requires a compatible lifting of $\mathcal{H}$ into the *isohilbert space* $\hat{\mathcal{H}}$ with *isostates* $|\hat{\psi}\rangle, |\hat{\phi}\rangle, \ldots$, *isoinner product* and *isonormalization*

$$\langle\hat{\psi}| \hat{\times} |\hat{\psi}\rangle \times \hat{I} = \left[\hat{\int} d\hat{r}^3 \ \hat{\psi}^\dagger(\hat{r}) \times \hat{T} \times \hat{\psi}(\hat{r}) \right] \times \hat{I} \in \hat{\mathcal{C}}, \tag{3.4.8a}$$

$$\langle\hat{\psi}| \hat{\times} |\hat{\psi}\rangle = 1, \tag{3.4.8b}$$

first introduced by Myung and Santilli in 1982 [45] (see also monographs [6,7] for a comprehensive study).

It is easy to see that the isoinner product is still inner (because $\hat{T} > 0$). Thus, $\hat{\mathcal{H}}$ is still Hilbert and the lifting $\mathcal{H} \rightarrow \hat{\mathcal{H}}$ is an isotopy. Also, it is possible to prove that *iso-Hermiticity coincides with conventional Hermiticity*,

$$\langle\hat{\psi}|\hat{\times}(\hat{H}\hat{\times}|\hat{\psi}\rangle) \equiv (\langle\hat{\psi}|\hat{\times}\hat{H}^{\hat{\dagger}})\hat{\times}|\hat{\psi}\rangle, \tag{3.4.9a}$$

$$\hat{H}^{\hat{\dagger}} \equiv \hat{H}^{\dagger} = \hat{H}. \tag{3.4.9b}$$

As a result, *all quantities that are observable for quantum mechanics remain so for hadronic mechanics.*

For consistency, the conventional eigenvalue equation $H \times |\psi\rangle = E \times |\psi\rangle$ must also be lifted into the *isoeigenvalue form* [7]

$$\hat{H}\hat{\times}|\hat{\psi}\rangle = \hat{H} \times \hat{T} \times |\hat{\psi}\rangle = \hat{E}\hat{\times}|\hat{\psi}\rangle = (E \times \hat{I}) \times \hat{T} \times |\hat{\psi}\rangle = E \times |\hat{\psi}\rangle, \tag{3.4.10}$$

where, as one can see, the final results are ordinary numbers.

Note the *necessity* of the isotopic action $\hat{H}\hat{\times}|\hat{\psi}\rangle$, rather than $\hat{H} \times |\hat{\psi}\rangle$. In fact, only the former admits $\hat{I}$ as the correct unit,

$$\hat{I}\hat{\times}|\hat{\psi}\rangle = \hat{T}^{-1} \times \hat{T} \times |\hat{\psi}\rangle \equiv |\hat{\psi}\rangle. \tag{3.4.11}$$

It is possible to prove that *the isoeigenvalues of isohermitian operators are isoreal*, i.e., they have the structure $\hat{E} = E \times \hat{I}$, $E \in R(n, +, \times)$. As a result all real eigenvalues of quantum mechanics remain real for hadronic mechanics.

We also recall the notion of *isounitary operators* as the isooperators $\hat{U}$ on $\hat{\mathcal{H}}$ over $\hat{C}$ satisfying the isolaws

$$\hat{U}\hat{\times}\hat{U}^{\hat{\dagger}} = \hat{U}^{\hat{\dagger}}\hat{\times}\hat{U} = \hat{I}, \tag{3.4.12}$$

where we have used the identity $\hat{U}^{\hat{\dagger}} \equiv \hat{U}^{\dagger}$.

We finally indicate the notion of *isoexpectation value* of an isooperators $\hat{H}$ on $\hat{\mathcal{H}}$ over $\hat{C}$

$$\langle\hat{H}\rangle = \frac{\langle\hat{\psi}|\hat{\times}\hat{H}\hat{\times}|\hat{\psi}\rangle}{\langle\hat{\psi}|\hat{\times}|\hat{\psi}\rangle}. \tag{3.4.131}$$

It is easy to see that *the isoexpectation values of isohermitian operators coincide with the isoeigenvalues*, as in the conventional case.

Note also that *the isoexpectation value of the isounit is the isounit*,

$$\langle\hat{I}\rangle = \hat{I}, \tag{3.4.14}$$

provided, of course, that one uses the isoquotient (otherwise $\langle\hat{I}\rangle = I$).

The isotopies of quantum mechanics studied in the next sections are based on the following novel invariance property of the conventional Hilbert space [xxx], here expressed in term of a non-null scalar n independent from the integration variables,

$$\langle\hat{\phi}| \times |\hat{\psi}\rangle \times I \equiv \langle\hat{\phi}| \times n^{-2} \times |\hat{\psi}\rangle \times (n^2 \times I) = \langle\phi|\hat{\times}|\psi\rangle \times \hat{I}. \tag{3.4.15}$$

Note that new invariances (3.4.15) remained undetected throughout the 20-th century because they required the prior discovery of *new numbers*, those with arbitrary units.

3.4.4 Structure of Operator Isomechanics and its Isodual

The structure of operator isomechanics is essentially given by the following main steps [47]:

1) The description of closed-isolated systems is done via *two* quantities, the Hamiltonian representing all action-at-a-distance potential interactions, plus the isounit representing all nonlinear, nonlocal and non-Hamiltonian effects,

$$H(t, r, p) = p^2/2m + V(r), \tag{3.4.16a}$$

$$\hat{I} = \hat{I}(t, r, p, \psi, \nabla\psi, \ldots). \tag{3.4.16b}$$

The explicit form of the Hamiltonian is that conventionally used in quantum mechanics although written on isospaces over isofields,

$$\hat{H} = \hat{p}\hat{\times}\hat{p}\hat{/}\hat{2}\hat{\times}\hat{m} + \hat{V}(\hat{r}). \tag{3.4.17}$$

A generic expression of the isounit for the representation of two spinning particles with point-like change (such as the electrons) in conditions of deep penetration of their wavepackets (as occurring in chemical valence bonds and many other cases) is given by

$$\hat{I} = \exp\left[\Gamma(\psi, \psi^\dagger) \times \int dv\, \psi_\downarrow^\dagger(r)\psi_\uparrow(r)\right], \tag{3.4.18}$$

where the nonlinearity is expressed by $\Gamma(\psi, \psi^\dagger)$ and the nonlocality is expressed by the volume integral of the deep wave-overlappings $\int dv\, \psi_\downarrow^\dagger(r)\psi_\uparrow(r)$. All isounits will be restricted by the conditions of being positive-definite (thus everywhere invertible) as well as of recovering the trivial unit of quantum mechanics for sufficiently big mutual distances r,

$$\lim_{r\to\infty} \int dv\, \psi_\downarrow^\dagger(r)\psi_\uparrow(r) = 0. \tag{3.4.19}$$

2) The lifting of the multiplicative unit $I > 0 \to \hat{I} = 1/\hat{T} > 0$ requires the reconstruction of the entire formalism of quantum mechanics into such a form to

admit $\hat{I}$ as the correct left and right unit at all levels of study, including numbers and angles, conventional and special functions, differential and integral calculus, metric and Hilbert spaces, algebras and groups, etc., without any exception known to the authors. This reconstruction is "isotopic" in the sense of being axiom-preserving. Particularly important is the preservation of all conventional quantum laws as shown below.

3) The mathematical structure of nonrelativistic hadronic mechanics is characterized by [6]:

3a) The isofield $\hat{C} = \hat{C}(\hat{c}, +, \hat{\times})$ with *isounit* $\hat{I} = 1/\hat{T} > 0$, *isocomplex numbers* and related *isoproduct*

$$\hat{c} = c \times \hat{I} = (n_1 + i \times n_2) \times \hat{I},\ \hat{c}\hat{\times}\hat{d} = (c \times d) \times \hat{I},\ \ \hat{c}, \hat{d} \in \hat{\mathcal{C}},\ \ c, d \in C, \quad (3.4.20)$$

the isofield $\hat{R}(\hat{n}, +, \hat{\times})$ of *isoreal numbers* $\hat{n} = n \times \hat{I},\ n \in R$, being a particular case;

3b) The iso-Hilbert space $\hat{\mathcal{H}}$ with isostates $|\hat{\psi}\rangle, |\hat{\phi}\rangle, \ldots,$ *isoinner product* and *isonormalization*

$$\langle\hat{\phi}|\hat{\times}|\hat{\psi}\rangle \times \hat{I}\ \in \hat{S}, \quad \langle\hat{\psi}|\hat{\times}|\hat{\psi}\rangle = 1, \quad (3.4.21)$$

and related theory of isounitary operators;

3c) The Euclid-Santilli isospace $\hat{E}(\hat{r}, \hat{\delta}, \hat{\mathrm{R}})$ with *isocoordinates, isometric and isoinvariant* respectively given by

$$\hat{r} = \{r^k\} \times \hat{I}, \quad (3.4.22a)$$

$$\hat{\delta} = \hat{T}(t, r, p, \psi, \nabla\psi, \ldots) \times \delta, \quad (3.4.22b)$$

$$\delta = Diag.(1, 1, 1), \quad (3.4.22c)$$

$$\hat{r}^{\hat{2}} = (r^i \times \hat{\delta}_{ij} \times r^j) \times \hat{I} \in \hat{\mathrm{R}}; \quad (3.4.22d)$$

3d) The isodifferential calculus and the isofunctional analysis (see Section 3.2);

3e) The Lie-Santilli isotheory with enveloping isoassociative algebra $\hat{\xi}$ of operators $\hat{A},\ \hat{B},\ \ldots,$ with isounit $\hat{I}$, isoassociative product $\hat{A}\hat{\times}\hat{B} = \hat{A} \times \hat{T} \times \hat{B}$, *Lie-Santilli isoalgebra with brackets and isoexponentiation*

$$[\hat{A}\hat{,}\hat{B}] = \hat{A}\hat{\times}\hat{B} - \hat{B}\hat{\times}\hat{A}, \quad (3.4.23a)$$

$$\hat{U} = \hat{e}^X = (e^{X \times \hat{T}}) \times \hat{I} = \hat{I} \times (e^{\hat{T} \times X}), \quad X = X^\dagger, \quad (3.4.13b)$$

and related isosymmetries characterizing groups of isounitary transforms on $\hat{\mathcal{H}}$ over $\hat{C}$,

$$\hat{U}\hat{\times}\hat{U}^\dagger = \hat{U}^\dagger\hat{\times}\hat{U} = \hat{I}. \quad (3.4.24)$$

As we shall see in Sections 3.4.8 and 3.4.9, the above entire mathematical structure can be achieved in a truly elementary way via nonunitary transforms of quantum formalisms. Their isotopic reformulations then proves the invariance of hadronic mechanics, namely, its capability of predicting the same numbers for the same conditions at different times.

Under the above outlined structure we have the following main features:

I) Hadronic mechanics is a covering of quantum mechanics, because the latter theory is admitted uniquely and unambiguously at the limit when the isounit recovers the conventional unit, $\hat{I} \rightarrow I$;

II) Said covering is further characterized by the fact that hadronic mechanics coincides with quantum mechanics everywhere except for (as we shall see, generally small) non-Hamiltonian corrections at short mutual distances of particles caused by deep mutual overlapping of the wavepackets and/or charge distributions of particles;

III) Said covering is finally characterized by the fact that the indicated non-Hamiltonian corrections are restricted to verify all abstract axioms of quantum mechanics, with consequential preservation of is basic laws for closed non-Hamiltonian systems as a whole, as we shall see shortly.

Note that *composite hadronic systems*, such as *hadrons, nuclei, isomolecules, etc.*, are represented via the tensorial product of the above structures. This can be best done via the identification first of the *total isounit, total isofields, total isohilbert spaces, etc.*,

$$\hat{I}_{\text{tot}} = \hat{I}_1 \times \hat{I}_2 \times \ldots, \; \hat{C}_{\text{tot}} = \hat{C}_1 \times \hat{C}_2 \times \ldots, \; \hat{\mathcal{H}}_{\text{tot}} = \hat{\mathcal{H}}_1 \times \hat{\mathcal{H}}_2 \times \ldots . \quad (3.4.25)$$

Note also that some of the units, fields and Hilbert spaces in the above tensorial products can be *conventional*, namely, the composite structure may imply *local-potential long range interactions* (e.g., those of Coulomb type), which require the necessary treatment via *conventional* quantum mechanics, and *nonlocal-nonpotential short range interactions* (e.g., those in deep wave-overlappings), which require the use of operator isomechanics.

3.4.5 Dynamical Equations of Operator Isomechanics and their Isoduals

The formulations of the preceding sections permit the identification of the following fundamental dynamical equations of the Lie-isotopic branch of hadronic mechanics, known under the name of *iso-Heisenberg equations* or *Heisenberg-Santilli isoequetions*that were identified in the original proposal of 1978 to build hadronic mechanics [5], are can be presented in their finite and infinitesimal forms,

$$\hat{A}(\hat{t}) = \hat{U} \hat{\times} \hat{A}(\hat{0}) \hat{\times} \hat{U}^{\hat{\dagger}} = \{\hat{e}^{\hat{i} \hat{\times} \hat{H} \hat{\times} \hat{t}}\} \hat{\times} \hat{A}(\hat{0}) \hat{\times} \{\hat{e}^{-\hat{i} \hat{\times} \hat{t} \hat{\times} \hat{H}}\}, \quad (3.4.26a)$$

$$\hat{i} \hat{\times} \hat{d} \hat{A} \hat{/} \hat{d} \hat{t} = [\hat{A} \hat{,} \hat{H}] = A \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{A} = \hat{A} \times \hat{T} \times \hat{H} - \hat{H} \times \hat{T} \times \hat{A}, \quad (3.4.26b)$$

with the corresponding *fundamental hadronic isocommutation rules*

$$[\hat{b}^\mu \hat{,} \hat{b}^\nu] = \hat{i} \hat{\times} \hat{\omega}^{\mu\nu} = i \times \omega^{\mu\nu} \times \hat{I}_6, \quad \hat{b} = (\hat{r}^k, \hat{p}_k), \quad (3.4.27)$$

with corresponding *iso-Schrödinger equations* for the energy, also known as *Schrödinger-Santilli isoequations* identified by Myung and Santilli [45] and Mignani [48] in 1982 over conventional fields and first formulated in an invariant way by Santilli in monograph [7] of 1995

$$\hat{i}\hat{\times}\hat{\partial}_{\hat{t}}|\hat{\psi}\rangle = \hat{H}\hat{\times}|\hat{\psi}\rangle = \hat{H}\times\hat{T}\times|\hat{\psi}\rangle = \hat{E}\hat{\times}|\hat{\psi}\rangle = E\times|\hat{\psi}\rangle, \tag{3.4.28a}$$

$$|\hat{\psi}(\hat{t})\rangle = \hat{U}\hat{\times}|\hat{\psi}(\hat{0})\rangle = \{\hat{e}^{i\hat{H}\hat{\times}\hat{t}}\}\hat{\times}|\hat{\psi}(\hat{0})\rangle, \tag{3.4.28b}$$

and *isolinear momentum* first identified by Santilli in Ref. [7] of 1995 thanks to the discovery of the isodifferential calculus

$$\hat{p}_k\hat{\times}|\hat{\psi}\rangle = \hat{p}_k\times\hat{T}\times|\hat{\psi}\rangle - \hat{i}\hat{\times}\hat{\partial}_k|\hat{\psi}\rangle = -i\times\hat{I}^i_k\times\partial_i|\hat{\psi}\rangle, \tag{3.4.29}$$

It is evident that the iso-Heisenberg equations in their infinitesimal and exponentiated forms are a realization of the Lie-Santilli isotheory of Section 3.2, which is therefore the algebraic and group theoretical structure of the isotopic branch of hadronic mechanics.

Note that Eqs. (3.4.26) and (3.4.28) automatically bring into focus the general need for a *time isounit* and related characterization of the time isodifferential and isoderivative

$$\hat{I}_t(t, r, \psi, \ldots) = \hat{T}_t > 0, \tag{3.4.30a}$$

$$\hat{d}\hat{t} = \hat{I}_t\times d\hat{t}, \quad \hat{\partial}_{\hat{t}} = \hat{I}_t\times\partial_t. \tag{3.4.30b}$$

Note also that $\omega^{\mu\nu}$ in Eqs. (3.4.xxx) is the *conventional* Lie tensor, namely, the same tensor appearing in the conventional canonical commutation rules, thus confirming the axiom-preserving character of isomechanics.

The limited descriptive capabilities of quantum models should be kept in mind, purely Hamiltonian and, as such, they can only represent systems which are linear, local and potential. By comparison, we can write Eq. (3.4.28a) in its explicit form

$$\begin{gathered}\hat{i}\hat{\times}\hat{\partial}_{\hat{t}}\hat{\psi} = i\times\hat{I}_t\times\partial_{\hat{t}}|\hat{\psi}\rangle = \hat{H}\hat{\times}|\hat{\psi}\rangle = \hat{H}\times\hat{T}\times|\hat{\psi}\rangle = \\ = \{\hat{p}_k\times\hat{p}_k/\hat{2}\hat{\times}\hat{m} + \hat{U}_k(\hat{t},\hat{r})\hat{\times}\hat{v}^k + \\ +\hat{U}_0(\hat{t},\hat{r})\}\times\hat{T}(\hat{t},\hat{r},\hat{p},\hat{\psi},\nabla\psi,\ldots)\times|\hat{\psi}(\hat{t},\hat{r})\rangle = \\ = \hat{E}\hat{\times}|\hat{\psi}(t,\hat{x})\rangle = E\times|\hat{\psi}(\hat{t},\hat{x})\rangle,\end{gathered} \tag{3.4.31}$$

thus proving the following

THEOREM 3.4.1 [7]: Hadronic mechanics id "directly universal" for all infinitely possible, sufficiently smooth and regular, closed non-Hamiltonian systems, *namely, it can represent in the fixed coordinates of the experimenter all infinitely possible closed-isolated systems with linear and nonlinear, local and nonlocal, and potential as well as nonpotential internal forces verifying the conservation of the total energy.*

A consistent formulation of the isolinear momentum (3.4.29) escaped identification for two decades, thus delaying the completion of the construction of hadronic mechanics, as well as its practical applications. The consistent and invariant form (3.4.29) with consequential isocanonical commutation rules were first identified by Santilli in the second edition of Vol. II of this series, Ref. [7] of 1995 and memoir [10], following the discovery of the isodifferential calculus.

3.4.6 Preservation of Quantum Physical Laws

As one can see, the fundamental assumption of isoquantization is the lifting of the basic unit of quantum mechanics, Planck's constant $\hbar$, into a matrix $\hat{I}$ with nonlinear, integro-differential elements which also depend on the wavefunction and its derivatives

$$\hbar = I > 0 \rightarrow \hat{I} = \hat{I}(t, r, p, \psi, \hat{\psi}, \ldots) = \hat{I}^{\dagger} > 0. \tag{3.4.32}$$

It should be indicated that the above generalization is only *internal* in closed non-Hamiltonian because, when measured from the outside, *the isoexpectation values and isoeigenvalues of the isounit recover Planck's constant identically* [46],

$$\langle \hat{I} \rangle = \frac{\langle \hat{\psi} | \hat{\times} \hat{I} \hat{\times} | \hat{\psi} \rangle}{\langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle} = 1 = \hbar, \tag{3.4.33a}$$

$$\hat{I} \hat{\times} | \hat{\psi} \rangle = \hat{T}^{-1} \times \hat{T} \times | \hat{\psi} \rangle = 1 \times | \hat{\psi} \rangle = | \hat{\psi} \rangle. \tag{3.4.33b}$$

Moreover, the isounit is the fundamental invariant of isomechanics, thus preserving all axioms of the conventional unit $I = \hbar$, e.g.,

$$\hat{I}^{\hat{n}} = \hat{I} \hat{\times} \hat{I} \hat{\times} \ldots \hat{\times} \hat{I} \equiv \hat{I}, \tag{3.4.34a}$$

$$\hat{I}^{\frac{1}{2}} \equiv \hat{I}, \tag{3.4.34b}$$

$$\hat{i} \hat{\times} \hat{d} \hat{I} \hat{/} \hat{d} t = [\hat{I} \hat{,} \hat{H}] = \hat{I} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{I} \equiv 0. \tag{3.4.34c}$$

Despite their generalized structure, *Eqs. (3.4.26) and (3.4.28) preserve conventional quantum mechanical laws under nonlinear, nonlocal and nonpotential interactions* [7].

To begin an outline, the preservation of Heisenberg's uncertainties can be easily derived from isocommutation rules (3.4.27):

$$\Delta x^k \times \Delta p_k \geq \frac{1}{2} \times \langle [\hat{x}^k \hat{,} \hat{p}_k] \rangle = \frac{1}{2}. \tag{3.4.35}$$

To see the preservation of Pauli's exclusion principle, recall that the regular (two-dimensional) representation of $SU(2)$ is characterized by the conventional *Pauli matrices* σ_k with familiar commutation rules and eigenvalues on $\mathcal{H}$ over C,

$$[\sigma_i, \sigma_j] = \sigma_i \times \sigma_j - \sigma_j \times \sigma_i = 2 \times i\varepsilon_{ijk} \times \sigma_k, \tag{3.4.36a}$$

$$\sigma^2 \times |\psi\rangle = \sigma_k \times \sigma^k \times |\psi\rangle = 3 \times |\psi\rangle, \tag{3.4.36b}$$

$$\sigma_3 \times |\psi\rangle = \pm 1 \times |\psi\rangle. \tag{3.4.36c}$$

The isotopic branch of hadronic mechanics requires the construction of *nonunitary images of Pauli's matrices* first constructed in Ref. [49] that, for diagonal nonunitary transforms and isounits, can be written (see also Section 3.3.6)

$$\hat{\sigma}_k = U \times \sigma_k \times U^\dagger, \quad U \times U^\dagger = \hat{I} \neq I, \tag{3.4.37a}$$

$$\begin{gathered} U = \begin{pmatrix} i \times n_1 & 0 \\ 0 & i \times n_2 \end{pmatrix}, \quad U^\dagger = \begin{pmatrix} -i \times n_1 & 0 \\ 0 & -i \times n_2 \end{pmatrix}, \\ \hat{I} = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix}, \end{gathered} \tag{3.4.37b}$$

where the n's are well behaved nowhere null functions, resulting in the *regular Pauli-Santilli isomatrices* [49]

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix}, \ \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1^2 \\ i \times n_2^2 & 0 \end{pmatrix}, \ \hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}. \tag{3.4.38}$$

Another realization is given by *nondiagonal unitary transforms* [*loc. cit.*],

$$\begin{gathered} U = \begin{pmatrix} 0 & n_1 \\ n_2 & 0 \end{pmatrix}, \quad U^\dagger = \begin{pmatrix} 0 & n_2 \\ n_1 & 0 \end{pmatrix}, \\ \hat{I} = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix}, \end{gathered} \tag{3.4.39}$$

with corresponding *regular Pauli-Santilli isomatrices*,

$$\begin{gathered} \hat{\sigma}_1 = \begin{pmatrix} 0 & n_1 \times n_2 \\ n_1 \times n_2 & 0 \end{pmatrix}, \ \hat{\sigma}_2 = \begin{pmatrix} 0 & -i \times n_1 \times n_2 \\ i \times n_1 \times n_2 & 0 \end{pmatrix}, \\ \hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \end{gathered} \tag{3.4.40}$$

or by more general realizations with Hermitian nondiagonal isounits $\hat{I}$ [15].

All Pauli-Santilli isomatrices of the above regular class verify the following isocommutation rules and isoeigenvalue equations on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$

$$[\hat{\sigma}_i \hat{,} \hat{\sigma}_j] = \hat{\sigma}_i \times \hat{T} \times \hat{\sigma}_j - \hat{\sigma}_j \times \hat{T} \times \hat{\sigma}_i = 2 \times i \times \varepsilon_{ijk} \times \hat{\sigma}_k, \tag{3.4.41a}$$

$$\hat{\sigma}^{\hat{2}} \hat{\times} |\hat{\psi}\rangle = (\hat{\sigma}_1 \hat{\times} \hat{\sigma}_1 + \hat{\sigma}_2 \hat{\times} \hat{\sigma}_2 + \hat{\sigma}_3 \hat{\times} \hat{\sigma}_3) \hat{\times} |\hat{\psi}\rangle = 3 \times |\hat{\psi}\rangle, \tag{3.4.41b}$$

$$\hat{\sigma}_3 \hat{\times} |\hat{\psi}\rangle = \pm 1 \times |\hat{\psi}\rangle, \tag{3.4.41c}$$

thus preserving conventional spin 1/2, and establishing the preservation in isochemistry of the Fermi-Dirac statistics and Pauli's exclusion principle.

It should be indicated for completeness that the representation of the isotopic $S\hat{U}(2)$ also admit *irregular isorepresentations*, that no longer preserve conventional values of spin [49]. The latter structures are under study for the characterization of spin under the most extreme conditions, such as for protons and electrons in the core of collapsing stars and, as such, they have no known relevance for isomechanics.

The preservation of the superposition principle under nonlinear interactions occurs because of the reconstruction of linearity on isospace over isofields, thus regaining the applicability of the theory to composite systems.

Recall in this latter respect that conventionally nonlinear models,

$$H(t, x, p, \psi, \ldots) \times |\psi\rangle = E \times |\psi\rangle, \tag{3.4.42}$$

violate the superposition principle and have other shortcomings (see Section 1.5). As such, they cannot be applied to the study of composite systems such as molecules. All these models can be *identically* reformulated in terms of the isotopic techniques via the embedding of all nonlinear terms in the isotopic element,

$$H(t, x, p, \psi, \ldots) \times |\psi\rangle \equiv H_0(t, x, p) \times \hat{T}(\psi, \ldots) \times |\psi\rangle = E \times |\psi\rangle, \tag{3.4.43}$$

by regaining the full validity of the superposition principle in isospaces over isofields with consequential applicability to composite systems.

The preservation of causality follows from the one-dimensional isounitary group structure of the time evolution (3.4.28) (which is isomorphic to the conventional one); the preservation of probability laws follows from the preservation of the axioms of the unit and its invariant decomposition as indicated earlier; the preservation of other quantum laws then follows.

The same results can be also seen from the fact that operator isomechanics coincides at the abstract level with quantum mechanics by conception and construction. As a result, hadronic and quantum versions are *different realizations of the same abstract axioms and physical laws.*

Note that the preservation of conventional quantum laws under nonlinear, nonlocal and nonpotential interactions is crucially dependent on the capability of isomathematics to reconstruct linearity, locality and canonicity-unitarity on isospaces over isofields.

The preservation of conventional physical laws by the isotopic branch of hadronic mechanics was first identified by Santilli in report [47]. It should be indicated that the same quantum laws *are not* generally preserved by the broader genomechanics, evidently because the latter must represent by assumption *non*-conservation laws and other *departures* from conventional quantum settings.

With the understanding that the theory does not receive the classical determinism, it is evident that isomechanics provides a variety of "completions" of quantum mechanics according to the celebrated E-P-R argument [50], such as:

1) Isomechanics "completes" quantum mechanics via the addition of nonpotential-nonhamiltonian interactions represented by nonunitary transforms.

2) Isomechanics "completes" quantum mechanics via the broadest possible (non-oriented) realization of the associative product into the isoassociative form.

3) Isomechanics "completes" quantum mechanics in its classical image.

In fact, as proved by well known procedures based on *Bell's inequalities*, quantum mechanics does not admit direct classical images on a number of counts. On the contrary, as studied in details in Refs. [51], the nonunitary images of Bell's inequalities permit indeed direct and meaningful classical limits which do not exist for the conventional formulations.

Similarly, it is evident that isomechanics constitutes a specific and concrete realization of "hidden variables" [52] λ which are explicitly realized by the isotopic element, $\lambda = \hat{T}$, and actually turned into an operator hidden variables. The "hidden" character of the realization is expressed by the fact that hidden variables are embedded in the unit and product of the theory.

In fact, we can write the iso-Schrödinger equation $\hat{H}\hat{\times}|\hat{\psi}\rangle = \hat{H} \times \lambda \times |\hat{\psi}\rangle = E \times |\hat{\psi}\rangle$, $\lambda = \hat{T}$. As a result, the "variable" λ (now generalized into the operator $\hat{T}$) is "hidden" in the modular associative product of the Hamiltonian $\hat{H}$ and the state $|\hat{\psi}\rangle$.

Alternatively, we can say that hadronic mechanics provides an explicit and concrete realization of hidden variables because all distinctions between $\hat{H}\hat{\times}|\hat{\psi}\rangle$ and $H \times |\psi\rangle$ cease to exist at the abstract realization-free level.

For studies on the above and related issues, we refer the interested reader to Refs. [51] and quoted literature.

3.4.7 Isoperturbation Theory and its Isodual

We are now sufficiently equipped to illustrate the computational advantages in the use of isotopies.

THEOREM 3.4.2 [7]: Under sufficient continuity conditions, all perturbative and other series that are conventionally divergent (weakly convergent) can be turned into convergent (strongly convergent) forms via the use of isotopies with sufficiently small isotopic element (sufficiently large isounit),

$$|\hat{T}| \ll 1, \quad |\hat{I}| \gg 1. \tag{3.4.44}$$

The emerging perturbation theory was first studied by Jannussis and Mignani [53], and then studied in more detail in monograph [7] under the name of *isoperturbation theory*.

Consider a Hermitian operator on $\mathcal{H}$ over C of the type

$$H(k) = H_0 + k \times V, \quad H_0 \times |\psi\rangle = E_0 \times |\psi\rangle, \tag{3.4.45a}$$

$$H(k) \times |\psi(k)\rangle = E(k) \times |\psi(k)\rangle, \quad k \gg 1. \tag{3.4.45b}$$

Assume that H_0 has a nondegenerate discrete spectrum. Then, conventional perturbative series are *divergent*, as well known. In fact, the eigenvalue $E(k)$ of $H(k)$ up to second order is given by

$$\begin{gathered} E(k) = E_0 + k \times E_1 + k^2 \times E_2 = \\ = E_0 + k \times \langle\psi| \times V \times |\psi\rangle + k^2 \times \sum_{p \neq n} \frac{|\langle\psi_p| \times V \times |\psi_n\rangle|^2}{E_{0n} - E_{0p}}. \end{gathered} \tag{3.4.46}$$

But under isotopies we have

$$H(k) = H_0 + k \times V, \quad H_0 \times \hat{T} \times |\tilde{\psi}\rangle = \tilde{E}_0 \times |\tilde{\psi}\rangle, \quad \tilde{E}_0 \neq E_0, \tag{3.4.47a}$$

$$H(k) \times \hat{T} \times |\hat{\psi}(k)\rangle = \tilde{E}(k) \times |\hat{\psi}(k)\rangle, \quad \tilde{E} \neq E, \quad k > 1. \tag{3.4.47b}$$

A simple lifting of the conventional perturbation expansion then yields

$$\tilde{E}(k) = \tilde{E}_0 + k \times \tilde{E}_1 + k^2 \times \tilde{E}_2 + \hat{O}(k^2) =$$

$$= \tilde{E}_0 + k \times \langle\tilde{\psi}| \times \hat{T} \times V \times \hat{T} \times |\tilde{\psi}\rangle + \tag{3.4.48a}$$

$$+k^2 \times \sum_{p \neq n} \frac{|\langle\hat{\psi}_p| \times \hat{T} \times V \times \hat{T} \times |\hat{\psi}_n\rangle|^{\hat{2}}}{\tilde{E}_{0n} - \tilde{E}_{0p}}, \tag{3.4.48b}$$

whose convergence can be evidently reached via a suitable selection of the isotopic element, e.g., such that $|\hat{T}| \ll k$.

As an example, for a positive-definite constant $\hat{T} \ll k^{-1}$, expression (3.4.46) becomes

$$\begin{gathered} \tilde{E}(k) = \tilde{E}_0 + k \times \hat{T}^2 \times \langle\hat{\psi}| \times V \times |\psi *\rangle + k^2 \times T^5 \times \\ \times \sum_{p \neq n} \frac{|\langle\psi_p| \times V \times |\psi_n\rangle|^2}{\tilde{E}_{0n} - \tilde{E}_{0p}}. \end{gathered} \tag{3.4.49}$$

This shows that the original divergent coefficients $1, k, k^2, \ldots$ are now turned into the manifestly convergent coefficients $1, k \times T^2, k^2 \times T^5, \ldots$, with $k > 1$ and $\hat{T} \ll 1/k$, thus ensuring isoconvergence for a suitable selection of $\hat{T}$ for each given k and V.

A more effective reconstruction of convergence can be seen in the algebraic approach. At this introductory stage, we consider a divergent canonical series,

$$A(k) = A(0) + k \times [A, H]/1! + +k^2 \times [[A, H], H]/2! + \ldots \to \infty, \quad k > 1, \quad (3.4.50)$$

where $[A, H] = A \times H - H \times A$ is the familiar Lie product, and the operators A and H are Hermitian and sufficiently bounded. Then, under the isotopic lifting the preceding series becomes [7]

$$\hat{A}(k) = \hat{A}(0) + k \times [A\hat{,}H]/1! + k^2 \times [[A\hat{,}H]\hat{,}H]/2! + \cdots \leq |N| < \infty, \quad (3.4.51a)$$

$$[A\hat{,}H] = A \times \hat{T} \times H - H \times \hat{T} \times A, \quad (3.4.51b)$$

which holds, e.g., for the case $T = \varepsilon \times k^{-1}$, where ε is a sufficiently small positive-definite constant.

In summary, the studies on the construction of hadronic mechanics have indicated that the apparent origin of divergences (or slow convergence) in quantum mechanics and chemistry is their lack of representation of nonlinear, nonlocal, and nonpotential effects because when the latter are represented via the isounit, full convergence (much faster convergence) can be obtained.

As we shall see, all known applications of hadronic mechanics verify the crucial condition $|\hat{I}| \gg 1$, $|\hat{T}| \ll 1$, by permitting convergence of perturbative series. For instance, in the case of chemical bonds, hadronic chemistry allows computations at least one thousand times faster than those of quantum chemistry, with evident advantages, e.g., a drastic reduction of computer time (see Chapter 9). Essentially the same results are expected for hadronic mechanics and hadronic superconductivity.

The reader should meditate a moment on the evident possibility that *hadronic mechanics offers realistic possibilities of constructing a convergent perturbative theory for strong interactions.* As a matter of fact, the divergencies that have afflicted strong interactions through the 20-th century originates precisely from the excessive approximation of hadrons as points, with the consequential sole potential interactions and related divergencies.

In fact, whenever hadrons are represented as they actually are in reality, extended and hyperdense particles, with consequential potential as well as nonpotential interactions, all divergencies are removed by the isounit.

3.4.8 Simple Construction of Operator Isomechanics and its Isodual

Despite their *mathematical equivalence*, it should be indicated that quantum and hadronic mechanics are *physically inequivalent*, or, alternatively, hadronic mechanics is outside the classes of equivalence of quantum mechanics because the former is a *nonunitary image* of the latter.

As we shall see in the next chapters, the above property provides means for the explicit construction of the new model of isomechanics bonds from the conventional model. The main requirement is that of identifying the *nonhamiltonian* effects one desires to represent, which as such, are necessarily *nonunitary*. The resulting nonunitary transform is then assumed as the fundamental space isounit of the new isomechanics [46]

$$U \times U^\dagger = \hat{I} \neq I, \tag{3.4.52}$$

under which transform we have the liftings of: the quantum unit into the isounit,

$$I \to \hat{I} = U \times I \times U^\dagger; \tag{3.4.53}$$

numbers into isonumbers,

$$a \to \hat{a} = U \times a \times U^\dagger = a \times (U \times U^\dagger) = a \times \hat{I}; \quad a = n, c; \tag{3.4.54}$$

associative products $A \times B$ into the isoassociative form with the correct isotopic element,

$$A \times B \to \hat{A} \hat{\times} \hat{B} = \hat{A} \times \hat{T} \times \hat{B}, \tag{3.4.55a}$$

$$\hat{A} = U \times A \times U^\dagger, \ \hat{B} = U \times B \times U^\dagger, \ \hat{T} = (U \times U^\dagger)^{-1} = T^\dagger; \tag{3.4.55b}$$

Schrödinger's equation into the isoschrödinger's equations

$$\begin{gathered} H \times |\psi\rangle = E \times |\psi\rangle \to U(H \times |\psi\rangle) = \\ = (U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi\rangle) = \\ = \hat{H} \times \hat{T} \times |\hat{\psi}\rangle = \hat{H} \hat{\times} |\hat{\psi}\rangle; \end{gathered} \tag{3.4.56}$$

Heisenberg's equations into their isoheisenberg generalization

$$\begin{gathered} i \times dA/dt - A \times H - H \times A = 0 \to \\ \to U \times (i \times dA/dt) \times U^\dagger - U(A \times H - H \times A) \times U^\dagger = \\ = \hat{i} \hat{\times} d\hat{A}/dt - \hat{A} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{A} = 0; \end{gathered} \tag{3.4.57}$$

the Hilbert product into its isoinner form

$$\begin{gathered} \langle\psi| \times |\psi\rangle \to U \times \langle\psi| \times |\psi\rangle \times U^\dagger = \\ = (\langle\psi| \times U^\dagger) \times (U \times U)^{-1} \times (U \times |\psi\rangle) \times (U \times U)^{-1} = \langle\hat{\psi}| \hat{\times} |\hat{\psi}\rangle \times \hat{I}; \end{gathered} \tag{3.4.58}$$

canonical power series expansions into their isotopic form

$$\begin{gathered} A(k) = A(0) + k \times [A, H] + k^2 \times [[A, H,], H] + \ldots \to U \times A(k) \times U^\dagger = \\ = U \times \Big[A(0) + k \times [A, H] + k^2 \times [[A, H], H] + \ldots\Big] \times U^\dagger = \\ = \hat{A}(\hat{k}) = \hat{A}(0) + \hat{k} \hat{\times} [\hat{A} \hat{,} \hat{H}] + \hat{k}^{\hat{2}} \hat{\times} [[\hat{A} \hat{,} \hat{H}] \hat{,} \hat{H}] + \ldots, \\ k > 1, \quad |\hat{T}| \ll 1; \end{gathered} \tag{3.4.59}$$

Schrödinger's perturbation expansion into its isotopic covering (where the usual summation over states $p \neq n$ is assumed)

$$\begin{gathered} E(k) = E(0) + k \times \langle\psi| \times V \times |\psi\rangle + k^2 \frac{|\langle\psi| \times V \times |\psi\rangle|^2}{E_{0n} - E_{0p}} + \ldots \to \\ \to U \times E(k) \times U^\dagger = U \times \Big[E(0) + k \times \langle\psi| \times V \times |\psi\rangle + \ldots\Big] \times U^\dagger = \\ = \hat{E}(\hat{k}) = \hat{E}(0) + \hat{k} \hat{\times} \langle\hat{\psi}| \times \hat{T} \times \hat{V} \times \hat{T} \times |\hat{\psi}\rangle + \ldots, \\ k > 1, \quad |\hat{T}| \ll 1; \end{gathered} \tag{3.4.60}$$

etc. All remaining aspects of operator isomechanics can then be derived accordingly, including the isoexponent, isologarithm, isodeterminant, isotrace, isospecial functions and transforms, *etc.* The isodual isomechanics can then be constructed via the now famil,kiar isodual map.

Note that the above construction via a nonunitary transform is the correct operator image of the derivability of the classical isohamiltonian mechanics from the conventional form via noncanonical transforms (Section 3.2.12).

The construction of hadronic mechanics via nonunitary transforms of quantum mechanics was first identified by Santilli in the original proposal [5e], and then worked out in subsequent contributions (see [12] for the latest presentation).

3.4.9 Invariance of Operator Isomechanics and of its Isodual

It is important to see that, in a way fully parallel to the classical case (Section 3.3.7), operator isomechanics is indeed invariant under the most general possible nonlinear, nonlocal and nonhamiltonian-nonunitary transforms, provided that, again, the invariance is treated via the isomathematics. In fact, any given nonunitary transform $U \times U^\dagger \neq I$ can always be decomposed into the form [12]

$$U = \hat{U} \times \hat{T}^{1/2},$$

under which nonunitary transforms on $\mathcal{H}$ over C are identically reformulated as isounitary transforms on the isohilbert space $\hat{\mathcal{H}}$ over the isofield $\hat{}$,

$$U \times U^\dagger \equiv \hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}. \tag{3.4.61}$$

The form-invariance of operator isomechanics under isounitary transforms then follows,

$$\hat{I} \to \hat{I}' = \hat{U} \hat{\times} \hat{I} \hat{\times} \hat{U}^\dagger \equiv \hat{I}, \ \hat{A} \hat{\times} \hat{B} \to \hat{U} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{U}^\dagger = \hat{A}' \hat{\times} \hat{B}', \ \textit{etc.}, \tag{3.4.62a}$$

$$\begin{gathered} \hat{H} \hat{\times} |\hat{\psi}\rangle = \hat{E} \hat{\times} |\hat{\psi}\rangle \to \hat{U} \times \hat{H} \hat{\times} |\hat{\psi}\rangle = \\ = (\hat{U} \times \hat{H} \times \hat{U}^\dagger) \hat{\times} (\hat{U} \hat{\times} |\hat{\psi}\rangle) = \hat{H}' \hat{\times} |\hat{\psi}'\rangle = \\ = \hat{U} \hat{\times} \hat{E} \hat{\times} |\hat{\psi}\rangle = \hat{E} \hat{\times} \hat{U} \hat{\times} |\hat{\psi}\rangle = \hat{E} \hat{\times} |\hat{\psi}'\rangle, \end{gathered} \tag{3.4.62b}$$

where one should note the preservation of the *numerical values* of the isounit, isoproducts and isoeigenvalues, as necessary for consistent applications. The invariance of isodual isomechanics then follows rather trivially.

Note that the invariance in quantum mechanics holds only for transformations $U \times U^\dagger = I$ *with fixed* I. Similarly, the invariance of isomechanics holds only for all nonunitary transforms such $\hat{U}\hat{\times}\hat{U}^\dagger = \hat{I}$ *with fixed* $\hat{I}$, and *not* for a transform $\hat{W}\hat{\times}\hat{W}^\dagger = \hat{I}' \neq \hat{I}$ because the change of the isounit $\hat{I}$ implies the transition to a *different physical system.*

The form-invariance of hadronic mechanics under isounitary transforms was first studied by Santilli in memoir [46].

3.5 SANTILLI ISORELATIVITY AND ITS ISODUAL

3.5.1 Limitations of Special and General Relativities

Special and general relativities are generally presented in contemporary academia as providing final descriptions of all infinitely possible conditions existing in the universe.

The scientific reality is basically different than the above academic posture. In Section 1.1 and Chapter 2, we have shown that special and general relativities *cannot* provide a consistent classical description of antiparticles because they admit no distinction between neutral matter and antimatter and, when used for charged antiparticles, they lead to inconsistent quantum images consisting of particles (rather than charge conjugated antiparticles) with the wrong sign of the charge. Hence, *the entire antimatter content of the universe cannot be credibly treated via special and/or general relativity.*[24]

A widespread academic posture, studiously conceived for adapting nature to preferred doctrines, is the belief that the university can be effectively reduced to point-particles solely under action-at-a-distance, potential interactions. This posture is dictated by the facts that: the mathematics underlying special and general relativities, beginning with their local-differential topology, can only represent (dimensionless) point-like particles; special and genera;l relativity are notoriously incompatible with the deformation theory (that is activated whenever extended particles are admitted); and said relativities are strictly Lagrangian or Hamiltonian, thus being only able to represent potential interactions.

However, in Section 1.3 and in this chapter, we have established the "No Reduction Theorems," according to which a macroscopic extended system in noncon-

[24]Particularly political is the academic posture that "antigravity does not exist because not predicted by Einstein's gravitation," when such a gravitational theory has no means for a credible representation of antimatter. As we shall see in Chapter 14, Volume II, when a credible quantitative representation of antimatter is included, antigravity (defined as gravitational repulsion) between matter and antimatter is unavoidable.

servative conditions (such as a satellite during re-entry in our atmosphere) cannot be consistently reduced to a finite number of point-particles all under potential forces and, vice versa, a finite number of quantum (that is, point-like) particles all under potential interactions cannot consistently recover a macroscopic nonconservative system. Hence, *all macroscopic systems under nonconservative forces, thus including all classical interior problems, cannot be consistently treated with special or general relativity.*[25]

Another posture in academia, also intended for adapting nature to a preferred doctrine, is that irreversibility is a macroscopic event that "disappears" (sic) when systems are reduced to their elementary constituents. This widespread academic belief is necessary because special and general relativities are *structurally reversible,* namely, their mathematical and physical axioms, as well as all known Hamiltonians are invariant under time reversal. This posture is complemented with manipulations of scientific evidence, such as the presentation of the probability of the synthesis of two nuclei into a third one, $n_1 + n_2 \rightarrow n_3$ while studiously suppressing the time reversal event that is simply unavoidable for a reversible theory, namely, the finite probability of the *spontaneous* decomposition $n_3 \rightarrow n_1 n_2$ following the synthesis. The latter probability is suppressed evidently because it would prove the inconsistency of the assumed basic doctrine. [26]

Unfortunately for mankind, the above academic postures are also used for all energy releasing processes despite the fact that they are irreversible. The vast majority of the research on energies releasing processes such as the "cold" and "hot" fusions, and the use of the vast majority of public fund,s are restricted to verify quantum mechanics and special relativity under the knowledge by experts that reversible theories cannot be exactly valid for irreversible processes/ In any case, the "No reduction theorems" prevent the consistent reduction of an

[25]Another political posture in academia is the treatment of the entire universe, thus including interior problems of stars, quasars and black-hole, with Einstein gravitation when it is well known that such a doctrine is purely "external," namely, can only represent point-like masses moving in vacuum in the gravitational field of a massive body. One can then understand the political backing needed for the credibility, e.g., of studies on black holes derived via a purely exterior theory.

[26]Serious physicists should not even redo the calculations for the probability of the spontaneous decay following the synthesis, because it is unavoidable under the assumption of the same Hilbert space for all initial and final nuclei and Heisenberg's uncertainty principle. In fact these assumptions imply that the nucleus n_1 or n_2 has a finite [probability of being outside of n_3 due to the coherence of the interior and interior Hilbert spaces. At this point, numerous additional manipulations of science are attempted to salvage preferred doctrines when inapplicable, rather than admitting their inapplicability and seeking covering theories. One of these manipulations is based on the "argument" that n_3 is extended, when extended sizes cannot be represented by quantum mechanics. Other manipulations are not worth reporting here. *The only scientific case of a rigorously proved, identically null probability of spontaneous disintegrations of a stable nucleus following its synthesis occurs when the initial and final Hilbert spaces are incoherent. This mandates the use of the conventional Hilbert space (quantum mechanics) for the initial states and the use of an incoherent iso-Hilbert space (hadronic mechanics) for the final state. This is the only possibility known to this author following half a century of studies of the problem.*

irreversible macroscopic event to an ideal ensemble of point-like abstractions of particles all in reversible conditions. Hence, *special and general relativities are inapplicable for any and all irreversible processes existing in the universe.*[27]

When restricting the arena applicability to those of the original conception (propagation of point particles and electromagnetic waves in vacuum), special relativity remains afflicted by still unresolved basic problems, such as the possibility that the relativity verifying one-way experiments on the propagation of light could be Galilean, rather than Lorentzian; the known incompatibility of special relativity with space conceived as a universal medium; and other unsettled aspects. Independently from that, we have shown in Section 1.4 that general relativity has no case of unequivocal applicability for numerous reasons, such as: curvature cannot possibly represent the free fall of a body along a straight radial line; the "bending of light" is due to Newtonian gravitation (and if curvature is assumed one gets double the bending experimentally measured); gravitation is a noncanonical theory, thus suffering of the Theorems of Catastrophic Inconsistencies of Section 1.5; etc.

In summary, on serious scientific grounds, and contrary to vastly popular political beliefs, special and general relativities have no uncontested arena of exact valid.

Far from pretending final knowledge, in this section we primarily claim the scientific honesty to have identified the above open problems and initiated quantitative studies for their resolution. Our position in regard to special relativity is pragmatic, in the sense that, under the conditions limpidly identified by Einstein, such as particles in accelerators, etc., special relativity works well. Additionally, special relativity has a majestic axiomatic structure emphasized various times by the author.

Hence, we shall assume special relativity at the foundation of this section and seek its *isotopic liftings,* namely, the most general possible formulations verifying at the abstract level the original axioms conceived by Lorentz, Poincaré, Einstein, Minkowski, Weyl and other founders. The first, and perhaps basic understanding of this section is the knowledge that *special relativity and isorelativity coincide at the abstract, realization-free level,* to such an extent that we could use the same formulae and identify the special or isotopic relativity via different meanings of the same symbols. Alternatively, to honor the memory of the founders, it is necessary to identify the widest possible applicability of their axioms before abandoning them for broader vistas.

[27]To honor the memory of Albert Einstein and other founders of our knowledge, it should be stressed that the use of the word "violation" would be nonscientific, since quantum mechanics and special relativity were not conceived for irreversible processes. Said disciplines have been applied to irreversible processes by Einstein's *followers* seeking money, prestige and power via the abuse of Einstein's name.

An additional, century-old, unresolved issue is the incompatibility of special relativity with the absolute reference frame at rest with the universal substratum (also called *ether*) that appears to be needed for the very characterization of all visible events in the Universe [54,55[. This latter aspect is fundamental for the studies of Volume II and are treated there to avoid unnecessary repetitions.

In regard to general relativity, our position is rather rigid: no research on general relativity can be considered scientifically serious unless the nine theorems s of catastrophic inconsistencies of Ref. [75] are disproved, not in academic corridors, but in refereed technical publications. Since this task appears to be hopeless, we assume the position that general relativity is catastrophically inconsistent and seek an alternative formulation.

As we shall see, *when the memory of the founders is honored in the above sense, the broadest possible realization of their axioms include gravitation and there is no need for general relativity as a separate theory.* Thus, another basic understanding of this section is the knowledge that we shall seek a *unification of special and general relativity into one single formulation based on the axioms of special relativity, known as Santilli isorelativity.* Needless to say, such a unification required several decades of research since it required the construction of the needed new mathematics, the achievement of the unification of the Minkowskian and Riemannian geometries, and the achievement of a universal invariance for all possible spacetime line elements prior to addressing the unification itself.

A further aspect important for the understanding of this section is that, *by no means isorelativity should be believed to be the final relativity of the universe because it is structurally reversible due to the Hermiticity of the isounit and isotopic element.*[28]

This creates the need for a yet broader relativity studied in the next chapte, and known under the name of *Santilli genorelativity,* this time, based on *genotopic liftings* of special relativity or isorelativity, namely, broadening requiring a necessary departure from the abstract axioms of special relativity into a form that is *structurally irreversible,* in the sense of possessing mathematical and physical axioms that are irreversible under all possible reversible Lagrangians or Hamiltonians.

The resolution of the above indicated problems for antimatter is achieved by the isodual image of the studies of this section.

[28]As we shall see in the next chapter, despite its Hermiticity, the isounit can depend on time in such a way that $\hat{I}(t,...) = \hat{I}^\dagger(t,...) \neq \hat{I}(-t,...)$. In this case isotopic theories represent systems verifying total conservation laws when isolated (because of the antisymmetry of the Lie-Santilli isobrackets), yet being structurally irreversible in their interior processes, as it is the case for all interior problems when considered isolated from the rest of the universe.

3.5.2 Minkowski-Santilli Isospaces and their Isoduals

As studied in Section 1.2, the "universal constancy of the speed of light" is a philosophical abstraction, particularly when proffered by experts without the additional crucial words "in vacuum", because the constancy of the speed of light has been solely proved in vacuum while, in general, experimental evidence establishes that *the speed of light is a local variable depending on the characteristics of the medium in which it propagates,* with well known expression

$$c = c_\circ / n, \tag{3.5.1}$$

where the familiar *index of refraction* n is a function of a variety of time t, coordinates r, density μ, temperature τ, frequency ω, etc., $n = n(t, r, \mu, \tau, \omega, \ldots)$.

In particular, the speed of light is generally smaller than that in vacuum when propagating within media of low density, such as atmospheres or liquids,

$$c \ll c_\circ, \quad n \gg 1, \tag{3.5.2}$$

while the speed of light is generally bigger than that in vacuum when propagating within special guides, or within media of very high density, such as the interior of stars and quasars,

$$c \gg c_\circ, \quad n \ll 1. \tag{3.5.3}$$

Academic claims of recovering the speed of light in water via photons scattering among the water molecules are afflicted by numerous inconsistencies studied in Section 1.2, and the same holds for other aspects.

Assuming that via some unknown manipulation special relativity is shown to represent consistently the propagation of light within physical media, such a representation would activate the catastrophic inconsistencies of Theorem 1.5.1.

This is due to the fact that *the transition from the speed of light in vacuum to that within physical media requires a noncanonical or nonunitary transform.*

This point can be best illustrated by using the metric originally proposed by Minkowski, which can be written

$$\eta = Diag.(1, 1, 1, -c_\circ^2). \tag{3.5.4}$$

Then, the transition from $c_\circ$ to $c = c_\circ/n$ in the metric can only be achieved via a noncanonical or nonunitary transform

$$\eta = Diag.(1, 1, 1, -c_\circ^2) \rightarrow \hat{\eta} =$$

$$= Diag.(1, 1, 1, -c_\circ/n^2) = U \times \eta \times U^\dagger, \tag{3.5.5a}$$

$$U \times U^\dagger = Diag.(1,\ 1,\ 1,\ 1/n^2) \neq I. \tag{3.5.5b}$$

An invariant resolution of the limitations of special relativity for closed and reversible systems of extended and deformable particles under Hamiltonian and non-Hamiltonian interactions has been provided by the lifting of special relativity into a new formulation today known as *Santilli isorelativity,* where: the prefix "iso" stands to indicate that relativity principles apply on isospacetime over isofields; and the characterization of "special" or "general" is inapplicable because, as shown below, *isorelativity achieves a geometric unification of special and general relativities.*

Isorelativity was first proposed by R. M. Santilli in Ref. [58] of 1983 via the first invariant formulation of *iso-Minkowskian spaces* and related *iso-Lorentz symmetry.* The studies were then continued in: Ref. [59] of 1985 with the first isotopies of the rotational symmetry; Ref. [49] of 1993 with the first isotopies of the SU(2)-spin symmetry; Ref. [60] of 1993 with the first isotopies of the Poincaré symmetry; Ref. [51] of 1998 with the first isotopies of the SU(2)-isospin symmetries, Bell's inequalities and local realism; and Refs. [61,62] on the first isotopies of the spinorial covering of the Poincaré symmetry.

The studies were then completed with memoir [26] of 1998 presenting a comprehensive formulation of the iso-Minkowskian geometry and its capability to unify the Minkowskian and Riemannian geometries, including its formulation via the mathematics of the Riemannian geometry (such iso-Christoffel's symbols, isocovariant derivatives, etc.). The author then dedicated various monographs to the field through the years.

Numerous independent studies on Santilli isorelativity are available in the literature, one can inspect in this respect Refs. [32–43] and papers quoted therein; Aringazin's proof [63] of the direct universality of the Lorentz-Poincaré-Santilli isosymmetry for all infinitely possible spacetimes with signature $(+, +, +, -)$; Mignani's exact representation [64] of the large difference in cosmological redshifts between quasars and galaxies when physically connected; the exact representation of the anomalous behavior of the meanlives of unstable particles with speed by Cardone et al. [65–66]; the exact representation of the experimental data on the Bose-Einstein correlation by Santilli [67] and Cardone and Mignani [68]; the invariant and exact validity of the iso-Minkowskian geometry within the hyperdense medium in the interior of hadrons by Arestov et al. [69]; the first known exact representation of molecular features by Santilli and Shillady [70,71]; and numerous other contributions.

Evidently we cannot review isorelativity in the necessary details to avoid a prohibitive length. Nevertheless, to achieve minimal self-sufficiency of this presentation, it is important to outline at least its main structural lines (see monograph [55] for detailed studies).

The central notion of isorelativity is the lifting of the basic unit of the Minkowski space and of the Poincaré symmetry, $I = Diag.(1, 1, 1, 1)$, into a 4×4-dimensional, nowhere singular and positive-definite matrix $\hat{I} = \hat{I}_{4\times 4}$ with an unrestricted functional dependence on local spacetime coordinates x, speeds v, accelerations a, frequencies ω, wavefunctions ψ, their derivative $\partial\psi$, and/or any other needed variables,

$$I = Diag.(1,1,1) \rightarrow \hat{I}(x, v, a, \omega, \psi, \partial\psi, \ldots) =$$
$$= 1/\hat{T}(x, v, \omega, \psi, \partial\psi, \ldots) > 0. \quad (3.5.6)$$

Isorelativity can then be constructed via the method of Section 3.4.6, namely, by assuming that the basic noncanonical or nonunitary transform coincides with the above isounit

$$U \times U^\dagger = \hat{I} = Diag.(g_{11}, g_{22}, g_{33}, g_{44}),$$
$$g_{\mu\mu} = g_{\mu\mu}(x, v, \omega, \psi, \partial\psi, \ldots) > 0, \quad \mu = 1, 2, 3, 4, \quad (3.5.7)$$

and then subjecting the *totality* of quantities and their operation of special relativity to the above transform.

This construction is, however, selected here only for simplicity in pragmatic applications, since the rigorous approach is the construction of isorelativity from its abstract axioms, a task we have to leave to interested readers for brevity (see the original derivations [7]).

This is due to the fact that the former approach evidently preserves the original eigenvalue spectra and does not allow the identification of anomalous eigenvalues emerging from the second approach, such as those of the $SU(2)$ and $SU(3)$ isosymmetries [51].

Let $M(x, \eta, R)$ be the Minkowski space with local coordinates $x = (x^\mu)$, metric $\eta = Diag.(1,1,1,-1)$ and invariant

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \in R. \quad (3.5.8)$$

The fundamental space of isorelativity is the *Minkowski-Santilli isospace* [58] and related topology [10,22–25], $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ characterized by the liftings

$$I = Diag.(1,1,1,1) \rightarrow U \times I \times U^\dagger = \hat{I} = 1/\hat{T}, \quad (3.5.9a)$$

$$\eta = Diag.(1,1,1,-1) \times I \rightarrow (U^{\dagger -1} \times \eta \times U^{-1}) \times \hat{I} = \hat{\eta} =$$
$$= \hat{T} \times \eta = Diag.(g_{11}, g_{22}, g_{33}, -g_{44}) \times \hat{I}, \quad (3.5.9b)$$

with consequential isotopy of the basic invariant

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \in R \rightarrow$$
$$\rightarrow U \times x^2 \times U^\dagger = \hat{x}^{\hat{2}} = (\hat{x}^\mu \hat{\times} \hat{m}_{\mu\nu} \times x^\nu) \times I \in R, \quad (3.5.10)$$

whose projection in conventional spacetime can be written

$$\hat{x}^{\hat{2}} = [x^\mu \times \hat{\eta}_{\mu\nu}(x, v, a, \omega, \psi, \partial\psi, \ldots) \times x^\nu] \times \hat{I}. \quad (3.5.11)$$

The nontriviality of the above lifting is illustrated by the following:[29]

THEOREM 3.5.1: The Minkowski-Santilli isospaces are directly universal, in the sense of admitting as particular cases all possible spaces with the same signature $(+,+,+,-)$*, such as the Minkowskian, Riemannian, Finslerian and other spaces (universality), directly in terms of the isometric within fixed local variables (direct universality).*

Therefore, the correct formulation of the *Minkowski-Santilli isogeometry* requires the isotopy of all tools of the Riemannian geometry, such as the iso-Christoffel symbols, isocovariant derivative, etc. (see for brevity Ref. [15]).

Despite that, one should keep in mind that, in view of the positive-definiteness of the isounit [34,79], *the Minkowski-Santilli isogeometry coincides at the abstract level with the conventional Minkowski geometry, thus having a null isocurvature* (because of the basic mechanism of deforming the metric η by the amount $\hat{T}(x,\ldots)$ while deforming the basic unit of the inverse amount $\hat{I} = 1/\hat{T}$).

The geometric unification of the Minkowskian and Riemannian geometries achieved by the Minkowski-Santilli isogeometry constitutes the evident geometric foundation for the unification of special and general relativities studied below.

It should be also noted that, following the publication in 1983 of Ref. [58], numerous papers on "deformed Minkowski spaces" have appeared in the physical and mathematical literature (generally without a quotation of their origination in Ref. [58]).

These "deformations" are ignored in these studies because they are formulated via conventional mathematics and, consequently, they all suffer of the catastrophic inconsistencies of Theorem 1.5.1.

By comparison, isospaces are formulated via isomathematics and, therefore, they resolve the inconsistencies of Theorem 1.5.1, as shown in Section 3.5.9. This illustrates again the necessity of lifting the basic unit and related field jointly with all remaining conventional mathematical methods.

3.5.3 Poincaré-Santilli Isosymmetry and its Isodual

Let $P(3.1)$ be the conventional Poincaré symmetry with the well known ten generators $J_{\mu\nu}, P_\mu$ and related commutation rules hereon assumed to be known.

The second basic tool of isorelativity is the *Poincaré-Santilli isosymmetry* $\hat{P}(3.1)$ studied in detail in monograph [55] that can be constructed via the isothe-

[29] Fabio Cardone, Roberto Mignani and Alessio Marrani have uploaded a number of papers in the section hep-th of Cornell University arXiv copying *ad litteram* the results of paper [83], including the use of the same symbols, without any quotation at all of Santilli's preceding vast literature in the field. Educators, colleagues and editors of scientific journals are warned of the existence on ongoing legal proceedings one can inspect in the web site http://www.scientificethics.org/

ory of Section 3.2, resulting in the isocommutation rules [58,60]

$$[J_{\mu\nu}\hat{,}J_{\alpha\beta}] = i \times (\hat{\eta}_{\nu\alpha} \times J_{\beta\mu} - \hat{\eta}_{\mu\alpha} \times J_{\beta\nu} - \hat{\eta}_{\nu\beta} \times J_{\alpha\mu} + \hat{\eta}_{\mu\beta} \times J_{\alpha\nu}), \quad (3.5.12a)$$

$$[J_{\mu\nu}\hat{,}P_{\alpha}] = i \times (\hat{\eta}_{\mu\alpha} \times P_{\nu} - \hat{\eta}_{\nu\alpha} \times P_{\mu}), \quad (3.5.12b)$$

$$[P_{\mu}\hat{,}P_{\nu}] = 0, \quad (3.5.12c)$$

where we have followed the general rule of the Lie-Santilli isotheory according to which isotopies leave observables unchanged (since Hermiticity coincides with iso-Hermiticity) and merely change the *operations* among them.

The *iso-Casimir invariants* of $\hat{P}(3.1)$ are given by

$$P^{\hat{2}} = P_{\mu}\hat{\times}P^{\mu} = P^{\mu} \times \hat{\eta}_{\mu\nu} \times P^{\nu} = P_k \times g_{kk} \times P_k - p_4 \times g_{44} \times P_4, \quad (3.5.13a)$$

$$W^{\hat{2}} = W_{\mu}\hat{\times}W^{\mu}, \;\; W_{\mu} = \hat{\epsilon}_{\mu\alpha\beta\rho}\hat{\times}J^{\alpha\beta}\hat{\times}P^{\rho}, \quad (3.5.13b)$$

and they are at the foundation of classical and operator *isorelativistic kinematics.*

Since $\hat{I} > 0$, it is easy to prove that *the Poincaré-Santilli isosymmetry is isomorphic to the conventional symmetry.* It then follows that *the isotopies increase dramatically the arena of applicability of the Poincaré symmetry, from the sole Minkowskian spacetime to all infinitely possible spacetimes.*

Next, the reader should be aware that *the Poincaré-Santilli isosymmetry characterizes "isoparticles" (and not particles) via its irreducible isorepresentations.*

A mere inspection of the isounit shows that the Poincaré-Santilli isosymmetry characterizes actual nonspherical and deformable shapes as well as internal densities and the most general possible nonlinear, nonlocal and nonpotential interactions.

Since any interaction implies a renormalization of physical characteristics, it is evident that *the transition from particles to isoparticles, that is, from motion in vacuum to motion within physical media, causes an alteration (called isorenormalization), in general, of all intrinsic characteristics, such as rest energy, magnetic moment, charge, etc.*

As we shall see later on, the said isorenormalization has permitted the first exact numerical representation of nuclear magnetic moments, molecular binding energies and other data whose exact representation resulted to be impossible for nonrelativistic and relativistic quantum mechanics despite all possible corrections conducted over 75 years of attempts.

The explicit form of the *Poincaré-Santilli isotransforms* leaving invariant line element (3.5.11) can be easily constructed via the Lie-Santilli isotheory and are given:

(1) The **isorotations** [11]

$$\hat{O}(3) : \hat{\mathbf{x}}' = \hat{\Re}(\hat{\theta})\hat{\times}\hat{\mathbf{x}}, \;\; \hat{\theta} = \theta \times \hat{I}_{\theta} \in \hat{R}_{\theta}, \quad (3.5.14)$$

that, for isotransforms in the (1, 2)-isoplane, are given by

$$x^{1'} = x^1 \times \cos[\theta \times (g_{11} \times g_{22})^{1/2}] - x^2 \times g_{22} \times g_{11}^{-1} \times \sin[\theta \times (g_{11} \times g_{22})^{1/2}], \quad (3.5.15a)$$

$$x^{2'} = x^1 \times g_{11} \times g_{22}^{-1} \times \sin[\theta \times (g_{11} \times g_{22})^{1/2}] + x^2 \times \cos[\theta \times (g_{11} \times g_{22})^{1/2}]. \quad (3.5.15b)$$

For the general expression in three dimensions interested reader can inspect Ref. [7] for brevity.

Note that, since $\hat{O}(3)$ is isomorphic to $O(3)$, Ref. [59] proved, contrary to a popular belief throughout the 20-th century, that

LEMMA 3.5.1: The rotational symmetry remains exact for all possible signature-preserving $(+, +, +)$ *deformations of the sphere.*

The rotational symmetry was believed to be "broken" for ellipsoidal and other deformations of the sphere merely due to insufficient mathematics for the case considered because, when the appropriate mathematics is used, the rotational symmetry returns to be exact, and the same holds for virtually all "broken" symmetries.

The above reconstruction of the exact rotational symmetry can be geometrically visualized by the fact that *all possible signature-preserving deformations of the sphere are perfect spheres in isospace called isosphere.*

This is due to the fact that ellipsoidal deformations of the semiaxes of the perfect sphere are compensated on isospaces over isofields by the *inverse* deformation of the related unit

$$Radius \quad 1_k \rightarrow 1/n_k^2, \quad (3.5.16a)$$

$$Unit \quad 1_k \rightarrow n_k^2. \quad (3.5.16b)$$

We recover in this way the perfect sphere on isospaces over isofields

$$\hat{r}^{\hat{2}} = \hat{r}_1^2 + \hat{r}_2^2 + \hat{r}_3^2 \quad (3.5.17)$$

with exact $\hat{O}(3)$ symmetry, while its projection on the conventional Euclidean space is the ellipsoid

$$r_1^2/n_1^2 + r_2^2/n_2^2 + r_3^2/n_3^2, \quad (3.5.18)$$

with broken $O(3)$ symmetry.

(2) The **Lorentz-Santilli isotransforms** [26,29]

$$\hat{O}(3.1) : \hat{x}' = \hat{\Lambda}(\hat{v}, \ldots)\hat{\times}\hat{x}, \ \hat{v} = v \times \hat{I}_v \in \hat{R}_v, \quad (3.5.19)$$

that, for isotransforms in the (3,4)-isoplane, can be written

$$x^{1'} = x^1, \quad (3.5.20a)$$

$$x^{2'} = x^2, \tag{3.5.20b}$$

$$x^{3'} = x^3 \times \cosh[v \times (g_{33} \times g_{44})^{1/2}] -$$
$$-x^4 \times g_{44} \times (g_{33} \times g_{44})^{-1/2} \times \sinh[v \times (g_{33} \times g_{44})^{1/2}] =$$
$$= \hat{\gamma} \times (x^3 - \hat{\beta} \times \frac{g_{44}^{1/2}}{g_{33}^{1/2}} \times x^4), \tag{3.5.20c}$$

$$x^{4'} = -x^3 \times g_{33} \times (g_{33} \times g_{44})^{-1/2} \times \sinh[v(g_{33} \times g_{44})^{1/2}] +$$
$$+x^4 \times \cosh[v \times (g_{33} \times g_{44})^{1/2}] =$$
$$= \hat{\gamma} \times (x^4 - \hat{\beta} \times \frac{g_{33}^{1/2}}{g_{44}^{1/2}} \times x^3), \tag{3.5.20b}$$

where

$$\hat{\beta}^2 = \frac{v_k \times g_{kk} \times v_k}{c_o \times g_{44} \times c_o}$$

$= 1\frac{}{(1-\hat{\beta}^2)^{1/2}.(3.5.21)}$

For the general expression interested readers can inspect Ref. [7].

Contrary to another popular belief throughout the 20-th century, Ref. [58] proved that

LEMMA 3.5.2: The Lorentz symmetry remains exact for all possible signature preserving $(+, +, +, -)$ *deformations of the Minkowski space.*

Again, the symmetry remains exact under the use of the appropriate mathematics.

The above reconstruction of the exact Lorentz symmetry can be geometrically visualized by noting that the light cone

$$x_2^2 + x_3^2 - c_o^2 \times t^2 = 0, \tag{3.5.22}$$

can only be formulated in vacuum, while within physical media we have the *light isocone*

$$\frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - \frac{c_o^2 \times t^2}{n^2(\omega, \ldots)} = 0, \tag{3.5.23}$$

that, when formulated on isospaces over isofield, is also a perfect cone, as it is the case for the isosphere. This property then explains how the Lorentz symmetry is reconstructed as exact according to Lemma 3.5.2 or, equivalently, that $\hat{O}(3.1)$ is isomorphic to $O(3.1)$.

(3) The **isotranslations** [29]

$$\hat{\mathcal{T}}(4) : \hat{x}' = \hat{\mathcal{T}}(\hat{a},\ldots)] \hat{\times} x = \hat{x} + \hat{A}(\hat{a}, x, \ldots), \ \ \hat{a} = a \times \hat{I}_a \in \hat{R}_a, \tag{3.5.24}$$

that can be written

$$x^{\mu'} = x^\mu + A^\mu(a, \ldots), \tag{3.5.25a}$$

$$A^\mu = a^\mu (g_{\mu\mu} + a^\alpha \times [g_{i\mu} \hat{,} P_\alpha]/1! + \ldots), \tag{3.5.25b}$$

and there is no summation on the μ indices.

We reach in this way the following important result:

LEMMA 3.5.3 [55]: Isorelativity permits an axiomatically correct extension of relativity laws to noninertial frames.

In fact, noninertial frames are transformed into frames that are inertial on isospaces over isofields, called *isoinertial,* as established by the fact that isotranslations (3.5.25) are manifestly nonlinear and, therefore, noninertial on conventional spaces while they are isolinear on isospaces, according to a process similar to the reconstruction of locality, linearity and canonicity.

The isoinertial character of the frames can also be seen from the isocommutativity of the linear momenta, Eqs. (3.5.12c), while such a commutativity is generally lost in the projection of Eqs. (3.5.12c) on ordinary spaces over ordinary fields, thus confirming the lifting of conventional noninertial frames into an isoinertial form.

This property illustrates again the origin of the name "isorelativity" to indicate that conventional relativity axioms are solely applicable in isospacetime.

(4) The novel **isotopic transformations** [60]

$$\hat{\mathcal{I}}(1) : \hat{x}' = \hat{w}^{-\hat{1}} \hat{\times} \hat{x} = w^{-1} \times \hat{x}, \ \hat{I}' = w^{-2} \times \hat{I}, \tag{3.5.26}$$

where w is a constant,

$$\hat{I} \to \hat{I}' = \hat{w}^{-2} \hat{\times} \hat{I} = w^{-2} \times \hat{I} = 1/\hat{T}', \tag{3.5.27a}$$

$$\hat{x}^{\hat{2}} = (x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I} \equiv \hat{x}'^{\hat{2}} =$$
$$= [x^\mu \times (w^2 \times \hat{\eta}_{\mu\nu}) \times x^\nu] \times (w^2 \times \hat{I}). \tag{3.5.27b}$$

Contrary to another popular belief throughout the 20-th century, we therefore have the following

THEOREM 3.5.2: The Poincaré-Santilli isosymmetry, hereon denoted with

$$\hat{P}(3.1) = \hat{O}(3.1) \hat{\times} \hat{\mathcal{T}}(4) \hat{\times} \hat{\mathcal{I}}(1), \tag{3.5.28}$$

and, therefore, the conventional Poincaré symmetry, are eleven dimensional.

The increase of dimensionality of the fundamental spacetime symmetry as, predictably, far reaching implications, including a basically novel and axiomatically consistent grand unification of electroweak and gravitational interactions studied in Chapter 5.

The simplest possible realization of the above formalism for isorelativistic kinematics can be outlined as follows. The first application of isorelativity is that of providing *an invariant description of locally varying speeds of light propagating within physical media.* For this purpose a realization of isorelativity requires the knowledge of the *density* of the medium in which motion occurs.

The simplest possible realization of the fourth component of the isometric is then given by the function

$$g_{44} = n_4^2(x, \omega, \ldots), \tag{3.5.29}$$

normalized to the value $n_4 = 1$ for the vacuum (note that the density of the medium in which motion occurs *cannot* be described by special relativity). The above representation then follows with invariance under $\hat{P}(3.1)$.

In this case the quantities n_k, $k = 1, 2, 3$, represent the *inhomogeneity and anisotropy of the medium considered.* For instance, if the medium is homogeneous and isotropic (such as water), all metric elements coincide, in which case

$$\hat{I} = Diag.(g_{11}, g_{22}, g_{33}, g_{44}) = n_4^2 \times Diag.(1, 1, 1, 1), \tag{3.5.30a}$$

$$\hat{x}^{\hat{2}} = \frac{x^2}{n_4^2} \times n_4^2 \times I \equiv x^2, \tag{3.5.30b}$$

thus confirming that *isotopies are hidden in the Minkowskian axioms,* and this may be a reason why they were not been discovered until recently.

Next, isorelativity has been constructed for the invariant description of systems of extended, nonspherical and deformable particles under Hamiltonian and non-Hamiltonian interactions.

Practical applications then require the knowledge of the actual shape of the particles considered, here assumed for simplicity as being spheroidal ellipsoids with semiaxes n_1^2, n_2^2, n_3^2.

Note that the minimum number of constituents of a closed non-Hamiltonian system is two. In this case we have shapes represented with $n_{\alpha k}$, $\alpha = 1, 2, , \ldots, n$.

Specific applications finally require the identification of the nonlocal interactions, e.g., whether occurring on an extended *surface* or *volume.* As an illustration, two spinning particles denoted 1 and 2 in condition of deep mutual penetration and overlapping of their wavepackets (as it is the case for valence bonds), can be described by the following Hamiltonian and total isounit

$$H = \frac{p_1 \times p_1}{2 \times m_1} + \frac{p_2 \times p_2}{2 \times m_2} + V(r), \tag{3.5.31a}$$

$$\hat{I}_{Tot} = Diag.(n_{11}^2, n_{12}^2, n_{13}^2, n_{14}^2) \times Diag.(n_{21}^2, n_{22}^2, n_{23}^2, n_{24}^2) \times$$
$$\times e^{N \times (\hat{\psi}_1/\psi_1 + \hat{\psi}_2/\psi_2) \times \int \hat{\psi}_{1\uparrow}(r)^\dagger \times \hat{\psi}_{2\downarrow}(r) \times dr^3}, \tag{3.5.31b}$$

where N is a positive constant.

The above realization of the isounit has permitted the first known *invariant and numerically exact* representation of the binding energy and other features of the hydrogen, water and other molecules [71,72] (see Chapter 9) for which a historical 2% has been missing for about one century. The above isounit has also been instrumental for a number of additional data on two-body systems whose representation had been impossible with quantum mechanics, such as the origin of the spin 1 of the ground state of the deuteron that, according to quantum axioms, should be zero.

Note in isounit (3.5.31) the nonlinearity in the wave functions, the nonlocal-integral character and the impossibility of representing any of the above features with a Hamiltonian.

From the above examples interested readers can then represent any other closed non-Hamiltonian systems.

3.5.4 Isorelativity and Its Isodual

The third important part of the new isorelativity is given by the following isotopies of conventional relativistic axioms that, for the case of motion along the third axis, can be written [29] as follows [60]:

ISOAXIOM I. The projection in our spacetime of the maximal causal invariant isospeed is given by:

$$V_{Max} = c_\circ \times \frac{g_{44}^{1/2}}{g_{33}^{1/2}} = c_\circ \frac{n_3}{n_4} = c \times n_3. \tag{3.5.32}$$

This isoaxiom resolves the inconsistencies of special relativity recalled earlier for particles and electromagnetic waves propagating within physical media such as water.

In fact, water is homogeneous and isotropic, thus requiring that

$$g_{11} = g_{22} = g_{33} = g_{44} = 1/n^2, \tag{3.5.33}$$

where n is the index of refraction.

In this case the maximal causal speed for a massive particle is $c_\circ$ as experimentally established, e.g., for electrons, while the local speed of electromagnetic waves is $c = c_\circ/n$, as also experimentally established.

Note that such a resolution requires *the abandonment of the speed of light as the maximal causal speed for motion within physical media, and its replacement with the maximal causal speed of particles.*

It happens that in vacuum these two maximal causal speeds coincide. However, even in vacuum the correct maximal causal speed remains that of particles and *not* that of light, as generally believed.

At any rate, physical media are generally opaque to light but not to particles. Therefore, the assumption of the speed of light as the maximal causal speed within media in which light cannot propagate would be evidently vacuous.

It is an instructive exercise for interested readers to prove that

LEMMA 3.5.4: The maximal causal isospeed of particles on isominkowski space over an isofield remains $c_\circ$.

In fact, on isospaces over isofields $c_\circ^2$ is deformed by the index of refraction into the form $c_\circ^2/n_4^2$, but the corresponding unit $\mathrm{cm}^2/\mathrm{sec}^2$ is deformed by the inverse amount, $n_4^2 \times \mathrm{cm}^2/\mathrm{sec}^2$, thus preserving the numerical value $c_\circ^2$ due to the structure of the isoinvariant studied earlier.

The understanding of isorelativity requires the knowledge that, when formulated on the Minkowski-Santilli isospace over the isoreals, Isoaxiom I coincides with the conventional axiom that is, the maximal causal speed returns to be c. The same happens for all remaining isoaxioms.

ISOAXIOM II. The projection in our spacetime of the isorelativistic addition of isospeeds within physical media is given by:

$$v_{Tot} = \frac{v_1 + v_2}{1 + \dfrac{v_1 \times g_{33} \times v_2}{c_\circ \times g_{44} \times c_\circ}} = \frac{v_1 + v_2}{1 + \dfrac{v_1 \times n_4^2 \times v_2}{c_\circ \times n_3^2 \times c_\circ}}. \tag{3.5.34}$$

We have again the correct result that *the sum of two maximal causal speeds in water,*

$$V_{max} = c_\circ \times (n_3/n_4), \tag{3.5.35}$$

yields the maximal causal speed in water, as the reader is encouraged to verify.

Note that such a result is impossible for special relativity. Note also that *the "relativistic" sum of two speeds of lights in water, $c = c_\circ/n$, does not* yield the speed of light in water, thus confirming that the speed of light within physical media, assuming that they are transparent to light, is not the fundamental maximal causal speed.

ISOAXIOM III. The projection in our spacetime of the isorelativistic laws of dilation of time $t_\circ$ and contraction of length $\ell_\circ$ and the variation of mass $m_\circ$ with speed are given respectively by:

$$t = \hat{\gamma} \times t_\circ, \tag{3.5.36a}$$

$$\ell = \hat{\gamma}^{-1} \times \ell_\circ, \tag{3.5.36b}$$

$$m = \hat{\gamma} \times m_\circ. \tag{3.5.36c}$$

$$\hat{\beta} = \frac{v_k \times g_{kk}}{c_o \times g_{44}} = \frac{v_k}{V_{Max}}, \ \ \hat{\gamma} = \frac{1}{(1 - \hat{\beta}^2)^{1/2}}, \tag{3.5.d}$$

where one should npote that, since the speed is always smaller than the maximal possible speed, $\hat{\gamma}$ cannot assume imaginary values.

Note that in water these values coincide with the relativistic ones as it should be since particles such as the electrons have in water the maximal causal speed $c_\circ$.

Note again the necessity of avoiding the interpretation of the local speed of light as the maximal local causal speed. Note also that the mass diverges at the maximal local causal speed, but *not* at the local speed of light.

ISOAXIOM IV. The projection in our spacetime of the iso-Doppler law is given by the isolaw (here formulated for simplicity for 90° *angle of aberration):*

$$\omega = \hat{\gamma} \times \omega_\circ. \tag{3.5.37}$$

This isorelativistic axioms permits an *exact, numerical and invariant representation* of the large differences in cosmological redshifts between quasars and galaxies when physically connected.

In this case light simply exits the huge quasar chromospheres already redshifted due to the decrease of the speed of light, while the speed of the quasars can remain the *same* as that of the associated galaxy. Note again as this result is impossible for special relativity.

Isoaxiom IV also permits a numerical interpretation of the internal blue- and redshift of quasars due to the dependence of the local speed of light on its frequency.

Finally, Isoaxiom IV predicts that *a component* of the predominance toward the red of sunlight at sunset is of iso-Doppler nature. This prediction is based on the different travel within atmosphere of light at sunset as compared to the zenith (evidently because of the travel within a comparatively denser atmosphere).

By contrast, the popular representation of the apparent redshift of sunlight at sunset is that via the scattering of light among the molecules composing our atmosphere. Had this interpretation be correct, the sky at the zenith should be red, while it is blue.

At any rate, the claim of representation of the apparent redshift via the scattering of light is political because of the impossibility of reaching the needed numerical value of the redshift, as serious scholars are suggested to verify.

ISOAXIOM V. The projection in our spacetime of the isorelativistic law of equivalence of mass and energy is given by:

$$E = m \times V_{Max}^2 = m \times c_\circ^2 \times \frac{g_{44}}{g_{33}} = m \times c_\circ^2 \times \frac{n_3^2}{n_4^2} = c \times n_3 \tag{3.5.38}$$

Note a crucial axiomatic difference between the conventional axiom $E = m \times c_c irc^2$ and isoaxiom V. They coincide in vacuum, water and other media transparent to light, but are otherwise structurally different. We should note that, in early references, the conventional axiom $E = m \times c_c irc^2$, where $c_\circ$ is the speed of light in vacuum, was lifted into the form $E = m \times c^2$ where c is the local speed of light within physical media. However, the latter form lead to inconsistencies in applications studied in Volume II (e.g., when the medium considered is opaque to light in which case both $c_\circ$ and c are meaningless) and had to be further lifted into Isoaxiom V.

Among various applications, *Isoaxiom V removes any need for the "missing mass" in the universe.* This is due to the fact that all isotopic fits of experimental data agree on values $g_{44} \gg 1$ within the hyperdense media in the interior of hadrons, nuclei and stars [7].

As a result, Isoaxiom V yields a value of the total energy of the universe dramatically bigger than that believed until now under the assumption of the universal validity of the speed of light in vacuum.

For other intriguing applications of Isoaxioms V, e.g., for the rest energy of hadronic constituents, we refer the interested reader to monographs [55,61].

The *isodual isorelativity* for the characterization of antimatter can be easily constructed via the isodual map of Chapter 2, and its explicit study is left to the interested reader for- brevity.

3.5.5 Isorelativistic Hadronic Mechanics and its Isoduals

The isorelativistic extension of relativistic hadronic mechanics is readily permitted by the Poincaré-Santilli isosymmetry. In fact, iso-invariant (3.5.13a) characterizes the following *iso-Gordon equation* on $\hat{\mathcal{H}}$ over $\hat{C}$ [55]

$$\hat{p}_\mu \hat{\times} |\hat{\psi}\rangle = -\hat{i} \hat{\times} \hat{\partial}_\mu |\hat{\psi}\rangle = -i \times \hat{I}_\mu^\nu \times \partial_\nu |\hat{\psi}\rangle, \tag{3.5.39a}$$

$$(\hat{p}_\mu \hat{\times} \hat{p}^\mu + \hat{m}_\circ^2 \hat{\times} \hat{c}^4) \hat{\times} |\hat{\psi}\rangle = (\hat{\eta}^{\alpha\beta} \times \partial_\alpha \times \partial_\beta + m_\circ^2 \times c^4) \times |\hat{\psi}\rangle = 0. \tag{3.5.39b}$$

The linearization of the above second-order equations into the *Dirac-Santilli isoequation* has been first studied in Refs. [60–62] and then by other authors (although generally without the use of isomathematics, thus losing the invariance).

By recalling the structure of Dirac's equation as the Kronecker product of a spin 1/2 massive particle and its antiparticle of Chapter 2, the Dirac-Santilli isoequation is formulated on the total isoselfadjoint isospace and related isosymmetry

$$\hat{M}^{tot} = [\hat{M}^{orb}(\hat{x}, \hat{\eta}, \hat{R}) \times \hat{S}^{spin}(2)] \times$$

$$\times [\hat{M}^{d\,orb}(\hat{x}^d, \hat{\eta}^d, \hat{R}^d) \times \hat{S}^{d\,spin}(2)] = \hat{M}^{d\,tot}, \tag{3.5.40a}$$

$$\hat{S}^{tot} = \hat{P}(3.1) \times \hat{P}^d(3.1) = \hat{S}^{d\,tot}, \tag{3.5.40b}$$

and can be written [29]

$$[\hat{\gamma}^\mu \hat{\times} (\hat{p}_\mu - \hat{e} \hat{\times} \hat{A}_\mu) + \hat{i} \hat{\times} \hat{m}] \hat{\times} |\phi(x)\rangle = 0, \tag{3.5.41a}$$

$$\hat{\gamma}^\mu = g^{\mu\mu} \times \gamma^\mu \times \hat{I}, \tag{3.5.41b}$$

where the γ's are the conventional Dirac matrices.

Note the appearance of the isometric elements directly in the structure of the isogamma matrices and their presence also when the equation is projected in the conventional spacetime.

The following generators

$$J_{\mu\nu} = (S_k,\, L_{k4}), P_\mu, \tag{3.5.42a}$$

$$S_k = (\hat{\epsilon}_{kij} \hat{\times} \hat{\gamma}_i \hat{\times} \hat{\gamma}_j)/2, \;\; L_{k4} = \hat{\gamma}_k \hat{\times} \hat{\gamma}_4/2, \;\; P_\mu = \hat{p}_\mu, \tag{3.5.42b}$$

characterize the *isospinorial covering of the Poincaré-Santilli isosymmetry.*

The notion of "isoparticle" can be best illustrated with the above realization because it implies that, *in the transition from motion in vacuum (as particles have been solely detected and studied until now) to motion within physical media, particles generally experience the alteration, called "mutation", of all intrinsic characteristics,* as illustrated by the following isoeigenvalues,

$$\hat{S}^{\hat{2}} \hat{\times} |\hat{\psi}\rangle = \frac{g_{11} \times g_{22} + g_{22} \times g_{33} + g_{33} \times g_{11}}{4} \times |\hat{\psi}\rangle, \tag{3.5.43a}$$

$$\hat{S}_3 \hat{\times} |\hat{\psi}\rangle = \frac{(g_{11} \times g_{22})^{1/2}}{2} \times |\hat{\psi}\rangle. \tag{3.5.43b}$$

The mutation of spin then characterizes a necessary mutation of the intrinsic magnetic moment given by [29]

$$\tilde{\mu} = \left(\frac{g_{33}}{g_{44}}\right)^{1/2} \times \mu, \tag{3.5.44}$$

where μ is the conventional magnetic moment for the same particle when in vacuum. The mutation of the rest energy and of the remaining characteristics has been identified before via the isoaxioms.

Note that the invariance under isorotations allows the rescaling of the radius of an isosphere. Therefore, for the case of the perfect sphere we can always have $g_{11} = g_{22} = g_{33} = g_{44}$ in which case the magnetic moment is not mutated. These results recover conventional classical knowledge according to which *the alteration of the shape of a charged and spinning body implies the necessary alteration of its magnetic moment.*

The construction of the isodual isorelativistic hadronic mechanics is left to the interested reader by keeping in mind that the iso-Dirac equation is isoselfdual as the conventional equation.

To properly understand the above results, one should keep in mind that *the mutation of the intrinsic characteristics of particles is solely referred to the constituents of a hadronic bound state under conditions of mutual penetration of their wave packets (such as one hadronic constituent) under the condition of recovering conventional characteristics for the hadronic bound state as a whole (the hadron considered),* much along Newtonian subsidiary constrains on non-Hamiltonian forces, Eqs. (3.1.6).

It should be also stressed that *the above indicated mutations violate the unitary condition when formulated on conventional Hilbert spaces, with consequential catastrophic inconsistencies, Theorem 1.5.2.*

As an illustration, the violation of causality and probability law has been established for all eigenvalues of the angular momentum M different than the quantum spectrum

$$M^2 \times |\psi\rangle = \ell(\ell + 1) \times |\psi\rangle, \quad \ell = 0, 1, 2, 3, \dots \ . \tag{3.5.45}$$

As a matter of fact, these inconsistencies are the very reason why the mutations of internal characteristics of particles for bound states at short distances could not be admitted within the framework of quantum mechanics.

By comparison, hadronic mechanics has been constructed to recover unitarity on iso-Hilbert spaces over isofields, thus permitting an invariant description of internal mutations of the characteristics of the constituents of hadronic bound states, while recovering conventional features for states as a whole.

Far from being mere mathematical curiosities, the above mutations permit basically new structure models of hadrons, nuclei and stars, with consequential, new clean energies and fuels (see Chapters 11, 12).

These new advances are prohibited by quantum mechanics precisely because of the preservation of the *intrinsic* characteristics of the constituents in the transition from bound states at large mutual distance, for which no mutation is possible, to the bound state of the same constituents in condition of mutual penetration, in which case mutations have to be admitted in order to avoid the replacement of a scientific process with unsubstantiated personal beliefs one way or the other (see Chapter 12 for details).

3.5.6 Isogravitation and its Isodual

As indicated in Section 1.4, there is no doubt that the classical and operator formulations of gravitation on a curved space have been the most controversial theory of the 20-th century because of an ever increasing plethora of problematic aspects remained vastly ignored. By contrast, as also reviewed in Section 1.4, special relativity in vacuum has a majestic axiomatic consistence in its *invariance* under the Poincaré symmetry.

Recent studies have shown that the formulation of gravitation on a curved space or, equivalently, the formulation of gravitation based on "covariance", is necessarily noncanonical at the classical level and nonunitary at the operator level, thus suffering of all catastrophic inconsistencies of Theorems 1.4.1 and 1.4.2.

These catastrophic inconsistencies can only be resolved via a new conception of gravity based on a *universal invariance,* rather than covariance.

Additional studies have identified profound axiomatic incompatibilities between gravitation on a curved space and electroweak interactions. These incompatibilities have resulted to be responsible for the lack of achievement of an axiomatically consistent grand unification since Einstein's times (see Chapter 14).

No knowledge of isotopies can be claimed without a knowledge that isorelativity has been constructed to resolve at least some of the controversies on gravitation. The fundamental requirement is *the abandonment of the formulation of gravity via curvature on a Riemannian space and its formulation instead on an iso-Minkowskian space* via the following steps characterizing *exterior isogravitation in vacuum,* first presented in Refs. [73,74]:

I) Factorization of any given Riemannian metric representing exterior gravitation $g^{ext}(x)$ into a nowhere singular and positive-definite 4×4-matrix $\hat{T}(x)$ times the Minkowski metric η,

$$g^{ext}(x) = \hat{T}^{ext}_{grav}(x) \times \eta; \tag{3.5.47}$$

II) Assumption of the inverse of $\hat{T}_{grav}$ as the fundamental unit of the theory,

$$\hat{I}^{ext}_{grav}(x) = 1/\hat{T}^{ext}_{grav}(x); \tag{3.5.48}$$

III) Submission of the totality of the Minkowski space and relative symmetries to the noncanonical/nonunitary transform

$$U(x) \times I^{\dagger}(x) = \hat{I}^{ext}_{grav}. \tag{3.5.49}$$

The above procedure yields the isominkowskian spaces and related geometry $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$, resulting in a new conception of gravitation, exterior isogravity, with the following main features [26]:

i) Isogravity is characterized by a universal *symmetry* (and not a covariance), the Poincaré-Santilli isosymmetry $\hat{P}(3.1)$ for the gravity of matter with isounit

$\hat{I}^{ext}_{grav}(x)$, the isodual isosymmetry $\hat{P}^d(3.1)$ for the gravity of antimatter, and the isoselfdual symmetry $\hat{P}(3.1) \times \hat{P}^d(3.1)$ for the gravity of matter-antimatter systems;

ii) All conventional field equations, such as the Einstein-Hilbert and other field equations, can be formulated via the Minkowski-Santilli isogeometry since the latter preserves all the tools of the conventional Riemannian geometry, such as the Christoffel's symbols, covariant derivative, etc. [15];

iii) Isogravitation is isocanonical at the classical level and isounitarity at the operator level, thus resolving the catastrophic inconsistencies of Theorems 1.5.1 and 1.5.2;

iv) An axiomatically consistent operator version of gravity always existed and merely crept in unnoticed through the 20-th century because gravity is imbedded where nobody looked for, in the *unit* of relativistic quantum mechanics, and it is given by isorelativistic hadronic mechanics outlined in the next section.

v) The basic feature permitting the above advances is the abandonment of curvature for the characterization of gravity (namely, curvature characterized by metric $g^{ext}(x)$ referred to the unit I) and its replacement with *isoflatness,* namely, the verification of the axioms of flatness in isospacetime, while preserving conventional curvature in its projection on conventional spacetime (or, equivalently, curvature characterized by the $g(x) = \hat{T}^{ext}_{grav}(x) \times \eta$ referred to the isounit $\hat{I}_{grav}(x)$ in which case curvature becomes null due to the inter-relation $\hat{I}^{ext}_{grav}(x) = 1/\hat{T}^{ext}_{grav}(x))$ [26].

A resolution of numerous controversies on classical formulations of gravity then follows from the above main features, such as:

a) The resolution of the century old controversy on the lack of existence of consistent total conservation laws for gravitation on a Riemannian space, which controversy is resolved under the universal $\hat{P}(3.1)$ symmetry by mere visual verification that the generators of the conventional and isotopic Poincaré symmetry are the same (since they represent conserved quantities in the absence and in the presence of gravity);

b) The controversy on the fact that gravity on a Riemannian space admits a well defined "Euclidean", but not "Minkowskian" limit, which controversy is trivially resolved by isogravity via the limit

$$\hat{I}^{ext}_{grav}(x) \rightarrow I; \tag{3.5.50}$$

c) The resolution of the controversy that Einstein's gravitation predicts a value of the bending of light that is twice the experimental value, one for curvature and one for newtonian attraction, which controversy is evidently resolved by the elimination of curvature as the origin of the bending, as necessary in any case for the free fall of a body along a straight radial line in which no curvature of any type is conceivably possible or credible; and other controversies.

A resolution of the controversies on quantum gravity can be seen from the property that relativistic hadronic mechanics of the preceding section *is* a quantum formulation of gravity whenever $\hat{T} = \hat{T}_{grav}$.

Such a form of operator gravity is as axiomatically consistent as conventional relativistic quantum mechanics because the two formulations coincide, by construction, at the abstract, realization-free level.

As an illustration, whenever

$$\hat{T}^{ext}_{grav} = Diag.(g^{ext}_{11}, g^{ext}_{22}, g^{ext}_{33}, g^{ext}_{44}), \quad g_{\mu\mu} > 0, \tag{3.5.51}$$

the Dirac-Santilli isoequation (3.5.41) provides a direct representation of the conventional electromagnetic interactions experienced by an electron, represented by the vector potential A_μ, plus gravitational interactions represented by the isogamma matrices.

Once curvature is abandoned in favor of the broader isoflatness, the axiomatic incompatibilities existing between gravity and electroweak interactions are resolved because:

i) isogravity possesses, at the abstract level, the *same* Poincaré invariance of electroweak interactions;

ii) isogravity can be formulated on the *same* flat isospace of electroweak theories; and

iii) isogravity admits positive and negative energies in the *same* way as it occurs for electroweak theories.

An axiomatically consistent *iso-grand-unification* then follows, as studied in Chapter 14.

Note that the above grand-unification requires the prior *geometric unification of the special and general relativities,* that is achieved precisely by isorelativity and its underlying iso-Minkowskian geometry.

In fact, special and general relativities are merely differentiated in isospecial relativity by the explicit realization of the unit. In particular, *black holes are now characterized by the zeros of the isounit* [7]

$$\hat{I}^{ext}_{grav}(x) = 0. \tag{3.5.52}$$

The above formulation recovers all conventional results on gravitational singularities, such as the singularities of the Schwarzschild's metric, since they are all described by the gravitational content $\hat{T}_{grav}(x)$ of $g(x) = \hat{T}_{grav}(x) \times \eta$, since η is flat.

This illustrates again that *all conventional results of gravitation, including experimental verifications, can be reformulated in invariant form via isorelativity.*

Moreover, the problematic aspects of general relativity mentioned earlier refer to the *exterior gravitational problem.* Perhaps greater problematic aspects exist in gravitation on a Riemannian space for *interior gravitational problems,* e.g., because of the lack of characterization of basic features, such as the density of the interior problem, the locally varying speed of light, etc.

These additional problematic aspects are also resolved by isorelativity due to the unrestricted character of the functional dependence of the isometric that, therefore, permits a direct geometrization of the density, local variation of the speed of light, etc.

The above lines constitute only the initial aspects of isogravitation since its most important branch is *interior isogravitation* as characterized by isounit and isotopic elements of the illustrative type

$$\hat{I}^{int}_{grav} = 1/\hat{T}^{int}_{grav} > 0, \tag{3.5.53a}$$

$$\hat{T}^{int}_{grav} = Diag.(g^{int}_{11}/n_1^2, g^{int}_{22}/n_2^2, g^{int}_{33}/n_3^2, g^{int}_{44}/n_4^2), \tag{3.5.53b}$$

permitting a *geometric representation directly via the isometric of the actual shape of the body considered, in the above case an ellipsoid with semiaxes n_1^2, n_2^2, n_3^2, as well as the (average) interior density n_4^2 with consequential representation of the (average value of the) interior speed of light $C = c/n_4$.*

A most important point is that the invariance of interior isogravitation under the Poincaré-Santilli isosymmetry persists in its totality since the latter symmetry is completely independent from the explicit value of the isounit or isotopic element, and solely depends on their positive-definite character.

Needless to say, isounit (3.4.53) is merely illustrative because a more accurate interior isounit has a much more complex functional dependence with a locally varying density, light speed and other characteristics as they occur in reality.

Explicit forms of these more adequate models depends on the astrophysical body considered, e.g., whether gaseous, solid or a mixture of both, and their study is left to the interested reader.

It should also be noted that *gravitational singularities should be solely referred to interior models* evidently because exterior descriptions of type (3.5.52) are a mere approximation or a geometric abstraction.

In fact, *gravitational singularities existing for exterior models are not necessarily confirmed by the corresponding interior formulations.* Consequently, the current views on black holes could well result to be pseudo-scientific beliefs because the only scientific statement that can be proffered at this time without raising issue of scientific ethics is that *the gravitational features of large and hyperdense aggregations of matter, whether characterizing a "black" or "brown" hole, are basically unresolved at this time.*

Needless to say, exterior isogravitation is a particular case of the interior formulation. Consequently, from now on, unless otherwise specified isogravitation will be referred to the interior form.

The cosmological implications are also intriguing and will be studied in Chapter 6. It should be indicated that numerous formulations of gravitation in flat

Minkowski space exist in the literature, such as Ref. [79] and papers quoted therein. However, these formulations have no connection with isogravity since the background space of the former is conventional, while that of the latter is a geometric unification of the Minkowskian and Riemannian spaces.

It is hoped that readers with young minds of any age admit the incontrovertible character of the limitations of special and general relativities and participate in the laborious efforts toward new vistas because any lack of participation in new frontiers of science, whether for personal academic interest or other reason, is a gift of scientific priorities to others.

Appendix 3.A
Universal Enveloping Isoassociative Algebras

The main structural component of Lie's theory is its *universal enveloping associative algebra* $\xi(L)$ of a Lie algebra L. In fact, Lie algebras can be obtained as the attached antisymmetric part $[\xi(L)]^- \approx L$; the infinite dimensional basis of $\xi(l)$ permit the exponentiation to a finite transformation group G; and the representation theory is crucially dependent on the right and/or left modular associative action originally defined on G.

In Section 3.2.9B we have reviewed the rudiments of the *universal enveloping isoassociative algebras* $\hat{\xi}(L)$ of a Lie-Santilli isoalgebra $\hat{L}$. It is easy to see that all features occurring for $\xi(L)$ carry over to the covering isoform $\hat{\xi}(L)$.[30]

In this appendix we would like to outline a more technical definition of universal enveloping isoassociative algebras since they are at the foundations of the unification of simple Lie algebras of dimension N into a single Lie-Santilli isoalgebra of the same dimension (Section 3.2.13).

With reference to Figure **??**, the envelop $\xi(L)$ can be defined as the (ξ, τ) where ξ is an associative algebra and τ is a homomorphism of L into the antisymmetric algebra ξ^- attached to ξ such that: if ξ' is another associative algebra and τ' is another homomorphism of L into ξ'^-, a unique isomorphism γ exists between ξ and ξ' in such a way that the diagram in the l.h.s of Figure **??** is commutative. The above definition evidently expresses the uniqueness of the Lie algebra L up to local isomorphism, and illustrates the origin of the name "universal" enveloping algebra of L.

With reference to the r.h.s. diagram of Figure **??**, the universal enveloping isoassociative algebra $\hat{\xi}(L)$ of a Lie algebra L was introduced in Ref. [4] as the set $\{(\xi, \tau), i, \hat{\xi}, \hat{\tau}\}$ where: (ξ, τ) is a conventional envelope of L; i is an isotopic mapping $L \to i(L) = \hat{L} \sim L$; $\hat{\xi}$ is an associative algebra generally nonisomorphic to ξ; $\hat{\tau}$ is a homomorphism of $\hat{L}$ into $\hat{\xi}^-$; such that: if $\hat{\xi}'$ is another associative algebra and $\hat{\tau}'$ is another homomorphism of $\hat{L}$ into $\hat{\xi}'^-$, there exists a unique

[30] We use the denomination $\hat{\xi}(L)$ rather than $\hat{\xi}(\hat{L})$ to stress the fact that the generators of ξ are those of L and not of $\hat{L}$, a requirement that is essential for consistent physical applications because the generators of L represent ordinary physical quantities (such as total energy, total linear momentum, etc.) that, as such, cannot be changed by isotopies.

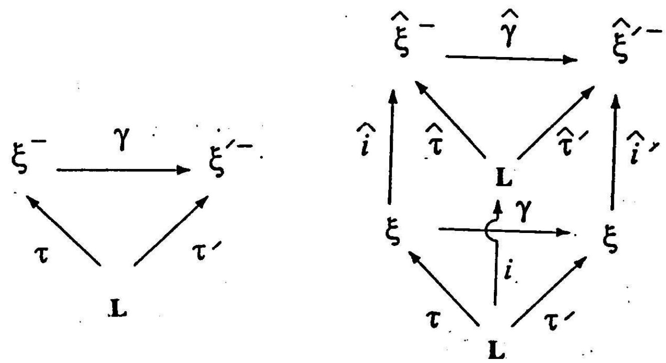

Figure 3.A.1. A schematic view of the universal enveloping associative algebra of a Lie algebra L and its lifting for the Lie-Santilli isoalgebra $\hat{L}$ according to the original proposal [4] of 1978.

isomorphism $\hat{\gamma}$ of $\hat{\xi}$ into $\hat{\xi}'$ with $\hat{\tau}' = \gamma(\hat{\tau})$ and two unique isotopies $i(\xi) = \hat{\xi}$ and $i(\hat{\xi}) = \hat{\xi}'$.

A primary objective of the above definition of isoenvelope is the *lack of uniqueness of the Lie algebra characterized by the isoenvelope* or, equivalently, the *characterization of a family of generally nonisomorphic Lie algebras via the use of only one basis.* The above definition of isoenvelope also explains in more details the variety of realization of the simple 3-dimensional Lie-Santilli isoalgebra $\hat{L}_3$ provided in Eq. (3.2.236), and may be of assistance in extending the same classification to other isoalgebras.

The above notion of isoenvelope represents the essential mathematical structure of hadronic mechanics, namely, the preservation of the conventional basis, i.e., the set of observables of quantum mechanics, and the generalization of the operations on them via an infinite number of isotopies so as to admit a new class of interactions structurally beyond the possibilities of quantum mechanics.

Appendix 3.B
Recent Advances in the TSSFN Isotopology

In Section 3.2.7 we introduced the elements of the *Tsagas-Sourlas-Santilli-Falcón-Núñez isotopology* (or TSSFN Isotopology for short). In this appendix we outline recent advances on the isotopology by the Spanish mathematicians R. M. Falcón Ganfornina and J. Núñez Valdés [24,25].

PROPOSITION 3.2.B1: Consider a mathematical structure

$$(E, +, \times, \circ, \bullet, \ldots),$$

if we construct an isotopic lifting such that:

a) Both primaries $*$, $\widehat{I}$ *and secondaries* $\star$, $\widehat{S}$ *isotopic elements are used.*

b) $(E, \star, *, \ldots)$ *is a structure of the same type as the initial, which is endowed with isounits* S, I, $\ldots$, *with respect to* $\star, *, \ldots$, *respectively.*

c) I *is an unit with respect to* $*$ *in the corresponding general set* V, *being* $T = \widehat{I}^{-I} \in V$ *the associated isotopic element.*

Then, by defining in the isotopic level the operations:

$$\widehat{a}\widehat{+}\widehat{b} = \widehat{a \star b}; \quad \widehat{a}\widehat{\times}\widehat{b} = \widehat{a * b}; \quad \ldots \tag{3.B.1}$$

And being defined in the projection level:

$$\overline{\widehat{a}} = a * \widehat{I}; \quad \alpha\overline{\widehat{+}}\beta = ((\alpha * T) \star (\beta * T)) * \widehat{I}; \quad \alpha\overline{\widehat{\times}}\beta = \alpha * T * \beta; \ \ldots \tag{3.B.2}$$

It is obtained that the isostructure $(\overline{\widehat{E}}, \overline{\widehat{+}}, \overline{\widehat{\times}}, \ldots)$ *is of the same type as the initial one.*

The study in Refs. [24,25] is made by taking into consideration both isotopic and projection levels. Equivalent results related to injective isotopies are also obtained. In the first place, Proposition 3.2.A1 is verified for topological spaces and for their elements and basic properties: isotopologies, isoclosed sets, isoopen sets, T_2, etc:

A *topological isospace* is every isospace endowed with a topological space structure. If, besides, such an isospace is an isotopic projection of a topological space, it is called *isotopological isospace.*

Similarly, they are defined concepts of *(iso)boundary isopoint, closure of a set, closed set, isointerior isopoint, interior of a set, open set, (iso)Hausdorff isospace and second countable isospace, among others.*

PROPOSITION 3.2.B2: The space from which any topological isospace in the isotopic level is obtained can be endowed with the final topology relative to the mapping **I**.

The isotopic projection of a topological space is an isotopological isospace in the projection level. If such a projection is injective, then every topological isospace in such a level is, in fact, isotopological.

Similar results are obtained for the concepts of (iso)boundary isopoint, isointerior isopoint and (iso)Hausdorff isospace.

Next, Refs. [24,25] generalize Kadeisvili's isocontinuity [19]. Particularly, the basic isofield can be endowed with an isoorder, according to the following procedure.

Let $\widehat{K}$ be an isofield associated with a field K, endowed with an order $\leq$, by using an isotopology which preserves the inverse element with respect to the addition. We define the *isoorder* $\widehat{\leq}$ as $\widehat{a}\widehat{\leq}\widehat{b}$ if and only if $a \leq b$. If the isotopy is injective, the isoorder $\overline{\widehat{\leq}}$ en $\overline{\widehat{K}}$ is defined in the same way.

PROPOSITION 3.2.B3: The isoorders $\widehat{\leq}$ *and* $\overline{\widehat{\leq}}$ *are orders over* $\widehat{K}$ *and* $\overline{\widehat{K}}$*, of the same type as* $\leq$.

Let $\widehat{U}$ be a $\widehat{R}$ isovectorspace with isonorm $\widehat{||}.\widehat{||} \equiv \widehat{||.||}$ and isoorder $\widehat{\leq}$, obtained from an isotopy compatible with respect to each one of the initial operations. It will be said that an isoreal isofunction $\widehat{f}$ of $\widehat{U}$ is *isocontinuous in* $\widehat{X} \in \widehat{U}$, if for all $\widehat{\epsilon}\widehat{>}\widehat{S}$, there exists $\widehat{\delta}\widehat{>}\widehat{S}$ such that for all $\widehat{Y} \in \widehat{U}$ with $\widehat{||}\widehat{X} - \widehat{Y}\widehat{||}\widehat{<}\widehat{\delta}$, it is verified that $\widehat{|}\widehat{f}(\widehat{X}) - \widehat{f}(\widehat{Y})\widehat{|}\widehat{<}\widehat{\epsilon}$. We will say that $\widehat{f}$ is *isocontinuous in* $\widehat{U}$ if it is isocontinuous in $\widehat{X}$, for all $\widehat{X} \in \widehat{U}$. Finally, when dealing with injective isotopies, the isocontinuity in the projection level is defined in a similar way.

PROPOSITION 3.2.B4: The isocontinuity in $\widehat{U}$ *is equivalent to the continuity in* U*. In the case of injective isotopies, both ones are equivalent to the one in* $\overline{\widehat{U}}$.

The isocontinuity on isotopological isospaces is also analyzed:

An *isocontinuous isomapping* in the isotopic level between two topological isospaces $\widehat{M}$ and $\widehat{N}$ is every isomapping $\widehat{f}:\widehat{M}\to\widehat{N}$ preserving closures. The definition in the projection level is given in a similar way.

PROPOSITION 3.2.B5: They are verified that:

a) $\widehat{f}$ *is isocontinuous if and only if the mapping* f *from which comes from is continuous. That result is similar in the projection level by using injective isotopies.*

b) Every isoconstant isomapping is isocontinuous.

c) Isocontinuity is preserved by both topological composition and product.

Finally, the analysis of (iso)(pseudo)metric isospaces is also concreted:

PROPOSITION 3.2.B6: Let $\widehat{M}$ *be a* $\widehat{K}$ *isovectorspace, isotopic lifting of a vectorspace* M*, endowed with a (pseudo)metric* d *defined on an ordered field* K*, by using an isotopy which preserves the inverse element and compatible with respect to the addition in* K*. Then, the isofunction* $\widehat{d}$ *is an iso(pseudo)metric.*

Let $(\widehat{M},d')$ be an (iso)(pseudo)metric $\widehat{K}$ isovectorspace, endowed with an isoorder $\widehat{\leq}$. $B_{d'}(\widehat{X_0},\widehat{\epsilon})=\{\widehat{X}\in\widehat{M}:d'(\widehat{X},\widehat{X_0})\widehat{<}\widehat{\epsilon}\}$ is called *metric ball* with center $\widehat{X_0}\in\widehat{M}$ and radius $\widehat{\epsilon}\widehat{>}\widehat{S}$. If M is endowed with a (pseudo)metric d, with $\widehat{d}=d'$, then every metric ball $B_{d'}=B_{\widehat{d}}=\widehat{B_d}$ in $\widehat{M}$, which is isotopic lifting of a metric ball B_d in M, is called *metric isoball* in $\widehat{M}$.

PROPOSITION 3.2.B7: Under conditions of Proposition XXX, if $B_d(X_0,\epsilon)$ *is a metric ball in* M*, then* $\widehat{B_d(X_0,\epsilon)}=B_{\widehat{d}}(\widehat{X_0},\widehat{\epsilon})$ *is a metric ball in* $\widehat{M}$*.*

A *metric neighborhood of an isopoint* $\widehat{X}\in\widehat{M}$ is a subset $\widehat{A}\subseteq\widehat{M}$ containing a metric ball centered in $\widehat{X}$. The set of metric neighborhoods of $\widehat{X}$ is denoted by $\widehat{\aleph}^{d'}_{\widehat{X}}$. Finally, if d' is the iso-Euclidean isodistance over $\widehat{R}^n$, the associated metric neighborhoods are called *iso-Euclidean neighborhoods*.

PROPOSITION 3.2.B8: Let d' *and* d'' *two (iso)(pseudo)metrics over an isovectorspace* $\widehat{M}$*. It is verified that* $\widehat{\aleph}^{d'}_{\widehat{X}}=\widehat{\aleph}^{d''}_{\widehat{X}}$ *if and only if every metric ball* $B_{d'}(\widehat{X},\widehat{\epsilon})$ *contains a ball* $B_{d''}(\widehat{X},\widehat{\rho})$ *and every ball* $B_{d''}(\widehat{X},\widehat{\delta})$ *contains a ball* $B_{d'}(\widehat{X},\widehat{\mu})$.

PROPOSITION 3.2.B9: Every isospace endowed with an (iso)(pseudo)metric is an isotopological isospace.

PROPOSITION 3.2.B10: Let $\widehat{f} : (\widehat{M}, d') \to (\widehat{N}, d'')$ *be an isomapping between* $\widehat{K}$*-isospaces endowed with (iso)(pseudo)metric and let us consider* $\widehat{X} \in \widehat{M}$*. Then,* $\widehat{f}$ *is isocontinuous in* $\widehat{X}$ *if and only if for all* $\widehat{\epsilon} \widehat{>} \widehat{S}$ *there exists* $\widehat{\delta} \in \widehat{K}$ *such that* $\widehat{\delta} \widehat{>} \widehat{S}$*, and if* $\widehat{Y} \in B_{d'}(\widehat{X}, \widehat{\delta})$*, then it is verified that* $\widehat{f}(\widehat{Y}) \in B_{d''}(\widehat{f}(\widehat{X}), \widehat{\epsilon})$*.*

PROPOSITION 3.2.B11: Let $\widehat{f} : \widehat{M} \to \widehat{N}$ *be an isomapping between two isotopological isospaces* $\widehat{M}$ *and* $\widehat{N}$*. If conditions of the definition of isocontinuity are satisfied, then* $\widehat{f}$ *is isocontinuous if and only if* $\widehat{f}^{-1}(\widehat{U})$ *is an isoopen of* $\widehat{M}$*, for all isoopen* $\widehat{U}$ *of* $\widehat{N}$*.*

Appendix 3.C
Recent Advances on the Lie-Santilli Isotheory

In Section 3.2.9 we have outlined the rudiments of the *Lie-Santilli isotheory.* It may be useful for the mathematically oriented reader to outline recent developments achieved by the Spanish mathematicians R. M. Falcón Ganfornina and J. Núñez Valdés [24,25,43] in the field beyond those presented in monographs [2,6,36,37].

Falcón and Núñez introduced in 2001 [37] a new construction model of isotopies which was similar to the one proposed by Santilli in 1978 although in its multivalued version presented by the same author later on [6] (see Chapter 4) because based on the use of several isolaws and isounits as operations existing in the initial mathematical structure. Such a model, which from now on will be called MCIM (*isoproduct construction model based on the multiplication*), was later generalized in Refs. [24,25,43]. In a schematic way, Santilli's isotopies can be described with the following diagram:

$$
\begin{array}{ccc}
 & & \text{General Level} \\
\text{Conventional Level} & \xrightarrow{\hspace{8em}} & (V,*,\star,\ldots) \\
 & & \cup \\
(E,+,\times,\ldots) & & (E,\star,*,\ldots) \\
\downarrow & \sharp & \downarrow \mathbf{I} \\
\text{Projection Level} & \xleftarrow[\hspace{8em}]{\pi} & \text{Isotopic level} \\
(\overline{\widehat{E}},\overline{\widehat{+}},\overline{\widehat{\times}},\ldots) & & (\widehat{E},\widehat{+},\widehat{\times},\ldots)
\end{array}
$$

where, by construction:

a) The mapping $\mathbf{I}:(E,\star,*,\ldots)\to(\widehat{E},\widehat{+},\widehat{\times},\ldots):X\to\widehat{X}$ is an isomorphism.

b) The *isotopic projection* is onto:
$\pi:(\widehat{E},\widehat{+},\widehat{\times},\ldots)\to(\overline{\widehat{E}},\overline{\widehat{+}},\overline{\widehat{\times}},\ldots):\widehat{a}\to\pi(\widehat{a})=\overline{\widehat{a}}=a*\widehat{I}.$

c) $\widehat{a}\widehat{+}\widehat{b}=\widehat{a\star b};\quad \widehat{a}\widehat{\times}\widehat{b}=\widehat{a*b};\ldots.$

d) $\overline{\widehat{a}}=a*\widehat{I};\quad \alpha\overline{\widehat{+}}\beta=((\alpha*T)\star(\beta*T))*\widehat{I};\quad \alpha\overline{\widehat{\times}}\beta=\alpha*T*\beta;\ldots.$

PROPOSITION 3.2.C1: The following properties are verified:

a) The isotopic projection associated with each injective isotopic lifting is an isomorphism.

b) If the isotopic lifting used is compatible with respect to all of initial operations, then the isostructure $\widehat{E}$ is isomorphic to the initial E.

c) The relation of being isotopically equivalents is of equivalence.

d) Every isotopy $\pi \circ \mathbf{I} : (E, +, \times, \circ, \bullet, \ldots) \to (\overline{\widehat{E}}, \overline{\widehat{+}}, \overline{\widehat{\times}}, \overline{\widehat{\circ}}, \overline{\widehat{\bullet}}, \ldots)$ can be considered as an isotopic lifting which follows the MCIM, that is, every mathematical isostructure is an isostructure with respect to the multiplication.

Then, it has a perfect sense to considerer each one of the isostructures which result of applying the MCIM to conventional structures. Particularly, we can consider the construction of *Santilli's isoalgebras* (as the isotopic lifting of each algebra, which is endowed with a structure of algebra).

PROPOSITION 3.2.C2: Let U be a K-algebra and let $\widehat{U}$ be a $\widehat{K}$-isovector-space. If a $K(a, \star, *)$-algebra $(U, \diamond, \Box, \cdot)$ is used in the general level, then the isotopic lifting $\widehat{U}$ corresponding to the isotopy of primary elements $\widehat{I}$ and $\Box$ and secondary ones $\widehat{S}$ and $\diamond$, when MCIM is used, has a structure of isoalgebra on $\widehat{K}$, and it preserves the initial type of the algebra.

A particular type of isoalgebra is the *Lie-Santilli isoalgebra* [4]. Particularly, if $\overline{\widehat{U}}$ is the isotopic projection of a Lie-Santilli isoalgebra,

$$\widehat{I} = \widehat{I}(x, dx, d^2x, t, T, \mu, \tau, \ldots)$$

is an isounit and a basis $\overline{\widehat{U}}$, $\{\overline{\widehat{e_1}}, \ldots, \overline{\widehat{e_n}}\}$ is fixed, where $\overline{\widehat{e_i}} \mathbin{\widehat{\cdot}} \overline{\widehat{e_j}} = \sum \overline{\widehat{c_{ij}^h}} \mathbin{\overline{\widehat{\bullet}}} \overline{\widehat{e_h}}$, $\forall\, 1 \leq i, j \leq n$, then coefficients $\overline{\widehat{c_{ij}^h}} \in \overline{\widehat{K}}$ are the *Maurer-Cartan coefficients* of the isoalgebra, which constitute a generalization of the conventional case, since they are not constants in general, but functions dependent of the factors of $\widehat{I}$.

Another interesting isoalgebra is the *Santilli's Lie-admissible algebra* [4], that is, the isoalgebra $\widehat{U}$ such that with the commutator bracket $[.,.]_{\widehat{U}} : [\widehat{X}, \widehat{Y}]_{\widehat{U}} = (\widehat{X} \mathbin{\widehat{\cdot}} \widehat{Y}) - (\widehat{Y} \mathbin{\widehat{\cdot}} \widehat{X})$ is an isotopic Lie isoalgebra. The following result is satisfied:

PROPOSITION 3.2.C3: Under conditions of Proposition XXX, let us suppose that the law $\widehat{\circ}$ of the isoalgebra $\widehat{U}$ is defined according $\widehat{X} \mathbin{\widehat{\circ}} \widehat{Y} = (X \circ Y) \Box \widehat{I}$, for all $X, Y \in U$. If U is a Lie (admissible) algebra, then $\widehat{U}$ is a Lie isoadmissible isoalgebra.

In this way, Santilli's Lie-admissible isoalgebras inherit the usual properties of conventional (admissible) Lie algebras. In the same way, usual structures related with such algebras have also their analogue ones when isotopies are used.

For instance, an *isoideal* of a Lie isoalgebra $\widehat{U}$ is every isotopic lifting of an ideal $\Im$ of U, which is by itself an ideal. In particular, the *center* of a Lie isoalgebra $\widehat{U}$, $\{\widehat{X} \in \widehat{U}$ such that $\widehat{X}\widehat{\cdot}\widehat{Y} = \widehat{S},\ \forall \widehat{Y} \in \widehat{U}\}$, is an isoideal of $\widehat{U}$. In fact, it is verified the following result:

PROPOSITION 3.2.C4: Let $\widehat{U}$ be a Lie isoalgebra associated with a Lie algebra U and let $\Im$ be an ideal of U. Then, the corresponding isotopic lifting $\widehat{\Im}$ is an isoideal of $\widehat{U}$.

An isoideal $\widehat{\Im}$ of a Lie isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, \widehat{\cdot})$, is called *isocommutative* if $\widehat{X}\widehat{\cdot}\widehat{Y} = \widehat{S}$, for all $\widehat{X} \in \widehat{\Im}$ and for all $\widehat{Y} \in \widehat{U}$, being $\widehat{U}$ *isocommutative* if it is so as an isoideal.

PROPOSITION 3.2.C5: $\widehat{U}$ is isocommutative if and only if U is commutative.

Lie-Santilli isoalgebras can also be introduced as follows. Given an $\widehat{K}$-isoassociative isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, \widehat{\cdot})$, the commutator in $\widehat{U}$ associated with $\widehat{\cdot}$: $[\widehat{X}, \widehat{Y}]_S = (\widehat{X}\widehat{\cdot}\widehat{Y}) - (\widehat{Y}\widehat{\cdot}\widehat{X})$, for all $\widehat{X}, \widehat{Y} \in \widehat{U}$ is denominated *Lie-Santilli bracket product* $[.,.]_S$ with respect to $\widehat{\cdot}$. The isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, [.,.]_S)$ is then denominated *Lie-Santilli algebra*.

DEFINITION 3.2.C6: Let $\widehat{U}$ be an $\widehat{K}$-isoassociative isoalgebra associated with a K-algebra U, under conditions of Proposition XXX. Then, the Lie-Santilli algebra associated with $\widehat{U}$ is a Lie isoalgebra if the algebra U is either associative or Lie admissible.

Apart from that, a Lie-Santilli isoalgebra $\widehat{U}$ is said to be *isosimple* if, being an isotopy of a simple Lie algebra, it is not isocommutative and the only isoideals which contains are trivial. In an analogous way, $\widehat{U}$ is called *isosemisimple* if, being an isotopy of a semisimple Lie algebra, it does not contain non trivial isocommutative isoideals. Note that, this definition involves that every isosemisimple Lie isoalgebra is also isosimple. Moreover, it is verified:

PROPOSITION 3.2.C7: Under conditions of Proposition XXX, the isotopic lifting of a (semi)simple Lie algebra is an iso(semi)simple Lie isoalgebra. Particularly, every isosemisimple Lie isoalgebra is a direct sum of isosimple Lie isoalgebras.

A lie-Santilli isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, \widehat{\cdot})$ is said to be *isosolvable* if, being an isotopy of a solvable Lie algebra, in the *isosolvability series*

$$\widehat{U}_1 = \widehat{U}, \quad \widehat{U}_2 = \widehat{U}\widehat{\cdot}\widehat{U}, \quad \widehat{U}_3 = \widehat{U}_2\widehat{\cdot}\widehat{U}_2, \ \ldots, \widehat{U}_i = \widehat{U}_{i-1}\widehat{\cdot}\widehat{U}_{i-1}, \ldots$$

there exists a natural integer n such that $\widehat{U}_n = \{\widehat{S}\}$. The minor of such integers is called *isosolvability index* of the isoalgebra.

PROPOSITION 3.2.C8: Under conditions of Proposition XXX, the isotopic lifting of a solvable Lie algebra is an isosolvable Lie isoalgebra.

An easy example of isosolvable Lie isoalgebras are the isocommutative isotopic Lie isoalgebras, since they verify, by definition, that $\widehat{U}\widehat{\cdot}\widehat{U} = \widehat{U}_2 = \{\widehat{S}\}$. It implies that every nonzero isocommutative Lie isoalgebra has an isosolvability index equals 2, being 1 the corresponding to the trivial isoalgebra $\{\widehat{S}\}$.

PROPOSITION 3.2.C9: Let $\widehat{U}$ be a Lie isoalgebra associated with a Lie algebra U. Under conditions of Proposition XXX, they are verified:

1) $\widehat{U}_i$ is an isoideal of $\widehat{U}$ and of $\widehat{U}_{i-1}$, for all $i \in N$.

2) If $\widehat{U}$ is isosolvable and U is solvable, then every isosubalgebra of $\widehat{U}$ is isosolvable.

3) The intersection and the product of a finite number of isosolvable isoideals of $\widehat{U}$ are isosolvable isoideals. Moreover, under conditions of Proposition XXX, the sum of a finite number of isosolvable isoideals is also an isosolvable isoideal.

By using this last result it can be deduced that the sum of all isosolvable isoideals of $\widehat{U}$ is another isosolvable isoideal, which is called *isoradical* of $\widehat{U}$. Note that it is different from the *radical* of $\widehat{U}$, which would be the sum of all solvable ideals of $\widehat{U}$. The isoradical is denoted by *isorad* $\widehat{U}$, not to be confused with *rad* $\widehat{U}$, and it will always contain $\{\widehat{S}\}$, because this last one is a trivial isosolvable isoideal of every isoalgebra. Note also that as every isosolvable isoideal of $\widehat{U}$ is a solvable ideal of $\widehat{U}$, then *isorad* $\widehat{U} \subset$ *rad* $\widehat{U}$. So, if $\widehat{U}$ is isosolvable, then $\widehat{U} =$ *isorad* $\widehat{U} =$ *rad* $\widehat{U}$, due to $\widehat{U}$ is solvable in particular.

PROPOSITION 3.2.C10: If $\widehat{U}$ is a semisimple Lie isoalgebra over a field of zero characteristic, then isorad $\widehat{U} = \{\widehat{S}\}$.

A Lie-Santilli isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, \widehat{\cdot})$ is called *isonilpotent* if, being an isotopy of a nilpotent Lie algebra, in the series

$$\widehat{U}^1 = \widehat{U}, \quad \widehat{U}^2 = \widehat{U}\widehat{\cdot}\widehat{U}, \quad \widehat{U}^3 = \widehat{U}^2\widehat{\cdot}\widehat{U}, \ldots, \widehat{U}^i = \widehat{U}^{i-1}\widehat{\cdot}\widehat{U}, \ldots$$

(which is called *isonilpotency series*), there exists a natural integer n such that $\widehat{U}^n = \{\widehat{S}\}$. The minor of such integers is denominated *nilpotency index* of the isoalgebra.

As an immediate consequence of this definition it is deduced that every isonilpotent Lie isoalgebra is isosolvable and that every nonzero isocommutative Lie isoalgebra has an isonilpotency index equals 2, being 1 the corresponding of the isoalgebra $\{\widehat{S}\}$. Moreover, they are verified:

PROPOSITION 3.2.C11: Under conditions of Proposition XXX, the isotopic lifting of a nilpotent Lie algebra is an isonilpotent isotopic Lie isoalgebra.

PROPOSITION 3.2.C12: Let $\widehat{U}$ be a Lie isoalgebra associated with a Lie algebra U. They are verified:

1) Under conditions of Proposition XXX, the sum of a finite number of isonilpotent isoideals of $\widehat{U}$ is another isonilpotent isoideal.

2) If $\widehat{U}$ is also isonilpotent and U is nilpotent, then

   (a) Every isosubalgebra of $\widehat{U}$ is isonilpotent.

   (b) Under conditions of Proposition XXX, if $\widehat{U}$ is nonzero isonilpotent, then its center is non null.

In a similar way as the case isosolvable, the result (1) involves that the sum of all isonilpotent isoideals of $\widehat{U}$ is another isonilpotent isoideal, which is denoted by *isonihil-radical* of $\widehat{U}$, to be distinguished from the nihil-radical of $\widehat{U}$, which is the sum of the radicals ideals. It will be represented by *isonil-rad* $\widehat{U}$, which allows to distinguish it from the *nil-rad* $\widehat{U}$. It is immediate that *isonil-rad* $\widehat{U} \subset$ *nil-rad* $\widehat{U} \cap$ *isorad* $\widehat{U} \subset$ *nil-rad* $\widehat{U} \subset$ *rad* $\widehat{U}$.

Apart from that, it is possible to relate an isosolvable isotopic Lie isoalgebra with its derived Lie isoalgebra, by using the following:

PROPOSITION 3.2.C13: Under conditions of Proposition XXX, a Lie isotopic isoalgebra is isosolvable if and only if its derived Lie isoalgebra is isonilpotent.

Finally, an isonilpotent Lie isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, \widehat{\cdot})$ is called *isofiliform* if, being an isotopy of a filiform Lie algebra, it is verified that

$$\dim \widehat{U}^2 = n - 2, \;\; \ldots, \;\; \dim \widehat{U}^i = n - i, \;\; \ldots, \;\; \dim \widehat{U}^n = 0,$$

where $\dim \widehat{U} = n$.

Note that the theory related with a filiform Lie algebra U is based on the use of a basis of such an algebra. So, starting from a basis $\{e_1, \ldots, e_n\}$ of U, which is preferably an *adapted basis*, we can deal with lots of concepts of it, such as dimensions of U and of elements of the nilpotency series, invariants i and j of U and, in general, the resting properties, starting from its structure coefficients, which are, in fact, responsible for the complete study of filiform Lie algebras.

Appendix 3.D
Lorentz versus Galileo-Roman Relativistic Symmetry

As indicated in Section 3.5.1, special relativity has remained unsettled after one century of studies, even in the arena of its original conception, namely, point–particles and electromagnetic waves propagating in vacuum. A reason of the ongoing debates is connected to the alternative of Lorentz invariance for the two-ways light experiments conducted to date, and the Galilean invariance expected for one-way light experiments.

The alternative of Lorentzian vs Galilean treatments is obscured by the fact that the former applies for relativistic speeds while the latter is not perceived as such. This limitation was resolved in the early 1970s by the *relativistic formulation of the Galilean symmetry and relativity* proposed by P. Roman, J. J. Aghassi and R. M. Santilli [76-78], and hereon called *Galileo-Roman symmetry and relativity.*[31]

In short, the alternative as to whether the ultimate relativity is of Lorentzian or Galilean type is far from being resolved. It is an easy prediction that such an alternative will not be resolved in these volumes. Consequently, in this appendix we can merely review the main ideas of the Galileo-Roman symmetry, and leave the resolution of the alternative to future generations.

By assuming an in depth knowledge of the Galileo symmetry and its scalar extension (that we cannot possibly review here), the Galileo-Roman symmetry is based on the following assumptions:

1) The carrier space is given by the Kronecker product of the conventional Minkowski space $M(x, \eta, R)$ times a one-dimensional space $U(u)$ where u represents the proper time normalized to the dimension of length for reason clarified below,

$$S_{tot} = M(3.1) \times U(u) \tag{3.D1}$$

[31] The name of "Galileo-Roman symmetry and relativity" is suggested because all basic concepts were originated by Paul Roman. In Refs. [75-77] and other papers, the author merely assisted Paul Roman in the technical elaboration of his views.

2) The Galileo-Roman symmetry is characterized by the following transformations

$$SO_o(3.1): \quad x^\mu \to x'^{\mu\,\prime} = \Lambda^\mu_\nu \times x^\nu, \quad \Lambda^B_\alpha eta \times \Lambda^\rho_\beta = \eta_a lpha^r ho, \tag{3.D2a}$$

$$T^a_4: \, : x^\mu \to x'^\mu = x^\mu + a^\mu, \tag{3.D2b}$$

$$T^b_4: \quad x^\mu \to x'^\mu = x^\mu + b^\mu \times u \tag{3.D2c}$$

$$T^\sigma_1: \quad u \to u' = u + \sigma, \tag{3.D2d}$$

where: Eqs. (3.D2a) are the (connected) conventional Lorentz transformations; Eqs. (3.D2b) are the conventional translations (with a^μ constants); Eqs. (3.D2c) and (3.D2d) are the new transformations with b^μ and σ non-null parameters, b^μ being dimensionless and σ having the dimension of length. Eqs. (3.D2c) were originally called *relativistic Galilean boosts,* [76] and here called *Galileo-Roman boosts,* since they are indeed a relativistic extension of the conventional nonrelativistic boosts. Eq. (3.D2d) was originally called the *relativistic Galilean time translation* [76],and it is here called the *Galileo-Roman time translation.*

3) The Galileo-Roman symmetry is then fifteen-dimensional and its connected component is written

$$GR = \{SO_o(3.1) \times T^b_4\} \times \{T^a_4 \times T^\sigma_1\}, \tag{3.D3}$$

where one should note: the presence of the Poincaré group as a subgroup; the presence of the conventional Galileo group as a subgroup; and the separation of conventional translations from the Lorentz symmetry and their association to the new variable u.

Group (3.D3) admits as an invariant subgroup the group $T^a_4 \times T^b_4 \times T^\sigma_1$. Hence, *the Galileo-Roman group (3.D3) is an extension of the restricted Lorentz group, but not of the Poincaré group,* even though the latter is also an extension of the Lorentz group. These are central features for the understanding of the differences between the Galileo symmetry, the Poincaré symmetry and the Galileo-Roman symmetry.

The conventional Galileo group requires a scalar extension for its dynamical application, and the same occurs for the Galileo-Roman group, thus leading to the covering

$$\widetilde{GR} = T^\theta_1 \times \{SL(2.C) \times T^b_4\} \times \{T^a_4 \times T^\sigma_1\}, \tag{3.D4}$$

where θ is the usual phase factor.

By denoting the generators of $SL(2.C)$ with $J_{\mu\nu}$, the generators of T^a_4 with $P_m u$, the generators of T^b_4 with Q_μ, and the generators of T^σ_1 with S, we have the following Lie algebra

$$[J_{\mu\nu}, J_{\rho\sigma}] = i \times (\eta_{\nu\rho} \times J_{\mu\sigma} - \eta_{mu\rho} \times J_{\nu\sigma} - \eta_{\mu\sigma} \times J_{\rho\nu} + \eta_{\nu\sigma} \times J_{\rho\mu}), \tag{3.D5a}$$

$$[P_\mu, J_{\rho\sigma}] = i \times (\eta_{\mu\rho} \times P_\sigma - \eta_{\mu\sigma} \times P_\rho), \quad (3.D5b)$$

$$[Q_\rho, J_{\mu\nu}] = i \times (\eta_{\mu\rho} \times Q_\nu - \eta_{\nu\rho} \times Q_\mu), \quad (3.D5c)$$

$$[P_\mu, Q_\nu] = i \times \eta_{\mu\nu} \times \ell^{-1}, \quad (3.D5d)$$

$$[S, Q_\nu] = i \times P_\nu, \quad (3.D5e)$$

$$[P_\mu, P_\nu] = [Q_\mu.Q_\nu] = [J_{\mu\nu}, S] = [P\mu, S] = 0, \quad (3.D5f)$$

where ℓ is the parameter originating from the scalar extension.

The physical interpretation is based on the following main aspects. Dynamics is assumed to verify the Galileo-Roman symmetry, with the Poincaré symmetry characterizing kinematics. Under such an assumption, the Galileo-Roman symmetry allows the introduction of a fully consistent *relativistic spacetime position operator* that is absent in relativistic quantum mechanics, with explicit expression

$$X_\mu = -\ell \times Q_\mu. \quad (3.D6)$$

In fact, the above interpretation is fully supported by commutation rules (3.D5).

Eq. (3.D6) introduces quite automatically a *universal length,* with the significant feature that *systems with different fundamental lengths are independent of each other.* The main dynamical invariant is no longer the familiar expression $P_\mu \times P^\mu = m^2$, but it is given instead by the following relativistic extension of the Galilean invariant

$$P_\mu \times P^\mu + 2 \times \ell{-}1 \times S = inv. \quad (3.D7)$$

By assuming the value

$$P_\mu \times P^\mu + 2 \times \ell{-}1 \times S = 0, \quad (3.D8)$$

the Galileo-Roman symmetry allows the introduction of the *relativistic mass operator*

$$\mathcal{M}^\in = \in \times \ell^{-\infty} \times \mathcal{S}. \quad (3.D9)$$

Note that the above definition is confirmed by commutation rules [3.D5) as well as from the fact that the above mass operator is invariant and a Lorentz scalar, as it should be. In particular, the eigenvalue of the above mass operator is the conventional scalar m^2 (see Ref. [76] for details). For a number of additional intriguing features of the Galileo-Roman symmetry, such as the nonlocality of the position operator "spread over" an area of radius ℓ, we have to refer the interested reader to paper [76] for brevity.

In closing with personal comments and recollections of these studies conducted some 37 years ago, there is no doubt that the Galileo-Roman group has dramatically more dynamical capabilities than the conventional Poincaré group. Also, to my best recollection, we could find no experimental data contradicting the Galileo-=Roman symmetry.

Yet, the novelty of the symmetry caused a real opposition furor among colleagues, namely, a reaction that has to be distinguished from proper scientific scrutiny. Part of the opposition was due to the political attachment to Einsteinian doctrines, but part was also due to the fact that the Galileo-Roman group required technical knowledge above the average of theoretical physicists of the time. [32]

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[32]The author would like to have a record of the following fully documented events.

The author initiated his Ph. D. research in the late 1960s at the Department of Physics of the University of Turin, Italy, with the Lie-admissible generalization of Lie's theory, A topic wastly unknown at thaty time in mathematics, let alone physics. Followeing the publication of his first paper in the field at il Nuovo Cimento of 1967, ref. [1] below, Tullio Regge, then head of that Physics Department and a self-qualified expert of Lie algebras, told in the author's face "you will never get an academic position in Italy." The author was subsequenrtly nominated by the Estonia Academy of Sciences for that paper amonmg the most illusttious applied mathematicians of all times (the only name of Italian origin in that list), but Regge's threat turned out to be true, and the author was forced to leave Italy after the publication of paper [1] for an academic job in the USA.

Being of Italian origin, the author filed his candidacy for the last session of the Italian "Libera Docenza" (Professorship in Physics) issued by the Italian Government, and did indeed participate in late 1974 jointly with a few other colleagues at this final session held at the University of "La Sapienza", Rome, Italy, and headed by V. De Alfaro of the University of Turin, Italy, P. Budini (now Budinovich) of the ICTP, Trieste, Italy, R. Gatto, of CERN and the University of Geneva, Switzerland (see other actions by Gatto quoted in Volume II), and others.

Even though the author constituted no threat for an academic job to their pupils in Italy (since at that time the author was Associate Professor of Physics at Boston University), and even though the author attempted to qualify himself as a scientists, thus requiring presentation of research jointly with a severe self-criticism for its limitations, the hostility by De Alfaro. Budini/Budinovich and Gatto against the research presented by the author, beginning with the relativistic extension of the Galilei group developed under the leadership of Paul Roman and with the collaboration of Jack J. Aghassi, was so furious that it turned into a rage rather inappropriate for heads of a governmental session.

In fact, De Alfaro, Budini/Budinovich and Gatto granted the "Libera Docenza" to all other participants except to the author, even though the author was the only one to held at that time an Associate Professorship in Physics at a major University in the U.S.A. and had a large record of publications, with no comparison by the other candidates, that are visible in the CV http://www.i-b-r.org/Ruggero-Maria-Santilli.htm and partly reproduced below. Additionally, by 1974 the author had received invitations for lectures at primary meetings in physics and mathematics; was teaching not only to a Graduate School in Physics in the U.S.A., but was also conducting post Ph. D. seminar courses for the colleagues in the Boston area in very advanced topics; was supervising Ph. D. students in the U.S.A. and had activity none of the other candidates could partially share. Among the publications available by 1974, we quote in chronological order:

1) The first formulation in 1967 of deformations of Lie algebras and first presentation in physics of Lie-admissible structures that subsequently lead to the construction of hadronic mechanics, Refs. [1,5,6,710] (note the publication by the Italian Physics Society and other major physics journals);

2) The relativistic extension of the Galilei group, with Paul Roman and Jack J. Aghassi, Refs. [14,-19] (note papers published by the American and Italian Physical Societies, among others);

3) The extension of the PCT Theorem to all discrete spacetime symmetries in quantum field theories, with Christos Ktorides, Refs. [22,23,25,27] (see the publication by the American Physical Society).

4) A severe, but gently written criticism of Einstein's gravitation showing in particular its incompatibility with quantum electrodynamics [24]

5) Various publications in in various fields with colleagues.

The author used to go (and continues to go) to Turin once or twice a year because of family ties. However, following the reception of (the equivalent at that time of today's) Ph. D. in physics in early 1967, the author never went back to the Department of Physics of the University of Turin, Italy, and he will never visit that department again for the rest of his life.

Another episode worth reporting is the following. The author had the privilege of frequent contacts with Nobel Laureate Abdus Salam, both personally and in his capacity as Director of the International Center for Theoretical Physics (ICTP) in Trieste, Italy. The author wants to honor his memory here with the view that Abdus Salam was one of the few "true scientists" of the 20-th century because of the dimension of his scientific vision combined with a serious commitment to scientific democracy for qualified inquiries.

In 1967, when the author had completed his Ph. D. studies and was about to leave Italy for the U.S.A., Abdus Salam invited him for a talk at the ICTP on paper [1] below dealing with the first presentation in physics of Lie-admissible covering (or deformations) of Lie algebras. At the end of the talk, Abdus Salam suggested the author to use the Lie-admissible algebras for possible advances in the hadronic structure, a suggestion that turned out to be prophetic.

The author kept periodically in contact with Abdus Salam and visited him at the ICTP through the years. In 1992 Abdus Salam was in the final stage of his unfortunate illness. Yet, he still managed the strength to invite the author for a series of joint mathematical and physical seminars entitled *Isotopic lifting of Galilei's relativity,* invitation that followed the appearance in 1991 of the two volumes by the author in the field. At the time of the first and and of the following seminars, the usually crowded lecture room at the ICTP was deserted, except for Abdus Salam, a scientist just arrived from Russia and two of the author friends (of the time). Subsequent investigations revealed that P. Budini/Budinovitch had requested the members of the ICTP and of the local university not to attend the author's seminars. The human and scientific difference in stature between Abdus Salam and the local crowd is set by the fact that the former, then unable to speak and at the edge of death, still had the scientific fire to listen to the last seminar of his life, while the local crowd abstained for fear of being muddied by new mathematics and physics.

Additional episodes have confirmed the existence in the Italian physics community of an unprecedented decay of scientific ethics that the author, being of Italian origin and education, feels obliged to denounce because occurring: without any visible denunciation by the Italian press; without the awareness of most Italian people; and without any visible containment whatsoever by responsible academic and political authorities, thanks to the complicity by the Italian press.

As indicated in Footnote 14 of Chapter 1, the author's works have been plagiarized so many times to generate the dubbing of the author as the "most plagiarized physicist of the 20-th century." Whether the occurrences were intentional or not, plagiarizing colleagues have been cooperative for corrections, essentially consisting in adding missed references of direct relevance in chronological order, *with the exception of Italian physicists* who have rejected the author's requests for simple quotation of prior works even when plagiarized identically including the symbols.

The lack of cooperation for corrections following plagiarisms, copyright infringements and paternity frauds was so incredible to force the author to file lawsuits in both the U. S. A. and Italy (see http://www.scientificethics.org). At any rate, evidence established that, among all scientists the world over, the author was forced to file lawsuits ONLY against Italian physicists and their backers.

The legal problems are escalating at this writing (November 7, 2007) because presidents and/or directors of the academic and governmental institutions involved in the lawsuits have refused any intervention in support of scientific ethics and accountability under public financial support, thus activating the Statute of *Respondeat Superior* for both, individual as well as institutions.

Additional serious shadows in the Italian physics community were caused by the take over in the early 1980s of the Italian Physical Society by Renato Angelo Ricci who systematically rejected hundreds of papers by the author and several independent colleagues in the various aspects presented herein. These systematic rejections lasted for over two decades, namely, from 1983 until the replacement of Renato Angelo Ricci as president at the turn of the century. The problems for the Italian physics community were not caused by the rejections *per se,* but by their motivation carrying Ricci's signature, such as "Your paper is rejected because the research is not accepted by Harvard University as your former affiliation." This established that Ricci was obeying orders from Harvard University and, in turn provided additional documentation, this time from Italy, of the scientific misconduct by Harvard University denounced and documented in Refs. [93,94].

As a result of all these extremely unpleasant experiences, the author is now reluctant to have any scientific exchange with Italian colleagues for fear that, following additional release of technical information, suggestions and material, there are additional plagiarisms, copyright infringements and paternity frauds forcing the filing of additional lawsuit, since the reported behavior appears to be normal in the current Italian physics community and, in any case, it is not denounced by the Italian press or opposed by Italian authorities, as documented in court beyond credible doubt.

Despite all the above, negative judgments are *a priori* wrong unless expressed with due exceptions, and this is particularly true for Italy due to the complexity and diversification of its culture. In fact, the author is sincerely pleased to report that his most important physics papers were published by the Italian Physical Society up to 1983 and then, again, new basic publications after the removal of Renato Angelo Ricci as president in the early 2000s. Similarly, the author is sincerely pleased to report that his most important mathematical papers were published by the Rendiconti Circolo Matematico di Palermo. If scientific ethics is implemented with the quotation of the original contributions in chronological order, other physical and mathematical societies have to follow the above identified leadership of the Italian societies.

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PUBLICATIONS BY R. M. SANTILLI AS OF 11974:

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[2] R. M. Santilli, Causality groups of the S-matrix, Nuovo Cimento 55B, 578-586 (l968) Seminar delivered at the University of S. Carolina, Columbia

[3] R. M. Santilli, Phase space symmetries of a relativistic plasma, Nuovo Cimento Vol. 56B, 323-326 (l968)

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Such huge an opposition essentially forced the author to abandon the studies in the field, a decision that he regretted later, but could not change at that time due to the need in the 1970s for the author to secure an academic position so as to feed and shelter two children in tender age and his wife.

During the 37 years that have passed since that time, the author discovered numerous theories published in the best technical journals that, in reality, did verify the Galileo-Roman symmetry, but were published as verifying the conventional Poincaré symmetry. All attempts by the author for editorial corrections turned out to be useless. That was unfortunate for the fully deserved continuation of Paul Romans name in science.

In this way, the author was exposed for to the academic rage caused by novelty and, in so doing, he acquired the necessary strength to resist academic disruptions when he proposed the construction of hadronic mechanics in 1978 [4]. In this way, the human experience gained by the author during his studies of the Galileo-Roman symmetry and relativity proved to be crucial for the proposal and continuation of the studies on hadronic mechanics again hardly credible obstructions, oppositions and disruptions.

Yet, the author hopes that studies on the Galileo-Roman symmetry and relativity are indeed continued by new generations of physicists, not only because of the dramatic richness of content compared to the Poincaré sub-symmetry, but also because the Galileo-Roman symmetry and its easily derivable isotopic extension appear to possess the necessary ingredients for a solution of the numerous

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unresolved problems of special relativity, including compatibility with the ultimate frontier of knowledge: space.[33]

[33]In the author's view, this may happen only when society will one future day understand the importance of scientific democracy for qualified inquiries.

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Chapter 4

LIE-ADMISSIBLE BRANCH OF HADRONIC MECHANICS AND ITS ISODUAL

NOTE; THIS CHAPTER MUST BE COMPLETED AND EDITED

4.1 INTRODUCTION

4.1.1 The Scientific Imbalance Caused by Irreversibility

As recalled in Chapter 1, physical, chemical or biological systems are called *irreversible* when their images under time reversal $t \rightarrow -t$ are prohibited by causality and/or other laws, as it is generally the case for nuclear transmutations, chemical reactions and organism growth.

Systems are called *reversible* when their time reversal images are as causal as the original ones, as it is the case for planetary and atomic structures when considered isolated from the rest of the universe, the structure of crystals, and other structures (see reprint volume [1] on irreversibility and vast literature quoted therein).

Another large scientific imbalance of the 20-th century studied in these monographs is the treatment of irreversible systems via the mathematical and physical formulations developed for reversible systems, since these formulations are themselves reversible, thus resulting in serious limitations in virtually all branches of science.

The problem is compounded by the fact that all used formulations are of Hamiltonian type, under the awareness that all known Hamiltonians are reversible over time (since all known potentials, such as the Coulomb potential $V(r)$, etc., are reversible).

This scientific imbalance was generally dismissed in the 20-th century with unsubstantiated statements, such as “irreversibility is a macroscopic occurrence that disappears when all bodies are reduced to their elementary constituents”.

Figure 4.1. All energy releasing processes are irreversible over time. By contrast, all formulations of the 20th century are fully reversible over time, a limitation that is apparently responsible for the lack of industrial development of any really new form of energy for over half a century, as well as the lack of resolution of the environmental problems caused by fossil fuels combustion depicted in this figure. A primary objective of hadronic mechanics is, firstly, identify formulations that are structurally irreversible (a task addressed in this chapter), as a necessary premise for their quantitative treatment of irreversible process and the search of basically new energies (a task address in Volume II).

These academic beliefs have been disproved by Theorem 1.3.3 according to which *a classical irreversible system cannot be consistently decomposed into a finite number of elementary constituents all in reversible conditions and, vice-versa, a finite collection of elementary constituents all in reversible conditions cannot yield an irreversible macroscopic ensemble.*

The implications of the above theorem are quite profound because it establishes that, contrary to popular beliefs, *irreversibility originates at the most primitive levels of nature, that of elementary particles, and then propagates all the way to our macroscopic environment.*

In this chapter we study the contribution by the author that originated the field, as well as contributions by a number of independent authors. The presentation will mainly follow the recently published memoir [32]. Nevertheless, an in depth knowledge of the topic requires the study of (at least some of) the author's monographs [18–23,29] and those by independent authors [33–39].

The author would like to express his sincere appreciation to the *Italian Physical Society* for publishing memoir [32] in *Il Nuovo Cimento B* as a final presentation of studies in the field initiated by the author in the same Journal in paper [7] forty years earlier.

4.1.2 The Forgotten Legacy of Newton, Lagrange and Hamilton

The scientific imbalance on irreversibility was created in the early part of the 20-th century when, to achieve compatibility with quantum mechanics and special relativity, the entire universe was reduced to potential forces. Jointly, the analytic equations were "truncated" with the removal of the external terms.

In reality, Newton [2] *did not* propose his celebrated equations restricted to forces derivable from a potential $F = \partial V/\partial r$, but proposed them for the most general possible forces,

$$m_a \times \frac{dv_{ka}}{dt} = F_{ka}(t, r, v), \quad k = 1, 2, 3; \quad a = 1, 2, \ldots, N, \tag{4.1.1}$$

where the conventional associative product of numbers, matrices, operators, etc. is continued to be denoted hereon with the symbol $\times$ so as to distinguish it from numerous other products needed later on.

Similarly, to be compatible with Newton's equations, Lagrange [3] and Hamilton [4] decomposed Newton's force into a potential and a nonpotential component,they represented all potential forces with functions today known as the Lagrangian and the Hamiltonian, and proposed their celebrated equations with external terms,

$$\frac{d}{dt}\frac{\partial L(t, r, v)}{\partial v_a^k} - \frac{\partial L(t, r, v)}{\partial r_a^k} = F_{ak}(t, r, v), \tag{4.1.2a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t, r, p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t, r, p)}{\partial r_a^k} + F_{ak}(t, r, p), \tag{4.1.2b}$$

$$L = \Sigma_a \frac{1}{2} \times m_a \times v_a^2 - V(t, r, v), \quad H = \Sigma_a \frac{\mathbf{p}_a^2}{2 \times m_a} + V(t, r, p), \tag{4.1.2c}$$

$$V = U(t, r)_{ak} \times v_a^k + U_o(t, r), \quad F(t, r, v) = F(t, r, p/m). \tag{4.1.2d}$$

More recently, Santilli [5] conducted comprehensive studies on the integrability conditions for the existence of a potential or a Lagrangian or a hamiltonian, called *conditions of variational selfadjointness.* These study permit the rigorous decomposition of Newtonian forces into a component that is variationally selfadjoint (SA) and a component that is not (NSA),

$$m_a \times \frac{dv_{ka}}{dt} = F_{ka}^{SA}(t, r, v) + F_{ka}^{NSA}(t, r, v). \tag{4.1.3}$$

Consequently, the true Lagrange and Hamilton equations can be more technically written

$$\Big[\frac{d}{dt}\frac{\partial L(t, r, v)}{\partial v_a^k} - \frac{\partial L(t, r, v)}{\partial r_a^k}\Big]^{SA} = F_{ak}^{NSA}(t, r, v), \tag{4.1.4a}$$

$$\Big[\frac{dr_a^k}{dt}-\frac{\partial H(t,r,p)}{\partial p_{ak}}\Big]^{SA}=0,\quad \Big[\frac{dp_{ak}}{dt}+\frac{\partial H(t,r,p)}{\partial r_a^k}\Big]^{SA}=F_{ak}^{NSA}(t,r,p). \quad (4.1.4b)$$

The *forgotten legacy of Newton, Lagrange and Hamilton is that irreversibility originates precisely in the truncated NSA terms,* because all known potential-SA forces are reversible. The scientific imbalance of Section 1.3 is then due to the fact that no serious scientific study on irreversibility can be done with the truncated analytic equations and their operator counterpart, since these equations can only represent reversible systems.

Being born and educated in Italy, during his graduate studies at the University of Torino, the author had the opportunity of studying in the late 1960s the original works by Lagrange that were written precisely in Torino and most of them in Italian.

In this way, the author had the opportunity of verifying *Lagrange's analytic vision of representing irreversibility precisely via the external terms,* due to the impossibility of representing all possible physical events via the sole use of the Lagrangian, since the latter was solely conceived for the representation of reversible and potential events. As the reader can verify, Hamilton had, independently, the same vision.

Consequently, the truncation of the basic analytic equations caused the impossibility of a credible treatment of irreversibility at the purely classical level. The lack of a credible treatment of irreversibility then propagated at the subsequent operator level.

It then follows that *quantum mechanics cannot possibly be used for serious studies on irreversibility* because the discipline was constructed for the description of reversible quantized atomic orbits and not for irreversible systems.

In plain terms, while the validity of quantum mechanics for the arena of its original conception and verification is beyond scientific doubt, the assumption of quantum mechanics as the final operator theory for all conditions existing in the universe is outside the boundaries of serious science.

This establishes the need for the construction of a broadening (or generalization here called *lifting*) of quantum mechanics specifically conceived for quantitative studies of irreversibility. Since reversible systems are a *particular case* of irreversible ones, the broader mechanics must be a *covering* of quantum mechanics, that is, admitting the latter under a unique and unambiguous limit.

It is easy to see that the needed broader mechanics must also be a covering of the isotopic branch of hadronic mechanics studied in the preceding chapter, thus being a new branch of hadronic mechanics. In fact, isomechanics is itself structurally reversible due to the Hermiticity of both the Hamiltonian, $\hat{H}=\hat{H}^\dagger$, and of the isotopic element, $\hat{T}=\hat{T}^\dagger$, while a serious study of irreversible processes requires a *structurally irreversible mechanics,* that is, a mechanics that is irreversible for all possible reversible Hamiltonians.[1]

4.1.3 Early Representations of Irreversible Systems

As reviewed in Section 1.5.2, the brackets of the time evolution of an observable $A(r, p)$ in phase space according to the analytic equations with external terms,

$$\frac{dA}{dt} = (A, H, F) = \frac{\partial A}{\partial r^k_a} \times \frac{\partial H}{\partial p_{ka}} - \frac{\partial H}{\partial r^k_a} \times \frac{\partial A}{\partial p_{ka}} + \frac{\partial A}{\partial r^k_a} \times F_{ka}, \tag{4.1.5}$$

violate the right associative and scalar laws.

Therefore, the presence of external terms in the analytic equations causes not only the loss of *all* Lie algebras in the study of irreversibility, but actually causes the loss of all possible algebras as commonly understood in mathematics.

To resolve this problem, the author initiated a long scientific journey beginning with his graduate studies at the University of Torino, Italy, following the reading of Lagrange's papers.

The original argument [7–9], still valid today, is to select analytic equations characterizing brackets in the time evolution verifying the following conditions:

(1) The brackets of the time evolution must verify the right and left associative and scalar laws to characterize an algebra;

(2) Said brackets must not be invariant under time reversal as a necessary condition to represent irreversibility *ab initio;*

(3) Said algebra must be a covering of Lie algebras as a necessary condition to have a covering of the truncated analytic equations, namely, as a condition for the selected representation of irreversibility to admit reversibility as a particular case.

Condition (1) requires that said brackets must be bilinear, e.g., of the form (A, B) with properties

$$(n \times A, B) = n \times (A, B), \quad (A, m \times B) = m \times (A, B); \quad n, m \in C, \tag{4.1.6a}$$

$$(A \times B, C) = A \times (B, C), \quad (A, B \times C) = (A, B) \times C. \tag{4.1.6b}$$

Condition (2) requires that brackets (A, B) should not be totally antisymmetric as the conventional Poisson brackets,

$$(A, B) \neq -(B, A), \tag{4.1.7}$$

because time reversal is realized via the use of Hermitian conjugation.

[1] An exception to this general rule we shall study later on occurs when the isotopic elements is indeed Hermitian, but explicitly dependent on time and such that $\hat{T}(t, \ldots) \neq \hat{T}(-t, \ldots)$.

Condition (3) then implies that brackets (A, B) characterize *Lie-admissible algebras* in the sense of Albert [10], namely, the brackets are such that the attached antisymmetric algebra is Lie.[2]

$$[A, B]^* = (A, B) - (B, A) = Lie. \tag{4.1.8}$$

In particular, the latter condition implies that the new brackets are formed by the superposition of totally antisymmetric and totally symmetric brackets,

$$(A, B) = [A, B]^* + \{A, B\}^*. \tag{4.1.9}$$

It should be noted that the operator realization of brackets (A, B) is also *Jordan-admissible* in the sense of Albert [10], namely, the attached symmetric brackets $\{A, B\}^*$ characterize a *Jordan algebra.* Consequently, *hadronic mechanics provides a realization of Jordan's dream, that of seeing his algebra applied to physics.*

However, the reader should be aware that, for certain technical reasons beyond the scope of this monograph, the classical realizations of brackets (A, B) are Lie-admissible but not Jordan-admissible. Therefore, Jordan-admissibility appears to emerge exclusively for operator theories.[3]

After identifying the above lines, Santilli [9] proposed in 1967 the following *generalized analytic equations*

$$\frac{dr_a^k}{dt} = \alpha \times \frac{\partial H(t, r, p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\beta \times \frac{\partial H(t, r, p)}{\partial r_a^k}, \tag{4.1.10}$$

(where α and β are real non-null parameters) that are manifestly irreversible. The brackets of the time evolution are then given by

$$i \times \frac{dA}{dt} = (A, H) =$$

---

[2]More technically, a generally nonassociative algebra U with elements a, b, c, ... and abstract product ab is said to be Lie-admissible when the attached algebra U^- characterized by the product $[a, b] = ab - ba$ verifies the *Lie axioms*

$$[a, b] = -[b, a],$$
$$[[a, b], c] + [[b, c], a] + [[c, b], a] = 0.$$

[3]More technically, a generally nonassociative algebra U with elements a, b, c, ... and abstract product ab is said to be Jordan-admissible when the attached algebra U^+ characterized by the product $\{a, b\} = ab + ba$ verifies the *Jordan axioms*

$$\{a, b\} = \{b, a\},$$
$$\{\{a, b\}, a^2\} = \{a, \{b, a^2\}\}.$$

In classical realizations of the algebra U the first axiom of Jordan-admissibility is generally verified but the second is generally violated, while in operator realizations both axioms are generally verified.

$$= \alpha \times \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}} - \beta \times \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ka}}, \tag{4.1.11}$$

whose brackets are manifestly Lie-admissible, but *not* Jordan-admissible as the interested reader is encouraged to verify.

The above analytic equations characterize the time-rate of variation of the energy

$$\frac{dH}{dt} = (\alpha - \beta) \times \frac{\partial H}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}}. \tag{4.1.12}$$

Also in 1967, Santilli [7,8] proposed an operator counterpart of the preceding classical setting consisting in the first known *Lie-admissible parametric generalization of Heisenberg's equation,* also called *deformed Heisenberg equations,*[4] in the following infinitesimal form

$$i \times \frac{dA}{dt} = (A, B) = p \times A \times H - q \times H \times A =$$

$$= m \times (A \times B - B \times A) + n \times (A \times B + B \times A), \tag{4.1.13a}$$

$$m = p + q, \quad n = q - p, \tag{4.1.13b}$$

where $p, q, p \pm q$ are non-null parameters, with finite counterpart

$$A(t) = e^{i \times H \times q} \times A(0) \times e^{-i \times p \times H}. \tag{4.1.14}$$

Brackets (A, B) are manifestly Lie-admissible with attached antisymmetric part

$$[A, B]^* = (A, B) - (B, A) = (p - q) \times [A, B]. \tag{4.1.15}$$

The same brackets are also Jordan-admissible in view of the property

$$\{A, B\}^* = (A, B) + (B, A) = (p + q) \times \{A, B\}, \tag{4.1.16}$$

The resulting time evolution is then manifestly irreversible (for $p \neq q$) with nonconservation of the energy

$$i \times \frac{dH}{dt} = (H, H) = (p - q) \times H \times H \neq 0, \tag{4.1.17}$$

as necessary for an open system.

Subsequently, Santilli realized that the above formulations are not invariant under their own time evolution (4.1.14) because Eqs. (4.1.11) are manifestly *nonunitary.*

[4]As we shall soon see, the term "deformed" is used for formulations that are catastrophically inconsistent because dreaming to treat new theories with the mathematics of the old ones.

The application of nonunitary transforms to brackets (4.1.12) then led to the proposal in memoir [11,12] of 1978 of the following *Lie-admissible operator generalization of Heisenberg equations* in their infinitesimal form

$$i \times \frac{dA}{dt} = A \times P \times H - H \times Q \times A = (A, H)^*, \tag{4.1.18}$$

with finite counterpart

$$A(t) = e^{i \times H \times Q} \times A(0) \times e^{-i \times P \times H}, \tag{4.1.19}$$

under the subsidiary conditions needed for consistency, as we shall see,

$$P = Q^\dagger, \tag{4.1.20}$$

where P, Q and $P \pm Q$ are now nonsingular operators (or matrices), and Eq. (4.1.16b) is a basic consistency condition explained later in this section.

Eqs. (4.1.18)–(4.1.19) are the *fundamental equations of hadronic mechanics.* Their basic brackets are manifestly Lie-admissible and Jordan admissible with structure

$$(A, B)^* = A \times P \times B - B \times Q \times A =$$

$$= (A \times T \times B - B \times T \times A) + (A \times R \times B + B \times R \times A), \tag{4.1.21a}$$

$$T = P + Q, \quad R = Q - P. \tag{4.1.21b}$$

As indicated in Section 1.5.2, it is easy to see that the application of a nonunitary transform to the parametric brackets of Eqs. (4.1.11) leads precisely to the operator brackets of Eqs. (4.1.17),

$$U \times (p \times A \times B - q \times B \times A) \times U^\dagger = \hat{A} \times P \times \hat{B} - \hat{B} \times Q \times \hat{A}, \tag{4.1.22a}$$

$$U \times U^\dagger \neq I, P = p \times (U \times (U^\dagger)^{-1}, Q = q \times (U \times U^\dagger)^{-1}, \hat{A} = U \times A \times U^\dagger. \tag{4.1.22b}$$

In particular, the application of any (nonsingular) nonunitary transforms preserves the Lie-admissible and Jordan-admissible characters. Consequently, fundamental equations (4.1.18), (4.1.19) are "directly universal" in the sense of Lemma 1.5.2.

However, the above equations *are not invariant* under their own (nonunitary) time evolution,

$$U \times (\hat{A} \times P \times \hat{B} - \hat{B} \times Q \times \hat{A},) \times U^\dagger = \hat{A}' \times P' \times \hat{B}' - \hat{B}' \times Q' \times \hat{A}', \tag{4.1.23}$$

where the lack of invariance is expressed by the lack of preservation of the numerical values of the P, Q operators because, as we shall see shortly, these operators characterize new multiplications.

By comparison, quantum mechanical brackets are indeed invariant under the class of admitted transformations, the unitary transforms

$$W \times A \times B - B \times A) \times W^{\dagger} = A' \times B' - B' \times A', \tag{4.1.24a}$$

$$W \times W^{\dagger} = W^{\dagger} \times W = I, A' = W \times A \times W^{\dagger}, B' = W \times B \times W^{\dagger}, \tag{4.1.24b}$$

where the invariance we are here referring to is expressed by the preservation of the associative product, namely, $A \times B$ *is not* mapped into a different product, say $A' * B'$.

As known to experts of quantum mechanics (to qualify as such), simple invariance (4.1.24) is at the foundations of the majestic axiomatic consistency of quantum mechanics, including: the prediction of the same numerical values under the same conditions at different times; the preservation of Hermiticity and, thus, of observables over time; and other basic features.

Consequently, Lie-admissible and Jordan admissible equations (4.1.18)–(4.1.19) are afflicted by the catastrophic inconsistencies of Theorem 1.5.2, as it is the fate for all nonunitary theories some of which are listed in Section 1.5. In particular, said equations do not preserve numerical predictions under the same conditions but at different times, do not preserve Hermiticity, thus do not admit observables, and have other catastrophic inconsistencies studied in detail in Section 1.5.

Moreover, in the form presented above, the dynamical equations are not derivable from a variational principle. Consequently, they admit no known unique map from classical into operator formulations.

In view of these insufficiencies, said equations cannot be assumed in the above given form as the basic equations of any consistent physical theory.

4.2 ELEMENTS OF SANTILLI GENOMATHEMATICS AND ITS ISODUAL

4.2.1 Genounits, Genoproducts and their Isoduals

The "direct universality" of Eqs. (4.1.18), (4.1.19) voids any attempt at seeking further generalizations in the hope of achieving invariance, since any nontrivial generalization would suffer the loss of any algebra in the brackets of the time evolution, with consequential inability to achieve any physically meaningful theory, e.g., because of the inability to treat the spin of a proton under irreversible conditions.

This occurrence leaves no alternative other than that of seeking a yet *new mathematics* permitting Eqs. (4.1.18), (4.1.19) to achieve the needed invariance.

After numerous attempts and a futile search in the mathematical literature of the Cantabrigian area,[5] Santilli proposed in Refs. [11,12] of 1978 the construction of a new mathematics specifically conceived for the indicated task, that eventually reached mathematical maturity for numbers only in paper [13] of 1993, mathematical maturity for the new differential calculus only in memoir [14] of 1996, and, finally, an invariant formulation of Lie-admissible equations only in paper [15] of 1997.

The new Lie-admissible mathematics is today known as *Santilli genomathematics*, where the prefix "geno" suggested in the original proposal [11,12] is used in the Greek meaning of "inducting" new axioms (as compared to the prefix "iso" of the preceding chapter denoting the preservation of the axioms).

The basic idea is to lift the isounits of the preceding chapter into a form that is still nowhere singular, but *non-Hermitian*, thus implying the existence of *two* different generalized units, today called *Santilli genounits* for the description of matter, that are generally written [13]

$$\hat{I}^{>} = 1/\hat{T}^{>}, \quad {}^{<}\hat{I} = 1/{}^{<}\hat{T}, \tag{4.2.1a}$$

$$\hat{I}^{>} \neq {}^{<}\hat{I}, \quad \hat{I}^{>} = ({}^{<}\hat{I})^{\dagger}, \tag{4.2.1b}$$

with two additional *isodual genounits* for the description of antimatter [14]

$$(\hat{I}^{>})^{d} = -(\hat{I}^{>})^{\dagger} = -{}^{<}\hat{I} = -1/{}^{<}\hat{T}, \quad ({}^{<}\hat{I})^{d} = -\hat{I}^{>} = -1/\hat{T}^{>}. \tag{4.2.2}$$

Jointly, all conventional and/or isotopic products $A\hat{\times}B$ among generic quantities (numbers, vector fields, operators, etc.) are lifted in such a form admitting the genounits as the correct left and right units at all levels, i.e.,

$$A > B = A \times \hat{T}^{>} \times B, \quad A > \hat{I}^{>} = \hat{I}^{>} > A = A, \tag{4.2.3a}$$

$$A < B = A \times {}^{<}\hat{T} \times B, \quad A < {}^{<}\hat{I} = {}^{<}\hat{I} < A = A, \tag{4.2.3b}$$

$$A >^{d} B = A \times \hat{T}^{>d} \times B, \quad A >^{d} \hat{I}^{>d} = \hat{I}^{>d} >^{d} A = A, \tag{4.2.3c}$$

$$A <^{d} B = A \times {}^{<}\hat{T}^{d} \times B, \quad A <^{d} {}^{<}\hat{I}^{d} = {}^{<}\hat{I}^{d} <^{d} A = A, \tag{4.2.3d}$$

for all elements A, B of the set considered.

As we shall see in Section 4.3, the above basic assumptions permit the representation of irreversibility with the most primitive possible quantities, the basic units and related products.

In particular, as we shall see in Section 4.3 and 4.4, genounits permit an invariant representation of the external forces in Lagrange's and Hamilton's

[5] Conducted in the period 1977–1978.

equations (4.1.2). As such, genounits are generally dependent on time, coordinates, momenta, wavefunctions and any other needed variable, e.g., $\hat{I}^> = \hat{I}^>(t^>, r^>, p^>, \psi^>, \ldots)$.

In fact, the assumption of all *ordered product to the right* $>$ represents matter systems moving forward in time, the assumption of all *ordered products to the left* $<$ represents matter systems moving backward in time, with the irreversibility being represented *ab initio* by the inequality $A > B \neq A < B$. Similar representation of irreversible antimatter systems occurs via isodualities.

4.2.2 Genonumbers, Genofunctional Analysis and Their Isoduals

Genomathematics began to reach maturity with the discovery made, apparently for the first time in paper [13] of 1993, that *the axioms of a field still hold under the ordering of all products to the right or, independently, to the left.*

This unexpected property permitted the formulation of *new numbers,* that can be best introduced as a generalization of the isonumbers [18], although they can also be independently presented as follows:

DEFINITION 4.2.1 [13]: Let $F = F(a, +, \times)$ be a field of characteristic zero as per Definitions 2.1.1 and 3.2.1. Santilli's forward genofields are rings $\hat{F}^> = \hat{F}(\hat{a}^>, \hat{+}^>, \hat{\times}^>)$ with elements

$$\hat{a}^> = a \times \hat{I}^>, \tag{4.2.4}$$

where $a \in F$, $\hat{I}^> = 1/\hat{T}^>$ is a non singular non-Hermitian quantity (number, matrix or operator) generally outside F and $\times$ is the ordinary product of F; the genosum $\hat{+}^>$ coincides with the ordinary sum $+$,

$$\hat{a}^> \hat{+}^> \hat{b}^> \equiv \hat{a}^> + \hat{b}^>, \ \forall \hat{a}^>, \ \hat{b}^> \in \hat{F}^>, \tag{4.2.5}$$

consequently, the additive forward genounit $\hat{0}^> \in \hat{F}$ coincides with the ordinary $0 \in F$; and the forward genoproduct $>$ is such that $\hat{I}^>$ is the right and left isounit of $\hat{F}^>$,

$$\hat{I}^> \hat{\times} \hat{a}^> = \hat{a}^> > \hat{I}^> \equiv \hat{a}^>, \ \forall \hat{a}^> \in \hat{F}^>. \tag{4.2.6}$$

Santilli's forward genofields verify the following properties:

1) For each element $\hat{a}^> \in \hat{F}^>$ there is an element $\hat{a}^{>-\hat{1}^>}$, called forward genoinverse, for which

$$\hat{a}^> > \hat{a}^{>-\hat{I}^>} = \hat{I}^>, \ \forall \hat{a}^> \in \hat{F}^>; \tag{4.2.7}$$

2) The genosum is commutative

$$\hat{a}^> \hat{+}^> \hat{b}^> = \hat{b}^> \hat{+}^> \hat{a}^>, \tag{4.2.8}$$

and associative

$$(\hat{a}^{>}\hat{+}^{>}\hat{b}^{>}) +^{>} \hat{c}^{>} = \hat{a}^{>}\hat{+}^{>}(\hat{b}^{>}\hat{+}^{>}\hat{c}^{>}), \;\; \forall \hat{a}, \hat{b}, \hat{c} \in \hat{F}; \tag{4.2.9}$$

3) The forward genoproduct is associative

$$\hat{a}^{>} > (\hat{b}^{>} > \hat{c}^{>}) = (\hat{a}^{>} > \hat{b}^{>}) > \hat{c}^{>}, \quad \forall \hat{a}^{>}, \hat{b}^{>}, \hat{c}^{>} \in \hat{F}^{>}, \tag{4.2.10}$$

but not necessarily commutative

$$\hat{a}^{>} > \hat{b}^{>} \neq \hat{b}^{>} > \hat{a}^{>}; \tag{4.2.11}$$

4) The set $\hat{F}^{>}$ is closed under the genosum,

$$\hat{a}^{>}\hat{+}^{>}\hat{b}^{>} = \hat{c}^{>} \in \hat{F}^{>}, \tag{4.2.12}$$

the forward genoproduct,

$$\hat{a}^{>} > \hat{b}^{>} = \hat{c}^{>} \in \hat{F}^{>}, \tag{4.2.13}$$

and right and left genodistributive compositions,

$$\hat{a}^{>} > (\hat{b}^{>}\hat{+}^{>}\hat{c}^{>}) = \hat{d}^{>} \in \hat{F}^{>}, \tag{4.2.14a}$$

$$(\hat{a}^{>}\hat{+}^{>}\hat{b}^{>}) > \hat{c}^{>} = \hat{d}^{>} \in \hat{F}^{>} \quad \forall \hat{a}^{>}, \hat{b}^{>}, \hat{c}^{>}, \hat{d}^{>} \in \hat{F}^{>}; \tag{4.2.14b}$$

5) The set $\hat{F}^{>}$ verifies the right and left genodistributive law

$$\hat{a}^{>} > (\hat{b}^{>}\hat{+}^{>}\hat{c}^{>}) = (\hat{a}^{>}\hat{+}^{>}\hat{b}^{>}) > \hat{c}^{>} = \hat{d}^{>}, \;\; \forall \hat{a}^{>}, \hat{b}^{>}, \hat{c}^{>}, \in \hat{F}^{>}. \tag{4.2.15}$$

In this way we have the forward genoreal numbers $\hat{R}^{>}$, the forward genocomplex numbers $\hat{C}^{>}$ and the forward genoquaternionic numbers $\hat{Q}C^{>}$ while the forward genooctonions $\hat{O}^{>}$ can indeed be formulated but they do not constitute genofields [14].

The backward genofields and the isodual forward and backward genofields are defined accordingly. Santilli's genofields are called of the first (second) kind when the genounit is (is not) an element of F.

The basic axiom-preserving character of genofields is illustrated by the following:

LEMMA 4.2.1 [13]: Genofields of first and second kind are fields (namely, they verify all axioms of a field).

Note that the conventional product "2 multiplied by 3" is not necessarily equal to 6 because, for isodual numbers with unit -1 it is given by -6 [13].

The same product "2 multiplied by 3" is not necessarily equal to +6 or −6 because, for the case of isonumbers, it can also be equal to an arbitrary number, or a matrix or an integrodifferential operator depending on the assumed isounit [13].

In this section we point out that "2 multiplied by 3" can be ordered to the right or to the left, and the result is not only arbitrary, but yielding different numerical results for different orderings, $2 > 3 \neq 2 < 3$, all this by continuing to verify the axioms of a field per each order [13].

Once the forward and backward genofields have been identified, the various branches of genomathematics can be constructed via simple compatibility arguments.

For specific applications to irreversible processes there is first the need to construct the *genofunctional analysis,* studied in Refs. [6,18] that we cannot review here for brevity. The reader is however warned that any elaboration of irreversible processes via Lie-admissible formulations based on conventional or isotopic functional analysis leads to catastrophic inconsistencies because it would be the same as elaborating quantum mechanical calculations with genomathematics.

As an illustration, Theorems 1.5.1 and 1.5.2 of catastrophic inconsistencies are activated unless one uses the ordinary differential calculus lifted, for ordinary motion in time of matter, into the following *forward genodifferentials and genoderivatives*

$$\hat{d}^{>}x = \hat{T}^{>}_{x} \times dx, \quad \frac{\hat{\partial}^{>}}{\hat{\partial}^{>}x} = \hat{I}^{>}_{x} \times \frac{\partial}{\partial x}, \text{ etc,} \tag{4.2.16}$$

with corresponding backward and isodual expressions here ignored.

Similarly, all conventional functions and isofunctions, such as isosinus, isocosinus, isolog, etc., have to be lifted in the genoform

$$\hat{f}^{>}(x^{>}) = f(\hat{x}^{>}) \times \hat{I}^{>}, \tag{4.2.17}$$

where one should note the necessity of the multiplication by the genounit as a condition for the result to be in $\hat{R}^{>}$, $\hat{C}^{>}$, or $\hat{O}^{>}$.

4.2.3 Genogeometries and Their Isoduals

Particularly intriguing are the *genogeometries* [16] (see also monographs [18] for detailed treatments). They are best characterized by a simple genotopy of the isogeometries, although they can be independently defined.

As an illustration, the *Minkowski-Santilli forward genospace* $\hat{M}^{>}(\hat{x}^{>}, \hat{\eta}^{>}, \hat{R}^{>})$ over the genoreal $\hat{R}^{>}$ is characterized by the following spacetime, *genocoordinates, genometric and genoinvariant*

$$\hat{x}^{>} = x\hat{I}^{>} = \{x^{\mu}\} \times \hat{I}^{>}, \quad \hat{\eta}^{>} = \hat{T}^{>} \times \eta, \quad \eta = Diag.(1,1,1,-1), \tag{4.2.18a}$$

$$\hat{x}^{>2^{>}} = \hat{x}^{>\mu} \hat{\times}^{>} \hat{\eta}^{>}_{\mu\nu} \hat{\times}^{>} \hat{x}^{>\nu} = (x^{\mu} \times \hat{\eta}^{>}_{\mu\nu} \times x^{\nu}) \times \hat{I}^{>}, \tag{4.2.18b}$$

where the first expression of the genoinvariant is on genospaces while the second is its projection in our spacetime.

Note that the Minkowski-Santilli genospace has, in general, an explicit dependence on spacetime coordinates. Consequently, it is equipped with the entire formalism of the conventional Riemannian spaces covariant derivative, Christoffel's symbols, Bianchi identity, etc. only lifted from the isotopic form of the preceding chapter into the genotopic form.

A most important feature is that *genospaces permit, apparently for the first time in scientific history, the representation of irreversibility directly via the basic genometric.* This is due to the fact that genometrics are nonsymmetric by conception, e.g.,

$$\hat{\eta}^{>}_{\mu\nu} \quad \neq \quad \hat{\eta}^{>}_{\nu\mu}. \tag{4.2.19}$$

Consequently, *genotopies permit the lifting of conventional symmetric metrics into nonsymmetric forms,*

$$\eta^{Minkow.}_{Symm} \quad \rightarrow \quad \hat{\eta}^{>}{}^{Minkow.-Sant.}_{NonSymm}. \tag{4.2.20}$$

Remarkably, *nonsymmetric metrics are indeed permitted by the axioms of conventional spaces* as illustrated by the invariance

$$\begin{aligned}(x^{\mu} \times \eta_{\mu\nu} \times x^{\nu}) \times I \equiv [x^{\mu} \times (\hat{T}^{>} \times \eta_{\mu\nu}) \times x^{\nu}] \times T^{>-1} \equiv \\ \equiv (x^{\mu} \times \hat{\eta}^{>}_{\mu\nu} \times x^{\nu}) \times \hat{I}^{>},\end{aligned} \tag{4.2.21}$$

where $\hat{T}^{>}$ is assumed in this simple illustration to be a complex number.

Interested readers can then work out backward genogeometries and the isodual forward and backward genogeometries with their underlying genofunctional analysis.

This basic geometric feature was not discovered until recently because hidden where nobody looked for, in the basic unit. However, this basic geometric advance in the representation of irreversibility required the prior discovery of basically new numbers, Santilli's genonumbers with nonsymmetric unit and ordered multiplication [14].

4.2.4 Santilli Lie-Admissible Theory and Its Isodual

Particularly important for irreversibility is the lifting of Lie's theory and Lie-Santilli's isotheories permitted by genomathematics, first identified by Ref. [11] of 1978 (and then studied in various works, e.g., [6,18–22]) via the following genotopies:

(1) The *forward and backward universal enveloping genoassociative algebra* $\hat{\xi}^{>}$, ${}^{<}\hat{\xi}$, with infinite-dimensional basis characterizing the *Poincaré-Birkhoff-Witt-Santilli genotheorem*

$$\hat{\xi}^{>} : \hat{I}^{>},\ \hat{X}_i,\ \hat{X}_i > \hat{X}_j,\ \hat{X}_i > \hat{X}_j > \hat{X}_k,\ \ldots,\ i \leq j \leq k, \tag{4.2.22a}$$

$$^{<}\hat{\xi} : \hat{I},\ ^{<}\hat{X}_i,\ \hat{X}_i < \hat{X}_j,\ \hat{X}_i < \hat{X}_j < \hat{X}_k,\ \ldots,\ i \leq j \leq k; \tag{4.2.22b}$$

where the "hat" on the generators denotes their formulation on genospaces over genofields and their Hermiticity implies that $\hat{X}^{>} =^{<} \hat{X} = \hat{X}$;

(2) The *Lie-Santilli genoalgebras* characterized by the universal, jointly Lie- and Jordan-admissible brackets,

$$^{<}\hat{L}^{>} : (\hat{X}_i \hat{,} \hat{X}_j) = \hat{X}_i < \hat{X}_j - \hat{X}_j > \hat{X}_i = C_{ij}^k \times \hat{X}_k, \tag{4.2.23}$$

here formulated in an invariant form (see below);

(3) The *Lie-Santilli genotransformation groups*

$$^{<}\hat{G}^{>} : \hat{A}(\hat{w}) = (\hat{e}^{\hat{i}\hat{\times}\hat{X}\hat{\times}\hat{w}})^{>} > \hat{A}(\hat{0}) <^{<} (\hat{e}^{-\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}}) =$$

$$= (e^{i\times\hat{X}\times\hat{T}^{>}\times w}) \times A(0) \times (e^{-i\times w\times^{<}\hat{T}\times\hat{X}}), \tag{4.2.24}$$

where $\hat{w}^{>} \in \hat{R}^{>}$ are the *genoparameters;* the *genorepresentation theory,* etc.

4.2.5 Genosymmetries and Nonconservation Laws

The implications of the Santilli Lie-admissible theory are significant mathematically and physically. On mathematical grounds, the Lie-Santilli genoalgebras are "directly universal" and include as particular cases all known algebras, such as Lie, Jordan, Flexible algebras, power associative algebras, quantum, algebras, supersymmetric algebras, Kac-Moody algebras, etc. (Section 1.5).

Moreover, when computed on the *genobimodule*

$$^{<}\hat{B}^{>} =^{<} \hat{\xi} \times \hat{\xi}^{>}, \tag{4.2.25}$$

Lie-admissible algebras verify all Lie axioms, while deviations from Lie algebras emerge only in their *projection* on the conventional bimodule

$$^{<}B^{>} =^{<} \xi \times \xi^{>} \tag{4.2.26}$$

of Lie's theory (see Ref. [17] for the initiation of the genorepresentation theory of Lie-admissible algebras on bimodules).

This is due to the fact that the computation of the left action $A < B = A \times^{<} \hat{T} \times B$ on $^{<}\hat{\xi}$ (that is, with respect to the genounit $^{<}\hat{I} = 1/^{<}\hat{T}$) yields the save value as the computation of the conventional product $A \times B$ on $^{<}\xi$ (that is, with respect to the trivial unit I), and the same occurs for the value of $A > B$ on $\hat{\xi}^{>}$.

The above occurrences explain the reason the structure constant and the product in the r.h.s. of Eq. (4.2.23) are those of a conventional Lie algebra.

In this way, thanks to genomathematics, *Lie algebras acquire a towering significance in view of the possibility of reducing all possible irreversible systems to primitive Lie axioms.*

The physical implications of the Lie-Santilli genotheory are equally far reaching. In fact, Noether's theorem on the reduction of reversible conservation laws to primitive Lie symmetries can be lifted to the *reduction, this time, of irreversible nonconservation laws to primitive Lie-Santilli genosymmetries.*

As a matter of fact, this reduction was the very first motivation for the construction of the genotheory in memoir [12] (see also monographs [6,18,19,20]). The reader can then foresee similar liftings of all remaining physical aspects treated via Lie algebras.

The construction of the isodual Lie-Santilli genotheory is an instructive exercise for readers interested in learning the new methods.

4.3 LIE-ADMISSIBLE CLASSICAL MECHANICS FOR MATTER AND ITS ISODUAL FOR ANTIMATTER

4.3.1 Fundamental Ordering Assumption on Irreversibility

Another reason for the inability during the 20-th century for in depth studies of irreversibility is the general belief that motion in time has only two directions, forward and backward (Eddington historical time arrows). In reality, motion in time admits *four* different forms, all essential for serious studies in irreversibility, given by: 1) *motion forward to future time* characterized by the forward genotime $\hat{t}^>$; 2) *motion backward to past time* characterized by the backward genotime $^<\hat{t}$; 3) *motion backward from future time* characterized by the isodual forward genotime $\hat{t}^{>d}$; and 4) *motion forward from past time* characterized by the isodual backward genotime $^<\hat{t}^d$.

It is at this point where the *necessity* of both time reversal and isoduality appears in its full light. In fact, time reversal is only applicable to matter and, being represented with Hermitian conjugation, permits the transition from motion forward to motion backward in time, $\hat{t}^> \rightarrow ^< \hat{t} = (\hat{t}^>)^\dagger$. If used alone, time reversal cannot identify all four directions of motions. The *only* additional conjugation known to this author that is applicable at all levels of study and is equivalent to charge conjugation, is isoduality [22].

The additional discovery of two complementary orderings of the product and related units, with corresponding isoduals versions, individually preserving the abstract axioms of a field has truly fundamental implications for irreversibility, since it permits the axiomatically consistent and invariant representation of irreversibility via the most ultimate and primitive axioms, those on the product and related unit. We, therefore, have the following:

FUNDAMENTAL ORDERING ASSUMPTION ON IRREVERSIBILITY [15]: *Dynamical equations for motion forward in time of matter (antimatter) sys-*

tems are characterized by genoproducts to the right and related genounits (their isoduals), while dynamical equations for the motion backward in time of matter (antimatter) are characterized by genoproducts to the left and related genounits (their isoduals) under the condition that said genoproducts and genounits are interconnected by time reversal expressible for generic quantities A, B with the relation,

$$(A > B)^\dagger = (A > \hat{T}^> \times B)^\dagger = B^\dagger \times (\hat{T}^>)^\dagger \times A^\dagger, \tag{4.3.1}$$

namely,

$$\hat{T}^> = ({}^<\hat{T})^\dagger \tag{4.3.2}$$

thus recovering the fundamental complementary conditions (4.1.17) or (4.2.2).

Unless otherwise specified, from now on physical and chemical expression for irreversible processes will have no meaning without the selection of one of the indicated two possible orderings.

4.3.2 Newton-Santilli Genoequations and Their Isoduals

Recall that, for the case of isotopies, the basic Newtonian systems are given by those admitting nonconservative internal forces restricted by certain constraints to verify total conservation laws called *closed non-Hamiltonian systems* [6b,18].

For the case of the genotopies under consideration here, the basic Newtonian systems are the conventional nonconservative systems without subsidiary constraints, known as*open non-Hamiltonian systems*, with generic expression (1.3), in which case irreversibility is entirely characterized by nonselfadjoint forces, since all conservative forces are reversible.

As it is well known, the above equations are not derivable from any variational principle in the fixed frame of the observer [6], and this is the reason all conventional attempts for consistently quantizing nonconservative forces have failed for about one century. In turn, the lack of achievement of a consistent operator counterpart of nonconservative forces lead to the belief that they are "illusory" because they "disappear" at the particle level.

The studies presented in this paper have achieved the first and only physically consistent operator formulation of nonconservative forces known to the author. This goal was achieved by rewriting Newton's equations (1.3) into an identical form derivable from a variational principle. Still in turn, the latter objective was solely permitted by the novel genomathematics.

It is appropriate to recall that Newton was forced to discover new mathematics, the differential calculus, prior to being able to formulated his celebrated equations. Therefore, readers should not be surprised at the need for the new genodifferential calculus as a condition to represent all nonconservative Newton's systems from a variational principle.

Recall also from Section 3.1 that, contrary to popular beliefs, there exist *four* inequivalent directions of time. Consequently, time reversal alone cannot represent all these possible motions, and isoduality results to be the only known additional conjugation that, when combined with time reversal, can represent all possible time evolutions of both matter and antimatter.

The above setting implies the existence of four different new mechanics first formulated by Santilli in memoir [14] of 1996, and today known as *Newton-Santilli genomechanics,* namely:

A) *Forward genomechanics* for the representation of forward motion of matter systems;

B) *Backward genomechanics* for the representation of the time reversal image of matter systems;

C) *Isodual backward genomechanics* for the representation of motion backward in time of antimatter systems, and

D) *Isodual forward genomechanics* for the representation of time reversal antimatter systems.

These new mechanics are characterized by:

1) Four different times, *forward and backward genotimes for matter systems and the backward and forward isodual genotimes for antimatter systems*

$$\hat{t}^{>} = t \times \hat{I}_t^{>}, \quad -\hat{t}^{>}, \quad \hat{t}^{>d}, \quad -\hat{t}^{>d}, \tag{4.3.3}$$

with (nowhere singular and non-Hermitian) *forward and backward time genounits and their isoduals* (Note that, to verify the condition of non-Hermiticity, the time genounits can be *complex valued.*),

$$\hat{I}_t^{>} = 1/\hat{T}_t^{>}, \quad -\hat{I}_t^{>}, \quad \hat{I}^{>d}_{t}, \quad -\hat{I}^{>d}_{t}; \tag{4.3.4}$$

2) The *forward and backward genocoordinates and their isoduals*

$$\hat{x}^{>} = x \times \hat{I}_x^{>}, \quad -\hat{x}^{>}, \quad \hat{x}^{>d}, \quad -\hat{x}^{>d}, \tag{4.3.5}$$

with (nowhere singular non-Hermitian) *coordinate genounit*

$$\hat{I}_x^{>} = 1/\hat{T}_x^{>}, \quad -\hat{I}_x^{>}, \quad \hat{I}^{>d}_{x}, \quad -\hat{I}^{>d}_{x}, \tag{4.3.6}$$

with *forward and backward coordinate genospace and their isoduals* $\hat{S}_x^{>}$, etc., and related *forward coordinate genofield and their isoduals* $\hat{R}_x^{>}$, etc.;

3) The *forward and backward genospeeds and their isoduals*

$$\hat{v}^{>} = \hat{d}^{>}\hat{x}^{>}/\hat{d}^{>}\hat{t}^{>}, \quad -\hat{v}^{>}, \quad \hat{v}^{>d}, \quad -\hat{v}^{>d}, \tag{4.3.7}$$

with (nowhere singular and non-Hermitian) *speed genounit*

$$\hat{I}_v^{>} = 1/\hat{T}_v^{>}, \quad -\hat{I}_v^{>}, \quad \hat{I}^{>d}_{v}, \quad -\hat{I}^{>d}_{v}, \tag{4.3.8}$$

with related *forward speed backward genospaces and their isoduals* $\hat{S}_v^{>}$, etc., over *forward and backward speed genofields* $\hat{R}_v^{>}$, etc.

The above formalism then leads to the *forward genospace for matter systems*

$$\hat{S}^{>}_{tot} = \hat{S}^{>}_{t} \times \hat{S}^{>}_{x} \times \hat{S}^{>}_{v}, \tag{4.3.9}$$

defined over the *forward genofield*

$$\hat{R}^{>}_{tot} = \hat{R}^{>}_{t} \times \hat{R}^{>}_{x} \times \hat{R}^{>}_{v}, \tag{4.3.10}$$

with *total forward genounit*

$$\hat{I}^{>}_{tot} = \hat{I}^{>}_{t} \times \hat{I}^{>}_{x} \times \hat{I}^{>}_{v}, \tag{4.3.11}$$

and corresponding expressions for the remaining three spaces obtained via time reversal and isoduality.

The basic equations are given by:

I) The *forward Newton-Santilli genoequations for matter systems* [14], formulated via the genodifferential calculus,

$$\hat{m}^{>}_{a} > \frac{\hat{d}^{>}\hat{v}^{>}_{ka}}{\hat{d}^{>}\hat{t}^{>}} = -\frac{\hat{\partial}^{>}\hat{V}^{>}}{\hat{\partial}^{>}\hat{x}^{>k}_{a}}; \tag{4.3.12}$$

II) The *backward genoequations for matter systems* that are characterized by time reversal of the preceding ones;

III) the *backward isodual genoequations for antimatter systems* that are characterized by the isodual map of the backward genoequations,

$${}^{<}\hat{m}^{d}_{a} < \frac{{}^{<}\hat{d}^{d<}\hat{v}^{d}_{ka}}{{}^{<}\hat{d}^{d<}\hat{t}^{d}} = -\frac{{}^{<}\hat{\partial}^{d<}\hat{V}^{d}}{{}^{<}\hat{\partial}^{d<}\hat{x}^{dk}_{a}}; \tag{4.3.13}$$

IV) the *forward isodual genoequations for antimatter systems* characterized by time reversal of the preceding isodual equations.

Newton-Santilli genoequations (4.3.12) are "directly universal" for the representation of all possible (well behaved) Eqs. (1.3) in the frame of the observer because they admit a multiple infinity of solution for any given nonselfadjoint force.

A simple representation occurs under the conditions assumed for simplicity,

$$N = \hat{I}^{>}_{t} = \hat{I}^{>}_{v} = 1, \tag{4.3.14}$$

for which Eqs. (3.12) can be explicitly written

$$\hat{m}^{>} > \frac{\hat{d}^{>}\hat{v}^{>}}{\hat{d}^{>}t} = m \times \frac{d\hat{v}^{>}}{dt} =$$

$$= m \times \frac{d}{dt}\frac{d(x \times \hat{I}^{>}_{x})}{dt} = m \times \frac{dv}{dt} \times \hat{I}^{>}_{x} + m \times x \times \frac{d\hat{I}^{>}}{dt} = \hat{I}^{>}_{x} \times \frac{\partial V}{\partial x}, \tag{4.3.15}$$

from which we obtain the genorepresentation

$$F^{NSA} = -m \times x \times \frac{1}{\hat{I}^>_x} \times \frac{d\hat{I}^>_x}{dt}, \tag{4.3.16}$$

that always admit solutions here left to the interested reader since in the next section we shall show a much simpler, universal, *algebraic* solution.

As one can see, in Newton's equations the nonpotential forces are part of the applied force, while in the Newton-Santilli genoequations nonpotential forces are represented by the genounits, or, equivalently, by the genodifferential calculus, in a way essentially similar to the case of isotopies.

The main difference between iso- and geno-equations is that isounits are Hermitian, thus implying the equivalence of forward and backward motions, while genounits are non-Hermitian, thus implying irreversibility.

Note also that the topology underlying Newton's equations is the conventional, Euclidean, local-differential topology which, as such, can only represent point particles.

By contrast, the topology underlying the Newton-Santilli genoequations is given by a genotopy of the isotopology studied in the preceding chapter, thus permitting the representation of extended, nonspherical and deformable particles via forward genounits, e.g., of the type

$$\hat{I}^> = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) \times \Gamma^>(t, r, v, \ldots), \tag{4.3.17}$$

where $n_k^2,\ k = 1, 2, 3$ represents the semiaxes of an ellipsoid, n_4^2 represents the density of the medium in which motion occurs (with more general nondiagonal realizations here omitted for simplicity), and $\Gamma^>$ constitutes a nonsymmetric matrix representing nonselfadjoint forces, namely, the contact interactions among extended constituents occurring for the motion forward in time.

4.3.3 Hamilton-Santilli Genomechanics and Its Isodual

In this section we show that, once rewritten in their identical genoform (4.3.12), Newton's equations for nonconservative systems are indeed derivable from a variational principle, with analytic equations possessing a Lie-admissible structure and Hamilton-Jacobi equations suitable for the first known consistent and unique operator map studied in the next section.

The most effective setting to introduce real-valued non-symmetric genounits is in the $6N$-dimensional *forward genospace (genocotangent bundle)* with local genocoordinates and their conjugates

$$\hat{a}^{>\mu} = a^\rho \times \hat{I}^{>\mu}_{1\ \rho},\ \ (\hat{a}^{>\mu}) = \begin{pmatrix} \hat{x}^{>k}_\alpha \\ \hat{p}^>_{k\alpha} \end{pmatrix} \tag{4.3.18}$$

and

$$\hat{R}^>_\mu = R_\rho \times \hat{I}^{>\rho}_{2\ \mu},\ (\hat{R}^>_\mu) = (\hat{p}_{k\alpha}, \hat{0}), \tag{4.3.19a}$$

$$\hat{I}_1^> = 1/\hat{T}_1^> = (\hat{I}_2^>)^T = (1/\hat{T}_2^>)^T, \tag{4.3.19b}$$

$$k = 1, 2, 3;\ \alpha = 1, 2, \ldots, N; \quad \mu,\ \rho = 1, 2, \ldots 6N,$$

where the superscript T stands for transposed, and nowhere singular, real-valued and non-symmetric genometric and related invariant

$$\hat{\delta}^> = \hat{T}^>_{1\,6N\times 6N}\,\delta_{6N\times 6N} \times \delta_{6N\times 6N}, \tag{4.3.20a}$$

$$\hat{a}^{>\mu} > \hat{R}^>_\mu = \hat{a}^{>\rho} \times \hat{T}_{1\,\rho}^{>\beta} \times \hat{R}^>_\beta = a^\rho \times \hat{I}_{2\,\rho}^{>\beta} \times R_\beta. \tag{4.3.20b}$$

In this case we have the following *genoaction principle* [14]

$$\hat{\delta}^> \hat{\mathcal{A}}^> = \hat{\delta}^> \hat{\int}^> [\hat{R}^>_\mu >_a \hat{d}^> \hat{a}^{>\mu} - \hat{H}^> >_t \hat{d}^> \hat{t}^>] =$$

$$= \delta \int [R_\mu \times \hat{T}_{1\,\nu}^{>\mu}(t, x, p, \ldots) \times d(a^\beta \times \hat{I}_{1\,\beta}^{>\nu}) - H \times dt] = 0, \tag{4.3.21}$$

where the second expression is the projection on conventional spaces over conventional fields and we have assumed for simplicity that the time genounit is 1.

It is easy to prove that the above genoprinciple characterizes the following *forward Hamilton-Santilli genoequations,* (originally proposed in Ref. [11] of 1978 with conventional mathematics and in Ref. [14] of 1996 with genomathematics (see also Refs. [18,19,20])

$$\hat{\omega}^>_{\mu\nu} > \frac{\hat{d}^> \hat{a}^{\nu >}}{\hat{d}^> \hat{t}^>} - \frac{\hat{\partial}^> \hat{H}^>(\hat{a}^>)}{\hat{\partial}^> \hat{a}^{\mu >}} =$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} dr/dt \\ dp/dt \end{pmatrix} - \begin{pmatrix} 1 & K \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} \partial H/\partial r \\ \partial H/\partial p \end{pmatrix} = 0, \tag{4.3.22a}$$

$$\hat{\omega}^> = \Big(\frac{\hat{\partial}^> R^>_\nu}{\hat{\partial}^> \hat{a}^{\mu >}} - \frac{\hat{\partial}^> \hat{R}^>_\mu}{\hat{\partial}^> \hat{a}^{\nu >}}\Big) \times \hat{I}^> = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \hat{I}^>, \tag{4.3.22b}$$

$$K = F^{NSA}/(\partial H/\partial p), \tag{4.3.22c}$$

where one should note the "direct universality" of the simple algebraic solution (3.22c).

The time evolution of a quantity $\hat{A}^>(\hat{a}^>)$ on the forward geno-phase-space can be written in terms of the following brackets

$$\frac{\hat{d}^> \hat{A}^>}{\hat{d}^> t^>} = (\hat{A}^>, \hat{H}^>) = \frac{\hat{\partial}^> \hat{A}^>}{\hat{\partial}^> \hat{a}^{>\mu}} > \hat{\omega}^{\mu\nu >} > \frac{\hat{\partial}^> \hat{H}^>}{\hat{\partial} \hat{a}^{>\nu}} =$$

$$= \frac{\partial \hat{A}^>}{\partial \hat{a}^{>\mu}} \times S^{\mu\nu} \times \frac{\partial \hat{H}^>}{\partial \hat{a}^{>\nu}} =$$

$$= \Big(\frac{\partial \hat{A}^>}{\partial \hat{r}^{>k}_\alpha} \times \frac{\partial \hat{H}^>}{\partial \hat{p}^>_{ka}} - \frac{\partial \hat{A}^>}{\partial \hat{p}^>_{ka}} \times \frac{\partial \hat{H}^>}{\partial \hat{r}^{>k}_a}\Big) + \frac{\partial \hat{A}^>}{\partial \hat{p}^>_{ka}} \times F^{NSA}_{ka}, \tag{4.3.23a}$$

$$S^{>\mu\nu} = \omega^{\mu\rho} \times \hat{I}^{2\mu}_\rho, \omega^{\mu\nu} = (||\omega_{\alpha\beta}||^{-1})^{\mu\nu}, \tag{4.3.23b}$$

where $\omega^{\mu\nu}$ is the conventional Lie tensor and, consequently, $S^{\mu\nu}$ is Lie-admissible in the sense of Albert [7].

As one can see, the important consequence of genomathematics and its genodifferential calculus is that of turning the triple system (A, H, F^{NSA}) of Eqs. (1.5) in the bilinear form $(A\hat{,}B)$, thus characterizing a consistent algebra in the brackets of the time evolution.

This is the central purpose for which genomathematics was built (note that the multiplicative factors represented by K are fixed for each given system). The invariance of such a formulation will be proved shortly.

It is an instructive exercise for interested readers to prove that the brackets $(A\hat{,}B)$ are Lie-admissible, although not Jordan-admissible.

It is easy to verify that the above identical reformulation of Hamilton's historical time evolution correctly recovers the *time rate of variations of physical quantities* in general, and that of the energy in particular,

$$\frac{dA^>}{dt} = (A^>, H^>) = [\hat{A}^>, \hat{H}^>] + \frac{\partial \hat{A}^>}{\partial \hat{p}^>_{k\alpha}} \times F^{NSA}_{k\alpha}, \tag{4.3.24a}$$

$$\frac{dH}{dt} = [\hat{H}^>, \hat{H}^>] + \frac{\partial \hat{H}^>}{\partial \hat{p}^>_{k\alpha}} \times F^{NSA}_{ka} = v^k_\alpha \times F^{NSA}_{ka}. \tag{4.3.24b}$$

It is easy to show that genoaction principle (4.3.21) characterizes the following *Hamilton-Jacobi-Santilli genoequations* [14]

$$\frac{\hat{\partial}^> \mathcal{A}^>}{\hat{\partial}^> \hat{t}^>} + \hat{H}^> = 0, \tag{4.3.25a}$$

$$\Big(\frac{\hat{\partial}^> \mathcal{A}^>}{\hat{\partial}^> \hat{a}^{>\mu}}\Big) = \Big(\frac{\hat{\partial}^> \mathcal{A}^>}{\hat{\partial}^> x^{>k}_a}, \frac{\hat{\partial}^> \mathcal{A}^>}{\hat{\partial}^> p^>_{ka}}\Big) = (\hat{R}^>_\mu) = (\hat{p}^>_{ka}, \hat{0}), \tag{4.3.25b}$$

which confirm the property (crucial for genoquantization as shown below) that the genoaction is indeed independent of the linear momentum.

Note the *direct universality* of the Lie-admissible equations for the representation of all infinitely possible Newton equations (1.3) (universality) directly in the fixed frame of the experimenter (direct universality).

Note also that, *at the abstract, realization-free level, Hamilton-Santilli genoequations coincide* with Hamilton's equations without external terms, yet represent those with external terms.

The latter are reformulated via genomathematics as the only known way to achieve invariance and derivability from a variational principle while admitting a consistent algebra in the brackets of the time evolution [38].

Therefore, Hamilton-Santilli genoequations (3.6.66) are indeed irreversible for all possible reversible Hamiltonians, as desired. The origin of irreversibility rests in the contact nonpotential forces F^{NSA} according to Lagrange's and Hamilton's teaching that is merely reformulated in an invariant way.

The above Lie-admissible mechanics requires, for completeness, *three* additional formulations, the *backward genomechanics* for the description of *matter moving backward in time,* and the isoduals of both the forward and backward mechanics for the description of *antimatter.*

The construction of these additional mechanics is left to the interested reader for brevity.

4.4 LIE-ADMISSIBLE OPERATOR MECHANICS FOR MATTER AND ITS ISODUAL FOR ANTIMATTER

4.4.1 Basic Dynamical Equations

A simple genotopy of the naive or symplectic quantization applied to Eqs. (3.24) yields the *Lie-admissible branch of hadronic mechanics* [18] comprising four different formulations, the *forward and backward genomechanics for matter and their isoduals for antimatter.* The forward genomechanics for matter is characterized by the following main topics:

1) The nowhere singular (thus everywhere invertible) non-Hermitian *forward genounit* for the representation of all effects causing irreversibility, such as contact nonpotential interactions among extended particles, etc. (see the subsequent chapters for various realizations)

$$\hat{I}^{>} = 1/\hat{T}^{>} \neq (\hat{I}^{>})^{\dagger}, \tag{4.4.1}$$

with corresponding ordered product and genoreal $\hat{R}^{>}$ and genocomplex $\hat{C}^{>}$ genofields;

2) The *forward genotopic Hilbert space* $\hat{\mathcal{H}}^{>}$ with *forward genostates* $|\hat{\psi}^{>}>$ and *forward genoinner product*

$$<^{<}\hat{\psi}|>|\hat{\psi}^{>}> \times \hat{I}^{>} = <^{<}\hat{\psi}| \times \hat{T}^{>} \times |\hat{\psi}^{>}> \times \hat{I}^{>} \in \hat{C}^{>}, \tag{4.4.2}$$

and fundamental property

$$\hat{I}^{>} > |\hat{\psi}^{>}> = |\hat{\psi}^{>}>, \tag{4.4.3}$$

holding under the condition that $\hat{I}^{>}$ is indeed the correct unit for motion forward in time, and *forward genounitary transforms*

$$\hat{U}^{>} > (^{<}\hat{U})^{\dagger >} = (^{<}\hat{U})^{\dagger >} > \hat{U}^{>} = \hat{I}^{>}; \tag{4.4.4}$$

3) The fundamental Lie-admissible equations, first proposed in Ref. [12] of 1974 (p. 783, Eqs. (4.18.16)) as the foundations of hadronic mechanics, formulated on conventional spaces over conventional fields, and first formulated in Refs. [14,18] of 1996 on genospaces and genodifferential calculus on genofields, today's known as *Heisenberg-Santilli genoequations*, that can be written in the finite form

$$\hat{A}(\hat{t}) = \hat{U}^{>} > \hat{A}(0) <^{<} \hat{U} = (\hat{e}_{>}^{\hat{i} \hat{\times} \hat{H} \hat{\times} \hat{t}}) > \hat{A}(\hat{0}) < ({}_{<}\hat{e}^{-\hat{i} \hat{\times} \hat{t} \hat{\times} \hat{H}}) =$$

$$= (e^{i \times \hat{H} \times \hat{T}^{>} \times t}) \times A(0) \times (e^{-i \times t \times {}^{<}\hat{T} \times \hat{H}}), \tag{4.4.5}$$

with corresponding infinitesimal version

$$\hat{i} \hat{\times} \frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = (\hat{A} \hat{;} \hat{H}) = \hat{A} < \hat{H} - \hat{H} > \hat{A} =$$

$$= \hat{A} \times^{<} \hat{T}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots .) \times \hat{H} - \hat{H} \times \hat{T}^{>}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots) \times \hat{A}, \tag{4.4.6}$$

where there is no time arrow, since Heisenberg's equations are computed at a fixed time;

4) The equivalent *Schrödinger-Santilli genoequations*, first suggested in the original proposal [12] to build hadronic mechanics (see also Refs. [17,23,24]), formulated via conventional mathematics and in Refs. [14,18] via genomathematics, that can be written

$$\hat{i}^{>} > \frac{\hat{\partial}^{>}}{\hat{\partial}^{>} \hat{t}^{>}} |\hat{\psi}^{>} >= \hat{H}^{>} > |\hat{\psi}^{>} >=$$

$$= \hat{H}(\hat{r}, \hat{v}) \times \hat{T}^{>}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \hat{\partial}\hat{\psi} \ldots) \times |\hat{\psi}^{>} >= E^{>} > |\psi^{>} >, \tag{4.4.7}$$

where the time orderings in the second term are ignored for simplicity of notation;

5) The *forward genomomentum* that escaped identification for two decades and was finally identified thanks to the genodifferential calculus in Ref. [14] of 1996

$$\hat{p}_k^{>} > |\hat{\psi}^{>} >= -\hat{i}^{>} > \hat{\partial}_k^{>} |\hat{\psi}^{>} >= -i \times \hat{I}_k^{> i} \times \partial_i |\hat{\psi}^{>} >; \tag{4.4.8}$$

6) The *fundamental genocommutation rules* also first identified in Ref. [14],

$$(\hat{r}^i \hat{;} \hat{p}_j) = i \times \delta^i_j \times \hat{I}^{>}, \ \ (\hat{r}^i \hat{;} \hat{r}^j) = (\hat{p}_i \hat{;} \hat{p}_j) = 0; \tag{4.4.9}$$

7) The *genoexpectation values* of an observable for the forward motion $\hat{A}^{>}$ [14,19]

$$\frac{<^{<} \hat{\psi}| > \hat{A}^{>} > |\hat{\psi}^{>} >}{<^{<} \hat{\psi}| > |\hat{\psi}^{>} >} \times \hat{I}^{>} \in \hat{C}^{>}, \tag{4.4.10}$$

under which the genoexpectation values of the genounit recovers the conventional Planck's unit as in the isotopic case,

$$\frac{<\hat{\psi}|>\hat{I}^{>}>|\hat{\psi}>}{<\hat{\psi}|>|\hat{\psi}>}=I. \tag{4.4.11}$$

The following comments are now in order. Note first in the genoaction principle the crucial independence of isoaction $\hat{\mathcal{A}}^{>}$ in form the linear momentum, as expressed by the Hamilton-Jacobi-Santilli genoequations (4.3.25). Such independence assures that genoquantization yields a genowavefunction solely dependent on time and coordinates, $\hat{\psi}^{>}=\hat{\psi}^{>}(t,r)$.

Other geno-Hamiltonian mechanics studied previously [7] do not verify such a condition, thus implying genowavefunctions with an explicit dependence also on linear momenta, $\hat{\psi}^{>}=\hat{\psi}^{>}(t,r,p)$ that violate the abstract identity of quantum and hadronic mechanics whose treatment in any case is beyond our operator knowledge at this writing.

Note that *forward geno-Hermiticity coincides with conventional Hermiticity.* As a result, *all quantities that are observables for quantum mechanics remain observables for the above genomechanics.*

However, unlike quantum mechanics, physical quantities are generally *nonconserved,* as it must be the case for the energy,

$$\hat{i}^{>}>\frac{\hat{d}^{>}\hat{H}^{>}}{\hat{d}^{>}\hat{t}^{>}}=\hat{H}\times({}^{<}\hat{T}-\hat{T}^{>})\times\hat{H}\neq 0. \tag{4.4.12}$$

Therefore, *the genotopic branch of hadronic mechanics is the only known operator formulation permitting nonconserved quantities to be Hermitian as a necessary condition to be observable.*

Other formulation attempt to represent nonconservation, e.g., by adding an "imaginary potential" to the Hamiltonian, as it is often done in nuclear physics [25]. In this case the Hamiltonian is non-Hermitian and, consequently, the nonconservation of the energy cannot be an observable.

Besides, said "nonconservative models" with non-Hermitian Hamiltonians are nonunitary and are formulated on conventional spaces over conventional fields, thus suffering all the catastrophic inconsistencies of Theorem 1.3.

We should stress the representation of irreversibility and nonconservation beginning with the most primitive quantity, the unit and related product. *Closed irreversible systems* are characterized by the Lie-isotopic subcase in which

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}}=[\hat{A}\hat{,}\hat{H}]=\hat{A}\times\hat{T}(t,\ldots)\times\hat{H}-\hat{H}\times\hat{T}(t,\ldots)\times\hat{A}, \tag{4.4.13a}$$

$${}^{<}\hat{T}(t,\ldots)=\hat{T}^{>}(t,\ldots)=\hat{T}(t,\ldots)=\hat{T}^{\dagger}(t,\ldots)\neq\hat{T}(-t,\ldots), \tag{4.4.13b}$$

for which the Hamiltonian is manifestly conserved. Nevertheless the system is manifestly irreversible. Note also the first and only known observability of the Hamiltonian (due to its iso-Hermiticity) under irreversibility.

As one can see, brackets (A, B) of Eqs. (4.6) are jointly Lie- and Jordan-admissible.

Note also that finite genotransforms (4.4.5) verify the condition of genohermiticity, Eq. (4.4).

We should finally mention that, as it was the case for isotheories, *genotheories are also admitted by the abstract axioms of quantum mechanics, thus providing a broader realization.* This can be seen, e.g., from the invariance under a complex number C

$$< \psi|x|\psi > \times I =< \psi|xC^{-1} \times |\psi > \times (C \times I) =< \psi| > |\psi > \times I^{>}. \tag{4.4.14}$$

Consequently, *genomechanics provide another explicit and concrete realization of "hidden variables" [26], thus constituting another "completion" of quantum mechanics in the E-P-R sense [27].* For the studies of these aspects we refer the interested reader to Ref. [28].

The above formulation must be completed with three additional Lie-admissible formulations, the backward formulation for matter under time reversal and the two additional isodual formulations for antimatter. Their study is left to the interested reader for brevity.

4.4.2 Simple Construction of Lie-Admissible Theories

As it was the case for the isotopies, a simple method has been identified in Ref. [44] for the construction of Lie-admissible (geno-) theories from any given conventional, classical or quantum formulation. It consists in *identifying the genounits as the product of two different nonunitary transforms,*

$$\hat{I}^{>} = ({}^{<}\hat{I})^{\dagger} = U \times W^{\dagger}, \quad {}^{<}\hat{I} = W \times U^{\dagger}, \tag{4.4.15a}$$

$$U \times U^{\dagger} \neq 1, \quad W \times W^{\dagger} \neq 1, \quad U \times W^{\dagger} = \hat{I}^{>}, \tag{4.4.15b}$$

and subjecting the totality of quantities and their operations of conventional models to said dual transforms,

$$I \rightarrow \hat{I}^{>} = U \times I \times W^{\dagger}, \quad I \rightarrow^{<} \hat{I} = W \times I \times U^{\dagger}, \tag{4.4.16a}$$

$$a \rightarrow \hat{a}^{>} = U \times a \times W^{\dagger} = a \times \hat{I}^{>}, \tag{4.4.16b}$$

$$a \rightarrow^{<} \hat{a} = W \times a \times U^{\dagger} =^{<} \hat{I} \times a, \tag{4.4.16c}$$

$$\begin{aligned} a \times b \rightarrow \hat{a}^{>} > \hat{b}^{>} = U \times (a \times b) \times W^{>} = \\ = (U \times a \times W^{\dagger}) \times (U \times W^{\dagger})^{-1} \times (U \times b \times W^{\dagger}), \end{aligned} \tag{4.4.16d}$$

$$\partial/\partial x \to \hat{\partial}^{>}/\hat{\partial}^{>}\hat{x}^{>} = U \times (\partial/\partial x) \times W^{\dagger} = \hat{I}^{>} \times (\partial/\partial x), \tag{4.4.16e}$$

$$< \psi| \times |\psi > \to <^{<} \psi| > |\psi^{>} > = U \times (< \psi| \times |\psi >) \times W^{\dagger}, \tag{4.4.16f}$$

$$H \times |\psi > \to \hat{H}^{>} > |\psi^{>} > =$$
$$= (U \times H \times W^{\dagger}) \times (U \times W^{\dagger})^{-1} \times (U \times \psi > W^{\dagger}), \text{ etc.} \tag{4.4.16g}$$

As a result, any given conventional, classical or quantum model can be easily lifted into the genotopic form.

Note that the above construction implies that *all conventional physical quantities acquire a well defined direction of time.* For instance, the correct genotopic formulation of energy, linear momentum, etc., is given by

$$\hat{H}^{>} = U \times H \times W^{\dagger}, \;\; \hat{p}^{>} = U \times p \times W^{>}, \text{ etc.} \tag{4.4.17}$$

In fact, under irreversibility, the value of a nonconserved energy at a given time t for motion forward in time is generally different than the corresponding value of the energy for $-t$ for motion backward in past times.

This explains the reason for having represented in this section energy, momentum and other quantities with their arrow of time $>$. Such an arrow can indeed be omitted for notational simplicity, but only after the understanding of its existence.

Note finally that a conventional, one dimensional, unitary Lie transformation group with Hermitian generator X and parameter w can be transformed into a covering Lie-admissible group via the following nonunitary transform

$$Q(w) \times Q^{\dagger}(w) = Q^{\dagger}(w) \times Q(w) = I, \; w \in R, \tag{4.4.18a}$$

$$U \times U^{\dagger} \neq I, \quad W \times W^{\dagger} \neq 1, \tag{4.4.18b}$$

$$A(w) = Q(w) \times A(0) \times Q^{\dagger}(w) = e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X} \to$$
$$\to U \times (e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X}) \times U^{\dagger} =$$
$$\equiv [U \times (e^{X \times w \times i}) \times W^{\dagger} \times (U \times W^{\dagger})^{-1} \times A \times A(0) \times$$
$$\times U^{\dagger} \times (W \times U^{\dagger})^{-1} \times [W \times (e^{-i \times w \times X}) \times U^{\dagger}] =$$
$$= (e^{i \times X \times X})^{>} > A(0) <^{<} (e^{-1 \times w \times X}) = \hat{U}^{>} > A(0) <^{<} \hat{U}, \tag{4.4.18c}$$

which confirm the property of Section 4.2, namely, that under the necessary mathematics *the Lie-admissible theory is indeed admitted by the abstract Lie axioms, and it is a realization of the latter broader than the isotopic form.*

4.4.3 Invariance of Lie-Admissible Theories

Recall that a fundamental axiomatic feature of quantum mechanics is the invariance under time evolution of all numerical predictions and physical laws, which invariance is due to the *unitary structure* of the theory.

However, quantum mechanics is reversible and can only represent in a scientific way beyond academic beliefs reversible systems verifying total conservation laws due to the antisymmetric character of the brackets of the time evolution.

As indicated earlier, the representation of irreversibility and nonconservation requires theories with a *nonunitary structure*. However, the latter are afflicted by the catastrophic inconsistencies of Theorem 1.3.

The only resolution of such a basic impasse known to the author has been the achievement of invariance under nonunitarity and irreversibility via the use of genomathematics, provided that such genomathematics is applied to the *totality* of the formalism to avoid evident inconsistencies caused by mixing different mathematics for the selected physical problem.

Let us note that, due to decades of protracted use it is easy to predict that physicists and mathematicians may be tempted to treat the Lie-admissible branch of hadronic mechanics with conventional mathematics, whether in part or in full. Such a posture would be equivalent, for instance, to the elaboration of the spectral emission of the hydrogen atom with the genodifferential calculus, resulting in an evident nonscientific setting.

Such an invariance was first achieved by Santilli in Ref. [15] of 1997 and can be illustrated by reformulating any given nonunitary transform in the *genounitary form*

$$U = \hat{U} \times \hat{T}^{>1/2}, W = \hat{W} \times \hat{T}^{>1/2}, \tag{4.4.19a}$$

$$U \times W^\dagger = \hat{U} > \hat{W}^\dagger = \hat{W}^\dagger > \hat{U} = \hat{I}^> = 1/\hat{T}^>, \tag{4.4.19b}$$

and then showing that genounits, genoproducts, genoexponentiation, etc., are indeed invariant under the above genounitary transform in exactly the same way as conventional units, products, exponentiations, etc. are invariant under unitary transforms,

$$\hat{I}^> \rightarrow \hat{I}^{>\prime} = \hat{U} > \hat{I}^> > \hat{W}^\dagger = \hat{I}^>, \tag{4.4.20a}$$

$$\begin{gathered} \hat{A} > \hat{B} \rightarrow \hat{U} > (A > B) > \hat{W}^\dagger = \\ = (\hat{U} \times \hat{T}^> \times A \times T^> \times \hat{W}^\dagger) \times (\hat{T}^> \times W^\dagger)^{-1} \times \hat{T}^> \times \\ \times (\hat{U} \times \hat{T}^>)^{-1} \times (\hat{U} \times T^> \times \hat{A} \times T^> \times \hat{W}^>) = \\ = \hat{A}' \times (\hat{U} \times \hat{W}^\dagger)^{-1} \times \hat{B} = \hat{A}' \times \hat{T}^> \times B' = \hat{A}' > \hat{B}', \text{ etc.}, \end{gathered} \tag{4.4.20b}$$

from which all remaining invariances follow, thus resolving the catastrophic inconsistencies of Theorem 1.3.

Note the *numerical invariances of the genounit* $\hat{I}^> \rightarrow \hat{I}^{>\prime} \equiv \hat{I}^>$, *of the genotopic element* $\hat{T}^> \rightarrow \hat{T}^{>\prime} \equiv \hat{T}^>$, *and of the genoproduct* $> \rightarrow >' \equiv >$ that are necessary to have invariant numerical predictions.

4.5 APPLICATIONS

4.5.1 Lie-admissible Treatment of Particles with Dissipative Forces

In this section we present a variety of classical and operator representations of nonconservative systems by omitting hereon for simplicity of notation all "hats" on quantities (denoting isotopies not considered in this section), omitting the symbol $\times$ to denote the conventional (associative) multiplication, but preserving the forward (backward) symbols $>$ ($<$) denoting forward (backward) motion in time for quantities and products. The content of this section was presented for the first time by the author in memoir [32].

Let us begin with a classical and operator representation of the simplest possible dissipative system, a massive particle moving within a physical medium, and being subjected to a linear, velocity-dependent resistive force

$$m\frac{dv}{dt} = F^{NSA} = -kv, \tag{4.5.1}$$

for which we have the familiar *variation (dissipation) of the energy*

$$\frac{d}{dt}(\frac{1}{2}mv^2) = -kv^2. \tag{4.5.2}$$

Progressively more complex examples will be considered below.

The representations of system (5.1) via the *Newton-Santilli genoequations* (3.12) is given by

$$m^> > \frac{d^> v^>}{d^> t^>} = 0. \tag{4.5.3}$$

As indicated in Section 3, the representation requires the selection of *three* generally different genounits, $I_t^>, I_r^>, I_v^>$. Due to the simplicity of the case and the velocity dependence of the applied force, the simplest possible solution is given by

$$I_t^> = I_r^> = 1, \quad I_v^>(t) = e^{\frac{k\times t}{m}} = 1/T_v^>(t) > 0, \tag{4.5.4a}$$

$$m^> > \frac{d^> v^>}{d^> t^>} = m\frac{d(vI_v^>)}{dt} = m\frac{dv}{dt}I^> + kv\frac{dI_v^>}{dt} = 0. \tag{4.5.4b}$$

The representation with *Hamilton-Santilli genoequations* (3.22) is also straightforward and can be written in disjoint $r^>$ and $p^>$ notations

$$H^> = \frac{p^{>2>}}{2^> > m^>} = \frac{p^2}{2m}I_p^>, \tag{4.5.5a}$$

$$v^> = \frac{\partial^> H^>}{\partial^> p^>} = \frac{p^>}{m}, \quad \frac{d^> p^>}{d^> t^>} = -\frac{\partial^> H^>}{\partial^> r^>} = 0. \tag{4.5.5b}$$

The last equation then reproduces equation of motion (5.1) identically under assumptions (5.4a).

The above case is instructive because the representation is achieved via the genoderivatives (Section 2.2). However, the representation exhibits no algebra in the time evolution. Therefore, we seek an alternative representation in which the dissipation is characterized by the Lie-admissible algebra, rather by the differential calculus.

This alternative representation is provided by the Hamilton-Santilli genoequations (3.22) in the unified notation $a^> = (r^{>k}, p^>_k)$ that become for the case at hand

$$\frac{da^{>\mu}}{dt} = \begin{pmatrix} dr^>/dt \\ dp^>/dt \end{pmatrix} = S^{>\mu\nu}\frac{\partial^> H^>}{\partial^> a^{>\nu}} = \begin{pmatrix} 0 & -1 \\ 1 & \frac{-kv}{(\partial H/\partial p)} \end{pmatrix} \begin{pmatrix} \partial^> H^>/\partial^> r^> \\ \partial^> H^>/\partial^> p^> \end{pmatrix}, \tag{4.5.6}$$

under which we have the genoequations

$$\frac{dr^>}{dt} = \frac{\partial^> H^>}{\partial^> p^>} = \frac{p^>}{m}, \quad \frac{dp^>}{dt} = -kv, \tag{4.5.7}$$

where one should note that the derivative can be assumed to be conventional, since the system is represented by the mutation of the Lie structure.

To achieve a representation of system (5.1) suitable for operator image, we need the following *classical, finite, Lie-admissible transformation genogroup*

$$A(t) = (e^{-t\frac{\partial H}{\partial a^\mu} S^{>\mu\nu} \frac{\partial}{\partial a^\nu}}) A(0) (e^{\frac{\partial}{\partial a^\nu} {}^< S^{\nu\mu} \frac{\partial H}{\partial a^\mu} t}), \tag{4.5.8}$$

defined in the 12−dimensional *bimodular genophasespace* ${}^<T^*M \times T^*M^>$, with *infinitesimal Lie-admissible time evolution*

$$\frac{dA}{dt} = \frac{\partial A}{\partial a^\mu}({}^< S^{\mu\nu} - S^{>\mu\nu})\frac{\partial H}{\partial a^\nu} =$$

$$= (\frac{\partial A}{\partial r^k}\frac{\partial H}{\partial p_k} - \frac{\partial H}{\partial r^k}\frac{\partial A}{\partial p_k}) - (\frac{kv}{(\partial H/\partial p)})\frac{\partial H}{\partial p}\frac{\partial A}{\partial p}) =$$

$$= [A, H] - kv\frac{\partial A}{\partial p}, \tag{4.5.9}$$

where we have dropped the forward arrow for notational convenience, and $\omega^{\mu\nu}$ is the canonical Lie tensor, thus proving the Lie-admissibility of the S-tensors. In fact, the attached antisymmetric brackets $[A, H]$ are the conventional Poisson brackets, while $\{A, H\}$ are indeed symmetric brackets (as requested by Lie-admissibility), but they do not characterize a Jordan algebra (Section 4.1.3).

It is easy to see that the time evolution of the Hamiltonian is given by

$$\frac{dH}{dt} = -kv\frac{\partial H}{\partial p} = -kv^2, \tag{4.5.10}$$

thus correctly reproducing behavior (5.2).

The operator image of the above dissipative system is straightforward. Physically, we are also referring to a first approximation of a massive and stable elementary particle, such as an electron, penetrating within hadronic matter (such as a nucleus). Being stable, the particle is not expected to "disappear" at the initiation of the dissipative force and be converted into "virtual states" due to the inability of represent such a force, but more realistically the particle is expected to experience a rapid dissipation of its kinetic energy and perhaps after that participate in conventional processes.

Alternatively, we can say that an electron orbiting in an atomic structure does indeed evolve in time with conserved energy, and the system is indeed Hamiltonian. By the idea that the same electron when in the core of a star also evolves with conserved energy is repugnant to reason. Rather than adapting nature to manifestly limited Hamiltonian theories, we seek their covering for the treatment of systems for which said theories were not intended for.

The problem is to identify forward and backward genounits and related genotopic elements $I^> = 1/T^>, {}^<I = 1/{}^<T$ for which the following *operator Lie-admissible genogroup* now defined on a genomodule ${}^<\mathcal{H} \times \mathcal{H}^>$

$$A(t) = (e^{iHT^>t})A(0)(e^{-it^<TH}), \tag{4.5.11}$$

and related infinitesimal form, the *Heisenberg-Santilli genoequations*

$$i\frac{dA}{dt} = A < H - H > A = A^<TH - HT^>A, \tag{4.5.12}$$

correctly represent the considered dissipative system.

By noting that the Lie-brackets in Eqs. (4.5.9) are conventional, we seek a realization of the genotopic elements for which the Lie brackets attached to the Lie-admissible brackets (5.12) are conventional and the symmetric brackets are Jordan-isotopic. A solution is then given by [32]

$$T^> = 1 - \Gamma, \quad {}^<T = 1 + \Gamma, \tag{4.5.13}$$

for which Eq. (5.12) becomes

$$i\frac{dA}{dt} = (AH - HA) - (A\Gamma H + H\Gamma A) =$$

$$= [A, H] - \{A\hat{,}H\}, \tag{4.5.14}$$

where $[A, H]$ are a conventional Lie brackets as desired, and $\{A\hat{,}H\}$ are Jordan-isotopic brackets. The desired representation then occurs for

$$I^{>} = e^{(k/m)H^{-1}} = 1/T^{>}, \quad {}^{<}I = e^{-H^{-1}(k/m)} = 1/{}^{<}T, \tag{4.5.15a}$$

$$i\frac{dH}{dt} = -\frac{kp^2}{m^2} = -kv^2. \tag{4.5.15b}$$

Note that the achievement of the above operator form of system (5.1) without the Lie-admissible structure would have been impossible, to our knowledge.

Despite its elementary character, the above illustration has deep implications. In fact, the above example constitutes the only known operator formulation of a dissipative system in which the *nonconserved* energy is represented by a *Hermitian* operator H, thus being an *observable* despite its nonconservative character. In all other cases existing in the literature the Hamiltonian is generally *non-Hermitian,* thus *non-observable.*

The latter occurrence may illustrate the reason for the absence of a consistent operator formulation of nonconservative systems throughout the 20-th century until the advent of the Lie-admissible formulations.

4.5.2 Direct Universality of Lie-Admissible Representations for Nonconservative Systems

We now show that the Lie-admissible formulations are "directly universal," namely, they provide a classical and operator representation of all infinitely possible (well behaved) nonconservative systems of N particles (universality)

$$m_n\frac{dv_{nk}}{dt} + \frac{\partial V}{\partial r^k_n} = F^{NSA}_{nk}(t, r, p, \dot{p}, ...), \quad n = 1, 2, 3, ..., N, \quad k = 1, 2, 3, \tag{4.5.16}$$

directly in the frame of the observer, i.e., without transformations of the coordinates of the experimenter to mathematical frames (direct universality).

An illustration is given by a massive object moving at high speed within a resistive medium, such as a missile moving in our atmosphere. In this case the resistive force is approximated by a power series expansion in the velocity truncated up to the 10-th power for the high speeds of contemporary missiles

$$m\frac{dv}{dt} = \Sigma_{\alpha=1,2,...,10}k_\alpha v^\alpha, \tag{4.5.17}$$

for which any dream of conventional Hamiltonian representation is beyond the boundary of science.

The direct universality of the Hamilton-Santilli genomechanics was proved in Section 3.3. The representation in geno-phase-space is characterized by the conventional Hamiltonian representing the physical total energy, and the genounit for forward motion in time representing the NSA forces, according to the equations

$$H = \Sigma_{n,k} \frac{p_{nk}^2}{2m_n} + V(r), \quad I^> = \begin{pmatrix} 1 & \frac{F^{NSA}}{(\partial H/\partial p)} \\ 1 & 0 \end{pmatrix} \tag{4.5.18}$$

under which we have the equations of motion (for $\mu, \nu = 1, 2, 3, ...6N$) [32]

$$\frac{da^{>\mu}}{dt} = \begin{pmatrix} dr_n^{>k}/dt \\ dp_{nk}^>/dt \end{pmatrix} = S^{>\mu\nu} \frac{\partial^> H^>}{\partial^> a^{>\nu}} = \begin{pmatrix} 0 & -1 \\ 1 & \frac{F^{NSA}}{(\partial H/\partial p)} \end{pmatrix} \begin{pmatrix} \partial^> H^> / \partial^> r_n^{>k} \\ \partial^> H^> / \partial^> p_{nk}^> \end{pmatrix}, \tag{4.5.19}$$

the classical, finite, Lie-admissible genosgenogroup

$$A(t) = \exp\left(-t \frac{\partial H}{\partial a^\mu} S^{>\mu\nu} \frac{\partial}{\partial a^\nu}\right) A(0) \exp\left(\frac{\partial}{\partial a^\nu} {}^< S^{\nu\mu} \frac{\partial H}{\partial a^\mu} t\right), \tag{4.5.20}$$

with infinitesimal time evolution

$$\begin{aligned} \frac{dA}{dt} &= \frac{\partial A}{\partial a^\mu} ({}^< S^{\mu\nu} - S^{>\mu\nu}) \frac{\partial H}{\partial a^\nu} = \\ &= \left(\frac{\partial A}{\partial r_n^k} \frac{\partial H}{\partial p_{nk}} - \frac{\partial H}{\partial r_n^k} \frac{\partial A}{\partial p_{nk}}\right) - \left(\frac{km}{(\partial H/\partial p)}\right)^{nk} \frac{\partial A}{\partial p_{nk}} \frac{\partial H}{\partial p_{nk}}) = \\ &= [A, H] + \{A, H\}, \end{aligned} \tag{4.5.21}$$

yielding the correct *nonconservation of the energy*

$$\frac{dH}{dt} = v^k F_k^{NSA}. \tag{4.5.22}$$

The operator image can be characterized by the genounits and related genotopic elements

$$I^> = e^\Gamma = 1/T^>, \quad {}^< I = e^{-\Gamma} = 1/{}^< T, \quad \Gamma = H^{-1}(v_n^k F_{nk}^{NSA}) H^{-1}, \tag{4.5.23}$$

with finite Lie-admissible time evolution

$$A(t) = \exp(iHe^{-\Gamma} t) A(0) \exp(-ite^{+\Gamma} H) \tag{4.5.24}$$

and related Heisenberg-Santilli genoequations

$$\begin{aligned} i\frac{dA}{dt} &= A < H - H > A = [A, H] + \{A \hat{,} H\} = \\ &= (AH - HA) + (A\Gamma H + H\Gamma A), \end{aligned} \tag{4.5.25}$$

that correctly represent the time rate of variation of the nonconserved energy,

$$i\frac{dH}{dt} = v_n^k F_{nk}^{NSA}. \tag{4.5.26}$$

The uninitiated reader should be incidentally aware that generally different genounits may be requested for different generators, as identified since Ref. [11].

In the latter operator case we are referring to an extended, massive and stable particle, such as a proton, penetrating at high energy within a nucleus, in which case the rapid decay of the kinetic energy is caused by contact, resistive, integrodifferential forces of nonlocal type, e.g., because occurring over the volume of the particle.

The advantages of the Lie-admissible formulations over pre-existing representation of nonconservative systems should be pointed out. Again, a primary advantage of the Lie-admissible treatment is the characterization of the *nonconserved* Hamiltonian with a *Hermitian,* thus *observable* quantity, a feature generally absent in other treatments.

Moreover, the "direct universality" of Lie-admissible representations requires the following comments. Recall that coordinates transformations have indeed been used in the representation of nonconservative systems because, under sufficient continuity and regularity, the Lie-Koening theorem assures the existence of coordinate transformations $(r, p) \rightarrow (r'(r, p),\ p'(r, p))$ under which a system that is non-Hamiltonian in the original coordinates becomes Hamiltonian in the new coordinates (see Ref. [6] for details). However, the needed transformations are necessarily nonlinear with serious physical consequences, such as:

1) Quantities with direct physical meaning in the coordinates of the experimenter, such as the Hamiltonian $H(r, p) = \frac{p^2}{2m} + V(r)$, are transformed into quantities that, in the new coordinates, have a purely mathematical meaning, such as $H'(r', p') = Nexp(Mr'^2/p'^3), N, M \in R$, thus preventing any physically meaningful operator treatment;

2) There is the loss of any meaningful experimental verifications, since it is impossible to place any measurement apparatus in mathematical coordinates such as $r' = KlogLr^3, p' = Pexp(Qrp), K, L, P, Q \in R$;

3) There is the loss of Galileo's and Einstein's special relativity, trivially, because the new coordinates (r', p') characterize a highly *noninertial* image of the original inertial system of the experimenter.

All the above, and other insufficiencies are resolved by the Lie-admissible treatment of nonconservative systems.

4.5.3 Pauli-Santilli Lie-Admissible Matrices

Following the study of the nonconservation of the energy, the next important topic is to study the behavior of the conventional quantum spin under contact nonconservative forces, a topic studied for the first time in memoir [32]. For this objective, it is most convenience to use the method of Suctions 4.4.2 and 4.4.3, namely, subject the conventional Pauli's matrices to two different nonunitary transforms. To avoid un-necessary complexity, we select the following two matrices

$$A = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}, \quad AA^\dagger \neq I, \ ] \ BB^\dagger \neq I, \tag{4.6.1}$$

where a and b are non-null real numbers, under which we have the following forward and backward genounits and related genotopic elements

$$I^> = AB^\dagger = \begin{pmatrix} 1 & b \\ a & 1 \end{pmatrix}, \quad T^> = \frac{1}{(1-ab)} \begin{pmatrix} 1 & -b \\ -a & 1 \end{pmatrix}, \tag{4.6.2a}$$

$$^<I = BA^\dagger = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}, \quad ^<T = \frac{1}{(1-ab} \begin{pmatrix} 1 & -a \\ -b & 1 \end{pmatrix}. \tag{4.6.2b}$$

The *forward and backward Pauli-Santilli genomatrices* are then given respectively by

$$\sigma_1^> = A\sigma_1 B^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & (a+b) \end{pmatrix}, \quad \sigma_2^> = A\sigma_2 B^\dagger = \begin{pmatrix} 0 & -i \\ i & (a+b) \end{pmatrix}, \tag{4.6.3a}$$

$$\sigma_3^> = A\sigma_3 B^\dagger = \begin{pmatrix} 1 & b \\ a & -1 \end{pmatrix}, \quad ^<\sigma_1 = B\sigma_1 A^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & (a+b) \end{pmatrix}, \tag{4.6.3b}$$

$$^<\sigma_2 = B\sigma_2 A^\dagger = \begin{pmatrix} 0 & -i \\ i & (a+b) \end{pmatrix}, \quad ^<\sigma_3 = A\sigma_3 B^\dagger = \begin{pmatrix} 1 & a \\ b & -1 \end{pmatrix}, \tag{4.6.3c}$$

in which the direction of time is embedded in the structure of the matrices.

It is an instructive exercise for the interested reader to verify that *conventional commutation rules and eigenvalues of Pauli's matrices are preserved under forward and backward genotopies,*

$$\sigma_i^> > \sigma_j^> - \sigma_j^> > \sigma_i^> = 2i\epsilon_{ijk}\sigma_k^>, \tag{4.6.4a}$$

$$\sigma_3^> > | >= \pm 1| >, \quad \sigma^{>2>} > | >= 2(2+1)| >, \tag{4.6.4b}$$

$$^<\sigma_i >^< \sigma_j -^< \sigma_j >^< \sigma_i = 2i\epsilon_{ijk}^< \sigma_k, \tag{4.6.4c}$$

$$< | <^< \sigma_3 =< | \pm 1, \ ; < | <^< \sigma^{2>} =< |(2(2+1). \tag{4.6.4d}$$

We can, therefore, conclude by stating that *Pauli's matrices can indeed be lifted in such an irreversible form to represent the direction of time in their very structure.* However, in so doing the conventional notion of spin is lost in favor of a covering notion in which the spin becomes a locally varying quantity, as expected to a proton in the core of a star.

Consequently, the Lie-admissible formulation of Pauli matrices confirms the very title of memoir [12] proposing the construction of hadronic mechanics.

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The argument is that, while special relativity and Pauli exclusion principle are unquestionably valid for the conditions of their original conception, particles at large mutual distances under action-at-a-distance interactions (such as for a point-like proton in a particle accelerator under long range electromagnetic interactions), by no means the same doctrines have to be necessarily valid for *one* hadronic constituent when considering all other constituents as external.[6]

The above analysis focuses the attention in an apparent fundamental structural difference between electromagnetic and strong interactions. Irrespective of whether considered part of the system (closed system) or external (open system), *electromagnetic interactions do verify Pauli principle,* as well known. The best example is given by Dirac's equation for the hydrogen atom that, as known to experts to qualify as such, represents one electron under the *external* electromagnetic field of the proton. The origin of the preservation of Pauli principle is that, whether electromagnetic interactions are closed or open, they are Hamiltonian. Lie's theory then applies with the conventional notion of spin, and Pauli principle follows.

By comparison, strong interactions are non-Hamiltonian for the numerous reasons indicated during tour analysis. Consequently, the conventional notion of spin cannot be preserved, and Pauli principle is inapplicable in favor of broader vistas. It is intriguing to note that the representation of a proton via isomechanics allows indeed a representation of its extended, nonspherical and deformable shape. Nevertheless, Pauli's principle is preserved under isotopies, as indicated in Chapter 3. Hence, the inapplicability of Pauli's principle is here referred to, specifically and solely, for open irreversible conditions at short mutual distances, exactly according to the original proposal to build hadronic mechanics [12].

The above distinction between electromagnetic and strong interactions is the conceptual foundation of monographs [40,41] suggesting the characterization of the hadronic constituents via *Lie-admissible,* rather than Lie or Lie-isotopic algebras, with the consequential inapplicability of the conventional. notion of spin. These basic issues will be studied in detail in Volume II in connection with explicit structure models of hadrons with physical constituents, that is, constituents that can be produced free in spontaneous decays while being compatible with the SU(3)-color Mendeleev-type *classification* of hadrons.

To conclude, *not only special relativity, but also Pauli principle is inapplicable (rather than violated) for a hadron under external strong interactions.* Needless to

[6]The reader should always keep in mind that, even though not stated in the technical literature for evident political reasons, quantum mechanics can only represent the proton as a dimensionless point.

say, when a particle with the open nonconservative spin under consideration here is "completed" with the inclusion of all remaining strong interacting particles here considered as external, Pauli principle is recovered in full for the center of mass of the ensemble as a whole because the "completion" is treated via isomechanics.

4.5.4 Minkowski-Santilli Irreversible Genospacetime

One of the fundamental axiomatic principles of hadronic mechanics is that irreversibility can be directly represented with the background geometry and, more specifically, with the metric of the selected geometry. This requires the necessary transition from the conventional *symmetric* metrics used in the 20-th century to covering *nonsymmetric* genometric.

To show this structure, we study in this section the genotopy of the conventional Minkowskian spacetime and related geometry with the conventional metric $\eta = Diag.(1,1,1,-1)$ and related spacetime elements $x^2 = x^\mu \eta_{\mu\nu} x^\nu,\ x = (x^1, x^2, x^3, x^4),\ x^4 = ct,\ c = 1$. For this purpose, we introduce the following four-dimensional non-Hermitian, nonsingular and real-valued forward and backward genounits

$$I^> = CD^\dagger = 1/T^>, \quad {}^<I = DC^\dagger = 1/{}^<T, \quad CC^\dagger \neq I, \quad DD^\dagger \neq I, \tag{4.6.5}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ p & 0 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ q & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4.6.6}$$

where $p \neq q$ are non-null real numbers, under which we have the following forward and backward genotopy of the Minkowskian line element

$$\begin{gathered} x^2 \to x^{>2>} = Cx^2D^\dagger = C(x^t\eta x)D^\dagger = \\ = (C^t x^t D^{t\dagger})(CD^\dagger)^{-1}(C\eta D^\dagger)(CD^\dagger)^{-1}(CxD^\dagger) = \\ = (x^t I^>)T^>\eta^>T^>(I^>x) = x^\mu \eta^>_{\mu\nu} x^\nu = \\ = (x^1x^1 + x^1qx^3 + x^2x^2 + x^3x^3 + x^1px^4 - x^4x^4), \end{gathered} \tag{4.6.7a}$$

$$\begin{gathered} Dx^2C^\dagger = D(x^t\eta x)C^\dagger = \\ = (x^{t<}I)^<T^<\eta^<T(^<Ix) = x^{\mu<}\eta_{\mu\nu}x^\nu = \\ = (x^1x^1 + x^1px^3 + x^2x^2 + x^3x^3 + x^1qx^4 - x^4x^4), \end{gathered} \tag{4.6.7b}$$

resulting in the forward and backward nonsymmetric genometrics

$$\eta^> = \begin{pmatrix} 1 & 0 & q & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ q & 0 & 0 & 1 \end{pmatrix}, \quad {}^<\eta = \begin{pmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ q & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4.6.8}$$

exactly as desired.

Note that irreversibility selects a mutation of the line elements along a preselected direction of space and time.

Note also that the quantities p and q can be functions of the local spacetime variables, in which case the resulting *Minkowskian genogeometry* can be equipped by a suitable lifting of the machinery of the Riemannian geometry (see Ref. [16] for the isotopic case and Chapter 3).

Note finally that the above genospacetime includes, as particular case, *an irreversible formulation of the Riemannian geometry,* where irreversibility is represented at the ultimate geometric foundations, the basic unit and the metric.

It should be indicated that the above irreversible formulation of spacetime has intriguing implications for the mathematical model known as *geometric locomotion* studied in detail in monograph [73] via the isotopies of the Minkowskian geometry. In fact, a main unresolved problems is the directional deformation of the geometry as needed to permit the geometric locomotion in one preferred direction of space. An inspection of the mutated line elements (4.6.7) clearly shows that the genotopies are preferable over the isotopies for the geometric locomotion, as well as, more generally, for a more realistic geometric characterization of irreversible processes.

The construction of the *Lorentz-Santilli genotransformations* is elementary, due to their formal identify with the isotopic case of Chapter 3, and its explicit construction left as an instructive exercise for the interested reader.

4.5.5 Dirac-Santilli Irreversible Genoequation

To complete the illustrations in particle physics, we now outline the simplest possible genotopy of Dirac's equation via the genotopies of the preceding two suctions, one for the spin content of Dirac's equation and the other for its spacetime structure. Also, we shall use Dirac's equation in its isodual re-interpretation representing a direct product of one electron and one positron, the latter without any need of second quantization (see monograph [73] for detail). In turn, the latter re-interpretation requires the use of the *isodual transform* $A \to A^d = -A^\dagger$) as being distinct from Hermitian conjugation. Under the above clarifications, the *forward Dirac genoequation* here referred to can be written

$$\eta^{>\mu\nu}\gamma^>_\mu T^> p^>_\nu - im)T^>|\psi^> >= 0 \tag{4.6.9a}$$

$$p^>_\nu T^>|\psi^> >= -i\frac{\partial^>}{\partial^> x^{>\nu}}|\psi^> >= -iI^>\frac{\partial}{\partial x^>}|\psi^> >, \tag{4.6.9b}$$

with *forward genogamma matrices*

$$\gamma^>_4 = \begin{pmatrix} A & 0 \\ 0 & B^d \end{pmatrix}\begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}\begin{pmatrix} A^d & 0 \\ 0 & B \end{pmatrix} = \begin{pmatrix} AA^d & 0 \\ 0 & -B^dB \end{pmatrix}, \tag{4.6.10a}$$

$$\gamma_k^{>} = \begin{pmatrix} A & 0 \\ 0 & B^d \end{pmatrix} \begin{pmatrix} 0 & \sigma_k \\ \sigma_k^d & 0 \end{pmatrix} \begin{pmatrix} A^d & 0 \\ 0 & B \end{pmatrix} = \tag{4.6.10b}$$

$$= \begin{pmatrix} 0 & A\sigma_k B^\dagger \\ B\sigma_k^d A^d & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma_k \\ \sigma_k^d & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sigma_k^{>} \\ {}^{<}\sigma_k^d & 0 \end{pmatrix}, \tag{4.6.10c}$$

$$\{\gamma_\mu^{>}\hat{,}\gamma_\nu^{>}\} = \gamma_\mu^{>} T^{>} \gamma_\nu^{>} + \gamma_\nu^{>} T^{>} \gamma_\mu^{>} = 2\eta_{\mu\nu}^{>}, \tag{4.6.10d}$$

where $\eta_{\mu\nu}^{>}$ is given by the same genotopy of Eqs. (4.6.10a).

Interested readers can then construct the backward genoequation. They will discover in this way a new fundamental symmetry of Dirac's equation that remained undiscovered throughout the 20-th century, its *isoselfduality* (invariance under isoduality.) This new symmetry is now playing an increasing role for realistic cosmologies, those inclusive of antimatter, or for serious unified theories that must also include antimatter to avoid catastrophic inconsistencies [73] (see Volume II).

It is an instructive exercise for the interested reader to verify a feature indicated earlier, the inapplicability of the conventional notion of spin and, consequently, of Pauli principle for the Dirac-Santilli genoequation. As we shall see in Volume II, the conventional Dirac equation represents the electron in the structure of the hydrogen atom. By comparison, the Dirac-Santilli genoequation represents the same electron when totally immersed in the hyperdense medium inside a proton, thus characterizing the structure of the neutron according to hadronic mechanics..

Note that, while the electron is moving forward, the positron is moving backward in time although referred to a negative unit of time, as a necessary condition to avoid the inconsistencies for negative energies that requested the conjecture of the "hole theory" (see monograph xxx for brevity).

4.5.6 Dunning-Davies Lie-Admissible Thermodynamics

A scientific imbalance of the 20-th century has been the lack of interconnections between thermodynamics, on one side, and classical and quantum mechanics, on the other side. This is due to the fact that the very notion of entropy,
indexEntropy let alone all thermodynamical laws, are centrally dependent on irreversibility, while classical and quantum Hamiltonian mechanics are structurally reversible (since all known potentials are reversible in time).

As recalled in Section 4.1, said lack of interconnection was justified in the 20-th century on the belief that the nonconservative forces responsible for irreversibility according to Lagrange and Hamilton, are "fictitious" in the sense that they only exist at the classical level and they "disappear" when passing to elementary particles, since the latter were believed to be completely reversible. In this way, thermodynamics itself was turned into a sort of "fictitious" discipline.

This imbalance has been resolved by hadronic mechanics beginning from its inception. In fact, Theorems 1.3.3 has established that, far from being "fictitious," nonconservative forces originate at the ultimate level of nature, that of elementary particles in conditions of mutual penetration causing contact nonpotential (NSA) interactions. The insufficiency rested in the inability by quantum mechanics to represent nonconservative forces, rather than in nature. In fact, hadronic mechanics was proposed and developed precisely to reach an operator representation of the nonconservative forces originating irreversibility along the legacy of Lagrange and Hamilton.

As a result of the efforts presented in this chapter, we now possess not only classical and operator theories, but more particularly we have a *new mathematics,* the genomathematics, whose basic axioms are not invariant under time reversal beginning from the basic units, numbers and differentials.

Consequently, hadronic mechanics does indeed permit quantitative studies of the expected interplay between thermodynamics and classical as well as operator mechanics. These studies were pioneered by J. Dunning Davies [30] who introduced the first known study of thermodynamics via methods as structurally irreversible as their basic laws, resulting in a formulation we hereon call *Dunning-Davies Lie-admissible thermodynamics.* This section is dedicated to a review of Dunning-Davies studies.

Let us use conventional thermodynamical symbols, a classical form of thermodynamics, and the simple construction of irreversible formulations via two different complex valued quantities A and B. Then, the first law of thermodynamics can be lifted from its conventional formulation, that via reversible mathematics, into the form permitted by genomathematics

$$Q \to Q^> = AQB^\dagger = QI^>, \quad U \to U^> = AUB^\dagger = UI^>, \quad \text{etc.}, \tag{4.6.11a}$$

$$dQ = dU + pdV \quad \to \quad d^>Q^> = d^>U^> + p^> > d^>V^>, \tag{4.6.11b}$$

where, in the absence of operator forms, Hermitian conjugation is complex conjugation. For the second law we have

$$dQ = TdS \quad \to \quad d^>Q^> = T^> > d^>S^>, \tag{4.6.12}$$

thus implying that

$$TdS = dU + pdV \quad \to \quad T^> > d^>S^> = d^>U^> + p^> > d^>V^>. \tag{4.6.13}$$

As one can see, genomathematics permits the *first known formulation of entropy with a time arrow,* the only causal form being that forward in time. When the genounit does not depend on the local variables, the above genoformulation reduces to the conventional one identically, e.g.,

$$\begin{aligned} T^> > d^>S^> &= (TI^>)I^{>-1}[I^{>-1}d(SI^>) = TdS = \\ &= I^{>-1}d(VI^>) + (pI^>)I^{>-1}d(VI^>) = dU + pdV. \end{aligned} \tag{4.6.14}$$

This confirms that genomathematics is indeed compatible with thermodynamical laws.

However, new vistas in thermodynamics are permitted when the genounit is dependent on local variables, in which case reduction (4.6.13) is no longer possible. An important case occurs when the genounit is explicitly dependent on the entropy. In this case the l.h.s. of Eq. (4.6.13) becomes

$$TdS + TS(I^{>-1}dI^{>}) = dU + pdV. \tag{4.6.15}$$

We then have new thermodynamical models of the type

$$I^{>} = e^{f(S)}, \;\; T^{>} > d^{>}S^{>} = T\left(1 + S\frac{\partial f(S)}{\partial S}\right)dS = dU + pdV, \tag{4.6.16}$$

permitting thermodynamical formulations of the behavior of anomalous gases (such as magnegases [21]) via a suitable selection of the $f(S)$ function and its fit to experimental data. Needless to say, equivalent models can be constructed for an explicit dependence of the genounit from the other variables. For these and other aspects we have to refer the interested reader to Volume II.

4.5.7 Ongoing Applications to New Clean Energies

A primary objective of Volume II is to study industrial applications of hadronic mechanics to new clean energies that are under development at the time of writing this first volume (2002). Hence, we close this chapter with the following preliminary remarks.

The societal, let alone scientific implications of the proper treatment of irreversibility are rather serious. Our planet is afflicted by increasingly catastrophic climactic events mandating the search for basically new, environmentally acceptable energies, for which scope the studies reported in these monographs were initiated.

All known energy sources, from the combustion of carbon dating to prehistoric times to the nuclear energy, are based on irreversible processes. By comparison, all established doctrines of the 20-th century, such as quantum mechanics and special relativity, are reversible, as recalled in Section 4.1.

It is then easy to see that *the serious search for basically new energies requires basically new theories that are as structurally irreversible as the process they are expected to describe.* At any rate, all possible energies and fuels that could be predicted by quantum mechanics and special relativity were discovered by the middle of the 20-th century. Hence, the insistence in continuing to restrict new energies to verify preferred reversible doctrines may cause a condemnation by posterity due to the environmental implications.

An effective way to illustrate the need for new irreversible theories is given by nuclear fusions. All efforts to date in the field, whether for the "cold fusion" or the "hot fusion," have been mainly restricted to verify quantum mechanics and special relativity. However, *whether "hot" or "cold," all fusion processes are irreversible, while quantum mechanics and special relativity are reversible.*

It has been shown in Ref. [31] that the failure to date by both the "cold" and the "hot" fusions to achieve industrial value is primarily due to the treatment of irreversible nuclear fusions with reversible mathematical and physical methods.

In the event of residual doubt due to protracted use of preferred theories, it is sufficient to compute the quantum mechanical probability for two nuclei to "fuse" into a third one, and then compute its time reversal image. In this way the serious scholar will see that special relativity and quantum mechanics may predict a fully causal *spontaneous disintegration of nuclei following their fusion,* namely, a prediction outside the boundary of science.

The inclusion of irreversibility in quantitative studies of new energies suggests the development, already partially achieved at the industrial level (see Chapter 8 of Ref. [20]), of the new, controlled "intermediate fusion" of light nuclei [31], that is, a fusion occurring at minimal threshold energies needed: 1) To verify conservation laws; 2) To expose nuclei as a pre-requisite for their fusion (a feature absent in the "cold fusion" due to insufficient energies), and 3) To prevent uncontrollable instabilities (as occurring at the very high energies of the "hot fusion").

It is hoped that serious scholars will participate with independent studies on the irreversible treatment of new energies, as well as on numerous other open problems, because in the final analysis lack of participation in basic advances is a gift of scientific priorities to others.

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Chapter 5

HYPERSTRUCTURAL BRANCH OF HADRONIC MECHANICS AND ITS ISODUAL

5.1 The Scientific Imbalance in Biology

In our view, the biggest scientific imbalance of the 20-th century has been the treatment of biological systems (herein denoting DNA, cells, organisms, etc.) via conventional mathematics, physics and chemistry because of various reasons studied in detail in Chapter 1.1.

We here limit ourselves to recall that biological events, such as the growth of an organism, are irreversible over time, while the mathematics of the 20-th century and related formulations are structurally reversible, that is, reversible for all possible Hamiltonians. Therefore, any treatment of biological systems via reversible mathematics, physical and chemical formulations can indeed receive temporary academic acceptance, but cannot pass the test of time.

Quantum mechanics is ideally suited for the treatment of the structure of the hydrogen atom or of crystals, namely, systems that are fully reversible. These systems are represented by quantum mechanics as being ageless. Recall also that quantum mechanics is unable to treat deformations because of incompatibilities with basic axioms, such as that of the rotational symmetry.

Therefore, *the strict application to biological systems of the mathematics underlying quantum mechanics and chemistry implies that all organisms from cells to humans are perfectly reversible, totally rigid and fully eternal.*

5.2 The Need in Biology of Irreversible Multi-Valued Formulations

It is possible to see that, despite their generality, the invariant irreversible genoformulations studied in the preceding chapter are insufficient for in depth treatments of biological systems.

In fact, recent studies conducted by Illert [1] have pointed out that the *shape* of sea shells can certainly be represented via conventional mathematics, such as the Euclidean geometry.

However, the latter conventional geometries are inapplicable for a representation of the *growth over time* of sea shells. Computer simulations have shown that the imposition to sea shell growth of conventional geometric axioms causes the lack of proper growth, such as deformations and cracks, as expected, because said geometries are strictly reversible over time, while the growth of sea shells is strictly irreversible.

The same studies by Illert [1] have indicated the need of a mathematics that is not only structurally irreversible, but also *multi-dimensional.* As an example, Illert achieved a satisfactory representation of sea shells growth via the *doubling of the Euclidean reference axes,* namely, via a geometry appearing to be six-dimensional.

A basic problem in accepting such a view is the lack of compatibility with our sensory perception. When holding sea shells in our hands, we can fully perceive their shape as well as their growth with our three Eustachian tubes. Hence, any representation of sea shells growth with more than three dimensions is incompatible with our perception of reality.

Similarly, our sensory perception can indeed detect curvature. Thus, any representation of sea shell growth with the Riemannian geometry would equally be incompatible with our sensory perception. At any rate, any attempt at the use of the Riemannian geometry for sea shell growth would be faced with fatal inconsistencies, such as the inability to represent bifurcations and other aspects since such representations would be prohibited by curvature.

These occurrences pose a rather challenging problem, the construction of yet another *new mathematics* that is

(1) Structurally irreversible over time (as that of the preceding section);

(2) Capable to represent deformations;

(3) Invariant under the time evolution in the sense of predicting the same number under the same conditions but at different times;

(4) Multi-dimensional; and, last but not least,

(5) Compatible with our sensory perception.

The only solution known to the author is that of building an irreversible *multi-valued* (rather than multi-dimensional) new mathematics, in the sense that the basic axioms of the space representation can remain three-dimensional to achieve compatibility with our sensory perception, but each axis can have more than one value, thus being multi-valued.

A search in the mathematical literature soon revealed that a mathematics verifying all the above requirements did not exist and had to be constructed.

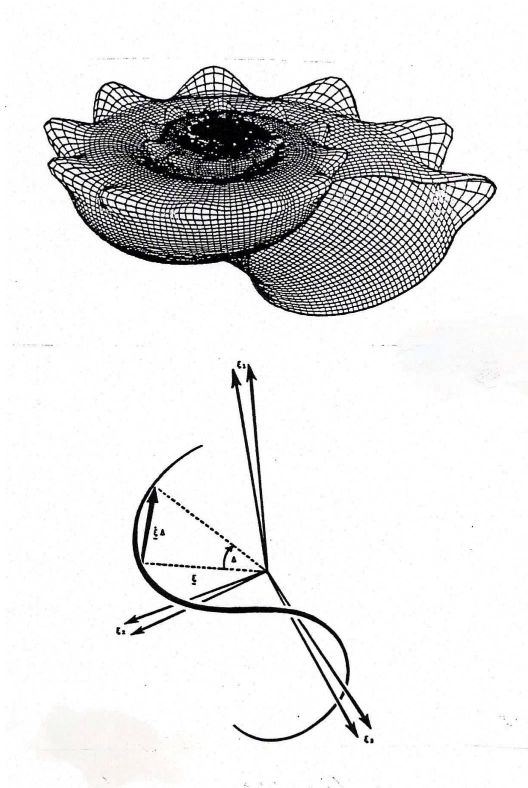

Figure 5.1. A schematic view of Illert [1] has shown that a representation of the growth over time of a seashell can be effectively done by doubling the number of reference axes. However, seashell growth is perceived by our sensory perception as occurring in three-dimensional space. The multi-valued hyperstructural branch of hadronic mechanics studied in this chapter provides a solution of these seemingly discordant requirements because, on side, it is as multi-valued as desired while, on the other side, remains three-dimensional at the abstract, realization-free level.

As an example, in their current formulations, *hyperstructures* (see, e.g., Ref. [2]) lack a well defined left and right unit thus lacking the applicability to the measurements; they do not have conventional operations, but rather the so-called *weak operations*, thus lacking applicability to experiments; they are not structurally irreversible; and they lack invariance. Consequently, conventional hyperstructures are not suitable for applications in biology.

5.3 Rudiments of Santilli Hyper-Mathematics and Hypermechanics

After a number of trials and errors, a yet broader mathematics verifying the above five conditions was identified by R. M. Santilli in monographs [3] of 1995 and in works [4,5], and subsequently studied by R. M. Santilli and the mathematician T. Vougiouklis in paper [6] of 1996 (see also mathematical study [7]). These studies resulted in a formulation today known as *Santilli hypermathematics.*

For an in depth study, including the all crucial *Lie-Santilli hypertheory,* we refer the reader to the mathematical treatments [4–7]. By assuming an in depth knowledge of genomathematics of the preceding chapter, we here limit ourselves to indicate that the selected hypermathematics is based on the assumption that the single-valued forward and backward genounits of the preceding chapter although replaced with the following *multi-valued hyperunits*

$$\hat{I}^{>}(t, x, v, \psi, \partial_x\psi, ...) = Diag.(\hat{I}_1^{>}, \hat{I}_2^{>}, \hat{I}_3^{>}) =$$

$$= Diag.\Big[(\hat{I}_{11}^{>}, \hat{I}_{12}^{>}, ..., \hat{I}_{1m}^{>}), (\hat{I}_{21}^{>}, \hat{I}_{22}^{>}, ..., \hat{I}_{2m}^{>}), (\hat{I}_{31}^{>}, \hat{I}_{32}^{>}, ..., \hat{I}_{3m}^{>})\Big], \tag{5.1a}$$

$$^{<}\hat{I}(t, x, v, \psi, ...) = Diag.(^{<}\hat{I}_1, ^{<}\hat{I}_2, ^{<}\hat{I}_3) =$$

$$= Diag.\Big[(^{<}\hat{I}_{11}, ^{<}\hat{I}_{12}, ..., ^{<}\hat{I}_{1m}), (^{<}\hat{I}_{21}, ^{<}\hat{I}_{22}, ..., ^{<}\hat{I}_{2m}),$$

$$(^{<}\hat{I}_{31}, ^{<}\hat{I}_{32}, ..., ^{<}\hat{I}_{3m})\Big], \tag{5.1b}$$

with corresponding *ordered hyperproducts to the right and to the left*

$$A > B = A \times \hat{T}^{>} \times B, A < B = A \times^{<} \hat{T} \times B, \tag{5.2a}$$

$$\hat{I}^{>} > A = A > \hat{I}^{>} = A, \quad ^{<}\hat{I} < AA <^{<} \hat{I} = A, \tag{5.2b}$$

$$\hat{I}^{>} = (^{<}\hat{I})^{\dagger} = 1/\hat{T}^{>}. \tag{5.2c}$$

Following the hyperlifting of the methods of the preceding chapter, we reach the following basic equations of the *multi-valued hyperstructural branch of hadronic mechanics,* first proposed by Santilli in monographs [3] of 1995 (see also the mathematical works [4–6], here written in the finite and infinitesimal forms

$$i\ dA/dt = A \lhd H - H \rhd A, \tag{5.3a}$$

$$A(t) = \hat{e}^{i \times H \times t} \lhd A(0) \rhd \hat{e}^{-i \times t \times H}, \tag{5.3b}$$

quoted in Footnote 15 of Chapter 1, where the multivalued character of all quantities and their operations is assumed.

In the above expressions the reader should recognize the diagonal elements of the genounits of the preceding chapter and then identify the multi-valued character for each diagonal element. Consequently, the above mathematics *is not* $3m$-dimensional, but rather it is 3-dimensional and m-multi-valued, namely, each axis in three-dimensional space can assume m different values.

Such a feature permits the increase of the reference axes, e.g., for $m = 2$ we have six axes as used by Illert [1], while achieving compatibility with our sensory perception because at the abstract, realization-free level hypermathematics characterized by hyperunit is indeed 3-dimensional.

It is instructive for readers interested in learning the new mathematics to prove the following

LEMMA 5.1 [3]: All rings of elements $a \times \hat{I}^>$ ($^<\hat{I} \times a$), where a is an ordinary (real, complex or quaternionic) number and $\hat{I}^>$ ($^<\hat{I}$) is the forward (backward) multivalued hyperunit, when equipped with the forward (backward) hyperproduct, verify all axioms of a field.

A good understanding of the above property can be reached by comparison with the preceding studies. The discovery of isofields [8] studied in Chapter 3 was made possible by the observation that *the axioms of a field are insensitive to the value of the unit.* As a result of which we have isoproducts of the type

$$\hat{I} = 1/3 = 1/\hat{T}, \quad 2\hat{\times}3 = 2 \times \hat{T} \times 3 = 18. \tag{5.4}$$

The discovery of genofields also in Ref. [8] was due to the observation that *the axioms of a field are additionally insensitive to the ordering of a product to the right or to the left, provided that all operations are restricted to one selected order.* This lead to *two* inequivalent multiplications, one to the right and one to the left, as necessary to represent irreversibility, such as

$$\hat{I}^> = 1/3 = \hat{T}, \;\; 2 > 3 = 18, {}^<\hat{I} = 3, \;\; 2 < 3 = 2. \tag{5.5}$$

Lemma 5.1 essentially reflects the additional property according to which *the axioms of a field are also insensitive as to whether, in addition to the selection of an ordering as per genofields, the units and (ordered) products are multivalued,* e.g.,

$$\hat{I}^> = \{1/3, 1/5\}, \;\; 2\hat{>}3 = \{18, 30\}, \;\; {}^<\hat{I} = \{3, 2\}, \;\; 2\hat{<}3 = \{2, 3\}, \tag{5.6}$$

where the results of the hypermultiplications should be interpreted as an ordered set.

Once the notion of hyperfield is understood, the construction of all remaining aspects of hypermathematics can be conducted via simple compatibility arguments, thus leading in this way to *hyperspaces, hyperfunctional analysis, hyperdifferential calculus, hyperalgebras, etc.*

Note that the resulting hyperformulations are invariant as it is the case for genomathematics. The proof of such an invariance is here omitted for brevity, but recommended to readers interested in a serious study of the field.

The above features serve to indicate that the biological world has a complexity simply beyond our imagination, and that studies of biological problems conducted in the 20-th century, such as attempting an understanding the DNA code via numbers dating back to biblical times, are manifestly insufficient.

The above features appear to be necessary for the representation of biological systems. As an example, consider the association of two atoms in a DNA producing an organ composed by a very large number of atoms, such as a liver. A quantitative treatment of this complex event is given by representing the two atoms with α and β and by representing their association in a DNA with the hyperproduct. The resulting large number of atoms γ_k in the organ is then represented by the ordered multi-valued character of the hyperproduct, such as

$$\alpha \hat{>} \beta = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \ldots, \gamma_n, \}. \tag{5.7}$$

The above attempt at decybrings the DNA code is another illustration of our view that the complexity of biological systems is simply beyond our comprehension at this time. A mathematical representation will eventually be achieved in due time. However, any attempt at its “understanding” would face the same difficulties of attempting to understand infinite-dimensional Hilbert space in quantum mechanics, only the difficulties are exponentially increased for biological structures.

5.4 Rudiments of Santilli Isodual Hypermathematics

The *isodual hypermathematics* can be constructed via the isodual map of Chapter 2 here expressed for an arbitrary operator $\hat{A}$,

$$\hat{A}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots) \to -\hat{A}^\dagger(-\hat{t}^\dagger, -\hat{r}^\dagger, -\hat{p}^\dagger, -\hat{\psi}^\dagger, \ldots) = \hat{A}^d(\hat{t}^d, \hat{r}^d, \hat{p}^d, \hat{\psi}^d, \ldots), \tag{5.8}$$

applied to the *totality* of hypermathematics, including its operations, with no exception (to avoid inconsistencies), thus yielding *isodual hyperunits, isodual hypernumbers, isodual hyperspaces, etc.*

Consequently, the formulations here considered have *four* different hyperunits, the forward and backward hyperunits and their isoduals,

$$\hat{I}^>,\ {}^<\hat{I},\ \hat{I}^{>d},\ {}^{<d}\hat{I}, \tag{5.9}$$

that, in turn, have to be specialized into forward and backward *space and time hyperunits* and their isoduals.

Consequently, the formulations herein considered have *four* different *hypercoordinates*

$$\hat{x}^>,\ {}^<\hat{x},\ \hat{x}^{>d},\ {}^<\hat{x}^d, \tag{5.10}$$

and *four* different *hypertimes,*

$$\hat{t}^>,\ {}^<\hat{t},\ \hat{t}^{>d},\ {}^<\hat{t}^d. \tag{5.11}$$

In chapter 2 (see also Figure 2.2) we have studied the need for four different times. We now have the four different hypertimes for: 1) Motion forward to future times characterized by $\hat{t}^>$; 2) Motion backward to past time characterized by $^<\hat{t}$; 3) Motion backward from future times characterized by $\hat{t}^{>d}$, and 4) motion forward from past times characterized by $^{<d}\hat{t}$. The main difference between the four times of Chapter 2 and the four hypertimes of this chapter is that the former are single-valued while the latter are multi-valued.

Note again the *necessity of the isodual map to represent all four possible time evolutions.* In fact, the conventional mathematics, such as that underlying special relativity, can only represent two our four possible time evolutions, motion forward to future time and motion backward to past time, the latter reached via the conventional time reversal operation.

The following intriguing and far reaching aspect emerges in biology. Until now we have strictly used isodual theories for the sole representation of antimatter. However, Illert [1] has shown that *the representation of the bifurcations in sea shells requires the use of all four directions of time.*

The latter aspect is an additional illustration of the complexity of biological system. In fact, the occurrence implies that the "intrinsic time" of a seashell, that is, the time perceived by a sea shells as a living organism, is so complex to be beyond our comprehension at this writing. Alternatively, we can say that the complexity of hypertimes is intended to reflect the complexity of biological systems.

In conclusion, the achievement of invariant representations of biological structures and their behavior can be one of the most productive frontiers of science, with far reaching implications for other branches, including mathematics, physics and chemistry.

As an illustration, a mathematically consistent representation of the non-Newtonian propulsion of sap in trees, all the way up to big heights, automatically provides a model of *geometric propulsion* studied in Volume II, namely propulsion caused by the alteration of the local geometry without any external applied force.

5.5 Santilli Hyperrelativity and Its Isodual

All preceding formulations can be embodied into one single axiomatic structure submitted in monographs [3,5] and today known as *Santilli hyperrelativity and its isodual,* that are characterized by:

1) The *irreversible, multi-valued, forward and backward, Minkowski-Santilli hyperspace* with the following *forward and backward spacetime hypercoordinates* and *forward and backward hyperintervals* over *forward and backward hyperfields*, and their isoduals

$$\hat{M}^>(\hat{x}^>, \hat{\eta}^>, \hat{R}^>), \hat{x}^{>2} = \hat{x}^{>\mu} \hat{>} \hat{\eta}^>_{\mu\nu} \hat{>} \hat{x}^{>\nu} \in \hat{R}^>, \tag{5,12a}$$

$$^<\hat{M}(^<\hat{x}, ^<\hat{\eta}, ^<\hat{R}), ^<\hat{x}^2 =^< \hat{x}^{>\mu} \hat{<} ^<\hat{\eta}_{\mu\nu} \hat{<} ^<x^\nu \in ^< \hat{R}, \tag{5.12b}$$

$$\hat{m}^{>d}(\hat{x}^{>d}, \hat{\eta}^{>d}, \hat{R}^{>d}, ^<\hat{m}^d(^<\hat{x}^d, ^<\hat{\eta}^d, ^<\hat{R}^d; \tag{5.12c}$$

2) The corresponding *irreversible, multi-valued, forward and backward Poincaré-Santilli hypersymmetry* and their isoduals here written via the Kronecker product

$$\hat{P}^{>}(3,1)time^{<}\hat{P}(3.1) \times \hat{P}^{>d}(3,1)time^{<}\hat{P}^{d}(3.1), \tag{5.13}$$

essentially given by the Poincaré-Santilli genosymmetry of the preceding chapter under a multi-valued realization of the local coordinates and their operations;

3) The corresponding *forward and backward hyperaxioms* and their isoduals:

FORWARD HYPERAXIOM I. The projection in our spacetime of the maximal causal invariant speed on forward Minkowski-Santilli hyperspace in $(3,4)$*-dimensions is given by:*

$$\hat{V}_{Max} = c_o \times \frac{b_4^{>}}{b_3^{>}} = c_o \times \frac{n_3^{>}}{n_4^{>}} = \hat{c}^{>}/\hat{b}_3^{>}, \quad \hat{c}^{>} = c_o \times b_4^{>} = \frac{c_o}{\hat{n}_4^{>}}, \tag{5.14}$$

FORWARD HYPERAXIOM II. The projection in our spacetime of the hyperrelativistic addition of speeds within MULTI-VALUED physical media represented by the forward Minkowski-Santilli hyperspace is given by:

$$\hat{V}_{tot}^{>} = \frac{\hat{v}_1^{>} + \hat{v}_2^{>}}{\hat{1}^{>} + \frac{\hat{v}_1^{>} \times b_3^{>2} \times \hat{v}_2^{>}}{c_o \times b_4^{>2} \times c_o}} = \frac{\hat{v}_1^{>} + \hat{v}_2^{>}}{\hat{1}^{>} + \frac{\hat{v}_1^{>} \times \hat{n}_4^{>2} \times \hat{v}_2^{>}}{c_o \times \hat{n}_3^{>2} \times c_o}}. \tag{5.15}$$

FORWARD HYPERAXIOM III. The projection in our spacetime of the forward hyperdilation of forward hypertime, forward hypercontraction of forward hyperlength and the variation of forward hypermass with the forward hyperspeed are given respectively by

$$\hat{t}^{>} = \hat{\gamma}^{>} \times \hat{t}_{\circ}^{>}, \tag{5.16a}$$

$$\hat{\ell}_{\circ}^{>} = \hat{\gamma}^{>} \times \hat{\ell}, \tag{5.16b}$$

$$\hat{m}^{>} = \hat{\gamma}^{>} \times \hat{m}_{\circ}^{>}. \tag{5.16c}$$

FORWARD HYPERAXIOM IV. The projection in our spacetime of the Doppler-Santilli forward hyperlaw is given by the expression (here formulated for simplicity for 90° *angle of aberration):*

$$\hat{\omega}^{>} = \hat{\gamma} \times^{>} \hat{\omega}_{\circ}^{>}. \tag{5.17}$$

ISOAXIOM V. The projection in our spacetime of the hyperrelativistic law of equivalence of forward hypermass and the forward hyperenergy is given by:

$$\hat{E}^{>} = \hat{m}^{>} \times \hat{V}_{\max}^{2>} = \hat{m}^{>} \times c_o^2 \times \frac{\hat{b}_4^{>\ 2}}{\hat{b}_3^{>\ 2}} = \hat{m}^{>} \times c_o^2 \times \frac{\hat{n}_3^{>\ 2}}{\hat{n}_4^{>\ 2}}. \tag{5.18}$$

Figure 5.2. Samples of sliced seashells showing the complexity of their structure. Illert [1] has shown that a mathematical representation of their four-lobes bifurcations requires all four directions of times, namely, the knowledge by the seashell of motions forward in future and past times as well as motions backward from future and in past times. The need for multi-valued methods, plus these four different time arrows then identify our hyperstructures and their isoduals quite uniquely. Whatever the appropriate theory, it can be safely stated that the complexity of the "intrinsic time" of biological structure (that perceived by said structures rather than by us) can be safely stated to be beyond our comprehension at this writing.

In the above expressions we have used the following notations: *hypergamma and hyperbeta* are given by

$$\hat{\gamma}^{>} = (1-\hat{\beta}^{2>})^{-1/2}, \quad \hat{\beta}^{>} = \hat{v}^{2>} \times \hat{n}_4^{>\ 2}/c_o^2 \times \hat{n}_3^{>\ 2} = \hat{v}^{2>} \times \hat{b}_3^{>\ 2}/c_o^2 \times \hat{b}_4^{>\ 2}; \quad (5.19)$$

the upper symbol $^{>}$ denotes motion forward to future times; the upper symbol $\hat{x}$, etc., denotes multivalued character; and all multiplications are conventional (rather than being hyperproducts) since the hyperaxioms are expressed in their projection in our spacetime to avoid excessive complexity.

The study of the backward and isodual hyperaxioms is left to the interested reader.

A few comments are now inn order:

i) Hyperrelativity and its isodual are the most general forms of relativities known at this writing that can be formulated on numbers verifying the axioms of a field, thus admitting a well defined left and right unit with consequential applicability to measurements;

ii) Hyperrelativity and its isodual are invariant under their respective time hyperevolutions, thus predicting the same numerical results at different time, and being applicable to experiments;

iii) Hyperrelativity and its isodual are multi-valued rather than multi-dimensional, namely, they permit the representation of multi-universes in a form compatible with our sensory perception of spacetime;

iv) The speed of light in vacuum c_o has been assumed to remain unchanged under hyperlifting, thus meaning that the speed of light is the same for all vacuum foliations of spacetime.

v) Like all other quantities, hyperspeeds in general and, in particular, the hyperspeed of light must necessarily be multi-valued for consistency, namely, assume different values for different foliations of spacetime.

Note the covering character of hyperrelativity in the sense of admitting as particular cases the genorelativity of Chapter 4, the isorelativity of Chapter 3 and the conventional special relativity whenever all units return to have the value 1 dating back to biblical times.

As we shall see in Volume II, hyperrelativity and its isodual, with particular reference to the 44-multi-valued hyperdimensional hypersymmetry (5.13)[1], will allow the formulation of the most general known, thus the most complex known, cosmology that includes, for the first time, biological structure as a condition for the appropriate use of the word "cosmology" in its Greek sense.

[1]The reader should recall that the Poincaré symmetry is *eleven*-dimensional and not ten dimensional as popularly believed because of the discovery permitted by isomathematics of the additional, 11-th dimensional isoscalar isoinvariance studied in Section 3.5.

Appendix 5.A
Eric Trell's Hyperbiological Structures TO BE COMPLETED AND EDITED.

A new conception of biological systems providing a true advance over rather primitive prior conceptions, has been recently proposed by Erik Trell (see Ref. (164) and contributions quoted therein). It is based on representative blocks which appear in our space to be next to each other, thus forming a cell or an organism, while having in reality hypercorrelations, thus having the structure of hypernumbers, hypermathematics and hyperrelativity, with consequential descriptive capacities immensely beyond those of pre-existing, generally single-valued and reversible biological models. Regrettably, we cannot review Trell's new hyperbiological model to avoid an excessive length, and refer interested readers to the original literature (164).

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Postscript

In the history of science some basic advances in physics have been preceded by basic advances in mathematics, such as Newtons invention of calculus and general relativity relying on Riemannian geometry. In the case of quantum mechanics the scientific revolution presupposed the earlier invention of complex numbers. With new numbers and more powerful mathematics to its disposition, physics could be lifted to explain broader and more complex domains of physical reality.

The recent and ongoing revolution of physics, initiated by Prof. Ruggero Maria Santilli, lifting the discipline from quantum mechanics to hadronic mechanics, is consistent with this pattern, but in a more far-reaching and radical way than earlier liftings of physics made possible from extensions of mathematics.

Santilli realized at an early stage that basic advances in physics required invention of new classes of numbers and more adequate and powerful mathematics stemming from this. His efforts to develop such expansions of mathematics started already in 1967, and this enterprise went on for four decades. Its basic novelties, architecture and fruits are presented in the present volume. During this period a few dozen professional mathematicians world wide have made more or less significant contributions to fill in the new Santilli fields of mathematics, but the honor of discovering these vast new continents and work out their basic topology is Santillis and his alone. These new fields initiated by Santilli made possible realization of so-called Lie-admissible physics. For this achievement Santilli in 1990 received the honor from Estonia Academy of Science of being appointed as mathematician number seven after world war two considered a landmark in the history of algebra.

With regard to Sophus Lie it may be of some interest to note that the Norwegian examiners of his groundbreaking doctoral thesis in 1871 were not able to grasp his work, due to its high degree of novelty and unfamiliarity. However, due to Lie already being highly esteemed among influential contemporary mathematicians at the continent, it was not an option to dismiss his thesis. As in other disciplines, highly acknowledged after Thomas Kuhns publication of The structure of scientific revolutions in 1962, sufficiently novel mathematics implies some paradigmatic challenge. Therefore, it is not strange that some mathematicians and physicists have experienced difficulties taking the paradigmatic leap necessary to grasp the basics of hadronic mathematics or to acknowledge its far-reaching implications. Such a challenge is more demanding when scientific novelty implies a reconfiguration of conventional basic notions in the discipline. This is, as Kuhn noted, typically easier for younger and more emergent scientific minds.

Until Santilli the number 1 was silently taken for granted as the primary unit of mathematics. However, as noted by mathematical physicist Peter Rowlands at University of Liverpool, the number 1 is already loaded with assumptions, that can be worked out from a lifted and broader mathematical framework. A partial and rough analogy might be linguistics where it is obvious that a universal science of language must be worked out from a level of abstraction that is higher than having to assume the word for mother to be the first word.

Santilli detrivialized the choice of the unit, and invented isomathematics where the crux was the lifting of the conventional multiplicative unit (i.e. conservation of its topological properties) to a matrix isounit with additional arbitrary functional dependence on other needed variables. Then the conventional unit could be described as a projection and deformation from the isounit by the link provided by the so-called isotopic element inverse of the isounit. This represented the creation of a new branch of mathematics sophisticated and flexible enough to treat systems entailing sub-systems with different units, i.e. more complex systems of nature.

Isomathematics proved necessary for the lifting of quantum mechanics to hadronic mechanics. With this new mathematics it was possible to describe extended particles and abandon the point particle simplification of quantum mechanics. This proved highly successful in explaining the strong force by leaving behind the non-linear complexities involved in quantum mechanics struggle to describe the relation between the three baryon quarks in the proton. Isomathematics also provided the mathematical means to explain the neutron as a bound state of a proton and an electron as suggested by Rutherford. By means of isomathematics Santilli was also able to discover the fifth force of nature (in cooperation with Professor Animalu), the contact force inducing total overlap between the wave packets of the two touching electrons constituting the isoelectron. This was the key to understanding hadronic superconductivity which also can take place in fluids and gases, i.e. at really high temperatures. These advances from hadronic mechanics led to a corresponding lifting of quantum chemistry to hadronic chemistry and the discovery of the new chemical species of magnecules with non-valence bounds. Powerful industrial-ecological technology exploiting these theoretical insights was invented by Santilli himself from 1998 on.

Thus, the development of hadronic mathematics by Santilli was not only motivated by making advances in mathematics per se, but also of its potential to facilitate basic advances in physics and beyond. These advances have been shown to be highly successful already. Without the preceding advances in mathematics, the new hadronic technology would not have been around. The mere existence of this technology is sufficient to demonstrate the significance of hadronic mathematics. It is interesting to note that the directing of creative mathematics into this path was initiated by a mathematical physicist, not by a pure mathematician. In general this may indicate the particular potential for mathematical advances by relating the mathematics to unsolved basic problems in other disciplines, as well as to real life challenges.

In the history of mathematics it is not so easy to find parallels to the achievements made by Santilli, due to hadronic mathematics representing a radical and general lifting, relegating the previous mathematics to a subclass of isomathematics, in some analogy to taking the step from the Earth to the solar system. However, the universe also includes other solar systems as well as galaxies.

In addition to isonumbers Santilli invented the new and broader class of genonumbers with the possibility of asymmetric genounits for forward vs. backward genofields, and designed to describe and explain irreversibility, characteristic for more complex systems of nature. Quantum mechanical approaches to biological systems never achieved appreciable success, mainly due to being restricted by a basic symmetry and hence reversibility in connected mathematical axioms. It represented an outstanding achievement of theoretical biology when Chris Illert in the mid-1990s was able to find the universal algorithm for growth of sea shells by applying hadronic geometry. Such an achievement was argued not to be possible for more restricted hyperdimensional geometries as for example the Riemannian. This specialist study in conchology was the first striking illustration of the potency as well as necessity of iso- and genomathematics to explain irreversible systems in biology.

Following the lifting from isomathematics to genomathematics, Santilli also established one further lifting, by inventing the new and broader class of hyperstructural numbers or Santilli hypernumbers. Such hypernumbers are multivalued and suitable to describe and explain even more complex systems of nature than possible with genonumbers. Due to its irreversible multivalued structure hypermathematics seems highly promising for specialist advances in fields such as genetics, memetics and communication theory. By the lifting to hypermathematics hadronic mathematics as a whole may be interpreted as a remarkable step forward in the history of mathematics, in the sense of providing the essential and sufficiently advanced and adequate tools for mathematics to expand into disciplines such as anthropology, psychology and sociology. In this way it is possible to imagine some significant bridging between the two cultures of science: the hard and the soft disciplines, and thus amplifying a tendency already represented to some extent by complexity science.

The conventional view of natural scientists has been to regard mathematics as a convenient bag of tools to be applied for their specific purposes. Considering the architecture of hadronic mathematics, this appears more as only half of the truth or one side of the coin. Besides representing powerful new tools to study nature, hadronic mathematics also manifests with a more intimate and inherent connection to physics (and other disciplines), as well as to Nature itself. In this regard hadronic geometry may be of special interest as an illustration:

Isogeometry provided the new notions of a supra-Euclidean isospace as well as its anti-isomorphic isodual space, and the mathematics to describe projections and deformations of geometrical relations from isospace and its isodual into Euclidean space. However, these appear as more than mere mathematical constructs. Illert showed that the universal growth pattern of sea shells could be found only by looking for it as a trajectory in a hidden isospace, a trajectory which is projected into Euclidean space and thereby manifest as the deformed growth patterns humans observe by their senses. Further, the growth pattern of a certain class of sea shells (with bifurcations) could only be understood from the addition and recognition of four new, non-trivial time categories (predicted to be discovered by hadronic mechanics) which manifest as information jumps back and forth in Euclidean space. With regard to sea shell growth, one of this non-trivial time flows could only be explained as a projection from isodual spacetime. This result was consistent with the physics of hadronic mechanics, analyzing masses at both operator and classical level from considering matter and anti-matter (as well as positive and negative energy) to exist on an equal footing in our universe as a whole and hence with total mass (as well as energy and time) cancelling out as zero for the total universe. To establish a basic physical comprehension of Euclidean space constituted as a balanced combination of matter and antimatter, it was required to develop new mathematics with isonumbers and isodual numbers basically mirroring each other. Later, corresponding anti-isomorphies were achieved for genonumbers and hypernumbers with their respective isoduals.

Thus, there is a striking and intimate correspondence between the isodual architecture of hadronic mathematics and the isodual architecture of hadronic mechanics (as well as of hadronic chemistry and hadronic biology). Considering this, one might claim that the Santilli inventions of new number fields in mathematics represent more than mere inventions or constructs, namely discoveries and reconstructions of an ontological architecture being for real also outside the formal landscapes created by the imagination of mathematics and logic. This opens new horizons for treating profound issues in cosmology and ontology.

One might say that with the rise of hadronic mathematics the line between mathematics and other disciplines has turned more blurred or dotted. In some respect this represents a revisit to the Pythagorean and Platonic foundations of mathematics in the birth of western civilization. Hadronic mathematics has provided much new food for thought and further explorations for philosophers of science and mathematics.

If our civilization is to survive despite its current problems, it seems reasonable to expect Santilli to be honored in future history books not only as a giant in the general history of science, but also in the specific history of mathematics. Hadronic mathematics provided the necessary fuel for rising scientific revolutions in other hadronic sciences. This is mathematics that matters for the future of our world, and hopefully Santillis extraordinary contributions to mathematics will catch fire among talented and ambitious young mathematicians for further advances to be made. The present mellowed volume ought to serve as an excellent appetizer in this regard.

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October 8, 2007

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