This excellent agreement confirms our hypothesis that the joint origin of the dark matter and dark energy is based on the sterile neutrinos [1-4] as well as their breakup and decay products (see above and Ref. [1]). This result supports also the introduction of the normal (Ω_Λ^{**}) and the total (Ω_Λ^{***}) dark energy in the present work (see the second half of Sec. 4).

Using the data of Tables III to VI, the corresponding results, derived in Sec. 7 for the massive universe, are also valid for the massive anti-universe.

8. The time dependence of the cosmological "constant"

The cosmological "constant" problem has a complex history [21]. In this work, for the total (massless and massive) universe, the vacuum energy densities or cosmological "constants", introduced already in Refs. [1, 2] as variable quantities, are compared with the prediction of the quantum field theory. For this goal, they are written in their time-dependent form, whereat the considerations are initially restricted to the results of Sec. 3.1. Firstly, we take into account only the 3 limiting cases (Hubble time (τ_{H}), Planck time (t_{Pl}) and big bang (t_{BB})). Secondly, we treat generally the case $t \le t_{\mathrm{Pl}}$. Thirdly, we consider the case $t \ge t_{\mathrm{Pl}}$.

Now, we describe firstly the 3 limiting cases (Hubble time, Planck time and big bang). For the Hubble time $\tau_{\mathrm{H}} = 1/H_0 = 4.585 \cdot 10^{17}\,\mathrm{s}$, we have

$$\rho_{\mathrm{vac},\Lambda}\, c^2 = \Omega_\Lambda\, \rho_{0\mathrm{C}}\, c^2 = \Omega_\Lambda \frac{3\, H_0^2 c^2}{8\pi\, G_N} =$$

$$= \Omega_\Lambda \frac{3\, c^2}{8\pi\, G_N\, \tau_{\mathrm{H}}^2} = 3.27 \cdot 10^3\ \mathrm{eV\,cm^{-3}} \qquad (8.1)$$

or

$$\Lambda = \Lambda_\Lambda = \frac{3\,\Omega_\Lambda}{R_0^2} = \frac{3\,\Omega_\Lambda}{c^2/H_0^2} =$$

$$= \frac{3\,\Omega_\Lambda}{c^2 \tau_{\mathrm{H}}^2} = 1.087 \cdot 10^{-52}\ \mathrm{m^{-2}}. \qquad (8.2)$$

For the Planck units as the limits between massless and massive universe, according to Eqs. (2.30) and (2.31), because of $N(T) = 1/2\Omega_\gamma$ [1, 2], we obtain

$$\rho_{vac}(T_{Pl})\,c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT_{Pl})^4}{(\hbar c)^3}\frac{1}{N(T)} = \rho_{vac}(t_{Pl})\,c^2 =$$

$$= \frac{1}{3}\frac{\pi^2}{15}\frac{\hbar}{c^3 t_{Pl}^4}\frac{1}{N(T)} = 6.93\cdot 10^{121}\,\mathrm{eV\,cm^{-3}} \tag{8.3}$$

or

$$\Lambda = \Lambda_{Pl} = \frac{8\pi^3}{45}\frac{1}{R_{Pl}^2\,N(T)} =$$

$$= \frac{8\pi^3}{45}\frac{1}{c^2 t_{Pl}^2\,N(T)} = 2.30\cdot 10^{66}\,\mathrm{m^{-2}}\,. \tag{8.4}$$

At the big bang (see Eqs. (3.23) to (3.26)), we get

$$\rho_{vac}(R_{BB})\,c^2 = \frac{2}{3}\Omega_\gamma\frac{\pi^2}{15}\frac{E_{Pl}^2}{\hbar c\,R_{BB}^2} = \rho_{vac}(t_{BB})\,c^2 =$$

$$= \frac{2}{3}\Omega_\gamma\frac{\pi^2}{15}\frac{E_{Pl}^2}{\hbar c^3 t_{BB}^2} = 4.227\cdot 10^{247}\,\mathrm{eV\,cm^{-3}} \tag{8.5}$$

or

$$\Lambda = \Lambda_{BB} = \frac{16\pi^3}{45}\frac{\Omega_\gamma}{R_{BB}^2} = \frac{16\pi^3}{45}\frac{\Omega_\gamma}{c^2 t_{BB}^2} = 1.406\cdot 10^{192}\,\mathrm{m^{-2}}\,. \tag{8.6}$$

Using Eqs. (8.1) to (8.4) at $t = t_{Pl}$ and $t = \tau_H$ as well as Eqs. (8.3) to (8.6) at $t = t_{BB}$ and $t = t_{Pl}$, we can introduce

$$\frac{\rho_{vac}(t_{Pl})}{\rho_{vac,\Lambda}} = \frac{\Lambda_{Pl}}{\Lambda_\Lambda} = \frac{16\pi^3}{135}\frac{\Omega_\gamma}{\Omega_\Lambda}\frac{\tau_H^2}{t_{Pl}^2} \cong 2.119\cdot 10^{118} \tag{8.7}$$

as well as

$$\frac{\rho_{vac}(t_{BB})}{\rho_{vac}(t_{Pl})} = \frac{\Lambda_{BB}}{\Lambda_{Pl}} \cong 6.10\cdot 10^{125}\,, \tag{8.8}$$

respectively. We summarize correspondingly these calculated data (8.1) to (8.6) in the Tables VIII and IX.

Table VIII. The calculated time-dependent vacuum energy densities or cosmological "constants" for $t_{BB} \le t \le t_{Pl}$.

State	kT (eV)	Time (s)	$\rho_{vac}c^2$ (eV cm^{-3})	Λ (m^{-2})
Big bang	$1.563 \cdot 10^{-35}$	$6.901 \cdot 10^{-107}$	$4.23 \cdot 10^{247}$	$1.41 \cdot 10^{192}$
E_0(e)	$5.110 \cdot 10^{5}$	$2.256 \cdot 10^{-66}$	$3.95 \cdot 10^{166}$	$1.32 \cdot 10^{111}$
$E_0(H^0)$	$1.260 \cdot 10^{11}$	$5.564 \cdot 10^{-61}$	$1.49 \cdot 10^{156}$	$2.16 \cdot 10^{100}$
E_0(X, Y)	$2.675 \cdot 10^{25}$	$1.181 \cdot 10^{-46}$	$1.44 \cdot 10^{127}$	$4.80 \cdot 10^{71}$
Planck	$1.221 \cdot 10^{28}$	$5.391 \cdot 10^{-44}$	$6.93 \cdot 10^{121}$	$2.30 \cdot 10^{66}$

Because of the new inflation model [1, 2], we have a discontinuity in the evolution of the universe, so that Eqs. (8.7) and (8.8) do not agree with the result of the quantum field theory, which predicts a value of about 10^{122} for the result (8.7) at a continuous evolution of the universe [11]. However, we can simulate a continuous evolution for the total (massless and massive) universe by $(\Lambda_{BB}\Lambda_{\Lambda})^{1/2}/\Lambda_{\Lambda} = (\Lambda_{BB}/\Lambda_{\Lambda})^{1/2} = 1.137 \cdot 10^{122}$, so that we obtain an excellent agreement with the quantum field theory. In other words, the expression $(\Lambda_{BB}/\Lambda_{\Lambda})^{1/2} = 1.137 \cdot 10^{122}$ corresponds to the geometric mean of the results (8.7) and (8.8).

Secondly, we treat now generally the case $t_{BB} \le t \le t_{Pl}$, which is also tabulated in Table VIII. Here, as examples, we estimate still the corresponding data for the X and Y gauge bosons as well as Higgs boson and the electron, given also in Table VIII.

For $t_{BB} \le t \le t_{Pl}$, according to Eqs. (2.41) and (2.42), for $R = \overline{R}$ (see, e.g., Eq. (6.16)), because of $R = ct$ [see Eq. (2.43)), we have the variable (time-dependent) vacuum energy density or cosmological "constant:

$$\rho_{\text{vac}}(R)\,c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\text{Pl}}^2}{\hbar c\,R^2} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\text{Pl}}^2}{\hbar c^3 t^2} \tag{8.9}$$

or

$$\Lambda = \frac{16}{45}\pi^3\,\Omega_\gamma \frac{1}{R^2} = \frac{16}{45}\pi^3\,\Omega_\gamma \frac{1}{c^2 t^2}\,. \tag{8.10}$$

For example, at the X and Y gauge bosons, we have $\overline{R}_{\text{X, Y}} = 3.541\cdot 10^{-36}$ cm (see Eq. (6.16)), i.e. we obtain $t = \overline{R}_{\text{X, Y}}/c = 1.181\cdot 10^{-46}$ s, so that we get

$$\rho_{\text{vac}}(\text{X, Y})\,c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\text{Pl}}^2}{\hbar c\,\overline{R}_{\text{X, Y}}^2} =$$

$$= \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\text{Pl}}^2}{\hbar c^3 t^2} = 1.443\cdot 10^{127}\ \text{eV cm}^{-3} \tag{8.11}$$

or

$$\Lambda_{\text{X, Y}} = \frac{16}{45}\pi^3\,\Omega_\gamma \frac{1}{\overline{R}_{\text{X, Y}}^2} =$$

$$= \frac{16}{45}\pi^3\,\Omega_\gamma \frac{1}{c^2 t^2} = 4.80\cdot 10^{71}\ \text{m}^{-2}\,. \tag{8.12}$$

The data (8.11) and (8.12) are tabulated in Table VIII.

By the results (8.9) and (8.10), we have estimated also the corresponding data for the Higgs boson and the electron, given also in Table VIII.

Thirdly, we describe now generally the case $t \ge t_{Pl}$ (tabulated in Table IX), whereat the considerations are here restricted particularly to the radiation-dominated universe ($z \ge 10^5$, $T \ge 3\cdot 10^5$ K or $t \le 3\cdot 10^9$ s). Here, according to Eqs. (2.30) and (2.31), via the results (2.10) and (2.12), we have generally the variable vacuum energy density or cosmological "constant

$$\rho_{\mathrm{vac}}(T)\,c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} \tag{8.13}$$

or

$$\Lambda = \frac{8\pi G_N}{c^2}\rho_{\mathrm{vac}}(T) = \frac{8}{45}\pi^3\frac{(kT)^4}{E_{\mathrm{Pl}}^2(\hbar c)^2}\frac{1}{N(T)}. \tag{8.14}$$

i.e. explicitly for the radiation-dominated universe their time dependence is considered according to Refs. [1-5] by the connection

$$t = \frac{1}{2}\left(\frac{90\,\hbar^3 c^5}{8\,\pi^3 G_N}\right)^{1/2}\frac{1}{\sqrt{N(T)}\,(kT)^2} =$$

$$= \frac{(2.42035 \pm 0.00015)}{\sqrt{N(T)}}\left(\frac{\mathrm{MeV}}{kT}\right)^2 \mathrm{s}. \tag{8.15}$$

Table IX. The calculated time-dependent vacuum energy densities or cosmological "constants" for $t \geq t_{\mathrm{Pl}}$.

State	$N(T)$	kT (eV)	Time (s)	$\rho_{\mathrm{vac}}c^2$ (eV cm^{-3})	Λ (m^{-2})
Planck	$1/2\Omega_\gamma$	$1.221\cdot10^{28}$	$5.39\cdot10^{-44}$	$6.93\cdot10^{121}$	$2.30\cdot10^{66}$
$E_0(\mathrm{X,Y})$	160.75	$2.675\cdot10^{25}$	$2.67\cdot10^{-40}$	$9.09\cdot10^{112}$	$3.02\cdot10^{57}$
$E_0(\mathrm{H}^0)$	385/4	$1.260\cdot10^{11}$	$1.55\cdot10^{-11}$	$7.48\cdot10^{55}$	2.49
$E_0(\mathrm{e})$	43/4	$5.110\cdot10^{5}$	2.83	$1.81\cdot10^{35}$	$6.02\cdot10^{-25}$
$\Omega_\Lambda\rho_{0C}c^2$	3.362644	$4.430\cdot10^{-3}$	$6.65\cdot10^{15}$	$3.27\cdot10^{3}$	$1.09\cdot10^{-52}$
final	$(4/11)^{1/3}$	$5.811\cdot10^{-5}$	$2.17\cdot10^{20}$	$4.57\cdot10^{-4}$	$1.52\cdot10^{-59}$

Using the results (8.13) to (8.15), we get as their time dependence

$$\rho_{\text{vac}}(t)\,c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{5.85809\,\text{MeV}^4}{(\hbar c)^3[N(T)]^2}\frac{\text{s}^2}{t^2} \tag{8.16}$$

or

$$\Lambda = \frac{8}{45}\pi^3\frac{5.85809\,\text{MeV}^4}{(\hbar c)^2 E_{\text{Pl}}^2[N(T)]^2}\frac{\text{s}^2}{t^2}\,. \tag{8.17}$$

Now, by the results (8.13) to (8.17), we estimate also the corresponding data for the X and Y gauge bosons as well as the Higgs boson and the electron, given also in Table IX.

For example, at the X and Y gauge bosons, we have $N(T) = 160.75$ (see Refs. [1-3, 5, 10]) and $kT = 2.675\cdot 10^{25}\,\text{eV}$ (see Eq. (6.15)), i.e. we can determine $t = 2.67\cdot 10^{-40}\,\text{s}$ via Eq. (8.15), so that we get the results

$$\rho_{\text{vac}}(\text{X},\text{Y})\,c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} =$$

$$= \frac{1}{3}\frac{\pi^2}{15}\frac{5.85809\,\text{MeV}^4}{(\hbar c)^3[N(T)]^2}\frac{\text{s}^2}{t^2} = 9.09\cdot 10^{112}\,\text{eV cm}^{-3} \tag{8.18}$$

or

$$\Lambda_{\text{X, Y}} = \frac{8}{45}\pi^3\frac{(kT)^4}{E_{\text{Pl}}^2(\hbar c)^2}\frac{1}{N(T)} =$$

$$= \frac{8}{45}\pi^3\frac{5.85809\,\text{MeV}^4}{(\hbar c)^2 E_{\text{Pl}}^2[N(T)]^2}\frac{\text{s}^2}{t^2} = 3.02\cdot 10^{57}\,\text{m}^{-2}\,, \tag{8.19}$$

tabulated in Table IX.

Analogous to the X and Y gauge bosons (see Eqs. (8.18) and (8.19)), we have calculated still the corresponding data for the Higgs boson H^0 ($N(T) = 385/4$) and the electron ($N(T) = 43/4$), for which the rest energy is given in Sec. 7 and their value $N(T)$ is defined in Refs. [1-3, 5, 10]. They are also tabulated in Table IX, where we have also summarized all calculated values for the time-dependent vacuum energy densities or cosmological "constants" for $t \geq t_{\mathrm{Pl}}$.

At a temperature of $T \approx 5 \cdot 10^9\ \mathrm{K}$, the neutrinos decouple from the photons [6], i.e. from this point we have $N(T) = 3.362644$ [6, 8] in formula (8.15), which is valid for $T \geq 3 \cdot 10^5\ \mathrm{K}$. However, from $T \approx 5 \cdot 10^9\ \mathrm{K}$, we have $T_\nu = (4/11)^{1/3} T$ (see also Sec. 2), where T is again the photon temperature, i.e. the photons and neutrinos dominate now the universe. Then, by $N(T) = 3.362644$, for $T \leq 3 \cdot 10^5\ \mathrm{K}$, via the formulae (8.13) and (8.14), the corresponding time of the variable vacuum energy densities or cosmological "constants" can be determined only by the redshift $1 + z = T/T_0$ (see Eq. (2.10)), using the result (2.12) for the time calculation.

Then, for Table IX, the corresponding time of the limiting values (2.32) and (2.33) is again evaluated by $1 + z = T/T_0 = 18.86$ (see Eq. (2.34)) via the expression (see Eq. (7.7))

$$t = t(z) = \frac{2}{3\, H_0\, \Omega_\Lambda^{1/2}} \ln \frac{\sqrt{\Omega_\Lambda (1+z)^{-3}} + \sqrt{\Omega_\Lambda (1+z)^{-3} + \Omega_{\mathrm{m}}}}{\sqrt{\Omega_{\mathrm{m}}}} . \tag{8.20}$$

Using Eqs. (3.8) and (3.38) as well as interpreting because of Eq. (8.1) the result (3.75 a) as vacuum energy density of the final state of the massive universe, by Eqs. (3.10) and (8.13), for $N(T) = (4/11)^{1/3}$ (see Eq. (3.73)), we can write

$$\rho_{\mathrm{vac}}(T)\, c^2 = \frac{1}{3} \frac{\pi^2}{15} \frac{(kT)^4}{(\hbar c)^3} \frac{1}{N(T)} = \rho_{\mathrm{f1}}\, c^2 = \Omega_\Lambda\, \rho_{0\mathrm{C}}\, c^2 \left(\frac{d_{\mathrm{eff}}}{c t_{\mathrm{f2}}} \right)^3 \left(\frac{T_1}{T_2} \right)^3 =$$

$$= \frac{E_{\mathrm{d}}}{\frac{4}{3} \pi (c t_{\mathrm{f2}})^3} \left(\frac{3\, N(T)}{2} \right)^{3/4} \cong 4.56 \cdot 10^{-4}\ \mathrm{eV\, cm^{-3}} , \tag{8.21}$$

so that via the middle term of Eq. (8.14) we can estimate the corresponding cosmological "constant" to

$$\Lambda_{\mathrm{f}} = \frac{8\pi\, G_N}{c^2}\,\rho_{\mathrm{f}1} = 8\pi\,\frac{\hbar c}{E_{\mathrm{Pl}}^2}\,\frac{E_{\mathrm{d}}\{3\,N(T)/2\}^{3/4}}{4/3\,\pi\,(ct_{\mathrm{f}2})^3} = 1.519\cdot 10^{-59}\,\mathrm{m}^{-2}\,, \qquad (8.22)$$

where the time $t_{\mathrm{f}2} = 2.172\cdot 10^{20}\,\mathrm{s}$ is defined by Eq. (3.36).

Then, using again $N(T) = (4/11)^{1/3}$ and the connection (8.21), we can estimate the energy kT of the state $\rho_{\mathrm{f}1}\,c^2$ of the massive universe to

$$kT = \left[\frac{45}{\pi^2}\,(\hbar c)^3\,\rho_{\mathrm{f}1}\,c^2\,N(T)\right]^{1/4} = 5.811\cdot 10^{-5}\,\mathrm{eV}\,. \qquad (8.23)$$

The data (8.21) to (8.23) are also tabulated in Table IX.

Via the geometric means $(\Lambda_{\mathrm{BB}}\Lambda_{\mathrm{f}})^{1/2}$ and $(\Lambda_{\Lambda}\Lambda_{\mathrm{f}})^{1/2}$, we can once more simulate a continuous evolution of the total (massless and massive) universe by $(\Lambda_{\mathrm{BB}}\Lambda_{\mathrm{f}})^{1/2}/(\Lambda_{\Lambda}\Lambda_{\mathrm{f}})^{1/2} = (\Lambda_{\mathrm{BB}}/\Lambda_{\Lambda})^{1/2} = 1.137\cdot 10^{122}$ as a consequence of the geometric mean $\left\{\left[(\Lambda_{\mathrm{BB}}\Lambda_{\mathrm{f}})/\Lambda_{\mathrm{Pl}}^2\right]\times\left[\Lambda_{\mathrm{Pl}}^2/(\Lambda_{\Lambda}\,\Lambda_{\mathrm{f}})\right]\right\}^{1/2} = (\Lambda_{\mathrm{BB}}/\Lambda_{\Lambda})^{1/2} \cong 10^{122}$, so that we obtain again an excellent agreement with the quantum field theory (see above).

We mention still that because of the result (2.59) for the magnetic monopoles their rest energy $E_0(\mathrm{M}) = 6.849\cdot 10^{17}\,\mathrm{GeV}$ (see Eq. (2.58)) must be correct. However, their assumed statistical quantity $N(T) = 1/2\Omega_\gamma$ [1-3] is only correct for the estimation of their relativistic maximum energy E_{Pl}. Its real value must lie in the range $1/2\Omega_\gamma > N(T) > 160.75$. Because this "unknown" value $N(T)$ is necessary for corresponding calculations, we have not considered the magnetic monopoles for Tables VIII and IX.

With that, we have solved the problem of the time-dependent vacuum energy densities or cosmological "constants" on the basis of our quantum gravity [1, 2].

Thus, we have shown that the energy density (8.1) of the dark energy and the cosmological "constant" (8.2), which describe the present accelerated expansion of the universe, can be identified with the vacuum energy density or cosmological "constant" of the quantum field theory, so that we have confirmed the derivation of the quantum gravity [1, 2] by the gravitation via the particle-antiparticle pairs of the quantum vacuum [1, 2].

Besides, this solution corroborates the correctness of the estimation of the parameters of the big bang [1, 2], the new inflation model [1-5], the (light [1-5], heavy [1-4] and sterile [1-4]) neutrinos as well as the SUSY GUT particles (X and Y gauge bosons [1-5] as well as magnetic monopoles [1-5]). Consequently, the conception of the SUSY GUT is now well established within the theories of the early universe.

Using the data of Tables III to V, the results, derived in this Sec. 8 for the massless and massive universe, are also valid for the massless and massive anti-universe.

9 Summary

We have derived the transition from the final state of the universe and anti-universe to the big bang (origin) via the complete sterile (anti)neutrino decay and the quantum gravity. With that, we have solved precisely and uniquely this fundamental problem, whereat we have confirmed the explanation that the present dark matter and dark energy can be attributed to the invisible decay and breakup products of the sterile neutrinos. We have proved that the massless universe and anti-universe exist by zero-point oscillations. Finally, we have also shown that the massive universe and anti-universe can be explained reasonably by zero-point oscillations.

By aid of the time-dependent vacuum energy densities or cosmological "constants", we confirm the predictions of the quantum field theory.

The final age of the universe and the anti-universe was confirmed to $t_{f2} = 6883\,\text{Gyr}$, derived in Refs. [1, 2]. The lifetime $\tau_{\hat{\nu}}$ of the sterile neutrinos [1, 2] was also confirmed to $\tau_{\hat{\nu}} = 35.11\,\text{Gyr}$ [1, 2]. The rest energy of the photons was estimated to $E_0(\gamma) \cong 1.563 \cdot 10^{-35}\,\text{eV}$. This estimation is confirmed by the measured general galactic magnetic field. It was assumed that the rest energy of the gravitons has probably the same value as at the photons.