Exceptionally, because of the new introduction of the normal ($\hat{\nu}_\Lambda$, $\hat{\nu}_{\rm dm}$, $\hat{\nu}_{\rm b}$) and the total ($\hat{\nu}_\Lambda$, $\hat{\nu}_{\rm dm}$, $\hat{\nu}_{\rm b}$, $\hat{\nu}_{\rm CMB}$) dark energy (see above), we use the improved data of Table VI in Eqs. (7.19) to (7.29) for the important confirmation of the far-reaching conclusion in Sec. 7.

Consequently, using the data of Tables III to V including VI, the results, derived in this Sec. 4 for the universe, are also valid for the anti-universe.

5 The new thermal equilibrium and the light neutrinos

Via the results (2.47) to (2.50), we can directly support the introduction of the new thermal equilibrium by the assumptions

$$kT = kT_0(\nu_{\rm e}) = E_0(\nu_{\rm e}) \cong (1.589^{+0.078}_{-0.098})\cdot 10^{-3}\,{\rm eV}, \tag{5.1}$$

$$kT = kT_0(\nu_\mu) = E_0(\nu_\mu) \cong (8.85^{+0.14}_{-0.16})\cdot 10^{-3}\,{\rm eV}, \tag{5.2}$$

$$kT = kT_0(\nu_\tau) = E_0(\nu_\tau) \cong (4.93^{+0.12}_{-0.10})\cdot 10^{-2}\,{\rm eV} \tag{5.3}$$

$$kT = kT_0(\textstyle\sum_i \nu_i) = \sum_i E_0(\nu_i) \cong (5.97^{+0.14}_{-0.13})\cdot 10^{-2}\,{\rm eV}. \tag{5.4}$$

Then, the assumptions (5.1) to (5.4) yield the redshift conditions

$$1 + z_0(\nu_{\rm e}) = \frac{E_0(\nu_{\rm e})}{kT_0} = 6.766, \tag{5.5}$$

$$1 + z_0(\nu_\mu) = \frac{E_0(\nu_\mu)}{kT_0} = 37.68, \tag{5.6}$$

$$1 + z_0(\nu_\tau) = \frac{E_0(\nu_\tau)}{kT_0} = 209.9 \tag{5.7}$$

and

$$1 + z_0(\textstyle\sum_i \nu_i) = \frac{\sum_i E_0(\nu_i)}{kT_0} = 254.2. \tag{5.8}$$

For $N(T) = 3.362644$ (see Eq. (2.6)), the expression (2.5) yields

$$\Omega_{\rm r} = \frac{1}{2} N(T)\,\Omega_\gamma = 9.18\cdot 10^{-5}. \tag{5.9}$$

Then, taking Eqs. (2.61) to (2.67) as well as (5.5) to (5.9), we get semi-empirically the identical connections

$$\alpha_{\mathrm{GUT}} = \left[\Omega_{\mathrm{r}} \frac{1+z_0(\sum_i \nu_i)}{1+z_{\mathrm{reion}}(\sum_i \nu_i)}\right]^{1/2} = \left[\Omega_{\mathrm{r}} \frac{\sum_i E_0(\nu_i)}{kT(\sum_i \nu_i)}\right]^{1/2} = 0.0391\,, \qquad (5.10)$$

$$\alpha_{\mathrm{GUT}} = \left[\Omega_{\mathrm{r}} \frac{1+z_0(\nu_\tau)}{1+z_{\mathrm{reion}}(\nu_\tau)}\right]^{1/2} = \left[\Omega_{\mathrm{r}} \frac{E_0(\nu_\tau)}{kT(\nu_\tau)}\right]^{1/2} = 0.0391\,, \qquad (5.11)$$

$$\alpha_{\mathrm{GUT}} = \left[\Omega_{\mathrm{r}} \frac{1+z_0(\nu_\mu)}{1+z_{\mathrm{reion}}(\nu_\mu)}\right]^{1/2} = \left[\Omega_{\mathrm{r}} \frac{E_0(\nu_\mu)}{kT(\nu_\mu)}\right]^{1/2} = 0.0391 \qquad (5.12)$$

and

$$\alpha_{\mathrm{GUT}} = \left[\Omega_{\mathrm{r}} \frac{1+z_0(\nu_{\mathrm{e}})}{1+z(\nu_{\mathrm{e}})}\right]^{1/2} = \left[\Omega_{\mathrm{r}} \frac{E_0(\nu_{\mathrm{e}})}{kT(\nu_{\mathrm{e}})}\right]^{1/2} = 0.0391 \qquad (5.13)$$

in excellent agreement with Eq. (2.59).

The results (5.10) to (5.13) can be considered also as a reasonable argument for the introduction of the new thermal equilibrium in Refs. [1-5].

Using the data of Tables III to V, these considerations, derived in this Sec. 5 for the universe, are also valid for the anti-universe.

6 Rest energy of photons and gravitons including conclusions

To this day, it was assumed that the rest energies of the photons and the gravitons are zero. Therefore, we expect for their rest energies greater than zero also the same values. Then, the results (3.108) and (3.109) permit unique conclusions about the rest energy of the gravitons (G) and the photons (γ). Consequently, by Eq. (3.108), we have the first condition

$$E_0(\mathrm{G}) = kT_{\mathrm{BB}} = \frac{E_{\mathrm{H}}}{\frac{1}{2} N_0} = \frac{\rho_{0\mathrm{C}}\, c^2\, \frac{4}{3}\pi R_0^3}{\frac{1}{2} N_0} \cong 1.563 \cdot 10^{-35}\ \mathrm{eV}\,, \qquad (6.1\mathrm{a})$$