

Because of the assumption $E_0(\gamma) = E_0(G)$ [see Eq. (6.2)], the results (3.107) and (3.108) mean that for the universe we must assume the proper energy

$$\begin{aligned} 2E_H &\cong 2\rho_{0C} c^2 \frac{4}{3} \pi R_0^3 \cong \rho_{0C} c^2 \frac{4}{3} \pi (R_0 2^{\frac{1}{3}})^3 \cong \\ &\cong \rho_{0C} c^2 \frac{4}{3} \pi (R_0 r)^3 \cong 1.039 \cdot 10^{89} \text{ eV}, \end{aligned} \quad (6.65)$$

where $r = 2^{\frac{1}{3}} \cong 1.26$ is again the dimensionless time-independent comoving coordinate of the proper distance $d = R_0 r$.

Using the data of Tables III to V, the results, derived from Eqs. (6.44) to (6.65) for the massive universe, are also valid for the massive anti-universe.

7 Hubble “constants” as a function of cosmic evolution epochs

The Hubble “constants” determine how fast the universe expands over the time. Because of Eq. (6.10), they must possess discontinuities in the cosmic evolution.

By Eqs. (6.6) to (6.9), we have derived the values of the Hubble parameters $\bar{H}_{\text{BB}} = z_0(\gamma)/t_{\text{BB}} = 9.643 \cdot 10^{74} \text{ s}^{-1} = 2.976 \cdot 10^{94} \text{ km s}^{-1} \text{ Mpc}^{-1}$ (see Eq. (6.7)) and $\bar{H}_{\text{PI}} = z_0(\gamma)/t_{\text{PI}} = 1.234 \cdot 10^{12} \text{ s}^{-1} = 3.808 \cdot 10^{31} \text{ km s}^{-1} \text{ Mpc}^{-1}$ (see Eq. (6.9)) for the massless universe ($R_{\text{BB}} \leq R \leq R_{\text{PI}}$). They are the limiting values of the continuous function (6.12). However, between the Hubble parameters $\bar{H}_{\text{PI}} = z_0(\gamma)/t_{\text{PI}} = 1.234 \cdot 10^{12} \text{ s}^{-1} = 3.808 \cdot 10^{31} \text{ km s}^{-1} \text{ Mpc}^{-1}$ (see above) and $H_{\text{PI}} = 1/2t_{\text{PI}} = 9.275 \cdot 10^{42} \text{ s}^{-1} = 2.862 \cdot 10^{62} \text{ km s}^{-1} \text{ Mpc}^{-1}$ [lower limiting value of the early massive universe ($R_{\text{PI}} \leq R \leq \tilde{R}_0$) according to the new inflation model (see Eqs. (2.14) to (2.18))], we have a discontinuity (see Eq. (6.10)). These Hubble parameters are father than that of the Planck observations 2013 [12], which yield $H_0 = 2.181 \cdot 10^{-18} \text{ s}^{-1} = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (see Table I). The connection between all these values is given by Eq. (6.10). Consequently, instead of Eqs. (6.7) and (6.10), we can also write

$$\bar{H}_{\text{BB}} = \frac{z_0(\gamma)}{t_{\text{BB}} t_{\text{Pl}} (1+z_{\text{M}})^2 H_0} = 2.976 \cdot 10^{94} \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (7.1)$$

$$\bar{H}_{\text{Pl}} = z_0(\gamma) [1+z_{\text{M}}]^2 H_0 = 3.808 \cdot 10^{31} \text{ km s}^{-1} \text{ Mpc}^{-1} \quad (7.2)$$

and

$$H_{\text{Pl}} = 1/2 (1+z_{\text{M}})^2 H_0 = 2.862 \cdot 10^{62} \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (7.3)$$

i.e. Eqs. (7.1) and (7.2) show clearly a continuous connection by $\bar{H} = z_0(\gamma)/t$ for $t_{\text{BB}} \leq t \leq t_{\text{Pl}}$, whereas between Eqs. (7.2) and (7.3) we see again a large discontinuity.

According to Ref. [1], in the radiation-dominated early ($R_{\text{Pl}} \leq R \leq \tilde{R}_0$) and late ($\tilde{R}_0 \leq R \leq R_0$) massive universe $\{z \geq 10^5$ for the new inflation model [see Eqs. (2.14) to (2.18)]}, because of $t = 1/(2N(t)\Omega_\gamma)^{1/2}(1+z)^2 H_0$ as well as $\tilde{R} \propto t^{1/2}$ and $R \propto t^{1/2}$ (see, e.g., Ref. [1]), we have the continuous connection

$$H = \dot{R}/R = 1/2t = 1/2(2N(t)\Omega_\gamma)^{1/2}(1+z)^2 H_0. \quad (7.4)$$

For $t = t_{\text{Pl}}$, because of $N(T) = 1/2\Omega_\gamma$ and $z = z_{\text{M}}$ [1, 2], Eq. (7.4) gives again the expression (7.3).

Because Eqs. (7.2) and (7.3) yield a discontinuity at the Hubble parameters \bar{H}_{Pl} and H_{Pl} , we expect also a similar discontinuity between H_0 of the Planck observations (see above) and H_{acc} (see below) of the accelerated expansion (2.27). This assumption is clear because H_0 was derived from measurements of the CMB, which was formed $3.72 \cdot 10^5$ years after the big bang [7], whereas the accelerated expansion began $7.70 \cdot 10^9$ years after the big bang (see Eq. (2.35) and also below), i.e. at two very different epochs of the evolution of the universe. Therefore, the Hubble expansion rate $H_0 = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which in Ref. [7] was assumed as the present Hubble “constant” of the universe, is interpreted as the present “Hubble constant” of the CMB, so that this new Hubble “constant” $H_0 = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ can be used further as basis for all hitherto existing considerations for the evolution of the universe, i.e. it must also determine all Hubble “constants” of the universe. Thus, we expect that this

present CMB value $H_0 = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ must also yield a new Hubble “constant” for the beginning of the “present” accelerated expansion of the universe, so that this “constant” has a larger value $H_{\text{acc}} > H_0$ (see Eq. (7.6)).

Then, the accelerated expansion can be described uniquely and continuously via Eq. (2.38). Then, from the beginning (2.35) of the accelerated expansion, we can write

$$H_{\text{acc}} = \frac{(c^2 \tilde{\Lambda}/3)}{\Omega_{\Lambda}^{1/2}} = \frac{t_0 - t}{t_0 - \tilde{t}} H_0. \quad (7.5)$$

Then, taking the data at the expression (2.38), we can apply $t_0 - t = 2.72 \cdot 10^{17} \text{ s}$ and $t_0 - \tilde{t} = 1.928 \cdot 10^{17} \text{ s}$ ($t_0 = (4.358 \pm 0.016) \cdot 10^{17} \text{ s}$ see Table I), so that Eq. (7.5) gives

$$H_{\text{acc}} = \frac{t_0 - t}{t_0 - \tilde{t}} H_0 = 3.08 \cdot 10^{-18} \text{ s}^{-1} = 95.0 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (7.6)$$

i.e. the epoch of Eq. (7.6) begins at $\tilde{t} = 2.43 \cdot 10^{17} \text{ s} = 7.70 \text{ Gyr}$.

The result (7.6) means an extremely rapid expansion in contrast to the value $H_0 = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$. To this problem, we will return below.

In Eq. (7.6), for $1 + z_{\text{acc}} = 1.632$ (beginning of the accelerated expansion [1, 2]), by the result (2.15), via the accelerated expansion (2.27), we have determined the time $t = 1.638 \cdot 10^{17} \text{ s}$, whereas for $z_{\text{eq}} \gg z \geq 1$ (z_{eq} see Table I) according to Refs. [1, 2, 6] the time $\tilde{t} = 2.43 \cdot 10^{17} \text{ s}$ is defined by

$$\tilde{t} = t(z) = \frac{2}{3 H_0 \Omega_{\Lambda}^{1/2}} \ln \frac{\sqrt{\Omega_{\Lambda}(1+z)^{-3}} + \sqrt{\Omega_{\Lambda}(1+z)^{-3} + \Omega_m}}{\sqrt{\Omega_m}}, \quad (7.7)$$

whereat the corresponding scale factor [1, 2, 6] is given by

$$\begin{aligned} \frac{R}{R_0} &= \frac{1}{1+z} = \left(\frac{\Omega_m}{\Omega_{\Lambda}} \right)^{1/3} \left[\sinh\left(\frac{3}{2} \Omega_{\Lambda}^{1/2} H_0 \tilde{t}\right) \right]^{2/3} = \\ &= \left(\frac{\Omega_m}{\Omega_{\Lambda}} \right)^{1/3} \left[\frac{e^{3/2 \Omega_{\Lambda}^{1/2} H_0 \tilde{t}} - e^{-3/2 \Omega_{\Lambda}^{1/2} H_0 \tilde{t}}}{2} \right]^{2/3}. \end{aligned} \quad (7.8)$$

Analogously, for $1+z=1.05$, we have estimated $t=4.088 \cdot 10^{17}$ s and $\tilde{t}=4.139 \cdot 10^{17}$ s, so that instead of Eq. (7.6) we find now

$$H_{\text{acc}} = \frac{t_0 - t}{t_0 - \tilde{t}} H_0 = 2.69 \cdot 10^{-18} \text{ s}^{-1} = 83.0 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (7.9)$$

whereat the value $1+z=1.05$ represents a lower limit because of the accuracy of the data. Consequently, the result (7.9) means a decrease of the accelerated expansion if it is compared with the value (7.6).

Then, for $z=0$, because of $t=t_0$ and $\tilde{t}=t_0$, Eqs. (7.6) and (7.9) yield the indefinable value $H_{\text{acc}}=(0/0)H_0$. Instead of this value, we apply the results (3.42) according to Refs. [1, 2] i.e. we can now estimate the present Hubble parameter by the semi-empirical expression

$$H_{\text{acc},0} = \frac{(c^2 \tilde{\Lambda}/3)}{\Omega_{\Lambda}^{1/2}} = \frac{t_{\text{eff}} - t_0}{(\tau_{\hat{\nu}2} \ln 2) - t_0} H_0. \quad (7.10)$$

Consequently, taking the data, applied in the result (3.42), we have $t_{\text{eff}} - t_0 = 3.676 \cdot 10^{17}$ s and $(\tau_{\hat{\nu}2} \ln 2) - t_0 = 3.322 \cdot 10^{17}$ s, so that Eq. (7.10) yields (via the cosmological parameters of Table I) as present Hubble parameter

$$H_{\text{acc},0} = \frac{t_{\text{eff}} - t_0}{(\tau_{\hat{\nu}2} \ln 2) - t_0} H_0 = 2.41 \cdot 10^{-18} \text{ s}^{-1} = 74.4 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (7.11)$$

i.e. the epoch of Eq. (7.11) begins at $t_0 = 4.358 \cdot 10^{17}$ s = 13.81 Gyr (see Table I) after the big bang. Thus, we need no new physical assumptions.

Then, for $t \geq t_0$, at $t_{\text{eff}} = 8.034 \cdot 10^{17}$ s and $\tau_{\hat{\nu}2} \ln 2 = 7.680 \cdot 10^{17}$ s [1, 2], we assume semi-empirically

$$H_{\text{acc}}(t) = \left(\frac{t_{\text{eff}} - t_0}{(\tau_{\hat{\nu}2} \ln 2) - t_0} \right)^2 \frac{(\tau_{\hat{\nu}2} \ln 2) - t}{t_{\text{eff}} - t} H_0. \quad (7.12)$$

Thus, for $H_{\text{acc}}(t) = H_0$, we obtain

$$\begin{aligned} t &= \frac{([t_{\text{eff}} - t_0] / [(\tau_{\hat{\nu}2} \ln 2) - t_0])^2 \tau_{\hat{\nu}2} \ln 2 - t_{\text{eff}}}{([t_{\text{eff}} - t_0] / [(\tau_{\hat{\nu}2} \ln 2) - t_0])^2 - 1} = \\ &= 6.103 \cdot 10^{17} \text{ s} = 19.34 \text{ Gyr}. \end{aligned} \quad (7.13)$$

Therefore, in future, from $t = 19.34$ Gyr, for $H_{acc}(t) < H_0$, the expansion of the universe is still slower, i.e. the accelerated expansion is decelerated.

Consequently, the results (7.6) as well as (7.9) and (7.11) mean a faster expansion than the value $H_0 = 2.181 \cdot 10^{-18} \text{ s}^{-1} = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (see above). However, the Hubble parameters (7.9) and (7.11) yield a slower expansion than the value (7.6). Because of the result (7.13), the accelerated expansion of the universe is extremely reduced at $t = 19.34$ Gyr. This result agrees with the derivation of a slow linear expansion [1, 2], which is characterized by the expression [1, 2]

$$\frac{1}{\text{AU}} \frac{d}{dt} \text{AU} = 2.733 \cdot 10^{-20} \text{ s}^{-1}, \quad (7.14)$$

where $\text{AU} = 1.49597870700(3) \cdot 10^{11} \text{ m}$ describes the astronomical unit [10], whereas $d\text{AU}/dt$ [1, 2] represents the astronomical unit changing [1, 2]

$$\begin{aligned} \frac{d}{dt} \text{AU} &= c \left[\Omega_m (1 + z_{\text{MNA}})^2 + \Omega_\Lambda / (1 + z_{\text{MNA}}) \right] \frac{R_{\text{earth}} + R_{\text{sun}}}{R_0} = \\ &= (12.9_{-1.3}^{+1.2}) \text{ cm yr}^{-1} \end{aligned} \quad (7.15)$$

in accordance with the observation [19]

$$\frac{d}{dt} \text{AU} = (15 \pm 4) \text{ cm yr}^{-1}. \quad (7.16)$$

The values $R_{\text{earth}} = 6.378137 \cdot 10^6 \text{ m}$ and $R_{\text{sun}} = (6.9551 \pm 0.0004) \cdot 10^8 \text{ m}$ are the equatorial radii of the earth and the sun [10].

Then, by Eqs. (7.14) and (7.15), because of $R_0 = c/H_0$ (see Table I), for the slow linear (lin) expansion, we can assume its Hubble parameter to

$$\begin{aligned} H_{\text{lin}} &= \frac{1}{\text{AU}} \frac{d}{dt} \text{AU} = \\ &= \left[\Omega_m (1 + z_{\text{MNA}})^2 + \Omega_\Lambda / (1 + z_{\text{MNA}}) \right] \frac{R_{\text{earth}} + R_{\text{sun}}}{\text{AU}} H_0 = \\ &= 0.843 \text{ km s}^{-1} \text{ Mpc}^{-1}. \end{aligned} \quad (7.17)$$

Consequently, in future, the result (7.17) means a slow linear expansion of the late massive universe (see above) in comparison with the Hubble

parameters $95.0 \text{ km s}^{-1} \text{ Mpc}^{-1} \geq H_{\text{acc}} > 0.843 \text{ km s}^{-1} \text{ Mpc}^{-1}$ {see Eqs. (7.6), (7.9), (7.11) and (7.17)}.

Thus, at $H_{\text{acc}}(t) = H_{\text{lin}}$, we get

$$t = \frac{(H_0/H_{\text{lin}})([t_{\text{eff}} - t_0]/[(\tau_{\hat{\nu}2} \ln 2) - t_0])^2 \tau_{\hat{\nu}2} \ln 2 - t_{\text{eff}}}{(H_0/H_{\text{lin}})([t_{\text{eff}} - t_0]/[(\tau_{\hat{\nu}2} \ln 2) - t_0])^2 - 1} =$$

$$= 7.676 \cdot 10^{17} \text{ s} = 24.32 \text{ Gyr}. \quad (7.18)$$

Therefore, in future, at $H_{\text{acc}}(t) = H_{\text{lin}}$, i.e. from $t = 24.32 \text{ Gyr}$, the slow linear expansion of the universe dominates up to the final state of the massive universe.

Indeed, the present Hubble parameter $H_{\text{acc},0} = 74.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (see Eq. (7.11)) is confirmed by the observations of Riess et al. [20], which yield a value for the present Hubble “constant” of $74.03 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The deviation between these two expansion rates is about 0.5%. Thus, we cannot more denote the Hubble parameter $H_0 = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ as present expansion rate of the universe, since it was interpreted as the present Hubble “constant” of the CMB, so that we have assumed that we can express all Hubble parameters as a function of the present CMB value $H_0 = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ of the Planck observations 2013, since the present CMB value is slower in comparison with the present Hubble “constant” $H_{\text{acc},0} = 74.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Thus, we have used the present CMB value H_0 as basis for the description of the evolution of the universe, since it was reasonably derived via data [12] of the CMB formed at $3.72 \cdot 10^5$ years after the big bang. Because of these assumptions, in this work, all considerations are correct.

Consequently, our result of a slower expansion of the massive universe is in contrast to the interpretation of Riess et al. [20], in which the universe expands always faster, i.e. this interpretation contradicts the known physics and is only understandable in the framework of a new physics [20], whereas our far-reaching result agrees with the known physics [Λ cold dark matter model (Λ CDM) confirmed experimentally].

Because of a better explanation of the dark energy in the second half of Sec. 4, for the confirmation of this far-reaching conclusion, we use the new particle-defined cosmological parameter values of Table VI, since the accelerated expansion is determined by the normal dark energy $\Omega_{\Lambda}^{**} = 0.683_{-0.028}^{+0.038}$ (introduced by Eq. (4.24) and Table VI) via the vacuum energy density $\rho_{\text{vac}}^* c^2 = \Omega_{\Lambda}^{**} \rho_{0C}^* c^2 = (3.29_{-0.14}^{+0.15}) \cdot 10^3 \text{ eV cm}^{-3}$, which is again defined by the 3 sterile neutrinos $\hat{\nu}_{\Lambda}$, $\hat{\nu}_{\text{dm}}$ and $\hat{\nu}_{\text{b}}$ (see Sec. 4), i.e. we expect that we can estimate a still better present Hubble "constant" $H_{\text{acc},0}^*$ than $H_{\text{acc},0} = 74.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$, using again the result (7.10). Therefore, we apply following procedure in 3 steps.

Firstly, via this vacuum energy density $\rho_{\text{vac}}^* c^2 = (3.29_{-0.14}^{+0.15}) \cdot 10^3 \text{ eV cm}^{-3}$, according to Eq. (3.8), we determine the distance

$$d_{\text{eff}}^* = \left(\frac{E_{\text{Pl}}^2}{\hbar c \frac{1}{2} \Omega_{\Lambda}^{**} \rho_{0C}^* c^2 \frac{4}{3} \pi} \right)^{1/2} = (3.311_{-0.075}^{+0.070}) \cdot 10^{26} \text{ m}, \quad (7.19)$$

so that because of Eqs. (3.15) and (3.42) including (7.10) the lifetime $\tau_{\hat{\nu}}^* \rightarrow \tau_{\hat{\nu}2}^*$ of the sterile neutrinos is found to

$$\tau_{\hat{\nu}2}^* = \frac{d_{\text{eff}}^*}{c} = (1.104_{-0.025}^{+0.023}) \cdot 10^{18} \text{ s} = 34.98_{-0.79}^{+0.73} \text{ Gyr}. \quad (7.20)$$

Thus, the corresponding expression $(\tau_{\hat{\nu}2}^* \ln 2) - t_0^*$ yields

$$(\tau_{\hat{\nu}2}^* \ln 2) - t_0^* = (3.30_{-0.15}^{+0.14}) \cdot 10^{17} \text{ s}. \quad (7.21)$$

Secondly, using Eq. (2.30) with $N(T) = 3.362644$, we can form the condition

$$\begin{aligned} \rho_{\text{vac}}(T^*) c^2 &= \frac{1}{3} \frac{\pi^2 (kT^*)^4}{15 (\hbar c)^3} \frac{1}{N(T)} = \rho_{\text{vac}}^* c^2 = \\ &= \Omega_{\Lambda}^{**} \rho_{0C}^* c^2 = (3.29_{-0.14}^{+0.15}) \cdot 10^3 \text{ eV cm}^{-3}, \end{aligned} \quad (7.22)$$

i.e. the temperature T^* can be determined to

$$T^* = 51.49_{-0.55}^{+0.59} \text{ K}, \quad (7.23)$$

so that we obtain the redshift condition

$$1 + z_{\Lambda}^* = \frac{T^*}{T_0} = 18.89_{-0.20}^{+0.22}, \quad (7.24)$$

where now the influence of the normal dark matter Ω_{Λ}^{**} begins (see Eq. (2.20)). Then, taking the blueshift condition $1 + z(\nu_e) = 0.406_{-0.025}^{+0.020}$ of the electron neutrino (see Eq. (2.67)), analogous to Eq. (2.22), we estimate now the mean redshift condition $1 + z_{\text{MNA}}^*$ to

$$1 + z_{\text{MNA}}^* = \left([1 + z_{\Lambda}^*] [1 + z(\nu_e)] \right)^{1/2} = 2.769_{-0.100}^{+0.084}, \quad (7.25)$$

where it defines the mean negative acceleration \ddot{R}_{MNA}^* via Eqs. (2.23) and (2.24), i.e. we obtain this mean negative acceleration to

$$\begin{aligned} \ddot{R}_{\text{MNA}}^* &= -\frac{1}{2} c H_0^* \left[\Omega_{\text{m}}^* (1 + z_{\text{MNA}}^*)^2 + \Omega_{\Lambda}^{**} / (1 + z_{\text{MNA}}^*) \right] = \\ &= (-8.64_{-0.69}^{+0.60}) \cdot 10^{-8} \text{ cm s}^{-2}, \end{aligned} \quad (7.26)$$

if the transformations $H_0 \rightarrow H_0^* = 2.191 \cdot 10^{-18} \text{ s}^{-1}$, $\Omega_{\text{m}} \rightarrow \Omega_{\text{m}}^* = 0.311$ and $\Omega_{\Lambda} \rightarrow \Omega_{\Lambda}^* = 0.683$ are used according to Tables V and VI. Consequently, via Eq. (2.26), the effective scale factor R_{eff}^* is given by

$$R_{\text{eff}}^* = \frac{-\ddot{R}_{\text{MNA}}^*}{\Omega_{\Lambda}^{**} H_0^{*2}} = (2.63_{-0.21}^{+0.18}) \cdot 10^{26} \text{ m}, \quad (7.27)$$

so that via Eqs. (2.27) or (2.28) by the corresponding values of Table VI we can determine the time difference

$$t_{\text{eff}}^* - t_0^* = \frac{1}{\Omega_{\Lambda}^{**1/2} H_0^*} \ln \frac{R_{\text{eff}}^*}{R_0^*} = (3.61_{-0.23}^{+0.19}) \cdot 10^{17} \text{ s} = 11.44_{-0.73}^{+0.60} \text{ Gyr}. \quad (7.28)$$

Thirdly, using the results (7.21) and (7.28), via the corresponding expression (7.10), we find more exactly the present Hubble "constant" $H_{\text{acc},0}^*$ to

$$H_{\text{acc},0}^* = \frac{t_{\text{eff}}^* - t_0^*}{(\tau_{\nu 2}^* \ln 2) - t_0^*} H_0^* = 74.0_{-2.6}^{+2.0} \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (7.29)$$

i.e. the deviation between the observed value $74.03 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (see above) and the result (7.29) is very small.

This excellent agreement confirms our hypothesis that the joint origin of the dark matter and dark energy is based on the sterile neutrinos [1-4] as well as their breakup and decay products (see above and Ref. [1]). This result supports also the introduction of the normal (Ω_{Λ}^{**}) and the total (Ω_{Λ}^{***}) dark energy in the present work (see the second half of Sec. 4).

Using the data of Tables III to VI, the corresponding results, derived in Sec. 7 for the massive universe, are also valid for the massive anti-universe.

8. The time dependence of the cosmological “constant”

The cosmological “constant” problem has a complex history [21]. In this work, for the total (massless and massive) universe, the vacuum energy densities or cosmological “constants”, introduced already in Refs. [1, 2] as variable quantities, are compared with the prediction of the quantum field theory. For this goal, they are written in their time-dependent form, whereat the considerations are initially restricted to the results of Sec. 3.1. Firstly, we take into account only the 3 limiting cases (Hubble time (τ_H), Planck time (t_{Pl}) and big bang (t_{BB})). Secondly, we treat generally the case $t \leq t_{Pl}$. Thirdly, we consider the case $t \geq t_{Pl}$.

Now, we describe firstly the 3 limiting cases (Hubble time, Planck time and big bang). For the Hubble time $\tau_H = 1/H_0 = 4.585 \cdot 10^{17}$ s, we have

$$\begin{aligned} \rho_{\text{vac}, \Lambda} c^2 &= \Omega_{\Lambda} \rho_{0C} c^2 = \Omega_{\Lambda} \frac{3 H_0^2 c^2}{8\pi G_N} = \\ &= \Omega_{\Lambda} \frac{3 c^2}{8\pi G_N \tau_H^2} = 3.27 \cdot 10^3 \text{ eV cm}^{-3} \end{aligned} \quad (8.1)$$

or

$$\begin{aligned} \Lambda = \Lambda_{\Lambda} &= \frac{3 \Omega_{\Lambda}}{R_0^2} = \frac{3 \Omega_{\Lambda}}{c^2 / H_0^2} = \\ &= \frac{3 \Omega_{\Lambda}}{c^2 \tau_H^2} = 1.087 \cdot 10^{-52} \text{ m}^{-2}. \end{aligned} \quad (8.2)$$