Then, taking Eqs. (2.61) to (2.67) as well as (5.5) to (5.9), we get semi-empirically the identical connections

$$\alpha_{\text{GUT}} = \left[\Omega_{\text{r}} \frac{1+z_0(\sum_i \nu_i)}{1+z_{\text{reion}}(\sum_i \nu_i)}\right]^{1/2} = \left[\Omega_{\text{r}} \frac{\sum_i E_0(\nu_i)}{kT(\sum_i \nu_i)}\right]^{1/2} = 0.0391\,, \qquad (5.10)$$

$$\alpha_{\text{GUT}} = \left[\Omega_{\text{r}} \frac{1+z_0(\nu_\tau)}{1+z_{\text{reion}}(\nu_\tau)}\right]^{1/2} = \left[\Omega_{\text{r}} \frac{E_0(\nu_\tau)}{kT(\nu_\tau)}\right]^{1/2} = 0.0391\,, \qquad (5.11)$$

$$\alpha_{\text{GUT}} = \left[\Omega_{\text{r}} \frac{1+z_0(\nu_\mu)}{1+z_{\text{reion}}(\nu_\mu)}\right]^{1/2} = \left[\Omega_{\text{r}} \frac{E_0(\nu_\mu)}{kT(\nu_\mu)}\right]^{1/2} = 0.0391 \qquad (5.12)$$

and

$$\alpha_{\text{GUT}} = \left[\Omega_{\text{r}} \frac{1+z_0(\nu_{\text{e}})}{1+z(\nu_{\text{e}})}\right]^{1/2} = \left[\Omega_{\text{r}} \frac{E_0(\nu_{\text{e}})}{kT(\nu_{\text{e}})}\right]^{1/2} = 0.0391 \qquad (5.13)$$

in excellent agreement with Eq. (2.59).

The results (5.10) to (5.13) can be considered also as a reasonable argument for the introduction of the new thermal equilibrium in Refs. [1-5].

Using the data of Tables III to V, these considerations, derived in this Sec. 5 for the universe, are also valid for the anti-universe.

6 Rest energy of photons and gravitons including conclusions

To this day, it was assumed that the rest energies of the photons and the gravitons are zero. Therefore, we expect for their rest energies greater than zero also the same values. Then, the results (3.108) and (3.109) permit unique conclusions about the rest energy of the gravitons (G) and the photons (γ). Consequently, by Eq. (3.108), we have the first condition

$$E_0(\text{G}) = kT_{\text{BB}} = \frac{E_{\text{H}}}{\tfrac{1}{2} N_0} = \frac{\rho_{0\text{C}}\, c^2\, \tfrac{4}{3}\pi R_0^3}{\tfrac{1}{2} N_0} \cong 1.563 \cdot 10^{-35}\ \text{eV}\,, \qquad (6.1\text{a})$$

whereas Eq. (3.109) yields the second condition

$$E_0(\gamma) = kT_{\mathrm{BB}} = \frac{\rho_{\mathrm{vac}}(R_{\mathrm{BB}})\,c^2}{n_\gamma(R_{\mathrm{f}})} \cong 1.563\cdot 10^{-35}\ \mathrm{eV}\,. \tag{6.1 b}$$

Thus, we can assume that the results (6.1 a) and (6.1 b) describe the smallest possible thermal (particle) energies in the total (massless and massive) universe. Therefore, we suggest that because of the new thermal equilibrium these smallest possible thermal (particle) energies (6.1 a) and (6.1 b) of the big bang should be responsible for the rest energies $E_0(\mathrm{G})$ and $E_0(\gamma)$ of the gravitons and photons, respectively, i.e. we assume also

$$E_0(\gamma) = E_0(\mathrm{G}) = kT_{\mathrm{BB}} \cong 1.563\cdot 10^{-35}\mathrm{eV}\,. \tag{6.2}$$

Thus, they are practically massless. The suggestion (6.2) is supported by the gravitational potential energy (see Refs. [1, 2])

$$E_0(\gamma) = \frac{G_N}{c^4}\,\frac{E_0(\gamma)\times E_0(\gamma)}{R_{\mathrm{BB}}} =$$

$$= E_0(\mathrm{G}) = \frac{G_N}{c^4}\,\frac{E_0(\mathrm{G})\times E_0(\mathrm{G})}{R_{\mathrm{BB}}} \cong 1.563\cdot 10^{-35}\mathrm{eV}\,, \tag{6.3}$$

where R_{BB} is given by Eq. (3.23). These results (6.1 a) to (6.3) agree with the observed limits $E_0(\mathrm{G}) < 9.0\cdot 10^{-34}$ eV and $E_0(\gamma) < 3\cdot 10^{-27}$ eV (see Ref. [16]).

Now, the considerations are restricted to photons. Then, assuming the present relativistic thermal energy $E(\gamma) = kT_0 + E_0(\gamma)$ (see Table I for T_0), the redshift condition for photons yields

$$1 + z_0(\gamma) = \frac{E(\gamma)}{kT_0} = 1 + \frac{E_0(\gamma)}{kT_0}\,, \tag{6.4}$$

i.e. we find the redshift of the photons to

$$z_0(\gamma) = \frac{E_0(\gamma)}{kT_0} \cong 6.655\cdot 10^{-32}\,. \tag{6.5}$$

Assuming also for the big bang at $R_{\mathrm{BB}} = c\,t_{\mathrm{BB}} = 2.069\cdot 10^{-96}$ cm (see Eqs. (3.23) and (3.24)) because of the Robertson-Walker metric the validity of the Hubble relation $v = cz = \overline{H}R$ as a result of the Doppler effect for $v << c$ (see, e.g., Ref. [9]), the quantum mechanical zero-point velocity v_{BB} is given by

$$v_{BB} = \dot{R}_{BB} = c\, z_0(\gamma) = \overline{H} R_{BB} = \overline{H}_{BB} R_{BB} \cong 1.995 \cdot 10^{-23}\,\mathrm{m\,s^{-1}}, \tag{6.6}$$

so that for the big bang we obtain

$$\overline{H}_{BB} = \frac{\dot{R}_{BB}}{R_{BB}} = \frac{z_0(\gamma)}{t_{BB}} \cong 9.643 \cdot 10^{74}\,\mathrm{s^{-1}}. \tag{6.7}$$

Introducing the limiting values

$$v_{BB} = \dot{R}_{BB} = c\, z_0(\gamma) = \overline{H}_{Pl} R_{Pl} \cong 1.995 \cdot 10^{-23}\,\mathrm{m\,s^{-1}}, \tag{6.8}$$

we find

$$\overline{H}_{Pl} = \frac{\dot{R}_{BB}}{R_{Pl}} = \frac{z_0(\gamma)}{t_{Pl}} \cong 1.234 \cdot 10^{12}\,\mathrm{s^{-1}}. \tag{6.9}$$

Using the result $H_{Pl} = 1/2t_{Pl}$ of Eq. (3.102), the expression (6.9) yields the connections

$$\overline{H}_{Pl} = \frac{z_0(\gamma)}{t_{Pl}} = 2\, z_0(\gamma) H_{Pl} = z_0(\gamma) H_0 (1 + z_M)^2. \tag{6.10}$$

By the limiting values of Eqs. (6.6) to (6.9), we can generally introduce

$$v_{BB} = \dot{R}_{BB} = c\, z_0(\gamma) = \overline{H} R \cong 1.995 \cdot 10^{-23}\,\mathrm{m\,s^{-1}}, \tag{6.11}$$

i.e. because of Eq. (2.43) for $t_{BB} \le t \le t_{Pl}$ we get

$$\overline{H} = \frac{\dot{R}_{BB}}{R} = \frac{z_0(\gamma)}{t}. \tag{6.12}$$

Because of $kT_0 >> E_0(\gamma)$, we have $E(\gamma) = kT_0$, so that via Eq. (3.104) we can estimate the present total energy $E_{tot}(\gamma)$ of the photons to

$$E_{tot}(\gamma) = E(\gamma) \times \tfrac{1}{2} N_0 = kT_0 \times \tfrac{1}{2} N_0 \cong 7.805 \cdot 10^{119}\,\mathrm{eV}. \tag{6.13}$$

The present total photon energy (6.13), multiplied by $z_0(\gamma) = 6.655 \cdot 10^{-32}$ (see Eq. (6.5)), yields again the Hubble energy E_H (see Eqs (3.107) and (3.108)) of the present universe as follows

$$E_H = z_0(\gamma) \times E_{tot}(\gamma) \cong 5.194 \cdot 10^{88}\,\mathrm{eV}. \tag{6.14}$$

Similar results can be found for all remaining extreme massive particles in the universe. For example, we consider the X and Y gauge bosons as well as the protons and the electron neutrino.

According to Eq. (2.57) or Refs. [1-3, 5], for the X and Y gauge bosons, if we introduce $E_0(\mathrm{X}) = E_0(\mathrm{X, Y})$ and $E_0(\mathrm{Y}) = E_0(\mathrm{X, Y})$, we find

$$E_0(\mathrm{X, Y}) = 2.675 \cdot 10^{25}\ \mathrm{eV} \quad \text{(see Eq. (2.57))}, \tag{6.15}$$

$$\overline{R}_{\mathrm{X, Y}} = \frac{G_N}{c^4} E_0(\mathrm{X, Y}) = 3.541 \cdot 10^{-36}\ \mathrm{cm} \quad \text{(see Eq. (2.40))}, \tag{6.16}$$

$$E_0(\mathrm{X, Y}) = \frac{G_N}{c^4} \frac{E_0(\mathrm{X, Y}) \times E_0(\mathrm{X, Y})}{\overline{R}_{\mathrm{X, Y}}} = 2.675 \cdot 10^{25}\ \mathrm{eV}, \tag{6.17}$$

$$R_{\mathrm{BB}} = \frac{E_0(\gamma)}{E_0(\mathrm{X, Y})} \overline{R}_{\mathrm{X, Y}} = 2.069 \cdot 10^{-96}\ \mathrm{cm}, \tag{6.18}$$

$$1 + z_{\mathrm{X, Y}} = \frac{E_0(\mathrm{X, Y})}{kT_0} = 1.139 \cdot 10^{29}, \tag{6.19}$$

$$R_{\mathrm{X, Y}} = \frac{R_0}{1 + z_{\mathrm{X, Y}}} = 1.207 \cdot 10^{-1}\ \mathrm{cm} \quad \text{(see Eqs. (2.15) and (2.17))}, \tag{6.20}$$

$$\tfrac{1}{2} N_{\mathrm{X, Y}} = \frac{\tfrac{1}{2} R_{\mathrm{X, Y}}}{R_{\mathrm{BB}}} = 2.917 \cdot 10^{94}, \tag{6.21}$$

$$E_{\mathrm{tot}}(\mathrm{X, Y}) = E_0(\mathrm{X, Y}) \times \tfrac{1}{2} N_{\mathrm{X, Y}} = 7.804 \cdot 10^{119}\ \mathrm{eV} \tag{6.22}$$

and

$$E_{\mathrm{H}} = z_0(\gamma) \times E_{\mathrm{tot}}(\mathrm{X, Y}) = 5.194 \cdot 10^{88}\ \mathrm{eV}. \tag{6.23}$$

For the Higgs boson (see Ref. [16]), we have

$$E_0(\mathrm{H}^0) \cong 1.26 \cdot 10^{11}\ \mathrm{eV}, \tag{6.24}$$

$$\overline{R}_{\mathrm{H}^0} = \frac{G_N}{c^4} E_0(\mathrm{H}^0) = 1.668 \cdot 10^{-50}\ \mathrm{cm}, \tag{6.25}$$

$$E_0(\mathrm{H}^0) = \frac{G_N}{c^4} \frac{E_0(\mathrm{H}^0) \times E_0(\mathrm{H}^0)}{\overline{R}_{\mathrm{H}^0}} = 1.260 \cdot 10^{11}\ \mathrm{eV}, \tag{6.26}$$

$$R_{\mathrm{BB}} = \frac{E_0(\gamma)}{E_0(\mathrm{H}^0)} \overline{R}_{\mathrm{H}^0} = 2.069 \cdot 10^{-96}\ \mathrm{cm}, \tag{6.27}$$

$$1 + z_{\mathrm{H}^0} = \frac{E_0(\mathrm{H}^0)}{kT_0} = 5.365 \cdot 10^{14}, \tag{6.28}$$

$$R_{H^0} = \frac{R_0}{1+z_{H^0}} = 2.563\cdot 10^{13}\ \text{cm}, \tag{6.29}$$

$$\tfrac{1}{2} N_{H^0} = \frac{\tfrac{1}{2} R_{H^0}}{R_{BB}} = 6.194\cdot 10^{108}, \tag{6.30}$$

$$E_{tot}(\mathrm{H}^0) = E_0(\mathrm{H}^0)\times \tfrac{1}{2} N_{H^0} = 7.804\cdot 10^{119}\ \text{eV} \tag{6.31}$$

and

$$E_H = z_0(\gamma)\times E_{tot}(\mathrm{H}^0) = 5.194\cdot 10^{88}\ \text{eV}. \tag{6.32}$$

For the electron [10], we obtain

$$E_0(\mathrm{e}) = 5.109989\cdot 10^5\ \text{eV}, \tag{6.33}$$

$$\overline{R}_e = \frac{G_N}{c^4} E_0(\mathrm{e}) = 6.764\cdot 10^{-56}\ \text{cm}, \tag{6.34}$$

$$E_0(\mathrm{e}) = \frac{G_N}{c^4}\frac{E_0(\mathrm{e})\times E_0(\mathrm{e})}{\overline{R}_e} = 5.110\cdot 10^5\ \text{eV}, \tag{6.35}$$

$$R_{BB} = \frac{E_0(\gamma)}{E_0(\mathrm{e})}\overline{R}_e = 2.069\cdot 10^{-96}\ \text{cm}, \tag{6.36}$$

$$1+z_e = \frac{E_0(\mathrm{e})}{kT_0} = 2.176\cdot 10^9, \tag{6.37}$$

$$R_e = \frac{R_0}{1+z_e} = 6.319\cdot 10^{18}\ \text{cm}, \tag{6.38}$$

$$\tfrac{1}{2} N_e = \frac{\tfrac{1}{2} R_e}{R_{BB}} = 1.527\cdot 10^{114}, \tag{6.39}$$

$$E_{tot}(\mathrm{e}) = E_0(\mathrm{e})\times \tfrac{1}{2} N_e = 7.804\cdot 10^{119}\ \text{eV} \tag{6.40}$$

and

$$E_H = z_0(\gamma)\times E_{tot}(\mathrm{e}) = 5.194\cdot 10^{88}\ \text{eV}. \tag{6.41}$$

Consequently, for the extremely massive particles, the total energies (see, Eqs. (6.22, (6.31) and (6.40)) exist also during the evolution of the massive universe at the different times $t = t(z)$, defined by Eq. (2.12) via the corresponding redshifts (see Eqs. (6.19), (6.28) and (6.37)) and values $N(T)$ [1-3, 10]. These total energies, multiplied by the redshift $z_0(\gamma) = 6.655\cdot 10^{-32}$

(see Eq. (6.5)), yield again the Hubble energy E_{H} (see Eq. (3.108)) of the present universe according to Eqs. (6.23), (6.32) and (6.41). Therefore, this fact means that the zero-point oscillations describe also completely the massive universe. Using Eqs. (3.102) to (3.109), this result is confirmed by the ratios

$$\tfrac{1}{2}(1+z_{\mathrm{M}})^2 = \frac{1}{2\,H_0\,t_{\mathrm{Pl}}} = \frac{\tfrac{1}{2}R_0}{R_{\mathrm{Pl}}} = \frac{\tfrac{1}{2}N_0}{N_{\mathrm{Pl}}} = \frac{E_{\mathrm{H}}}{E_{\mathrm{Pl}}} = 4.254\cdot 10^{60}\,, \qquad (6.42)$$

since the basis of the mean (maximum) energies (3.78) and (3.83) of the universe and the anti-universe is the Planck energy E_{Pl} (see Eqs. (3.79) and (3.84)) as the greatest possible relativistic particle energy. Thus, new results are possible by coupling of all corresponding expressions. For example, in this chapter, the coupling of Eqs. (6.10) and (6.42) leads to

$$(1+z_{\mathrm{M}})^2 = \frac{\overline{H}_{\mathrm{Pl}}}{H_0\,z_0(\gamma)} = 2\frac{E_{\mathrm{H}}}{E_{\mathrm{Pl}}}\,. \qquad (6.43)$$

Using the data of Tables III to V, the results, derived by (6.1 a) to (6.43) for the total (massless and massive) universe, are also valid for the corresponding anti-universe.

In Refs. [1, 2], for particles (P) and antiparticles ($\overline{\mathrm{P}}$), we have introduced the Schwarzschild (S) radius $R_{\mathrm{S}} = R'$, where R' is defined by

$$R' = \frac{G_N}{c^4}\left[E_0(\mathrm{P}) + E_0(\overline{\mathrm{P}})\right], \qquad (6.44)$$

so that because of $E_0(\mathrm{P}) = E_0(\overline{\mathrm{P}})$ we obtain as Schwarzschild radius [9, 11]

$$R_{\mathrm{S}} = 2\frac{G_N}{c^4}E_0(\mathrm{P}) \qquad (6.45)$$

as distance between particle-antiparticle pairs in the quantum vacuum [1, 2], which penetrates completely the space-time continuum of the total (massless and massive) universe and anti-universe (see Eqs. (6.46) to (6.48)).

For example, at Higgs boson-antiboson pairs, we have the Schwarzschild radius

$$R_{\mathrm{S}} = R_{\mathrm{S}}(\mathrm{H}^0) = 2\overline{R}_{\mathrm{H}^0} = 3.336\cdot 10^{-50}\ \mathrm{cm}\,, \qquad (6.46)$$

where $\overline{R}_{\mathrm{H}^0}$ is given by Eq. (6.25). In the case of photons, for the corresponding photon-photon pairs, we obtain as Schwarzschild radius

$$R_S = R_S(\gamma) = 2R_{BB} = 4.138 \cdot 10^{-96} \text{ cm}, \tag{6.47}$$

where the big bang distance $R_{BB} = 2.069 \cdot 10^{-96}$ cm (see Eq. (3.23)) is defined according to Eq. (2.40) by $R_{BB} = (G_N/c^4)\, E_0(\gamma)$ via Eqs. (6.1 b). For gravitons, we can assume an "analogous" situation.

According to Eqs. (3.108), for the Hubble energy E_H of the present ($z = 0$) massive universe and anti-universe, we find plausibly for their Schwarzschild radius

$$R_S = 2\frac{G_N}{c^4} E_H = R_0 = 2 \times \tfrac{1}{2} R_0 = 1.375 \cdot 10^{28} \text{ cm}. \tag{6.48}$$

Then, we find the escape (esc) velocity v_{esc} (see, e.g., Refs. [9, 11]) for the present massive universe or anti-universe to

$$v_{esc} = \left(\frac{2\, G_N\, (E_H/c^2)}{R_0} \right)^{1/2} = c\,, \tag{6.49}$$

i.e. they are black holes because the surfaces of the present massive universe or anti-universe are limited by their Schwarzschild radius $R_S = R_0$, so that even the light cannot escape from the present massive universe or anti-universe.

Because of $R_{BB} < R_S$ (see Eq. (6.47)), an analogous situation has the big bang, so that we get no information about the big bang, since from a mass with the surface within the range of the Schwarzschild radius no events can be observed.

A completely other situation is observed at the extremely massive particles and antiparticles, which form the corresponding universe and anti-universe. For example, at the Higgs bosons and Higgs antibosons, their scale factors $R_{H^0} = 2.563 \cdot 10^{13}$ cm (see Eq. (6.29)) and analogously $R_{\overline{H}^0} = 2.563 \cdot 10^{13}$ cm do not lie within the range of the Schwarzschild radius (6.46).

Now, we try to calculate the rest energy of the photons via the general galactic magnetic field, assuming a coupling with the magnetic neutrino moment.

In Sec. 4, we have shown that the joint origin of the dark matter and the dark energy is a result of the invisible decay and breakup products of the sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\rm dm}$ and $\hat{\nu}_{\rm b}$. Therefore, we can assume that their magnetic (mag) energy density $\hat{\varepsilon}_{\rm mag}$ must be equivalent to

$$\hat{\varepsilon}_{\rm mag} = (\Omega_{\rm dm} + \Omega_\Lambda)\, \Omega_\gamma \rho_{0\rm C}\, c^2 = (\Omega_{\rm dm} + \Omega_\Lambda)\, \rho_0(\gamma)\, c^2 \qquad (6.50)$$

because of the definition (see, e.g., Refs. [7, 8])

$$\Omega_\gamma = \frac{\rho_\gamma}{\rho_{0\rm C}} = \frac{\rho_0(\gamma)}{\rho_{0\rm C}} = 5.46 \cdot 10^{-5} \quad \text{(see Table I)}, \qquad (6.51)$$

where $\rho_\gamma = \rho_0(\gamma)$ is the electromagnetic mass density of the photons of the present cosmic microwave background (CMB) explained, for example, in Ref. [8]. Taking

$$\rho_0(\gamma)\, c^2 = \frac{\pi^2}{15} \frac{(kT_0)^4}{(\hbar c)^3} = 0.26057 \, \text{eV cm}^{-3} \qquad (6.52)$$

and

$$\rho_{0\rm C}\, c^2 = 4.77 \cdot 10^3 \, \text{eV cm}^{-3}, \qquad (6.53)$$

we obtain the magnetic energy density $\hat{\varepsilon}_{\rm mag}$ to

$$\hat{\varepsilon}_{\rm mag} = (\Omega_{\rm dm} + \Omega_\Lambda)\, \rho_0(\gamma)\, c^2 = 0.2475 \, \text{eV cm}^{-3}. \qquad (6.54)$$

The galactic magnetic (gmag) energy density $\varepsilon_{\rm gmag}$ is given by

$$\varepsilon_{\rm gmag} = \frac{1}{2\mu_0} B^2 = \frac{1}{8\pi \cdot 1.602176565 \cdot 10^{-20}} \left(\frac{B}{\rm T}\right)^2 \text{eV/cm}^3, \qquad (6.55)$$

where μ_0 describes the permeability of free space and B represents the general galactic magnetic field. Then, by the condition $\varepsilon_{\rm gmag} = \hat{\varepsilon}_{\rm mag}$, we find the present general galactic magnetic field B to

$$B = \left[(\Omega_{\rm dm} + \Omega_\Lambda)\, \rho_0(\gamma) c^2 \times 8\pi \times 1.6022 \cdot 10^{-20} \, \text{cm}^3/\text{eV}\right]^{1/2} \text{T} =$$
$$= 3.157 \cdot 10^{-10} \, \text{T} \qquad (6.56)$$

in excellent agreement with the observations [17], which lie in the small range $(1.4 \pm 0.2) \cdot 10^{-10} \, \text{T} \le B \le (4.4 \pm 0.9) \cdot 10^{-10} \, \text{T}$.

Consequently, the remaining energy density $\varepsilon_{\rm b} = \Omega_{\rm b}\, \rho_0(\gamma)\, c^2$ must provide the present luminosity density of all galaxies to

$$\Lambda_0 = \Omega_b \, H_\Lambda \, \rho_0(\gamma)\, c^2 = 4.544 \cdot 10^{-33} \, \mathrm{W/m^3} =$$
$$= 2.886 \cdot 10^8 \, L_{\mathrm{SOL}}/\mathrm{Mpc}^3 \,, \tag{6.57}$$

where the Hubble parameter $H_\Lambda = \Omega_\Lambda^{1/2} H_0 = 1.805 \cdot 10^{-18} \, \mathrm{s}^{-1}$, which characterizes the present day situation of the massive universe by the exponential expansion (2.27) for $t = t_0$, and the solar (SOL) luminosity $L_{\mathrm{SOL}} = 3.828 \cdot 10^{26} \, \mathrm{W}$ [10] were used. The result (6.57) agrees with the limiting value of the total luminosity $\Lambda_{\mathrm{tot}} \lesssim 3 \cdot 10^8 \, L_{\mathrm{SOL}}/\mathrm{Mpc}^3$ of all galaxies in Ref. [9]. This agreement supports the upper assumptions.

Because of the estimations (6.54) to (6.56), we can assume that the gravitons yield no contribution to this general galactic magnetic field.

Therefore, we assume that a connection must exist between the magnetic field (6.56) as well as the rest energy of the photons via the magnetic moments of the light neutrinos [1-3, 5, 10, 18] if we consider the sum of ratios of the corresponding rest energies [1-3] of the end products of the 3 transformation types [2, 3] (in form of light neutrinos ν into heavy neutrinos $\widetilde{\nu}$, heavy neutrinos $\widetilde{\nu}$ into sterile neutrinos $\hat{\nu}$ and light neutrinos ν into sterile neutrinos $\hat{\nu}$) as well as the neutrino number densities $n(\nu) = n(\widetilde{\nu}) = 112 \, \mathrm{cm}^{-3}$ and $n(\hat{\nu}) = 0.178 \, \mathrm{cm}^{-3}$ [1-3]. This sum provides $3\Omega_{\mathrm{dm}}$, since it contains once $\Omega_{\mathrm{dm}} = E_0(\widetilde{\nu}_{\mathrm{dm}})/\left[E_0(\widetilde{\nu}_\Lambda) + E_0(\widetilde{\nu}_{\mathrm{dm}}) + E_0(\widetilde{\nu}_{\mathrm{b}})\right] = 0.265$ and twice $\Omega_{\mathrm{dm}} = E_0(\hat{\nu}_{\mathrm{dm}})/\left[E_0(\hat{\nu}_\Lambda) + E_0(\hat{\nu}_{\mathrm{dm}}) + E_0(\hat{\nu}_{\mathrm{b}})\right] = 0.265$. These rest energies possess the values $E_0(\widetilde{\nu}_\Lambda) = 29.3 \, \mathrm{eV}$, $E_0(\widetilde{\nu}_{\mathrm{dm}}) = 11.31 \, \mathrm{eV}$, $E_0(\widetilde{\nu}_{\mathrm{b}}) = 2.08 \, \mathrm{eV}$, $E_0(\hat{\nu}_\Lambda) = 18436 \, \mathrm{eV}$, $E_0(\hat{\nu}_{\mathrm{dm}}) = 7120 \, \mathrm{eV}$ and $E_0(\hat{\nu}_{\mathrm{b}}) = 1309 \, \mathrm{eV}$ [1-4]. Thus, we find semi-empirically the constant of this process to $3\Omega_{\mathrm{dm}} \, n(\widetilde{\nu})/n(\hat{\nu})$, which must be distributed symmetrically among the two decay products of the sterile neutrinos, i.e. this process must be characterized presently by the constant factor

$$\left[3 \, \Omega_{\mathrm{dm}} \, n(\widetilde{\nu})/n(\hat{\nu})\right]^{1/2} \cong 22.37 \,. \tag{6.58}$$

Using the geometric mean from the product of the neutrino number density ratio $n(\widetilde{\nu})/n(\hat{\nu}) = 629.2$ (see above) and the ratio of the results (4.1) and (4.4), for the present universe, a similar factor is plausibly found to

$$\Omega_{\mathrm{dm}}\left[\frac{n(\tilde{\nu})}{n(\hat{\nu})}\frac{(1-\mathrm{e}^{-(\tau_{\hat{\nu}}-\tau_{\mathrm{H}})/\tau_{\hat{\nu}}})}{\mathrm{e}^{-(\tau_{\hat{\nu}}-\tau_{\mathrm{H}})/\tau_{\hat{\nu}}}}\right]^{1/2}\cong 22.39\,. \tag{6.59}$$

According to Refs. [1-3, 5, 10, 18], the sum of the magnetic moment of all light massive neutrinos can be derived to

$$\mu_\nu(\sum_i \nu_i)=\frac{3}{8\pi^2\sqrt{2}}\,e\,[G_F/(\hbar c)^3]\,\hbar c^2\sum_i E_0(\nu_i)=$$
$$=3.203\cdot 10^{-19}(\sum_i E_0(\nu_i))/\mathrm{eV})\,\mu_B \tag{6.60}$$

with the Bohr magneton [10]

$$\mu_B=\frac{e\,\hbar\,c^2}{2\,E_0(\mathrm{e})}=(5.7883818066(38)\cdot 10^{-11}\,\mathrm{MeV/T}\;, \tag{6.61}$$

where the new quantities e, $[G_F/(\hbar c)^3]$, $\sum_i E_0(\nu_i)=5.97\cdot 10^{-2}\mathrm{eV}$ and $E_0(\mathrm{e})$ describes the elementary charge, the Fermi coupling constant [10], the sum of the rest energies of the light neutrinos [1-3, 5] and the rest energy of the electron [10], respectively. The subscript $i=\mathrm{e},\mu,\tau$ characterizes the light e, μ and τ neutrino. Thus, for the magnetic moment (6.60), we can also write

$$\mu_\nu(\sum_i \nu_i)=1.107\cdot 10^{-24}\,\mathrm{eV\,T^{-1}}, \tag{6.62}$$

so that, for example, via the factor (6.58), we find semi-empirically the connection between the magnetic field (6.56) as well as the rest energy of the photons (see Eq. (6.1 b)) to

$$B=[3\,\Omega_{\mathrm{dm}}\,n(\tilde{\nu})/n(\hat{\nu})]^{1/2}\,\frac{E_0(\gamma)}{\mu_\nu(\sum_i \nu_i)} \tag{6.63}$$

Indeed, if we use $B=3.157\cdot 10^{-10}\,\mathrm{T}$, the expression (6.63) yields

$$E_0(\gamma)=\frac{B}{[3\,\Omega_{\mathrm{dm}}\,n(\tilde{\nu})/n(\hat{\nu})]^{1/2}}\,\mu_\nu(\sum_i \nu_i)=1.563\cdot 10^{-35}\,\mathrm{eV}\,. \tag{6.64}$$

With that, we have confirmed the existence of the photon rest energy by the experimental value of the general galactic magnetic field (see, e.g., Ref. [17]), i.e. by a convincing direct experimental observation.

Because of the assumption $E_0(\gamma) = E_0(\mathrm{G})$ [see Eq. (6.2)], the results (3.107) and (3.108) mean that for the universe we must assume the proper energy

$$2E_\mathrm{H} \cong 2\rho_{0\mathrm{C}}\, c^2\, \tfrac{4}{3}\pi R_0^3 \cong \rho_{0\mathrm{C}}\, c^2\, \tfrac{4}{3}\pi (R_0\, 2^{1/3})^3 \cong$$
$$\cong \rho_{0\mathrm{C}}\, c^2\, \tfrac{4}{3}\pi (R_0\, r)^3 \cong 1.039\cdot 10^{89}\ \mathrm{eV}\,, \qquad (6.65)$$

where $r = 2^{1/3} \cong 1.26$ is again the dimensionless time-independent comoving coordinate of the proper distance $d = R_0 r$.

Using the data of Tables III to V, the results, derived from Eqs. (6.44) to (6.65) for the massive universe, are also valid for the massive anti-universe.

7 Hubble "constants" as a function of cosmic evolution epochs

The Hubble "constants" determine how fast the universe expands over the time. Because of Eq. (6.10), they must possess discontinuities in the cosmic evolution.

By Eqs. (6.6) to (6.9), we have derived the values of the Hubble parameters $\overline{H}_\mathrm{BB} = z_0(\gamma)/t_\mathrm{BB} = 9.643\cdot 10^{74}\ \mathrm{s}^{-1} = 2.976\cdot 10^{94}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (see Eq. (6.7)) and $\overline{H}_\mathrm{Pl} = z_0(\gamma)/t_\mathrm{Pl} = 1.234\cdot 10^{12}\ \mathrm{s}^{-1} = 3.808\cdot 10^{31}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (see Eq. (6.9)) for the massless universe ($R_\mathrm{BB} \le R \le R_\mathrm{Pl}$). They are the limiting values of the continuous function (6.12). However, between the Hubble parameters $\overline{H}_\mathrm{Pl} = z_0(\gamma)/t_\mathrm{Pl} = 1.234\cdot 10^{12}\ \mathrm{s}^{-1} = 3.808\cdot 10^{31}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (see above) and $H_\mathrm{Pl} = 1/2t_\mathrm{Pl} = 9.275\cdot 10^{42}\ \mathrm{s}^{-1} = 2.862\cdot 10^{62}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ [lower limiting value of the early massive universe ($R_\mathrm{Pl} \le R \le \widetilde{R}_0$) according to the new inflation model (see Eqs. (2.14) to (2.18))], we have a discontinuity (see Eq. (6.10)). These Hubble parameters are father than that of the Planck observations 2013 [12], which yield $H_0 = 2.181\cdot 10^{-18}\ \mathrm{s}^{-1} = 67.3\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (see Table I). The connection between all these values is given by Eq. (6.10). Consequently, instead of Eqs. (6.7) and (6.10), we can also write