$d_{f21} = d_{f2} - d_{f1} \cong 1.08 \cdot 10^{29}$  cm of the distance  $d_{f1}$ , we obtain the energy uncertainty

$$E_{12} = 6 kT_{BB} = \frac{\hbar c}{2d_{f21}} \cong 9.14 \cdot 10^{-35} \,\text{eV}$$
(3.135)

for the decay products of the 3 sterile neutrino types of the dark energy (see Sec. 3.1). Therefore, in the final state  $d_{f2}$  of the (massive) universe, for  $E_0^{\hat{\nu}}(\gamma) = E_0^{\hat{\nu}}(\frac{1}{2}\hat{\nu})$ , the energy uncertainty  $6kT_{\rm BB} = 3 \times E_0^{\hat{\nu}}(\gamma) + 3 \times E_0^{\hat{\nu}}(\frac{1}{2}\hat{\nu})$  of these decay products is correspondingly identical with the triple sum of the rest energies of the photons  $E_0(\gamma) = kT_{BB} \cong 1.563 \cdot 10^{-35} \text{ eV}$  (see Eq. (6.1 b)) and of the gravitons  $E_0(G) = kT_{BB} \cong 1.563 \cdot 10^{-35} \text{ eV}$  (see Eq. (6.1 a))}. These two new assumptions are based on the existence of the sterile neutrino-antineutrino pairs in the final state of the universe because of the Dirac theory, since one sterile neutrino-antineutrino pair determines these decay properties of the sterile neutrinos. Thus, the final state  $d_{f2}$  of the universe is a result of these photons and gravitons in an unstable equilibrium, which is destroyed by the gigantic potential energy (3.134), so that it gives the transition from the final state of the universe in the direction to the big bang. Then, the proper distance  $d_{f2}$  is determined by the quantum fluctuation  $d_{f21} = d_{f2} - d_{f1} \cong 1.08 \cdot 10^{29} \text{ cm}$ , i.e. the assumption  $\hat{\rho}_{\text{vac}} = \rho_{\text{f1}} = \rho_{\text{f2}} \times (T_1/T_2)^3$  is a function of  $d_{\text{f2}} = c t_{\text{f2}}$  (see Eqs. (3.75 a) or (3.118)).

Using the data of Tables III to V, all these events are also valid for the transition from the final state of the anti-universe in direction to the big bang.

With that, we have precisely and uniquely proved the eternal cyclic evolution of the anti-universe and the universe.

## 4 The explanation of the present dark matter and dark energy

Because of several incorrect interpretations in Ref. [1], we perform here once more the explanation of the present dark matter and dark energy, whereat these results are used for the derivation of new cosmological parameters instead of that in Table V.

According to Table I, the present, (ionisable, visible or baryonic) matter is  $\Omega_b = 0.0499$ , whereas the present, dark matter  $\Omega_{dm}$  and dark energy  $\Omega_{\Lambda}$  are given by  $\Omega_{dm} = 0.265$  and  $\Omega_{\Lambda} = 0.685$ .

They possess the total relation  $\Omega_b + \Omega_{dm} + \Omega_{\Lambda} = 1$ , which was generally derived in Refs. [2-4].

The fluxes  $\Phi(\hat{\nu}_{\Lambda})$ ,  $\Phi(\hat{\nu}_{dm})$  and  $\Phi(\hat{\nu}_{b})$ , calculated correctly in Ref. [1], show uniquely that the joint origin of the dark matter and the dark energy is based on the three sterile neutrino types  $\hat{\nu}_{\Lambda}$ ,  $\hat{\nu}_{dm}$  and  $\hat{\nu}_{b}$  [1-4] without the consideration of the Dirac theory. The Dirac theory leads to the corresponding sterile neutrino-anti-neutrino pairs (see Sec. 3.6 or 3.7). Therefore, we assume the decay of the corresponding sterile neutrinos via the gravitation in one massless photon ( $\gamma$ ) with the energy of their half rest energy and one sterile neutrino relic ( $\frac{1}{2}\hat{\nu}$ ) with the energy of their half rest energy (see also Eq. (3.135)). Consequently, we must together consider the present dark matter and dark energy, i.e. the sum  $\Omega_{dm} + \Omega_{\Lambda} = 0.265 + 0.685 = 0.950$ .

The decay of these sterile neutrinos is again determined by the universal decay law  $\overline{N}(t) = \overline{N}_0 e^{-t/\tau_{\hat{v}}}$ , where  $\tau_{\hat{v}}$  is the lifetime " $\tau_{\hat{v}} = 35.11 \text{ Gyr}$ " of the sterile neutrinos (see Eq. (3.15)). Assuming  $\overline{N}_0 = \Omega_{\rm dm} + \Omega_{\Lambda}$ , the remaining dark matter and dark energy  $\overline{N}(t)$  is given by  $\overline{N}(t) = (\Omega_{\rm dm} + \Omega_{\Lambda}) e^{-t/\tau_{\hat{v}}}$ . Using the Hubble (H) time  $\tau_{\rm H} = 1/H_0 = 14.53 \text{ Gyr}$  (see Table I), which is valid for all galaxies (which, for example, expand by the velocity  $v = H_0 R$  at very small redshifts, i.e. for v << c), we can assume the time  $t = \tau_{\hat{v}} - \tau_{\rm H} = 20.58 \text{ Gyr}$ , so that we obtain  $e^{-(\tau_{\hat{v}} - \tau_{\rm H})/\tau_{\hat{v}}} = 0.5565$ .

This assumption is supported by the fact that according to Eq. (2.17) this Hubble time (see Table I) defines also the scale factor  $R_0 = c\tau_{\rm H} = c/H_0$  of the present (massive) universe for z = 0, using Eq. (2.15).

Then, the decay products of the dark matter and dark energy (see above) are defined via the slow decay process

$$\Omega_{\rm dm} + \Omega_{\Lambda} \,(1 - e^{-(\tau_{\hat{\nu}} - \tau_{\rm H})/\tau_{\hat{\nu}}}) = 0.4213 \tag{4.1}$$

(

$$\left\{\Omega_{\rm dm}(\gamma) + \Omega_{\Lambda}(\gamma)\right\}_{\rm decay} = 0.2107 \tag{4.2}$$

and

$$\left\{\Omega_{\rm dm}(\frac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu})\right\}_{\rm decay} = 0.2107.$$
(4.3)

For these decay products, the dark radiation energy (4.2) lies in an invisible range of the radiation spectrum, whereas the neutral dark sterile neutrino relics (4.3) are thus also invisible.

Because of Eqs. (4.1) to (4.3), the remaining dark matter and dark energy (see above) is determined to

$$(\Omega_{\rm dm} + \Omega_{\Lambda}) e^{-(\tau_{\hat{\nu}} - \tau_{\rm H})/\tau_{\hat{\nu}}} = 0.5287.$$
(4.4)

Now, we assume that the repulsive (rep) force of the dark energy leads to a breakup of the remaining dark matter and dark energy (4.4) into two equivalent invisible parts of the massive sterile breakup neutrinos

$$\left\{\Omega_{\rm dm} + \Omega_{\Lambda}\right\}_{\rm ren} = 0.2644 \tag{4.5}$$

and of the sum of the two equal massless sterile breakup neutrinos products

 $\left\{\Omega_{\rm dm}(\gamma) + \Omega_{\Lambda}(\gamma)\right\}_{\rm rep} + \left\{\Omega_{\rm dm}(\frac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu})\right\}_{\rm rep} = 0.2644, \quad (4.6)$ 

so that results (4.5) and (4.6) are in equilibrium.

For Eq. (4.6), on the left hand side, the first term  $\{\Omega_{dm}(\gamma) + \Omega_{\Lambda}(\gamma)\}_{rep}$  gives the fraction of the breakup radiation, which lies in the invisible range of the radiation spectrum, whereas the second term  $\{\Omega_{dm}(\frac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu})\}_{rep}$  gives the fraction of the invisible massless sterile breakup neutrino relics.

These two fractions  $\{\Omega_{dm}(\gamma) + \Omega_{\Lambda}(\gamma)\}_{rep}$  and  $\{\Omega_{dm}(\frac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu})\}_{rep}$  exist independently on the corresponding decay products (4.2) and (4.3).

In this connection, by the slow decay process (4.1) to (4.3), we must consider that the massless photons (4.2) move with the velocity v = c, whereas the sterile massless neutrino relics (4.3) possess also the velocity v = c via Eq. (3.135), so that the results (4.2) and (4.3) cannot be connected with the (ionisable or visible) massive matter  $\Omega_{\rm b}$  because of v = c. A completely other situation exists for the breakup process (4.4) to (4.6), where we have the sterile massive breakup neutrino packet (4.5) with the velocity  $v \approx c$  and the two sterile massless breakup neutrino products (4.6) with the velocity v = c, so that only the massive  $(v \approx c)$  sterile breakup neutrino packet (4.5) can exist as the massive dark matter  $\Omega_{dm}$  together with the massive (ionisable or visible) matter  $\Omega_b$ , whereas the massless (v = c)sterile breakup neutrino products (4.6) cannot be connected with the massive (ionisable or visible) matter  $\Omega_b$ .

Therefore, for the total matter  $\Omega_{\rm m} = \Omega_{\rm b} + \{\Omega_{\rm dm} + \Omega_{\Lambda}\}_{\rm rep} = 0.3143$ , we obtain the ratio

$$\frac{\text{total matter}}{\text{matter}} = \frac{\Omega_{\text{m}}}{\Omega_{\text{b}}} = 6.30, \qquad (4.7)$$

which agrees excellently with the experimental value  $(\Omega_b + \Omega_{dm})/\Omega_b = 6.31$  (see above).

This agreement confirms our above-mentioned assumptions, i.e. the results (4.1) to (4.6) can be summarized correspondingly in the gigantic present universe after the process (4.4) to (4.6) because of the particle conservation to the massive dark matter

$$\Omega_{\rm dm} = \left\{ \Omega_{\rm dm} + \Omega_{\Lambda} \right\}_{\rm rep} = 0.2644 \tag{4.8}$$

and to the massless dark energy

 $\Omega_{\Lambda} = \left\{ \Omega_{dm}(\gamma) + \Omega_{\Lambda}(\gamma) \right\}_{rep} + \left\{ \Omega_{dm}(\frac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu}) \right\}_{rep} + \left\{ \Omega_{dm}(\frac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu}) \right\}_{rep} + \left\{ \Omega_{dm}(\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu}) \right\}_{rep} + \left\{ \Omega_{dm}(\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu}) \right\}_{rep} + \left\{ \Omega_{dm}(\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu$ 

 $+\left\{\Omega_{\rm dm}(\gamma) + \Omega_{\Lambda}(\gamma)\right\}_{\rm decay} + \left\{\Omega_{\rm dm}(\frac{1}{2}\hat{\nu}) + \Omega_{\Lambda}(\frac{1}{2}\hat{\nu})\right\}_{\rm decay} = 0.686 \quad (4.9)$ 

via the sum of the invisible massless sterile breakup neutrino products (see Eq. (4.6)) as well as the invisible massless decay products (see Eqs. (4.2) and (4.3)) of the sterile neutrinos.

With that, we have found a simple model for the explanation of the present dark matter and dark energy, i.e. it was proved that the invisible sterile breakup neutrinos  $\hat{v}_{\Lambda}$ ,  $\hat{v}_{dm}$  and  $\hat{v}_{b}$  as well as their invisible decay and breakup products form the joint origin of the dark matter and the dark energy.

Because of the two decay products of the 3 sterile neutrinos  $\hat{\nu}_{\Lambda}$ ,  $\hat{\nu}_{dm}$  and  $\hat{\nu}_{b}$ , we have already introduced the new factor  $(\Omega_{dm} + \Omega_{\Lambda})^{\frac{1}{6}} = 0.9915$  for the

excitation energy  $E_{\text{exc}}(N) = (\Omega_{\text{dm}} + \Omega_{\Lambda})^{\frac{1}{6}} E_{\text{BB}}$  at the zero-point oscillations in Sec. 3.6. Consequently, this factor describes the energy loss of the excitation energy for the zero-point oscillations, since they excite the decay of the 3 fundamental sterile neutrinos into the two decay products.

Then, via Eqs. (4.1) and (4.4), at the massless dark energy, because of the equivalence of the results (4.5) and (4.6), by the two massless decay (see Eqs. (4.2) and (4.3)) and the two massless breakup (see Eqs. (4.6)) products, i.e. because of 4 events, we can assume semi-empirically

$$\Omega_{\rm dm} + \Omega_{\Lambda} \simeq \left[\frac{1 - e^{-(\tau_{\hat{\varphi}} - \tau_{\rm H})/\tau_{\hat{\varphi}}}}{e^{-(\tau_{\hat{\varphi}} - \tau_{\rm H})/\tau_{\hat{\varphi}}}}\right]^{\frac{1}{4}} \simeq 0.945.$$
(4.10)

Indeed, the result (4.10) agrees well with the sum  $\Omega_{dm} + \Omega_{\Lambda} = 0.950$  of the present dark matter and dark energy (see above).

However, according to Refs. [1-4], we have 4 heavy neutrinos  $\tilde{\nu}_{\Lambda}$ ,  $\tilde{\nu}_{dm}$ ,  $\tilde{\nu}_{b}$ and  $\tilde{\nu}_{CMB}$ , which are derived by aid of the light neutrinos [1-4]. These 4 heavy neutrinos are again coupled correspondingly with the 4 sterile neutrinos  $\hat{\nu}_{\Lambda}$ ,  $\hat{\nu}_{dm}$ ,  $\hat{\nu}_{b}$  and  $\hat{\nu}_{CMB}$  [1-4]. In Refs. [1-4], we have also assumed that the fourth sterile neutrino  $\tilde{\nu}_{CMB}$  could be responsible for the photon decoupling. In Refs. [1, 2], we have in detail discussed the properties of these heavy and sterile neutrinos. However, in Refs. [1, 2], the value 629.2 of the semi-empirical explanation of the sterile neutrino calculation, estimated by the rest energies of the light and heavy neutrinos, for example, via formula (5.30) in Ref. [1], is useless, i.e. it is invalid more generally.

In Eqs. (4.1) to (4.10), we have shown that only the 3 sterile neutrinos  $\hat{\nu}_{\Lambda}$ ,  $\hat{\nu}_{dm}$  and  $\hat{\nu}_{b}$  describe the dark matter and the dark energy. Therefore, instead of the particle-defined present cosmological parameters of Table V, on the same way, we must introduce several new particle-defined present cosmological parameters of the universe.

The old particle-defined density parameters of Table V (see, e.g., Ref. [2]) are defined by

$$\Omega_{\nu}(\nu_{\rm e}) \cong (0.683^{+0.034}_{-0.042}) \,\Omega_{\gamma} = (1.69^{+0.09}_{-0.11}) \cdot 10^{-5} \, h^{-2} \,, \tag{4.11}$$

$$\begin{split} \Omega_{\tilde{\nu}}(\tilde{\nu}_{\rm CMB}) &= \Omega_{\nu}(\nu_{\rm e}) \, \frac{E_{\rm Pl}}{2\sqrt{2} \, E_{0}({\rm X})} = \\ &= (110, 2^{+5.5}_{-6.8}) \, \Omega_{\gamma} = (2.72^{+0.14}_{-0.17}) \cdot 10^{-3} \, h^{-2}, \qquad (4.12) \\ \Omega_{\rm b} &= \Omega_{\nu}(\nu_{\mu}) \, \frac{E_{\rm Pl}}{2E_{0}({\rm X})} + \Omega_{\nu} = \\ &= (894^{+35}_{-36}) \, \Omega_{\gamma} = (0.02211^{+0.00089}_{-0.0091}) \, h^{-2}, \qquad (4.13) \\ \Omega_{\rm dm} &= \Omega_{\nu}(\nu_{\tau}) \, \frac{E_{\rm Pl}}{2E_{0}({\rm X})} + \Omega_{\nu} = \\ &= (4862^{+234}_{-206}) \, \Omega_{\gamma} = (0.1202^{+0.0059}_{-0.0052}) \, h^{-2}, \qquad (4.14) \\ \Omega_{\rm m} &= \Omega_{\rm dm} + \Omega_{\rm b} = (5756^{+234}_{-206}) \, \Omega_{\gamma} = (0.1423^{+0.0059}_{-0.0052}) \, h^{-2} \end{split}$$

and

$$\Omega_{\Lambda} = \Omega_{\nu}(\nu_{\mu}) \frac{E_{\rm Pl}}{2E_0({\rm Y})} + \Omega_{\nu} \frac{E_{\rm Pl}}{E_0({\rm Y})} = = (12589^{+559}_{-505}) \Omega_{\gamma} = (0.311^{+0.015}_{-0.013}) h^{-2}, \qquad (4.16)$$

i.e. via  $\Omega_{tot} = 1 = \Omega_{\Lambda} + \Omega_m + 1.681322 \ \Omega_{\gamma} = (18347^{+560}_{-506}) \ \Omega_{\gamma}$  we obtain the radiation density parameter to

$$\Omega_{\gamma} = (5.45^{+0.15}_{-0.17}) \cdot 10^{-5} , \qquad (4.17)$$

so that because of  $\Omega_{\gamma} = (2.4728 \pm 0.0025) \cdot 10^{-5} h^{-2}$  [1-5] we can derive

$$h = 0.6736_{-0.0096}^{+0.0105} . \tag{4.18}$$

The applied neutrino density parameters of the universe are defined by Eqs. (2.51) to (2.56). The Planck energy has the value  $E_{\rm PI} = 1.220932 \cdot 10^{28}$ . For the X and Y gauge bosons, the rest energies  $E_0(X)$  and  $E_0(Y)$  are defined by Eq. (2.57). The corresponding values (4.13) to (4.18) are tabulated in Table V.

Then, in this work, instead of these old density parameters of the universe, we assume their new corresponding values to

$$\Omega_{\nu}^{*}(\nu_{\rm e}) \cong (0.683^{+0.034}_{-0.042}) \,\Omega_{\gamma}^{*} = (1.69^{+0.09}_{-0.11}) \cdot 10^{-5} h_{*}^{-2} \,, \tag{4.19}$$

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$$\Omega_{\tilde{\nu}}^{*}(\tilde{\nu}_{\text{CMB}}) = \Omega_{\nu}^{*}(\nu_{e}) \frac{E_{\text{Pl}}}{2\sqrt{2} E_{0}(X)} =$$

$$= (110, 2^{+8.1}_{-9.2}) \Omega_{\gamma}^{*} = (2.72^{+0.20}_{-0.23}) \cdot 10^{-3} h_{*}^{-2}, \qquad (4.20)$$

$$\Omega_{b}^{*} = \Omega_{\nu}^{*}(\nu_{\mu}) \frac{E_{\text{Pl}}}{2E_{0}(X)} + \Omega_{\nu}^{*} =$$

$$= (894^{+35}_{-36}) \Omega_{\gamma}^{*} = (0.02211^{+0.00089}_{-0.00091}) h_{*}^{-2}, \qquad (4.21)$$

$$\Omega_{dm}^{*} = \Omega_{\nu}^{*}(\nu_{\mu}) \frac{E_{\text{Pl}}}{-E_{\text{Pl}}} + \Omega_{\nu}^{*} =$$

$$= (4862^{+234}_{-206}) \Omega_{\gamma}^{*} = (0.1202^{+0.0059}_{-0.0052}) h_{*}^{-2}, \qquad (4.22)$$

$$\Omega_{\rm m}^* = \Omega_{\rm dm}^* + \Omega_{\rm b}^* = (5756_{-206}^{+234}) \,\Omega_{\gamma}^* = (0.1423_{-0.0052}^{+0.0059}) \,h_*^{-2} \,, \tag{4.23}$$

$$\Omega_{\Lambda}^{**} = \Omega_{\Lambda}^{*} + \Omega_{\nu}^{*} = \Omega_{\nu}^{*}(\nu_{\mu}) \frac{E_{\rm Pl}}{2E_{0}({\rm Y})} + \Omega_{\nu}^{*} \frac{E_{\rm Pl}}{E_{0}({\rm Y})} + \Omega_{\nu}^{*} = (12615^{+560}_{-506}) \Omega_{\nu}^{*} = (0.312^{+0.014}_{-0.013}) h_{*}^{-2}$$

$$(4.24)$$

and

$$\Omega_{\Lambda}^{***} = \Omega_{\Lambda}^{**} + \Omega_{\tilde{\nu}}^{*}(\tilde{\nu}_{CMB}) =$$
  
= (12725<sup>+565</sup><sub>-514</sub>)  $\Omega_{\nu}^{*} = (0.315^{+0.014}_{-0.013}) h_{*}^{-2},$  (4.25)

i.e. via  $\Omega_{tot}^* = 1 = \Omega_{\Lambda}^{**} + \Omega_m^* + 1.681322 \ \Omega_{\gamma}^* = (18483^{+566}_{-515}) \ \Omega_{\gamma}^*$  we have found the radiation density parameter to

$$\Omega_{\gamma}^{*} = (5.41^{+0.15}_{-0.17}) \cdot 10^{-5}, \qquad (4.26)$$

so that because of  $\Omega_{\gamma}^{*}$  = (2.4728 ± 0.0025)  $\cdot 10^{-5} h_{*}^{-2}$  we can determine

$$h_* = 0.676^{+0.011}_{-0.010} \,. \tag{4.27}$$

In Eqs. (4.24) and (4.25), instead of the old dark energy  $\Omega_{\Lambda}$  (see Eq. (4.16)), the two new values  $\Omega_{\Lambda}^{**} = \Omega_{\Lambda}^{*} + \Omega_{\nu}^{*}$  and  $\Omega_{\Lambda}^{***} = \Omega_{\Lambda}^{**} + \Omega_{\widetilde{\nu}}^{*}(\widetilde{\nu}_{\rm CMB})$  were defined. The values (4.20) to (4.27) are tabulated in Table VI. The first value  $\Omega_{\Lambda}^{**}$ , which describes alone the normal dark energy of the 3 sterile neutrinos  $\hat{\nu}_{\Lambda}$ ,  $\hat{\nu}_{\rm dm}$  and  $\hat{\nu}_{\rm b}$ , is improved by the term  $\Omega_{\nu}^{*}$  in analogy to the expressions (4.13) and (4.14), i.e.  $(\Omega_{\Lambda} + \Omega_{\nu})^* \rightarrow \Omega_{\Lambda}^{**}$ . At the second value  $\Omega_{\Lambda}^{***}$ , which yields the total dark energy of the 4 sterile neutrinos  $\hat{\nu}_{\Lambda}$ ,  $\hat{\nu}_{dm}$ ,  $\hat{\nu}_{b}$  and  $\hat{\nu}_{CMB}$ , the term  $\Omega_{\tilde{\nu}}^*(\tilde{\nu}_{CMB})$  could be responsible again for the decoupling of the photons [1, 2].

Then, by Refs. [1, 2, 6], the age of the present massive universe (z = 0), which now depends on the dark matter  $\Omega_m^*$  and the total dark energy  $\Omega_{\Lambda}^{***}$ , is defined by

$$t_0^* = \frac{2}{3 H_0^* \Omega_{\Lambda}^{***/2}} \ln \frac{\sqrt{\Omega_{\Lambda}^{***} + \sqrt{\Omega_{\Lambda}^{***} + \Omega_m^*}}}{\sqrt{\Omega_m^*}}.$$
 (4.28)

The values (4.20) to (4.28) are tabulated in Table VI. These values of Table VI agree well with the most recent data of Ref. [16], which are given in Table VII. According to Table VI, the new sum  $\Omega_{dm}^* + \Omega_{\Lambda}^{**} = 0.946$  confirms the value (4.10). Because of the heavy neutrino number density  $n(\tilde{\nu}_{CMB}) = 112 \text{ cm}^{-3}$  (see Refs. [1-4]), the above-mentioned assumption, where the fourth heavy neutrino  $\tilde{\nu}_{CMB}$  could be responsible for the decuopling of the photons, is supported by its rest energy  $E_0(\tilde{\nu}_{CMB})$  via the connection [1-4]

$$E_0(\tilde{\nu}_{\rm CMB}) = \frac{\Omega_{\tilde{\nu}}^*(\tilde{\nu}_{\rm CMB}) \ \rho_{0\rm C}^* \ c^2}{n(\tilde{\nu})} \cong 0.256^{+0.013}_{-0.016} \, \text{eV} , \qquad (4.29)$$

since by Eq. (4.29) we obtain the redshift of the photon decoupling to

$$z_{\rm dec}^* = \frac{E_0(\widetilde{\nu}_{\rm CMB})/k}{T_0} - 1 \cong 1089_{-68}^{+56} \tag{4.30}$$

in excellent agreement with the corresponding value  $z_* = 1089.9 \pm 0.4$  of the redshift (at which optical depth equals unity) in Table VII. Within the error limits, the cosmological parameter values of Tables I, II, V, VI and VII agree excellently. Thus, in this work, we do not correct generally all results derived by the data of Tables I and V, since the data of Table VII yield the same results for all considerations of the works [1-5] and the present paper within the error limits, i.e. we apply always the data of Table I (see also the paragraph before last in Sec. 2).

Symbol, equation	Value
h.	$0.676_{-0.010}^{+0.011}$
$H_0^*$	$100 h \text{ km s}^{-1} \text{ Mpc}^{-1} = 67.6^{+1.1}_{-1.0} \text{ km s}^{-1} \text{ Mpc}^{-1} =$
	$= h_* \times (9.777752 \text{Gyr})^{-1} = (2.191^{+0.034}_{-0.032}) \cdot 10^{-18} \text{s}^{-1}$
$t_{0}^{*}$	$(4.356^{+0.017}_{-0.015}) \cdot 10^{17} \text{ s} = 13.80 \pm 0.05 \text{ Gyr}$
$R_0^* = c \big/ H_0^*$	$(1.368^{+0.020}_{-0.021}) \cdot 10^{26} \text{ m}$
$\rho_{0C}^* = 3H_0^{*2} / 8\pi G_N$	$(4.82^{-0.16}_{-0.14}) \cdot 10^3 (\text{eV}/c^2) \text{ cm}^{-3}$
$\Omega^{*}_{ u}$ a)	$(6.35_{-0.14}^{+0.16}) \cdot 10^{-4} h_*^{-2} = (1.390_{-0.031}^{+0.035}) \cdot 10^{-3}$
$\Omega^*_{\widetilde{v}}(\widetilde{v}_{CMB})$	$(2.72^{+0.14}_{-0.17}) \cdot 10^{-3} h_{\bullet}^{-2} = (5.95^{+0.31}_{-0.37}) \cdot 10^{-3}$
$\Omega_{\rm b}^* = \rho_{\rm b}^* / \rho_{\rm 0C}^*$	$0.02211_{-0.00091}^{+0.00089} h_*^{-2} = 0.0484_{-0.0020}^{+0.0019}$
$\Omega_{\rm dm}^* = \rho_{\rm dm}^* \big/ \rho_{\rm 0C}^*$	$0.1202_{-0.0052}^{+0.0059} h_*^{-2} = 0.263_{-0.011}^{+0.013}$
$\Omega_m^* = \Omega_{dm}^* + \Omega_b^*$	$0.1423_{-0.0052}^{+0.0059} h_{\star}^{-2} = 0.311_{-0.011}^{+0.013}$
$\Omega_{\Lambda}^{**} = \rho_{\Lambda}^{**} \big/ \rho_{0C}^{*}$	$0.311_{-0.013}^{+0.014} h_{\star}^{-2} = 0.683_{-0.028}^{+0.031}$
$\Omega_{\Lambda}^{***} = \rho_{\Lambda}^{***} \big/ \rho_{00}^{*}$	$0.315_{-0.013}^{+0.014} h_{\star}^{-2} = 0.689_{-0.028}^{+0.031}$
$\Omega_{\gamma}^{*}=\rho_{\gamma}^{*}\big/\rho_{0\mathrm{C}}^{*}$	$(2.4728 \pm 0.0025) \cdot 10^{-5} h_{\star}^{-2} = (5.41^{+0.15}_{-0.17}) \cdot 10^{-5}$
$\Omega_{\rm tot}$	Î

1

Table VI. The estimated (present-day) values of the new particle-defined cosmological parameters for the universe.<sup>a)</sup> see Eq. (2.56).

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Table VII. The most recent (present-day) values of the cosmological parameters for the universe according to Ref. [16]

Symbol, equation	Value
T <sub>0</sub>	2.7255(6) K
h	$0.678 \pm 0.009$
$H_0$	$100 h \text{ km s}^{-1} \text{ Mpc}^{-1} = 67.8 \pm 0.9 \text{ km s}^{-1} \text{ Mpc}^{-1} =$
74 A	$= h \times (9.777752 \text{ Gyr})^{-1} = (2.197 \pm 0.029) \cdot 10^{-18} \text{ s}^{-1}$
<i>I</i> <sub>0</sub>	$13.80 \pm 0.04$ Gyr
$R_0 = c/H_0$	$0.9250629 \cdot 10^{26} h^{-1} m = (1.364 \pm 0.018) \cdot 10^{26} m$
$\rho_{0C} = 3H_0^2/8\pi G_N$	$1.05371 \cdot 10^4 h^2 (eV/c^2) cm^{-3} =$
	= $(4.84 \pm 0.13) \cdot 10^3 (\text{eV}/c^2) \text{ cm}^{-3}$
$\Omega_{\nu}$	$< 0.016$ (Planck CMB); $\geq 0.0012$ (mixing)
$\Omega_{\rm b} = \rho_{\rm b} / \rho_{\rm 0C}$	$0.02226 \pm 0.00023  h^{-2} = 0.0484 \pm 0.0010$
$\Omega_{\rm dm} = \rho_{\rm dm} / \rho_{\rm 0C}$	$0.1186 \pm 0.0020  h^{-2} = 0.258 \pm 0.011$
$\Omega_{\rm m} = \rho_{\rm m} / \rho_{\rm 0C}$	$0.308 \pm 0.012$
$\Omega_{\Lambda} = \rho_{\Lambda} / \rho_{0C}$	$0.692 \pm 0.012$
$\Omega_{\gamma} = \rho_{\gamma} / \rho_{0C}$	$(2.4728 \pm 0.0025) \cdot 10^{-5} h^{-2} = (5.38 \pm 0.15) \cdot 10^{-5}$
<i>Z</i> .	1089.9±0.4

э., т Exceptionally, because of the new introduction of the normal  $(\hat{v}_{\Lambda}, \hat{v}_{dm}, \hat{v}_{b})$ and the total  $(\hat{v}_{\Lambda}, \hat{v}_{dm}, \hat{v}_{b}, \hat{v}_{CMB})$  dark energy (see above), we use the improved data of Table VI in Eqs. (7.19) to (7.29) for the important confirmation of the far-reaching conclusion in Sec. 7.

Consequently, using the data of Tables III to V including VI, the results, derived in this Sec. 4 for the universe, are also valid for the anti-universe.

## 5 The new thermal equilibrium and the light neutrinos

Via the results (2.47) to (2.50), we can directly support the introduction of the new thermal equilibrium by the assumptions

$$kT = kT_0(\nu_e) = E_0(\nu_e) \cong (1.589^{+0.078}_{-0.098}) \cdot 10^{-3} \text{eV},$$
 (5.1)

$$kT = kT_0(\nu_{\mu}) = E_0(\nu_{\mu}) \cong (8.85^{+0.14}_{-0.16}) \cdot 10^{-3} \,\text{eV}\,, \tag{5.2}$$

$$kT = kT_0(\nu_\tau) = E_0(\nu_\tau) \cong (4.93^{+0.12}_{-0.10}) \cdot 10^{-2} \text{eV}$$
(5.3)

$$kT = kT_0(\sum_i \nu_i) = \sum_i E_0(\nu_i) \cong (5.97^{+0.14}_{-0.13}) \cdot 10^{-2} \,\text{eV}\,.$$
(5.4)

Then, the assumptions (5.1) to (5.4) yield the redshift conditions

$$1 + z_0(\nu_e) = \frac{E_0(\nu_e)}{kT_0} = 6.766, \qquad (5.5)$$

$$1 + z_0(\nu_\mu) = \frac{E_0(\nu_\mu)}{kT_0} = 37.68 , \qquad (5.6)$$

$$1 + z_0(\nu_\tau) = \frac{E_0(\nu_\tau)}{kT_0} = 209.9$$
(5.7)

and

$$1 + z_0(\sum_i v_i) = \frac{\sum_i E_0(v_i)}{kT_0} = 254.2.$$
(5.8)

For N(T) = 3.362644 (see Eq. (2.6)), the expression (2.5) yields

$$\Omega_{\rm r} = \frac{1}{2} N(T) \,\Omega_{\gamma} = 9.18 \cdot 10^{-5} \,. \tag{5.9}$$