(Table VI) as a result of a new interpretation of the dark matter and dark energy, which are needed in the second half of Sec. 7.

Using the data of Tables III to V, these considerations, derived in this Sec. 2 for the universe, are also valid for the anti-universe.

3 Final state and big bang as well as the time reversal solution and the eternal cyclic evolution of universe and anti-universe

In this Sec. 3, via results of Refs. [1, 2], we give a detailed review, whereat we show the relationship between the final state of the universe and the big bang.

3.1 The final state of the universe and the big bang

According to the results of Refs. [1, 2], in this chapter, via the relationship between the final state of the universe and the big bang (see also Sec. 3.11), we calculate their parameters.

At the time $t_{\text{eff}} = 8.034 \cdot 10^{17}\,\text{s} = 25.46\,\text{Gyr}$ (see Eq. (2.28)) or the scale factor $R_{\text{eff}} = 2.67 \cdot 10^{26}\,\text{m}$ (see Eq. (2.26)), we have the end of the "present" accelerated expansion because of the effective equilibrium (see Eq. (2.26)), so that here the final value of the dark (d) energy E_{d} is found via the vacuum energy density $\rho_{\text{vac},\Lambda}\, c^2 = \Omega_\Lambda \rho_{0\text{C}}\, c^2 = 3.27 \cdot 10^3\,\text{eV cm}^{-3}$ (see Eq. (2.32)) to

$$E_{\text{d}} = \Omega_\Lambda \rho_{0\text{C}} c^2\, \tfrac{4}{3} \pi\, d_{\text{eff}}^3\,, \tag{3.1}$$

where the proper distance d_{eff} is defined by

$$d_{\text{eff}} = R_{\text{eff}}\, r_{\text{eff}} = R_{\text{eff}}\, r \tag{3.2}$$

with $r = r_{\text{eff}}$ as the dimensionless, time-independent, comoving coordinate distance (see Refs. [1, 2, 6, 8, 11]).

Using the hypothesis of the joint origin of the dark matter and dark energy by the three sterile, neutrino types $\hat{\nu}_\Lambda$, $\hat{\nu}_{\text{dm}}$ and $\hat{\nu}_{\text{b}}$ (see, e.g., Ref. [1]), we can assume that this dark energy E_{d} must be distributed among the decay products of these sterile neutrinos ($\hat{\nu}$). Assuming initially the decay condition of these sterile neutrino types via the gravitation into one photon with the energy of their half rest energy $\frac{1}{2}E_0(\hat{\nu})$ and one sterile neutrino relic with the energy of their half rest energy $\frac{1}{2}E_0(\hat{\nu})$ (see Refs. [1-4]), all these photons $kT_\gamma = \frac{1}{2}E_0(\hat{\nu})$ must define in average the half dark energy $\frac{1}{2}E_\mathrm{d}$ as the greatest possible thermal photon energy kT according to

$$kT = \tfrac{1}{2}E_\mathrm{d} = \sum_\gamma \langle kT_\gamma \rangle = \sum_\gamma \langle \tfrac{1}{2}E_0(\hat{\nu}) \rangle . \tag{3.3}$$

Therefore, applying the new thermal equilibrium [1, 2] between the photons $kT_{\frac{1}{2}\hat{\nu}}$ and the sterile neutrino relics $\frac{1}{2}E_0(\hat{\nu})$, because of the energy conservation, all these sterile neutrino relics $\frac{1}{2}E_0(\hat{\nu}) = kT_{\frac{1}{2}\hat{\nu}}$ must also form in average the other half of the dark energy $\frac{1}{2}E_\mathrm{d}$ as the greatest possible thermal energy kT according to

$$kT = \tfrac{1}{2}E_\mathrm{d} = \sum_{\frac{1}{2}\hat{\nu}} \langle kT_{\frac{1}{2}\hat{\nu}} \rangle = \sum_{\frac{1}{2}\hat{\nu}} \langle \tfrac{1}{2}E_0(\hat{\nu}) \rangle , \tag{3.4}$$

since the corresponding energy $kT_{\frac{1}{2}\hat{\nu}}$ of the sterile neutrino relics must be equivalent to kT_γ of the corresponding photons. The nature of the decay product $\frac{1}{2}E_0(\hat{\nu}) = kT_{\frac{1}{2}\hat{\nu}}$ is explained by Eq. (3.135).

The gravitational potential energy $V_\mathrm{gr}(d_\mathrm{eff})$ of these decay products can be defined by

$$V_\mathrm{gr}(d_\mathrm{eff}) = \frac{G_N}{c^4}\frac{(\frac{1}{2}E_\mathrm{d})^2}{d_\mathrm{eff}} = \frac{\hbar c}{E_\mathrm{Pl}^2}\frac{(\frac{1}{2}E_\mathrm{d})^2}{d_\mathrm{eff}} . \tag{3.5}$$

Using $V_\mathrm{gr}(d_\mathrm{eff}) = \frac{1}{2}E_\mathrm{d}$, we obtain

$$d_\mathrm{eff} = \frac{G_N}{c^4}\tfrac{1}{2}E_\mathrm{d} = \frac{\hbar c}{E_\mathrm{Pl}^2}\tfrac{1}{2}E_\mathrm{d} . \tag{3.6}$$

By Eqs. (3.1) and (3.6), we find the condition

$$d_\mathrm{eff} = \frac{\hbar c}{E_\mathrm{Pl}^2}\tfrac{1}{2}\Omega_\Lambda \rho_{0\mathrm{C}} c^2 \tfrac{4}{3}\pi d_\mathrm{eff}^3 \tag{3.7}$$

with the solution of the proper distance

$$d_{\text{eff}} = \left(\frac{E_{\text{Pl}}^2}{\hbar c \, \tfrac{1}{2} \Omega_\Lambda \rho_{0\text{C}} c^2 \, \tfrac{4}{3} \pi} \right)^{1/2} = 3.321 \cdot 10^{26} \text{ m}, \tag{3.8}$$

so that Eq. (3.2) gives

$$r = r_{\text{eff}} = \frac{d_{\text{eff}}}{R_{\text{eff}}} = 1.2438. \tag{3.9}$$

Using Eqs. (3.1) and (3.8), we get quantitatively the dark energy

$$E_{\text{d}} = \Omega_\Lambda \rho_{0\text{C}} c^2 \, \tfrac{4}{3} \pi d_{\text{eff}}^3 = 5.017 \cdot 10^{89} \text{ eV}. \tag{3.10}$$

By the result (3.10), the assumptions (3.3) and (3.4) are trivially defined by

$$kT = \tfrac{1}{2} E_{\text{d}} = \sum_\gamma \langle kT_\gamma \rangle = \sum_\gamma \langle \tfrac{1}{2} E_0(\hat{\nu}) \rangle = 2.509 \cdot 10^{89} \text{ eV} \tag{3.11}$$

and

$$kT = \tfrac{1}{2} E_{\text{d}} = \sum_{\frac{1}{2}\hat{\nu}} \langle kT_{\frac{1}{2}\hat{\nu}} \rangle = \sum_{\frac{1}{2}\hat{\nu}} \langle \tfrac{1}{2} E_0(\hat{\nu}) \rangle = 2.509 \cdot 10^{89} \text{ eV}: \tag{3.12}$$

Then, according to Refs. [1, 2], via Eq. (3.11), the thermal photon number density $n_\gamma(R_{\text{eff}})$ is found to

$$n_\gamma(R_{\text{eff}}) = 2.701178 \, \frac{2.4041138}{\pi^2} \left(\frac{\tfrac{1}{2} E_{\text{d}}}{\hbar c} \right)^3 = 1.353 \cdot 10^{282} \text{ cm}^{-3}, \tag{3.13}$$

whereas because of Eqs. (3.4) or (3.12) for the sterile neutrino relics their thermal number density $n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}})$ must be equivalent to $n_\gamma(R_{\text{eff}})$, i.e. according to Eq. (3.13) we have the condition

$$n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) = n_\gamma(R_{\text{eff}}) = \frac{\pi^2}{15} \frac{(\tfrac{1}{2} E_{\text{d}})^3}{(\hbar c)^3} = 1.353 \cdot 10^{282} \text{ cm}^{-3}. \tag{3.14}$$

Thus, at $R_{\text{eff}} = 2.67 \cdot 10^{26}\,\text{m}$ (see above), in the massive universe, because of $n_\gamma(R_{\text{eff}}) = n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}})$, we have a stable equilibrium between the photons and the sterile neutrino relics, since still no complete decay of the sterile neutrinos has taken place.

Taking the result (3.8), the proper lifetime $\tau_{\hat{\nu}}$ of the sterile neutrinos can be assumed to

$$\tau_{\hat{\nu}} = \frac{d_{\text{eff}}}{c} = 1.108 \cdot 10^{18}\ \text{s} = 35.11\ \text{Gyr}. \tag{3.15}$$

Consequently, for $t_{\text{eff}} = 8.034 \cdot 10^{17}\ \text{s} = 25.46\ \text{Gyr}$ (see above), if we use the denotation $n(\hat{\nu})$ for the total number density of the sterile neutrinos, because of their constant volume $V(\hat{\nu})$ (see Sec. 3.3), by the universal decay law

$$n_{\hat{\nu}}(t_{\text{eff}}) = n(\hat{\nu})\ \text{e}^{-t_{\text{eff}}/\tau_{\hat{\nu}}}, \tag{3.16}$$

we have the decay of the half of all sterile neutrinos, since we observe

$$n_{\hat{\nu}}(t_{\text{eff}}) \approx \tfrac{1}{2} n(\hat{\nu}). \tag{3.17}$$

Therefore, via Eqs. (3.13) and (3.14), we assume

$$n_{\hat{\nu}}(t_{\text{eff}}) = n_{\gamma}(R_{\text{eff}}) = n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) = 1.353 \cdot 10^{282}\ \text{cm}^{-3}, \tag{3.18}$$

so that for the total number density of the sterile neutrinos we get

$$n(\hat{\nu}) = 2n_{\gamma}(R_{\text{eff}}) = 2n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) = $$

$$= 2 \times 2.701178 \frac{2.4041138}{\pi^2} \left(\frac{\frac{1}{2} E_{\text{d}}}{\hbar c} \right)^3 = 2.706 \cdot 10^{282}\ \text{cm}^{-3}. \tag{3.19}$$

Then, for the final state of the massive universe, at $R_{\text{f}} = 3.39 \cdot 10^{26}\ \text{m}$ (see Eq. (2.19)) or $t_{\text{f}} = 2.136 \cdot 10^{20}\ \text{s} = 6768\ \text{Gyr}$ (see Eq. (2.29)), we can assume a complete decay of all sterile neutrinos with $n(\hat{\nu}) = 2.706 \cdot 10^{282}\ \text{cm}^{-3}$, so that because of the complete conversion of the dark energy we get correspondingly an unstable equilibrium (see Eq. (3.135)) between the photons with

$$n_{\gamma}(R_{\text{f}}) = 2n_{\gamma}(R_{\text{eff}}) = n(\hat{\nu}) =$$

$$= 2 \times 2.701178 \frac{2.4041138}{\pi^2} \left(\frac{\frac{1}{2} E_{\text{d}}}{\hbar c} \right)^3 = 2.706 \cdot 10^{282}\ \text{cm}^{-3} \tag{3.20}$$

and the sterile neutrino relics with

$$n(\hat{\nu}) = n_{\gamma}(R_{\text{f}}) = n_{\frac{1}{2}\hat{\nu}}(R_{\text{f}}) = 2n_{\gamma}(R_{\text{eff}}) = 2n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) =$$

$$= 2 \times 2.701178 \frac{2.4041138}{\pi^2} \left(\frac{\frac{1}{2} E_{\text{d}}}{\hbar c} \right)^3 = 2.706 \cdot 10^{282}\ \text{cm}^{-3} \tag{3.21}$$

This unstable equilibrium leads to a transition from the final state of the massive universe in the direction of the big bang of the massless universe.

Therefore, for example, by Eqs. (2.46) and (3.20), because of particle conservation, we assume the condition

$$n_\gamma(R_{\mathrm{BB}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R_{\mathrm{BB}}^3} =$$

$$= n_\gamma(R_{\mathrm{f}}) = 2\times 2.701178\,\frac{2.4041138}{\pi^2}\left(\frac{\frac{1}{2}E_{\mathrm{d}}}{\hbar c}\right)^3 \tag{3.22}$$

with the solution for the big bang distance

$$R_{\mathrm{BB}} = \left(\frac{\pi^4\,\Omega_\gamma}{292.2273}\right)^{1/3}\frac{\hbar c}{\frac{1}{2}E_{\mathrm{d}}} = 2.069\cdot 10^{-98}\ \mathrm{m}\,. \tag{3.23}$$

Then, the remaining parameters of the big bang are defined as follows. By Eq. (2.43), the big bang has taken place at the time

$$t_{\mathrm{BB}} = \frac{R_{\mathrm{BB}}}{c} = \left(\frac{\pi^4\,\Omega_\gamma}{292.2273}\right)^{1/3}\frac{\hbar}{\frac{1}{2}E_{\mathrm{d}}} = 6.901\cdot 10^{-107}\ \mathrm{s}\,. \tag{3.24}$$

According to Eqs. (2.41) and (2.42), for the big bang, the vacuum energy density or the cosmological "constant" are given by

$$\rho_{\mathrm{vac}}(R_{\mathrm{BB}})\,c^2 = \widehat{\rho}_{\mathrm{vac}}\,c^2 = \frac{2}{3}\Omega_\gamma\frac{\pi^2}{15}\frac{E_{\mathrm{Pl}}^2}{\hbar c\,R_{\mathrm{BB}}^2} =$$

$$= \frac{2}{45}\frac{\Omega_\gamma^{1/3}}{\pi^{2/3}}(292.2273)^{2/3}\frac{E_{\mathrm{Pl}}^2}{(\hbar c)^3}(\tfrac{1}{2}E_{\mathrm{d}})^2 = 4.227\cdot 10^{247}\ \mathrm{eV\,cm^{-3}} \tag{3.25}$$

or

$$\Lambda = \widehat{\Lambda} = \Lambda_{\mathrm{BB}} = \frac{16}{45}\pi^3\Omega_\gamma\frac{1}{R_{\mathrm{BB}}^2} =$$

$$= \frac{16}{45}(\pi\,\Omega_\gamma)^{1/3}(292.2273)^{2/3}\left(\frac{\frac{1}{2}E_{\mathrm{d}}}{\hbar c}\right)^2 = 1.406\cdot 10^{192}\ \mathrm{m^{-2}}\,, \tag{3.26}$$

respectively.

Then, using Eq. (2.40) and the uncertainty relation, the smallest possible particle energy kT_{BB} and the energy uncertainty $\widetilde{E}_{\mathrm{BB}}$ (see Eqs. (3.128) and (3.129)) for the big bang in the universe are correspondingly found to

$$kT_{\rm BB} = \frac{E_{\rm Pl}^2}{\hbar c} R_{\rm BB} = \left(\frac{\pi^4\,\Omega_\gamma}{292.2273}\right)^{1/3} \frac{E_{\rm Pl}^2}{\tfrac{1}{2} E_{\rm d}} = 1.563\cdot 10^{-35}\ {\rm eV} \tag{3.27}$$

and

$$\widetilde{E}_{\rm BB} = k\widetilde{T}_{\rm BB} = \frac{\hbar}{2t_{\rm BB}} = \frac{\hbar c}{2R_{\rm BB}} = \frac{E_{\rm Pl}^2}{2kT_{\rm BB}} = 4.769\cdot 10^{90}\ {\rm eV}\,. \tag{3.28 a}$$

Then, Eq. (3.28 a), multiplied by a factor 2, gives the greatest possible relativistic energy $E_{\rm BB} = 2\widetilde{E}_{\rm BB}$ by the origin (see also Eq. (3.130)) to

$$E_{\rm BB} = 2\widetilde{E}_{\rm BB} = \frac{\hbar}{t_{\rm BB}} = \frac{\hbar c}{R_{\rm BB}} = \frac{E_{\rm Pl}^2}{kT_{\rm BB}} = 9.537\cdot 10^{90}\ {\rm eV}\,. \tag{3.28 b}$$

In Eq. (3.28 b), the expression $\hbar c/R_{\rm BB}$ is interpreted as the potential energy of a new attractive force, similar to the gravitational force (see Eq. (3.110)). To this new force, we will return in Eqs. (3.114) and (3.115).

With that, we have shown the relationship between the final state of the universe and the big bang (see also Sec. 3.11), since the transition, from the final state of the massive universe ($R_{\rm f} \geq R \geq R_{\rm Pl}$) in the direction to the big bang of the massless universe ($0 \to R_{\rm BB} \leq R \leq R_{\rm Pl}$), means the start of an eternal cyclic evolution of the total [massless ($R_{\rm BB} \leq R \leq R_{\rm Pl}$) and massive ($R_{\rm f} \geq R \geq R_{\rm Pl}$)] universe (see Sec. 3.11). More generally, when the universes begin by a hot big bang, they must have also an end and a fresh start.

Using the data of Tables III to V, all results, derived in this Sec. 3.1 for the total (massless and massive) universe, are also valid for the total (massless and massive) anti-universe and its cyclic evolution.

3.2 The lifetime of the sterile neutrinos in the total universe

According to Refs. [1, 2], in this chapter, we estimate the lifetime of the sterile neutrinos in the total universe. For this goal, we must still determine the initial sterile neutrino number density.

Therefore, we use the fact that the present day universe contains a relic neutrino (ν) background with the temperature $T_{\nu,0} = (4/11)^{1/3} T_0 = 1.945\,\mathrm{K}$ (see Eq. (2.7)), where ν characterizes the light neutrinos, which were assumed as nearly massless.

For the formation of the particle-defined cosmological parameters (see Refs. [1-4]), which were identified as the cosmological parameters of the heavy neutrinos (see Refs. [1, 2]), the necessary energy for the light neutrinos was taken from the early universe, so that the light and the heavy neutrinos have the same number density, i.e. $n(\nu) = n(\tilde{\nu}) = 112\,\mathrm{cm}^{-3}$ (see Refs. [1, 2]).

Consequently, the light (ν) and the heavy ($\tilde{\nu}$) neutrino background must possess the same temperature $T_{\nu,0} = T_{\tilde{\nu},0} = 1.945\,\mathrm{K}$ according to the ideal gas law because of the constant pressure.

At the pressure P, between the energy E and the volume V, for the expansion of the massive universe, the kinetic theory of gases yields

$$dE = -PdV\,. \tag{3.29}$$

At the formation of sterile neutrinos, the necessary energy is taken from the relic neutrino ($\tilde{\nu}$) background, whereat the number density of the sterile neutrinos ($\hat{\nu}$) decreases to $n(\hat{\nu}) = 0.178\,\mathrm{cm}^{-3}$ (see Refs. [1, 2]).

Therefore, according to Eq. (3.29), at constant pressure, for the heavy neutrino relic of $T_{\tilde{\nu},0} = 1.945\,\mathrm{K}$, we must here assume the small volume changing $-1/n(\hat{\nu})$, so that we get a cooling of this relic neutrino background from $T_{\tilde{\nu},0} = 1.945\,\mathrm{K}$ to $T_{\hat{\nu},0}$ in the large volume changing $-1/n(\tilde{\nu})$, so that the ideal gas law yields

$$T_{\hat{\nu},0} = \frac{n(\hat{\nu})}{n(\tilde{\nu})} T_{\tilde{\nu},0} = 3.09 \cdot 10^{-3}\,\mathrm{K}\,, \tag{3.30}$$

i.e. the corresponding thermal photon energy is given by

$$kT = k\,(11/4)^{1/3} T_{\hat{\nu},0} = 3.73 \cdot 10^{-7}\,\mathrm{eV}\,. \tag{3.31}$$

Then, using Eqs. (2.40) and (3.31), for the quantum gravity of the massless universe ($R \leq R_{\mathrm{Pl}}$), we obtain the distance

$$R = R_{\hat{\nu}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2} k\,(11/4)^{1/3} T_{\hat{\nu},0} = 4.938 \cdot 10^{-68}\,\mathrm{cm}\,, \tag{3.32}$$

so that Eqs. (2.44) or (2.46) yield the initial thermal number density of the sterile neutrinos, which must be identical with that of the sterile neutrino relics $n_{\hat{\nu}}(R_{\hat{\nu}})$ or of the massless photons $n_\gamma(R_{\hat{\nu}})$ from the decay of these initial sterile neutrinos in the massless universe, i.e. we have

$$n_{\hat{\nu}}(R_{\hat{\nu}}) = n_\gamma(R_{\hat{\nu}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R_{\hat{\nu}}^3} = 1.989\cdot 10^{197}\ \mathrm{cm}^{-3}. \qquad (3.33)$$

Therefore, by the universal decay law, for $n_\gamma(R_{\hat{\nu}}) = 1.989\cdot 10^{197}\,\mathrm{cm}^{-3}$ (see Eq. (3.33)) and $n_\gamma(R_\mathrm{f}) = 2.706\cdot 10^{282}\ \mathrm{cm}^{-3}$ (see Eq. (3.20)), using a constant sterile neutrino volume $V(\hat{\nu})$ for $n_\gamma(R_{\hat{\nu}})$ and $n_\gamma(R_\mathrm{f})$, we can assume

$$n_\gamma(R_{\hat{\nu}}) = n_\gamma(R_\mathrm{f})\,\mathrm{e}^{-t_\mathrm{f}/\tau_{\hat{\nu}}}, \qquad (3.34)$$

where t_f yields the age of the final state of the massive universe, whereas $\tau_{\hat{\nu}}$ describes the lifetime of the sterile neutrinos. Their constant volume $V(\hat{\nu})$ is determined in Sec. 3.3. Now, we have two possibilities.

Firstly, in the variant 1, we can thus calculate $\tau_{\hat{\nu}} = \tau_{\hat{\nu}1}$, using the age $t_\mathrm{f} = t_{\mathrm{f}1} = 2.136\cdot 10^{20}\,\mathrm{s} = 6768\,\mathrm{Gyr}$ (see Eq. (2.29)) for the final state of the massive universe.

Secondly, in the variant 2, we can estimate the age $t_\mathrm{f} = t_{\mathrm{f}2}$ for the final state of the massive universe, taking $\tau_{\hat{\nu}} = \tau_{\hat{\nu}2} = 1.108\cdot 10^{18}\,\mathrm{s} = 35.11\,\mathrm{Gyr}$ (see Eq. (3.15)).

Then, taking the results (3.20) and (3.33), via Eq. (3.34), the variant 1 yields

$$\tau_{\hat{\nu}1} = t_{\mathrm{f}1}\left(\ln\{n_\gamma(R_\mathrm{f})/n_\gamma(R_{\hat{\nu}})\}\right)^{-1} = 1.090\cdot 10^{18}\ \mathrm{s} = 34.52\,\mathrm{Gyr}, \qquad (3.35)$$

whereas the variant 2 provides

$$t_{\mathrm{f}2} = \tau_{\hat{\nu}2}\ln\{n_\gamma(R_\mathrm{f})/n_\gamma(R_{\hat{\nu}})\} = 2.172\cdot 10^{20}\ \mathrm{s} = 6883\,\mathrm{Gyr}. \qquad (3.36)$$

Now, we show that the variant 2 is correct. For this goal, we introduce the proper distances

$$d_{\mathrm{f}1} = R_\mathrm{f} r_{\mathrm{f}1} = ct_{\mathrm{f}1} = 6.404\cdot 10^{28}\ \mathrm{m} \qquad (3.37)$$

and

$$d_{\mathrm{f}2} = R_\mathrm{f} r_{\mathrm{f}2} = ct_{\mathrm{f}2} = 6.512\cdot 10^{28}\ \mathrm{m}, \qquad (3.38)$$

where r_{f1} and r_{f2} describe again the dimensionless time-independent comoving coordinate distances r (see Eqs. (3.2) and (3.9)). Thus, via Eqs. (3.1) and (3.8), because of the dark energy of Eq. (3.10), we can form its energy densities

$$\rho_{f1}c^2 = \Omega_\Lambda \rho_{0C} c^2 \left(\frac{d_{eff}}{d_{f1}}\right)^3 = 4.560 \cdot 10^{-4}\,\mathrm{eV\,cm^{-3}} \tag{3.39}$$

and

$$\rho_{f2}c^2 = \Omega_\Lambda \rho_{0C} c^2 \left(\frac{d_{eff}}{d_{f2}}\right)^3 = 4.337 \cdot 10^{-4}\,\mathrm{eV\,cm^{-3}}, \tag{3.40}$$

so that because of the proper volumes V_{f1} and V_{f2} we have the ratios

$$\frac{\rho_{f1}}{\rho_{f2}} = \frac{V_{f2}}{V_{f1}} = \left(\frac{d_{f2}}{d_{f1}}\right)^3 = \left(\frac{t_{f2}}{t_{f1}}\right)^3 = \left(\frac{\tau_{\hat{\nu}2}}{\tau_{\hat{\nu}1}}\right)^3 \cong 1.052\,. \tag{3.41}$$

The discrepancy between $t_f = t_{f1} = 2.136 \cdot 10^{20}\,\mathrm{s} = 6768\,\mathrm{Gyr}$ (see Eq. (2.29)) and $t_{f2} = 2.172 \cdot 10^{20}\,\mathrm{s} = 6883\,\mathrm{Gyr}$ (see Eq. (3.36)) has the plausible reason that the derivation of $t_f = t_{f1}$ is alone based on the vacuum energy density $\rho_{vac,\Lambda}c^2 = \Omega_\Lambda \rho_{0C} c^2 = 3.27 \cdot 10^3\,\mathrm{eV\,cm^{-3}}$ (see Eq. (2.32) and Refs. [1, 2]) or the cosmological "constant" $\Lambda_\Lambda = 3\,\Omega_\Lambda / R_0^2 = 1.087 \cdot 10^{-52}\mathrm{m^{-2}}$ (see Eq. (2.33) and Refs. [1, 2]), i.e. the limiting conditions (2.32) and (2.33) yield here the temperature $T = T_{f1} = 51.41\,\mathrm{K}$ (see Eq. (2.34)).

However, this discrepancy can be explained uniquely by Eq. (2.38), using the assumptions for the times $\tilde{t} = \tau_{\hat{\nu}2} \ln 2 = 7.680 \cdot 10^{17}\,\mathrm{s}$ ($\tau_{\hat{\nu}2}$ see Eqs. (3.15)) and $t = t_{eff} = 8.034 \cdot 10^{17}\,\mathrm{s}$ (see Eq. (2.28)), so that the result (2.38) provides

$$\left(\frac{c^2\tilde{\Lambda}}{3}\right)^{1/2} = \frac{t_{eff} - t_0}{(\tau_{\hat{\nu}2}\ln 2) - t_0}\,\Omega_\Lambda^{1/2} H_0 = 1.997 \cdot 10^{-18}\,\mathrm{s^{-1}} \tag{3.42}$$

as semi-empirical present limiting value. By Eq. (3.42), we obtain

$$\tilde{\Lambda} = 1.331 \cdot 10^{-52}\,\mathrm{m^{-2}}, \tag{3.43}$$

so that the temperature $T = T_{f2} = 54.07\,\mathrm{K}$ results from Eqs. (2.31) and (3.43) for $N(T) = 3.362644$ {see Eq. (2.6) or Refs. [6, 8]}.

For the constant pressure $P = -\Omega_\Lambda \rho_{0C} c^2$, the ideal gas law yields following connection between the temperatures $T = T_{f2} = 54.07\,\mathrm{K}$ (see above) and $T = T_{f1} = 51.41\,\mathrm{K}$ (see also above) as well as the corresponding volumes V_{f2} and V_{f1} for the state changing according to Eq. (3.41):

$$\frac{T_{f2}}{T_{f1}} = \frac{V_{f2}}{V_{f1}} = \left(\frac{d_{f2}}{d_{f1}}\right)^3 = \left(\frac{t_{f2}}{t_{f1}}\right)^3 = \left(\frac{\tau_{\hat{\nu}2}}{\tau_{\hat{\nu}1}}\right)^3 \cong 1.052\,. \tag{3.44}$$

Thus, we get

$$t_{f2} = t_{f1}\left(\frac{T_{f2}}{T_{f1}}\right)^{1/3} = 6883\ \mathrm{Gyr} \tag{3.45}$$

and

$$\tau_{\hat{\nu}2} = \tau_{\hat{\nu}1}\left(\frac{T_{f2}}{T_{f1}}\right)^{1/3} = 35.11\ \mathrm{Gyr}\,, \tag{3.46}$$

i.e. $t_{f2} = 6883\,\mathrm{Gyr}$ (see Eq. (3.36)) and $\tau_{\hat{\nu}2} = 35.11\,\mathrm{Gyr}$ (see Eq. (3.15)) are correct.

Using the data of Tables III to V, the results, derived in this Sec. 3.2 for the total (massless and massive) universe, are also valid for the total (massless and massive) anti-universe.

3.3 The constant volume of the sterile neutrinos

In this chapter, we derive the constant volume $V(\hat{\nu})$ of the sterile neutrinos (see also Sec. 3.2) in the total ("massless" and massive) universe.

For this goal, we assume that in the final state of the massive universe by the sterile neutrinos of the dark energy $E_d = 5.017 \cdot 10^{89}\,\mathrm{eV}$ (see Eq. (3.10)) the volume V_f is occupied, i.e. we have

$$V_{\rm f} = \frac{E_{\rm d}}{\Omega_\Lambda \rho_{0C} c^2} = 1.534 \cdot 10^{86}\ {\rm cm}^3 . \qquad (3.47)$$

Using the number density $n(\hat{\nu}) = 0.178\,{\rm cm}^{-3}$ of the sterile neutrinos (see Sec. 3.2), we can introduce their constant volume $V(\hat{\nu})$ to

$$V(\hat{\nu}) = \frac{{\rm const}(\hat{\nu})}{n(\hat{\nu})} , \qquad (3.48)$$

so that for the final state of the massive universe the number $N_{\rm f}$ of the sterile neutrinos is given by

$$N_{\rm f} = \frac{V_{\rm f}}{V(\hat{\nu})} = \frac{n(\hat{\nu})\, V_{\rm f}}{{\rm const}(\hat{\nu})} = \frac{2.731 \cdot 10^{85}}{{\rm const}(\hat{\nu})} . \qquad (3.49)$$

At the enormous age of the final state of the massive universe, we have assumed a complete decay of the $N_{\rm f}$ sterile neutrinos (see Sec. 3.1).

Consequently, in the massless universe, the beginning of the sterile neutrino decay, which must lead to the results (3.30) to (3.33), takes place in one photon and one sterile neutrino relic, so that instead of the volume $V_{\rm f}$ these 2 different decay products must yield the initial (i) volume $V_{\rm i}$ of the sterile neutrinos in the massless universe to

$$V_{\rm i} = \frac{2}{n(\hat{\nu})} = 11.24\,{\rm cm}^3 . \qquad (3.50)$$

Thus, the initial number $N_{\rm i}$ of the sterile neutrinos of the massless universe must be defined by

$$N_{\rm i} = \frac{V_{\rm i}}{V(\hat{\nu})} = \frac{2}{{\rm const}(\hat{\nu})} . \qquad (3.51)$$

Therefore, instead of Eq. (3.34), we can assume

$$N_{\rm i} = N_{\rm f}\, {\rm e}^{-t_{\rm f} / \tau_{\hat{\nu}}} , \qquad (3.52)$$

so that via the Eqs. (3.48) to (3.52) we can form the expressions

$$N_{\rm f} = V(\hat{\nu})\, n_\gamma(R_{\rm f}) = \frac{n(\hat{\nu})\, V_{\rm f}}{{\rm const}(\hat{\nu})} = \frac{{\rm const}(\hat{\nu})}{n(\hat{\nu})}\, n_\gamma(R_{\rm f}) \qquad (3.53)$$

and

$$N_{\mathrm{i}} = V(\hat{\nu})\, n_{\gamma}(R_{\hat{\nu}}) = \frac{2}{\mathrm{const}(\hat{\nu})} = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})}\, n_{\gamma}(R_{\hat{\nu}}). \tag{3.54}$$

Then, the expressions (3.53) and (3.54) provide the conditions

$$\frac{n(\hat{\nu})\, V_{\mathrm{f}}}{\mathrm{const}(\hat{\nu})} = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})}\, n_{\gamma}(R_{\mathrm{f}}) \tag{3.55}$$

and

$$\frac{2}{\mathrm{const}(\hat{\nu})} = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})}\, n_{\gamma}(R_{\hat{\nu}}). \tag{3.56}$$

Thus, the conditions (3.55) or (3.56) yield

$$\mathrm{const}(\hat{\nu}) = \left(\frac{n^2(\hat{\nu})\, V_{\mathrm{f}}}{n_{\gamma}(R_{\mathrm{f}})} \right)^{1/2} \cong 1.34 \cdot 10^{-99} \tag{3.57}$$

or

$$\mathrm{const}(\hat{\nu}) = \left(\frac{2\, n(\hat{\nu})}{n_{\gamma}(R_{\hat{\nu}})} \right)^{1/2} \cong 1.34 \cdot 10^{-99}, \tag{3.58}$$

so that Eq. (3.48) leads to the constant sterile neutrino volume

$$V(\hat{\nu}) = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})} \cong 7.528 \cdot 10^{-99}\ \mathrm{cm}^3. \tag{3.59}$$

Taking Eqs. (3.20) and (3.33) as well as (3.53) to (3.59), we obtain the particle numbers (see above):

$$N_{\mathrm{f}} = V(\hat{\nu})\, n_{\gamma}(R_{\mathrm{f}}) \cong 2.04 \cdot 10^{184} \tag{3.60 a}$$

and

$$N_{\mathrm{i}} = V(\hat{\nu})\, n_{\gamma}(R_{\hat{\nu}}) \cong 1.50 \cdot 10^{99}. \tag{3.60 b}$$

Consequently, by Eqs. (3.20) and (3.33), considering the expressions (3.47) as well as (3.49 to (3.51)), the results (3.60 a) and (3.60 b) yield the following connections for the constant volume $V(\hat{\nu})$ of the sterile neutrinos

$$V(\hat{\nu}) = \frac{V_{\mathrm{f}}}{N_{\mathrm{f}}} = \frac{N_{\mathrm{f}}}{n_{\gamma}(R_{\mathrm{f}})} \cong 7.53 \cdot 10^{-99}\ \mathrm{cm}^3 \tag{3.61 a}$$

or

$$V(\hat{\nu}) = \frac{V_\mathrm{i}}{N_\mathrm{i}} = \frac{N_\mathrm{i}}{n_\gamma(R_{\hat{\nu}})} \cong 7.53 \cdot 10^{-99}\ \mathrm{cm}^3 \tag{3.61 b}$$

in accordance with Eq. (3.59).

Thus, the expressions (3.60 a) to (3.61 b) confirm the results (3.35) and (3.36) because of Eq. (3.34), i.e. we have again

$$\frac{t_\mathrm{f}}{\tau_{\hat{\nu}}} = \ln \frac{n_\gamma(R_\mathrm{f})}{n_\gamma(R_{\hat{\nu}})} = \ln \frac{N_\mathrm{f}}{N_\mathrm{i}} \cong 196.03\,. \tag{3.62}$$

With that, the assumption (3.34) is proved.

Using the data of Tables III to V, the results, derived in this Sec. 3.3 for the total universe, are also valid for the total anti-universe.

3.4 Heat accumulation in the final state (big bang) of the universe as reason of the very high energy density

In this chapter, we explain the high number densities (3.20) and (3.21) of the decay products of the sterile neutrinos as a result of the very high energy densities in the final state of the universe.

For this goal, we use the expression (3.29), which for positive pressure defines an energy decrease at the expansion of the massive universe. However, if for the final state of the universe the negative pressure $-P = \rho_\mathrm{vac}\, c^2$ is assumed, we obtain a gigantic energy increase, which by the heat accumulation leads to an overheat, so that the consequence is a heat death of this final state of the universe, since its heat death means simultaneously a transition in direction to the "massless" universe and the big bang, so that we get a relationship of the final state of the universe and the big bang. The reason is the increased concentration of the decay products of the sterile neutrinos by their deceleration in the final state of the universe, so that the very high temperature of the vacuum energy density $\rho_\mathrm{vac}\, c^2 = \rho_\mathrm{vac}(T)\, c^2$ of the universe (see Eq. (2.30)) can be attributed to the half dark energy $\frac{1}{2} E_\mathrm{d}$ (see Eqs. (3.3) and (3.4) as well as (3.11) and (3.12)). This assumption confirms the hypothesis of a joint origin of the dark matter and the dark energy by sterile neutrinos (see, e.g., Ref. [1]).

Because of the expressions (3.16) to (3.21), for the final state of the massive universe, the total energy density of the sterile neutrinos can be assumed to $\rho(\hat{\nu})\,c^2 = 2\rho_\gamma(R_{\mathrm{eff}})\,c^2 + 2\rho_{½\hat{\nu}}(R_{\mathrm{eff}})\,c^2$.

Then, using Eq. (2.30), in the final state of the massive universe, because of the condition $2\rho_\gamma(R_{\mathrm{eff}})\,c^2 = 2\rho_{½\hat{\nu}}(R_{\mathrm{eff}})\,c^2$, for the decay products of the sterile neutrinos, we can assume

$$2\frac{\pi^2}{15}\frac{(½\,E_{\mathrm{d}})^4}{(\hbar c)^3} = 6.787\cdot 10^{371}\,\mathrm{eV\,cm^{-3}} =$$
$$= 2\rho_\gamma(R_{\mathrm{eff}})\,c^2 = 2\rho_{½\hat{\nu}}(R_{\mathrm{eff}})\,c^2 = ½\,\rho(\hat{\nu})\,c^2 = 2\,\rho_{\mathrm{vac}}\,c^2 =$$
$$= 2\frac{1}{3}\frac{\pi^2}{15}\frac{(kT_1)^4}{(\hbar c)^3}\frac{1}{N(T)}, \qquad (3.63)$$

so that for the final state of the massive universe we obtain the thermal energy

$$kT_1 = [3\,N(T)]^{¼}\,½\,E_{\mathrm{d}}, \qquad (3.64)$$

which leads to an overheat of the final state of the universe by the very high energy density (3.63), i.e. this universe meets one's heat death (see above).

Therefore, by the kinetic energy $E_{\mathrm{K}}(\gamma) = 2.70117\times ½\,E_{\mathrm{d}}$ (see Refs. [1, 2]), because of Eqs. (3.18) to (3.21), the energy densities (3.63) lead to the number density of the decay products of the sterile neutrinos

$$n(\hat{\nu}) = n_\gamma(R_{\mathrm{f}}) = 2n_\gamma(R_{\mathrm{eff}}) = 2.701178\,\frac{2\rho_\gamma(R_{\mathrm{eff}})\,c^2}{E_{\mathrm{K}}(\gamma)} =$$
$$= n_{½\hat{\nu}}(R_{\mathrm{f}}) = 2n_{½\hat{\nu}}(R_{\mathrm{eff}}) = 2.701178\,\frac{2\rho_{½\hat{\nu}}(R_{\mathrm{eff}})\,c^2}{E_{\mathrm{K}}(\gamma)} =$$
$$= 2\frac{\pi^2}{15}\left(\frac{½\,E_{\mathrm{d}}}{\hbar c}\right)^3 = 2.706\cdot 10^{282}\,\mathrm{cm^{-3}}. \qquad (3.65)$$

Similar to Eqs. (3.63) and (3.65), taking again Eqs. (3.18) to (3.21), we can express the energy densities $\frac{1}{2}\rho(\hat{\nu})c^2 = 2\rho_\gamma(R_{\text{eff}})c^2 = 2\rho_{\frac{1}{2}\hat{\nu}}(R_\text{f})c^2$ of the sterile neutrinos also as follows

$$\frac{\pi^2}{15}\frac{(kT_2)^4}{(\hbar c)^3} = \tfrac{1}{2}\rho(\hat{\nu})c^2 = 2\frac{\pi^2}{15}\frac{(\frac{1}{2}E_\text{d})^4}{(\hbar c)^3} = 6.787\cdot 10^{371}\,\text{eV cm}^{-3}, \quad (3.66)$$

so that in the final state of the universe we get the thermal energy

$$kT_2 = 2^{\frac{1}{4}}\,\tfrac{1}{2}E_\text{d}, \quad (3.67)$$

i.e. analogous to Eq. (3.65) we find

$$n(\hat{\nu}) = 2.701178\,\frac{\frac{1}{2}\rho(\hat{\nu})c^2}{E_\text{K}(\gamma)} =$$

$$= 2\frac{\pi^2}{15}\left(\frac{\frac{1}{2}E_\text{d}}{\hbar c}\right)^3 = 2.706\cdot 10^{282}\,\text{cm}^{-3}. \quad (3.68)$$

Indeed, by Eqs. (3.65) and (3.68), we can explain the high number densities (3.20) and (3.21) of the decay products of the sterile neutrinos. This result is confirmed by Eqs (3.64) and (3.67), since we can introduce the ratio

$$\frac{T_1}{T_2} = \left(\frac{3\,N(T)}{2}\right)^{\frac{1}{4}}, \quad (3.69)$$

so that for the final state of the universe (see Eqs. (3.37) to (3.41)) the entropy conservation (see, e.g., Refs. [6, 8]) yields

$$\left(\frac{T_1}{T_2}\right)^3 = \left(\frac{3\,N(T)}{2}\right)^{\frac{3}{4}} = \frac{V_{\text{f}2}}{V_{\text{f}1}} = \left(\frac{d_{\text{f}2}}{d_{\text{f}1}}\right)^3. \quad (3.70)$$

Using the condition (3.41) in the result (3.70), we obtain

$$\left(\frac{3\,N(T)}{2}\right)^{\frac{3}{4}} \cong 1.052 \quad (3.71)$$

with the solution

$$N(T) \cong 0.713 \quad (3.72)$$

in accordance with

$$N(T) = \left(\frac{4}{11}\right)^{1/3} \cong 0.713 \quad \text{(see Eq. (2.7))}. \qquad (3.73)$$

Then, by the conditions (3.41) as well as (3.70) to (3.73), the results (3.45) and (3.46) are confirmed because of

$$t_{f2} = t_{f1}\left(\frac{3\,N(T)}{2}\right)^{1/4} \cong 6883 \text{ Gyr} \qquad (3.74\text{ a})$$

and

$$\tau_{\hat{\nu}2} = \tau_{\hat{\nu}1}\left(\frac{3\,N(T)}{2}\right)^{1/4} \cong 35.11 \text{ Gyr}. \qquad (3.74\text{ b})$$

With that, the conception of a heat death of the final state of the massive universe, which simultaneously means the transition in direction to the massless universe and the big bang, is confirmed via the corresponding vacuum energy density (see Eq. (2.30)) of the quantum gravity for the massive universe. Then, taking again the conditions (3.41) as well as (3.70) to (3.73), for the dark energy density (3,39), we can introduce

$$\rho_{f1}c^2 = \Omega_\Lambda \rho_{0C} c^2 \left(\frac{d_{\text{eff}}}{c\,t_{f2}}\right)^3 \left(\frac{T_1}{T_2}\right)^3 = \rho_{f2}c^2 \left(\frac{T_1}{T_2}\right)^3 =$$

$$= \rho_{f2}c^2\left(\frac{3\,N(T)}{2}\right)^{3/4} \cong 4.56 \cdot 10^{-4} \text{ eV cm}^{-3}, \qquad (3.75\text{ a})$$

whereas the reversal

$$\rho_{f2}c^2 = \Omega_\Lambda \rho_{0C} c^2 \left(\frac{d_{\text{eff}}}{c\,t_{f1}}\right)^3 \left(\frac{T_2}{T_1}\right)^3 = \rho_{f1}c^2 \left(\frac{T_2}{T_1}\right)^3 =$$

$$= \rho_{f1}c^2\left(\frac{2}{3\,N(T)}\right)^{3/4} \cong 4.34 \cdot 10^{-4} \text{ eV cm}^{-3} \qquad (3.75\text{ b})$$

is forbidden, i.e. the dark energy density $\rho_{f1}c^2 \equiv 4.56 \cdot 10^{-4}$ eV cm^{-3} of the final state of the universe is a function of t_{f2} because of $T_1 > T_2$.

Considering the data of Tables III to V, the results, derived in this Sec. 3.4 for the total universe, are also valid for the total anti-universe.

3.5 Mean (maximum) energies of "massless" and massive universe

In this chapter, we explain the mean (maximum) energies of the total ("massless" and massive) universe, applying the relationship between its final state and the big bang (see Sec. 3.11). In work [1], the interpretation of these mean (maximum) energies is incorrect. Therefore, in this work, we correct this interpretation. For this goal, for the "massless" and massive universe (see also Refs. [1, 2]). we consider their joint boundaries $R = R_{\mathrm{Pl}}$ and $E = E_{\mathrm{Pl}}$.

As starting point, we use the massive universe. Then, the boundary $E = E_{\mathrm{Pl}}$ of the massive universe is the reason that we can assume the mean energy densities $E_{\mathrm{Pl}}\, n_{\gamma}(R_{\mathrm{f}})$ or $E_{\mathrm{Pl}}\, n_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{f}})$ for its final state, taking Eqs. (3.20) or (3.21). Thus, by the decay products of the sterile neutrinos, we have correspondingly the two identical mean energy densities

$$\bar{\rho}_1\, c^2 = E_{\mathrm{Pl}}\, n_{\gamma}(R_{\mathrm{f}}) = 3.304\cdot 10^{310}\,\mathrm{eV\,cm^{-3}} \qquad (3.76)$$

or

$$\bar{\rho}_1\, c^2 = E_{\mathrm{Pl}}\, n_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{f}}) = 3.304\cdot 10^{310}\,\mathrm{eV\,cm^{-3}}\,. \qquad (3.77)$$

Then, using the sterile neutrino volume $V(\hat{\nu}) = 7.528\cdot 10^{-99}\,\mathrm{cm^3}$ (see Eq. (3.59)) for the final state of the massive universe, we can evaluate the mean (maximum) energy

$$\bar{E}_1 = \bar{\rho}_1\, c^2 \times V(\hat{\nu}) \cong 2.49\cdot 10^{212}\,\mathrm{eV}\,. \qquad (3.78)$$

By the sterile neutrino number $N_{\mathrm{f}} = 2.04\cdot 10^{184}$ (see Eq. (3.60 a)), for the beginning of the massive universe, the greatest possible relativistic particle energy {Planck energy (see also Refs. [1, 2])} is again defined by

$$E_{\mathrm{Pl}} = \frac{\bar{E}_1}{N_{\mathrm{f}}} \cong 1.22\cdot 10^{28}\,\mathrm{eV}\,. \qquad (3.79)$$

In the next step, we consider the mean (maximum) energy of the "massless" universe, using the transition direction from the final state of the universe to the "massless" universe and the big bang as a result of the complete decay of the sterile neutrinos (see Sec. 3.1). Taking the expressions (3.63) and (3.66) as well as (3.69) and (3.73), via the ratio $4\rho_{\mathrm{vac}}/\rho(\hat{\nu})$, multiplied by $\rho(\hat{\nu})\, c^2 \times (T_2/T_1)^4$, because of $4\rho_{\text{vac}}/\rho(\hat{\nu})=1$, for the final state of the massive universe, we can derive the very high energy density

$$\rho_{\text{f}}(\hat{\nu})\,c^2=\rho(\hat{\nu})\,c^2\times\left(\frac{T_2}{T_1}\right)^4=4\rho_{\text{vac}}\,c^2\times\left(\frac{T_2}{T_1}\right)^4=$$

$$=4\,\frac{\pi^2}{15}\,\frac{(\tfrac{1}{2}E_{\text{d}})^4}{(\hbar c)^3}\,\frac{2}{3\,N(T)}\cong 1.268\cdot 10^{372}\ \text{eV cm}^{-3}\,. \tag{3.80}$$

Similar to the expression (3.80), the fourfold vacuum energy density (3.25), multiplied by $(T_2/T_1)^4=2/3N(T)$, permits the introduction of the high energy density

$$\rho_{\text{BB}}(\hat{\nu})\,c^2=4\rho_{\text{vac}}(R_{\text{BB}})\,c^2\times\left(\frac{T_2}{T_1}\right)^4=\frac{8}{3}\Omega_\gamma\,\frac{\pi^2}{15}\,\frac{E_{\text{Pl}}^2}{\hbar c\,R_{\text{BB}}^2}\left(\frac{T_2}{T_1}\right)^4=$$

$$=\frac{8}{3}\Omega_\gamma\,\frac{\pi^2}{15}\,\frac{E_{\text{Pl}}^2}{\hbar c\,R_{\text{BB}}^2}\,\frac{2}{3\,N(T)}\cong 1.579\cdot 10^{248}\ \text{eV cm}^{-3}\,. \tag{3.81}$$

Then, we can assume a mean energy density of the sterile neutrinos to

$$\bar{\rho}_2\,c^2=(\rho_{\text{f}}(\hat{\nu})\,c^2\times\rho_{\text{BB}}(\hat{\nu})\,c^2)^{1/2}\cong 1.415\cdot 10^{310}\ \text{eV cm}^{-3}\,, \tag{3.82}$$

so that via the volume $V=\tfrac{4}{3}\pi R_{\text{Pl}}^3=1.768\cdot 10^{-98}\ \text{cm}^3$ we obtain the mean (maximum) energy of the "massless" universe to

$$\bar{E}_2=\bar{\rho}_2\,c^2\times V\cong 2.50\cdot 10^{212}\ \text{eV}\,, \tag{3.83}$$

i.e. because of the sterile neutrino number $N_{\text{f}}=2.04\cdot 10^{184}$ (see Eq. (3.60 a)) we find again the Planck energy (see Eq. (3.79))

$$E_{\text{Pl}}=\frac{\bar{E}_2}{N_{\text{f}}}\cong 1.22\cdot 10^{28}\ \text{eV}\,. \tag{3.84}$$

It is plausible that the mean energies (3.78) and (3.83) must be identical, since the ratio of the mean densities $\bar{\rho}_1$ (see Eqs. (3.76) and (3.77)) and $\bar{\rho}_2$ (see Eq. (3.82)) is defined by

$$\frac{\bar{\rho}_1}{\bar{\rho}_2}=\frac{V}{V(\hat{\nu})}\cong 2.34\,. \tag{3.85}$$

We assume that we can interpret the mean (maximum) energy (3.83) as kinetic energy, which is necessary for the complete expansion from the big bang of the "massless" universe to the final state of the massive universe. In contrast to energy (3.83), for the mean (maximum) energy (3.78), we assume its interpretation as potential energy, which is responsible for the transition from the final state of the massive universe in direction to the "massless" universe and the big bang. We prove these assumptions in the chapters 3.8 and 3.11, respectively.

Using the data of Tables III to V, the results, derived in this Sec. 3.5 for the total universe, are also valid for the total anti-universe.

3.6 The zero-point oscillations for the massless universe

In this chapter, we demonstrate that the existence of the "massless" universe ($0 \leq R_{\mathrm{BB}} \leq R \leq R_{\mathrm{Pl}}$) can be attributed to the zero-point oscillations.

Therefore, at the distance $R_{\mathrm{BB}} = 2.069 \cdot 10^{-96}$ cm (see Eq. (3.23)), these zero-point oscillations must be formed via the greatest possible relativistic energy $E_{\mathrm{BB}} = 2\widetilde{E}_{\mathrm{BB}} = \hbar c / R_{\mathrm{BB}} = 9.537 \cdot 10^{90}$eV (see Eqs. (3.28 a) and (3.28 b)) because of the start of the universe at the distance $R_{\mathrm{BB}} = 2.069 \cdot 10^{-96}$ cm {see Eq. (3.23) and Sec. 3.10}.

Now, we prove the formation of these zero-point oscillations for the existence of the massless universe. For this goal, we use de Broglie waves. In the extremely relativistic case, between its momentum p of the particle as well as its relativistic energy E, we have the connection

$$p = \frac{E}{c}. \tag{3.86}$$

Then, the de Broglie wavelength λ of the sterile neutrinos $\hat{\nu}_{\Lambda}$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$, which describe the joint origin of the dark matter and dark energy (see Refs. [1, 2]), is defined by

$$\lambda = 2\pi \frac{\hbar}{p} = \frac{\hbar c.}{E/2\pi}. \tag{3.87}$$

For Eq. (3.87), we can write

$$\frac{\hbar c}{\lambda} = \frac{E}{2\pi}. \tag{3.88}$$

Because we have $N_{\hat{\nu}\uparrow} = 3$ excited sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\text{dm}}$ and $\hat{\nu}_{\text{b}}$ as a result of the zero-point oscillation via Dirac's fermion-antifermion theory, we can introduce $E(\hat{\nu}_\Lambda) + E(\hat{\nu}_{\text{dm}}) + E(\hat{\nu}_{\text{b}}) \to E$ for $E(\hat{\nu}_\Lambda) = E(\hat{\nu}_{\text{dm}}) = E(\hat{\nu}_{\text{b}})$ in Eqs. (3.87) and (3.88), so that analogous to Eq. (3.88) we get the condition $\hbar c/\lambda(\hat{\nu}_\Lambda) = \hbar c/\lambda(\hat{\nu}_{\text{dm}}) = \hbar c/\lambda(\hat{\nu}_{\text{b}})$ with $\lambda(\hat{\nu}_\Lambda) = \lambda(\hat{\nu}_{\text{dm}}) = \lambda(\hat{\nu}_{\text{b}}) \to \lambda$ for the "massless" universe. The arrow $\uparrow$ describes their spin direction.

Analogously, the zero-point oscillations must excite simultaneously $N_{\hat{\hat{\nu}}\uparrow} = 3$ sterile anti-neutrinos $\hat{\hat{\nu}}$ in the anti-universe, so that we obtain $N = 3$ sterile neutrino-antineutrino pairs for these "massless" universes. If we consider correspondingly still $N_{\hat{\nu}\downarrow} = 3$ sterile neutrinos and $N_{\hat{\hat{\nu}}\downarrow} = 3$ anti neutrinos with opposite spin direction (Pauli exclusions principle), we have together $N = 6$ excited sterile neutrino-antineutrino pairs.

Considering the simultaneous application of Dirac's fermion-antifermion theory for the universe and the anti-universe, we must introduce $N = 12$ sterile neutrino-antineutrino pairs. This N doubling is extremely important at the calculation of the properties of the massive universe and anti-universe {see the application of Eq. (3.90 c) in Eq. (3.132) to (3.134)}.

For the massive universe (anti-universe), we assume that the excitation energy of the zero-point oscillations has an energy loss in form of the factor $(\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6} = 0.9915$ by the induction of the decay of the 3 fundamental massive sterile neutrinos of the dark matter and dark energy into two massless decay products (see Sec. 4). In Ref. [1], we have neglected this influence of the dark matter and dark energy. In this work, we consider this effect.

Therefore, for the N excited sterile neutrino-antineutrino pairs, we must introduce the real excitation (exc) energy $E_{\text{exc}}(N) = (\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6} E_{\text{BB}}$, where $E_{\text{BB}} \equiv 2\widetilde{E}_{\text{BB}} = \hbar c/R_{\text{BB}} = 9.537 \cdot 10^{90}$ eV {see Eqs. (3.28 b) and (3.130)}.

Consequently, at N excited sterile neutrino-antineutrino pairs, by the transition $E \to E_{\text{exc}}(N)$, Eq. (3.88) must be transformed correspondingly to

$$\text{a)}\quad (\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6} N \frac{\hbar c}{\lambda} = \frac{E_{\text{exc}}(N)}{2\pi},$$

$$\text{b)}\quad \lambda = \frac{(\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6} \hbar c}{E^*(N)},$$

$$\text{c)}\quad E^*(N) = \frac{E_{\text{exc}}(N)}{N\,2\pi} = \frac{(\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6} E_{\text{BB}}}{N\,2\pi}. \qquad (3.89)$$

Thus, for the cases $N = 3, 6, 12$, Eq. (3.89 c) yields

$$\text{a)}\quad E^*(3) = (\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6} E_{\text{BB}}/6\pi = 5.017 \cdot 10^{89}\ \text{eV},$$

$$\text{b)}\quad E^*(6) = (\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6} E_{\text{BB}}/12\pi = 2.509 \cdot 10^{89}\ \text{eV},$$

$$\text{c)}\quad E^*(12) = (\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6} E_{\text{BB}}/24\pi = 1.254 \cdot 10^{89}\ \text{eV}. \qquad (3.90)$$

Consequently, the cases $N = 3$ and $N = 6$ are described by the energies (3.90 a) and (3.90 b), which agree with dark energy $E_{\text{d}} = 5.017 \cdot 10^{89}$ eV (see Eq. (3.10)) and with the half dark energy $\frac{1}{2} E_{\text{d}} = 2.509 \cdot 10^{89}$ eV (see Eqs. (3.11) and (3.12)) in the massive universe, respectively. The case $N = 12$, which is defined by the doubling of the number $N = 6$ of the sterile neutrino-antineutrino pairs (see above), is taken into account by the energy (3.90 c). Therefore, the energy (3.90 c) must be identical with $\frac{1}{4} E_{\text{d}} = 1.254 \cdot 10^{89}$ eV. To this energy (3.90 c), we will return in Eqs. (3.112) and (3.113) as well as (3.132 to (3.134)).

By Eqs. (3.89 b) and (3.90), we find the corresponding wavelength λ to

$$\text{a)}\quad \lambda = \frac{(\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6} \hbar c}{E^*(3)} = \frac{\hbar c}{E_{\text{BB}}/6\pi} = 3.90 \cdot 10^{-95}\ \text{cm},$$

$$\text{b)}\quad \lambda = \frac{(\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6} \hbar c}{E^*(6)} = \frac{\hbar c}{E_{\text{BB}}/12\pi} = 7.80 \cdot 10^{-95}\ \text{cm},$$

$$\text{c)}\quad \lambda = \frac{(\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6} \hbar c}{E^*(12)} = \frac{\hbar c}{E_{\text{BB}}/24\pi} = 1.56 \cdot 10^{-94}\ \text{cm}. \qquad (3.91)$$

Introducing the reduced wavelength $\bar{\lambda} = \lambda/2\pi$, Eq. (3.91) provides

$$\text{a)}\quad ƛ = \frac{(\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6}\hbar c}{2\pi E^*(3)} = \frac{\hbar c}{E_{\text{BB}}/3} = 6.21\cdot 10^{-96}\ \text{cm},$$

$$\text{b)}\quad ƛ = \frac{(\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6}\hbar c}{2\pi E^*(6)} = \frac{\hbar c}{E_{\text{BB}}/6} = 1.24\cdot 10^{-95}\ \text{cm},$$

$$\text{c)}\quad ƛ = \frac{(\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6}\hbar c}{2\pi E^*(12)} = \frac{\hbar c}{E_{\text{BB}}/12} = 2.48\cdot 10^{-95}\ \text{cm}. \tag{3.92}$$

By generalization of Eq. (3.92), we get $R_{\text{BB}} = ƛ/N = \hbar c/E_{\text{BB}}$, where N describes again the sterile neutrino-antineutrino pair number, i.e. we have $ƛ = N R_{\text{BB}}$ because of N as the basis of the zero-point oscillations.

Therefore, we can assume that the reduced wavelengths (3.92) lie in the wavelength range $ƛ_{\text{BB}} = R_{\text{BB}} < ƛ < ƛ_{\hat\nu} = R_{\hat\nu}$ of the massless universe, where the distance $R_{\hat\nu} = 4.938\cdot 10^{-68}\text{cm}$ is defined by Eq. (3.32). Thus, for $ƛ_{\hat\nu}$, we have the condition $ƛ_{\hat\nu} = N R_{\text{BB}} = R_{\hat\nu}$, whereat the number N of the particle-antiparticle pairs is determined by $N = R_{\hat\nu}/R_{\text{BB}} = 2.387\cdot 10^{28}$. Then, the wavelength $ƛ_{\text{Pl}} = N R_{\text{BB}} = R_{\text{Pl}}$ is also given by $N = R_{\text{Pl}}/R_{\text{BB}} = 7.812\cdot 10^{62}$.

Using the data of Tables III to V, the corresponding results, derived in this Sec. 3.6 for the massless universe, are also valid for the massless anti-universe.

With that, we have shown that the massless universe and anti-universe are connected with each other by the zero-point oscillations as a result of Dirac's fermion-antifermion theory.

We can thus assume that zero-point oscillations are also responsible for the massive and present universes, we will return to this problem in Sec. 3.7.

3.7 The particle horizon and the zero-point oscillations for the early and late massive universes (including the present universes)

In Ref. [1], the derivation of the particle horizon distances and of the sterile neutrino-antineutrino pair numbers are partly incorrect. Therefore, in this work, we correct now this inadmissibility.

According to Weinberg [8], the particle horizon distance $d_{\max}(t) = d_{\max}(z)$ of the known universe is defined by

$$d_{\max}(t) = d_{\max}(z) = R(t)\int_0^t \frac{c\,dt'}{R(t')}, \tag{3.93}$$

since the time t is a function of the redshift z (see Eq. (2.12)). This particle horizon limits the distance at which we can observe past events.

Using Eqs. (2.1) as well as (2.3) and (2.4), for the present (massive) universe ($z = 0$), by its age $t = t_0$ (see Tables I and V), in accordance with Weinberg, the value $d_{\max}(t_0) = d_{\max}(0)$ is given by

$$d_{\max}(t_0) = d_{\max}(0) = \frac{c}{H_0}\int_0^1 \frac{dx}{x^2\,(\Omega_\Lambda + \Omega_m x^{-3} + \Omega_r x^{-4})^{1/2}}, \tag{3.94}$$

where $x \equiv R/R_0 = 1/(1+z)$ (see Eq. (2.15)) and the corresponding quantities are explained by Tables I and V as well as by Eq. (2.5).

Then, for the known universe, we can introduce the general expression

$$d_{\max}(z) = \frac{R_0}{1+z}\int_0^{1/(1+z)} \frac{dx}{x^2\,(\Omega_\Lambda + \Omega_m x^{-3} + \Omega_r x^{-4})^{1/2}}. \tag{3.95}$$

However, for $x = 1/(1+z) \to 0$, the integrals (3.94) and (3.95) would be infinite. Consequently, we must determine a new lower integration limit. For this goal, via the big bang, we can assume as the lower integration limit $x = 1/(1+z_{BB})$, if we select $1+z_{BB} = R_0/R_{BB} = 6.646\cdot10^{123}$, so that because of $1+z_M = 2.916\cdot10^{30}$ {see the new inflation model (2.14) to (2.18) as well as Refs. [1-3]} we have $1+z_{BB} >> 1+z_M$, i.e., for example, Eq. (3.95) can be converted into its smallest possible value

$$d_{\max}(z_M) = \frac{R_0}{1+z_M}\int_{1/(1+z_{BB})}^{1/(1+z_M)} \frac{dx}{\Omega_r^{1/2}}. \tag{3.96}$$

Then, at the limit $R \to R_{Pl}$, because of this new inflation model, we must apply (see Eq. (2.5)) and $N(T) = 1/2\,\Omega_\gamma$ (see Refs. [1-3]), so that Eq. (3.96) yields

$$d_{\max}(z_M) = \frac{R_0}{(1+z_M)}\left[\frac{2}{1+z_M} - \frac{2}{1+z_{BB}}\right] = 2R_{Pl} = c2t_{Pl}. \tag{3.97}$$

For the quantum mechanical range $1+z_{BB} \geq 1+z \geq 1+z_M$ of the massless universe ($R_{BB} \leq R \leq R_{Pl}$ or $t_{BB} \leq t \leq t_{Pl}$) [1, 2], we must apply the result $R = ct$ (see Eq. (2.43)) of the quantum gravity [1, 2] in Eq. (3.93) with the lower integration limit $t' = t_{BB}$, so that the particle horizon (3.93) has the limiting expression

$$d(t) = c(t - t_{BB}), \tag{3.98}$$

which disappears for $t = t_{BB} = 6.901 \cdot 10^{-107}$ s (see Eq. (3.24)).

Consequently, because the results (3.97) and (3.98), we obtain the plausible condition

$$d(t) = c\,(t_{Pl} - t_{BB}) < d_{max}(z_M) = c2t_{Pl}\,. \tag{3.99}$$

Thus, for the total (massless and massive) universe, Eqs. (3.94) to (3.99) yield the connection

$$0 \leq d(t \leq t_{Pl}) < 2R_{Pl} \leq d_{max}(z \geq z_M) \leq d_{max}(0)\,. \tag{3.100}$$

According to the end of Sec. 3.6, the value $1+z_{BB} = R_0/R_{BB} = 6.646 \cdot 10^{123}$ is identical with the sterile neutrino-antineutrino pair number

$$N = N_0 = \frac{R_0}{R_{BB}} \tag{3.101}$$

for the universe and anti-universe, i.e. we must assume that the sterile neutrino (antineutrino) number $\frac{1}{2}N_0$ is alone responsible for the universe (anti-universe).

Now, we prove this assumption by the new inflation model (see Eqs. (2.14) to (2.18) or Refs. [1, 2]) via the early ($R_{Pl} \leq \widetilde{R} \leq \widetilde{R}_0$) for the late ($\widetilde{R}_0 \leq R \leq R_0$) massive universe [1, 2]. For this goal, by Eqs. (2.1) as well as (2.13) and (2.14), we can use $\dot{\widetilde{R}}/\widetilde{R} = (\frac{1}{2}N(T)\,\Omega_\gamma)^{\frac{1}{2}} H_0 (1+z)^2$, so that because of $N(T) = 1/2\,\Omega_\gamma$ at $t = t_{Pl}$ we can write

$$\frac{\dot{\widetilde{R}}(t_{Pl})}{\widetilde{R}(t_{Pl})} = H_{Pl} = \frac{1}{2\,t_{Pl}} = \tfrac{1}{2} H_0 (1+z_M)^2 \tag{3.102}$$

or

$$\frac{H_{Pl}}{H_0} = \frac{1}{2H_0\,t_{Pl}} = \frac{\frac{1}{2}R_0}{R_{Pl}} = \frac{\frac{1}{2}N_0}{N_{Pl}}\,. \tag{3.103}$$

Indeed, via the result (3.101), Eq. (3.103) yields the sterile neutrino (antineutrino) number alone for the universe (anti-universe) to

$$\tfrac{1}{2} N_0 = \tfrac{1}{2} R_0 N_{\mathrm{Pl}} / R_{\mathrm{Pl}} = \tfrac{1}{2} R_0 / R_{\mathrm{BB}} = 3.323 \cdot 10^{123}, \qquad (3.104)$$

whereat

$$N_{\mathrm{Pl}} = R_{\mathrm{Pl}} / R_{\mathrm{BB}} = 7.812 \cdot 10^{62} \quad \text{(see Sec. 3.6)}. \qquad (3.105)$$

By Eq. (3.104), we have supported the assumption to the end of Sec. 3.6 that the massive universe can be attributed also to zero-point oscillations, which are based on the sterile neutrino-antineutrino pairs.

Using the data of Tables III to V, the results, derived in this Sec. 3.7 for the massive universe, are also valid for the massive anti-universe.

Then, by Eq. (3.105), the Planck energy of the early massive universe or anti-universe is determined by

$$E_{\mathrm{Pl}} = N_{\mathrm{Pl}} \times kT_{\mathrm{BB}} \cong 1.221 \cdot 10^{28}\ \mathrm{eV}. \qquad (3.106)$$

The Hubble (H) energy E_{H} of the present late massive universe or anti-universe can be evaluated by the present critical energy density given by $\rho_{0\mathrm{C}}\, c^2 = 4.77 \cdot 10^3\ \mathrm{eV\,cm^{-3}}$ (see Tables I and V), and the Hubble length $R_0 = 1.375 \cdot 10^{28}\,\mathrm{cm}$ (see Tables I and V as well as Eq. (2.17)) to

$$E_{\mathrm{H}} \cong \rho_{0\mathrm{C}}\, c^2\, \tfrac{4}{3} \pi R_0^3 \cong 5.194 \cdot 10^{88}\ \mathrm{eV}. \qquad (3.107)$$

This energy (3.107) can be determined also by the smallest thermal (particle) energy $kT_{\mathrm{BB}} = 1.536 \cdot 10^{-35}\mathrm{eV}$ (see Eq. (3.27)), multiplied by the sterile neutrino (antineutrino) number (3.104), so that the Hubble energy of the present universe (anti-universe) is also defined by

$$E_{\mathrm{H}} = \tfrac{1}{2} N_0 \times kT_{\mathrm{BB}} \cong 5.194 \cdot 10^{88}\ \mathrm{eV}. \qquad (3.108)$$

The results (3.107) and (3.108) support again the assumption that the present late massive universe (anti-universe) can be described also by zero-point oscillations (see the end of Sec. 3.6).

Similarly, by Eqs. (3.20) and (3.27), we get

$$\rho_{\mathrm{vac}}(R_{\mathrm{BB}})\, c^2 = n_\gamma(R_{\mathrm{f}}) \times kT_{\mathrm{BB}} \cong 4.227 \cdot 10^{247}\ \mathrm{eV\,cm^{-3}} \qquad (3.109)$$

in excellent agreement with the result (3.25).

3.8 The greatest possible gravitational energy and the hypothetical superforce of the particle interactions

Using in the massless universe the Planck energy $E_{\text{Pl}} = (\hbar c^5/G_N)^{1/2}$ for the big bang at $R = R_{\text{BB}}$ (see Eq. (3.23)), we obtain the normal gravitational energy $\overline{E}_{\text{BB}}$ at the big bang for the massless universe and anti-universe by the gravitational potential energy to

$$\overline{E}_{\text{BB}} = \frac{G_N}{c^4}\,\frac{E_{\text{Pl}} \times E_{\text{Pl}}}{R_{\text{BB}}} = \frac{\hbar c}{R_{\text{BB}}} = 9.537 \cdot 10^{90}\ \text{eV}. \qquad (3.110)$$

The result (3.110) is equivalent to the energy (3.28 b). This energy (3.28 b) was interpreted as the energy $E_{\text{BB}} = \hbar c/R_{\text{BB}}$ via the potential of a new attractive force, which is now assumed as the attractive, hypothetical superforce of the particle interactions.

Therefore, we can assume that the normal gravitational energy (3.110) of the big bang corresponds to the highest value of the potential energy $E = \hbar c/R$ of the attractive, hypothetical superforce of the particle interactions.

At $R = R_{\text{Pl}} = (\hbar G_N/c^3)^{1/2}$, the normal potential energy of this hypothetical superforce has its lowest value E_{Pl}, where the unification of the strong and electroweak interaction (grand unification) begins, since the gravitational force and this hypothetical superforce are here again equivalent because of

$$E_{\text{Pl}} = \frac{G_N}{c^4}\,\frac{E_{\text{Pl}} \times E_{\text{Pl}}}{R_{\text{Pl}}} = \frac{\hbar c}{R_{\text{Pl}}} = 1.220932 \cdot 10^{28}\ \text{eV}. \qquad (3.111)$$

Because of Eq. (3.110), the normal gravitational energy $\overline{E}_{\text{BB}} = \hbar c/R_{\text{BB}}$ at the big bang and the highest energy value $E_{\text{BB}} = \hbar c/R_{\text{BB}}$ (see Eq. (3.110)) of this hypothetical superforce are conserved quantities with the same values.

Consequently, the greatest possible value $2\overline{E}_{\text{max}}$ of the gravitational energy at the big bang (origin) for the universe and anti-universe {distance $2R_{\text{BB}}$ (see Eqs. (3.28 a), (3.128) and (3.129))} is precisely and uniquely determined by the energy $E^*(12) = (\Omega_{\text{dm}} + \Omega_\Lambda)^{1/6}\, E_{\text{BB}}/24\pi = 1.254 \cdot 10^{89}$ eV (see Eq. (3.90 c)) to

$$2\overline{E}_{\max} = \frac{G_N}{c^4}\frac{\{E^*(12)\}\times\{E^*(12)\}}{2R_{\mathrm{BB}}} \cong 5.03\cdot10^{212}\,\mathrm{eV}\,, \tag{3.112}$$

so that for the universe (anti-universe) alone we find the greatest possible gravitational energy $\overline{E}_{\max}$ to

$$\overline{E}_{\max} = \frac{G_N}{c^4}\frac{\{E^*(12)\}\times\{E^*(12)\}}{2\times 2R_{\mathrm{BB}}} \cong 2.51\cdot10^{212}\,\mathrm{eV}\,. \tag{3.113}$$

This energy (3.113), which is interpreted as the kinetic energy for the complete expansion of the total ("massless" and massive) universe (anti-universe), is so identical with the mean (maximum) energies (3.83) {see also the end of Sec. 3.5}.

More generally, for the potential $V(R)$ of the hypothetical superforce of the particle interactions, in the massless universe, analogous to the gravitational potential, we can assume

$$V(R) = -\frac{\hbar c}{R}\,, \tag{3.114}$$

so that we find the law of this hypothetical superforce via the potential (3.114) to

$$\vec{K}(\vec{R}) = -\frac{\partial V(R)}{\partial \vec{R}} = -\frac{\vec{R}}{R}\frac{dV(R)}{dR} =$$

$$= -\frac{\hbar c}{R^2}\frac{\vec{R}}{R} \quad \text{for} \quad R_{\mathrm{BB}} \le R \le R_{\mathrm{Pl}}\,. \tag{3.115}$$

Using the data of Tables III to V, the results, derived in this Sec. 3.8 for the "massless" and massive universe, are also valid for the "massless" and massive anti-universe.

3.9 The very high temperature of the big bang

Using Eqs. (2.40) and (2.41), we can introduce the vacuum energy density $\widehat{\rho}_{\mathrm{vac}}\,c^2$ of the quantum gravity of the massless universe as a function of the temperature, i.e. we obtain

$$\rho_{\text{vac}}(R)\,c^2 \to \widehat{\rho}_{\text{vac}}\,c^2 = \widehat{\rho}_{\text{vac}}(T)\,c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\text{Pl}}^6}{(\hbar c)^3\,(kT)^2}\,. \qquad (3.116)$$

Because of the relationship of the final state of the massive universe and the big bang, at $t = t_{\text{BB}}$, in the massless universe, for the big bang, according to Eq. (3.129), the relativistic energy uncertainty

$$\widetilde{E}_{\text{BB}} = k\widetilde{T}_{\text{BB}} = \frac{\hbar}{2t_{\text{BB}}} = \frac{\hbar c}{2R_{\text{BB}}} = 4.769\cdot 10^{90}\ \text{eV} \qquad (3.117)$$

must be connected with the dark energy density of the final state of the massive universe (see Eq. (3.75 a))

$$\rho_{\text{f}1}c^2 = \Omega_\Lambda \rho_{0\text{C}}c^2\left(\frac{d_{\text{eff}}}{c\,t_{\text{f}2}}\right)^3\left(\frac{T_1}{T_2}\right)^3 = \Omega_\Lambda \rho_{0\text{C}}c^2\left(\frac{d_{\text{eff}}}{c\,t_{\text{f}2}}\right)^3\left(\frac{3\,N(T)}{2}\right)^{3/4} =$$

$$= \rho_{\text{f}2}c^2\left(\frac{3\,N(T)}{2}\right)^{3/4} = 4.56\cdot 10^{-4}\ \text{eV cm}^{-3}\,. \qquad (3.118)$$

Thus, taking the result (3.118), because of the relationship of the final state of the massive universe and the big bang, we can so assume the connection $\widehat{\rho}_{\text{vac}} = \rho_{\text{f}1} = \rho_{\text{f}2}\times(T_1/T_2)^3$ with $N(T) = (4/11)^{1/3}$, so that we obtain

$$\frac{8}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\text{Pl}}^6}{(\hbar c)^3\,(k\widetilde{T}_{\text{BB}})^2} = \rho_{\text{f}1}\,c^2 \qquad (3.119)$$

with the solution

$$k\widetilde{T}_{\text{BB}} = \widetilde{E}_{\text{BB}} = \left[\frac{8}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\text{Pl}}^6}{(\hbar c)^3\,\rho_{\text{f}1}\,c^2}\right]^{1/2} \cong 4.76\cdot 10^{90}\ \text{eV}\,, \qquad (3.120)$$

i.e. the result (3.28 a) is correct by $\widehat{\rho}_{\text{vac}} = \rho_{\text{f}1} = \rho_{\text{f}2}\times(T_1/T_2)^3$ (see above), since the energy $\widetilde{E}_{\text{BB}} \cong 4.76\cdot 10^{90}$ eV realizes also the state of $\rho_{\text{f}1} = \rho_{\text{f}2}\times(T_1/T_2)^3$ {see also Sec. 3.11}. Because of the result (3.120), the very high temperature of the start of the universe (see also Sec. 3.10 or 3.11) is given by

$$\widetilde{T}_{\text{BB}} = \widetilde{E}_{\text{BB}}/k \cong 5.52\cdot 10^{94}\ \text{K}\,. \qquad (3.121)$$

Using the data of Tables III to V, these considerations, derived in this Sec. 3.9 for the universe, are also valid for the anti-universe (see Sec. 3.10).

3.10 The time reversal solution for universe and anti-universe

At the 4-vectors and the tensors, the time reversal leads to a sign change of their time component. Consequently, for the universe and the time-reversal anti-universe, we must apply the results of Tables IV and V. However, because the parameters of Table I and V agree excellently, we use predominantly the parameters of Table I in this work. The universe is treated by the known way, for example, using the Friedmann equation (2.1). The time-reversal anti-universe is also determined, for example, by the Friedmann equation (2.1), whereat we must however consider the minus sign of the root as well as the results of Tables IV and V, i.e. we must take into account the sign "$-$" of the root together with $-\dot{R}$. Thus, we obtain the "same" solution for the scale factor of the anti-universe as at the universe. However, in contrast to the universe, at the anti-universe, we have a negative velocity and a negative time.

Then, using the result (2.43) of the quantum gravity [1, 2] for the "massless" ($R_{\mathrm{Pl}} \geq R \geq R_{\mathrm{BB}}$ and $R_{\mathrm{BB}} \leq R \leq R_{\mathrm{Pl}}$) anti-universe and universe as well as the general solutions [1-3, 5] of Eqs. (2.1) and (2.13) via the results (2.14) to (2.18) for the early ($\tilde{R}_0 \geq \tilde{R} \geq R_{\mathrm{Pl}}$ and $R_{\mathrm{Pl}} \leq \tilde{R} \leq \tilde{R}_0$) and, for example, the late ($R \geq \tilde{R}_0$ and $\tilde{R}_0 \leq R$) radiation-dominated massive anti-universe and universe, we have the positive distances and scale factors

$$R = c\,t \tag{3.122}$$

as well as

$$\tilde{R} = (2\,N(T)\,\Omega_\gamma)^{1/4}(R_{\mathrm{Pl}}\,c\,t)^{1/2} = c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\,(t_{\mathrm{pl}}\,t)^{1/2} \tag{3.123}$$

and

$$R = (2\,N(T)\,\Omega_\gamma)^{1/4}(R_0\,c\,t)^{1/2} = c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\left(\frac{t}{H_0}\right)^{1/2}, \tag{3.124}$$

respectively. Then, the corresponding velocities are given by

$$\dot{R} = \frac{dR}{dt} = \pm c \tag{3.125}$$

as well as

$$\dot{\tilde{R}} = \frac{d\tilde{R}}{dt} = \pm(2\,N(T)\,\Omega_\gamma)^{1/4}(R_{\rm Pl}\,c/t)^{1/2} =$$
$$= \pm c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\left(\frac{t_{\rm Pl}}{t}\right)^{1/2} \tag{3.126}$$

and

$$\dot{R} = \frac{dR}{dt} = \pm(2\,N(T)\,\Omega_\gamma)^{1/4}(R_0\,c/t)^{1/2} =$$
$$= \pm c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\left(\frac{1}{H_0 t}\right)^{1/2}, \tag{3.127}$$

where according to Table IV the sign "+" and the sign "−" are valid for the universe and the anti-universe, respectively. Therefore, we have the positive time direction (from the origin (big bang) to the future), which is valid for all fundamental physical processes in the universe, so that we have here an expanding universe with scale factors greater than zero as well as positive velocities $c \geq v > 0$. Correspondingly, for the anti-universe, we get a negative time direction (i.e. from the origin (big bang) to the past as a result of the time reversal), which is valid for all fundamental physical processes in the anti-universe, i.e. we have here an expanding anti-universe with scale factors greater than zero as well as negative velocities $-c \leq -v < 0$ and the transitions $H_0 \to -H_0$, $t_{\rm Pl} \to -t_{\rm Pl}$ and $t \to -t$. The velocities (3.125) to (3.127) of the anti-universe possess a negative sign because of the transition $t \to -t$ in the differential quotients of the velocities, so that we get here the negative velocities in contrast to the universe. Thus, we have simply explained the long-sought problem of the separation between matter (universe) and antimatter (anti-universe.

It is now clear that the results of the universe, derived in this work and the papers [1-5], are completely transferable to the anti-universe, using the data of Tables III to V.

Because of this time reversal solution, the beginning (big bang) of the anti-universe and the universe are a result of two equivalent energy uncertainties of the vacuum (origin), which arise from quantum fluctuations of the time or the distance according to the uncertainty relation. Then, according to Table IV, via the result (3.28 a), for the antimatter (anti-universe) and the matter (universe), these two equivalent energy uncertainties are given by

$$\widetilde{E}_{BB} = \frac{-\hbar}{2(-t_{BB})} = \frac{(-\hbar)(-c)}{2R_{BB}} \cong 4.769 \cdot 10^{90}\ \text{eV} \qquad (3.128)$$

and

$$\widetilde{E}_{BB} = \frac{\hbar}{2t_{BB}} = \frac{\hbar c}{2R_{BB}} \cong 4.769 \cdot 10^{90}\ \text{eV}\ , \qquad (3.129)$$

respectively. Thus, for a transition from the anti-universe to the universe, for the zero-point oscillations (see Sec. 3.6 or 3.7), we need the total energy (see also Eq. (3.28 b))

$$E_{BB} = 2\widetilde{E}_{BB} = \frac{\hbar}{t_{BB}} = \frac{\hbar c}{R_{BB}} = 9.537 \cdot 10^{90}\ \text{eV}\ . \qquad (3.130)$$

This total energy $E_{BB} = 9.537 \cdot 10^{90}$ eV, released by the vacuum (origin), is applied as the excitation energy for the zero-point oscillation which act again on the particles and antiparticles in the total {massless (see Sec. 3.6) and massive (see Sec. 3,7)} universe and anti-universe, respectively.

Therefore, at $t = 0$ (origin), the antimatter (3.128) and the matter (3.129) disappear by annihilation, so that the anti-universe (antimatter) and the universe (matter) expand in opposite time directions. These universes are based on the zero-point oscillations formed by the sterile neutrino-antineutrino pairs according to the chapters 3.6 and 3.7.

Now, we can explain the new inflation model {see Eqs. (2.14) to (2.18) or Refs. [1-5]} via a gigantic resonance as a result of the excitation of the magnetic monopoles [1-5] by the zero-point oscillations. This resonance, which is possible because of the binding of these monopoles to the early massive universe, ends at the rest energy of the X and Y gauge bosons [1-5], so that during a short inflationary phase [1-5] an enormous energy is released. This released energy is so large that it is enough for the inflation of the total early massive universe ($R_{Pl} \le \widetilde{R} \le \widetilde{R}_0$), i.e. it leads to an inflation, in which the scale factors of the early universe are enlarged by the enormous factor $1 + z_M = 2.916 \cdot 10^{30}$ (see Eq. (2.18)). However, for the excitation of this resonance, the expended energy is very small in comparison to the excitation energy $E_{BB} = 9.537 \cdot 10^{90}$ eV of the zero-point oscillations, so that the zero-point oscillation are not influenced.

Thus, similarly, at the beginning of the present accelerated expansion (see Eqs. (2.27) and (2.35)), we can assume the formation of a new very large resonance by excitation of the here dominating sterile neutrino-antineutrino pairs of the dark energy (see Eq. 3.132)) via the zero-point oscillations. Consequently, the very large energy of this new resonance could be responsible for the present accelerated expansion of the universe (see also Sec. 7).

The kinetic energy, which is necessary for the complete expansion of the total ("massless" and massive) universe, is given by the mean (maximum) energy (3.83), which is identical with the greatest possible gravitational energy $\overline{E}_{max} = 2.51 \cdot 10^{212}$ eV (see Eq. (3.113)).

In this Sec. 3.10, the results of the last four paragraphs are also valid for the anti-universe.

3.11 The eternal cyclic evolution for universe and anti-universe

For the final state of the universe, because of $\widehat{\rho}_{vac} = \rho_{f1} = \rho_{f2} \times (T_1/T_2)^3$, the result $k\widetilde{T}_{BB} = \widetilde{E}_{BB}$ of Eq. (3.120) gives again the big bang distance

$$R_{BB} = \frac{\hbar c}{2k\widetilde{T}_{BB}} = \frac{\hbar c}{2\widetilde{E}_{BB}} \cong 2.069 \cdot 10^{-96} \text{ cm} \quad \text{(see Eq. (3.23))}. \qquad (3.131)$$

Thus, this expression (3.131) proves uniquely the relationship between the final state of the universe and the big bang.

Via the dark energy $E_d = 5.017 \cdot 10^{89}$ eV (see Eq. (3.10)), the case $N = 12$, which is based on the simultaneous application of the Dirac theory for the universe and the anti-universe (see Sec. 3.6), leads to the connection

$$\tfrac{1}{4} E_d = E^*(12) = (\Omega_{dm} + \Omega_\Lambda)^{1/6} E_{BB}/24\pi = 1.254 \cdot 10^{89} \text{ eV}, \qquad (3.132)$$

considering the results of the zero-point oscillations (see Eq. (3.90 c)).

Consequently, via Eq. (3.132), we obtain the greatest possible gravitational energy for the universe and the anti-universe to

$$2\overline{E}_{\text{max}} = \frac{G_N}{c^4}\frac{\{\frac{1}{4}E_{\text{d}}\}\times\{\frac{1}{4}E_{\text{d}}\}}{2R_{\text{BB}}} \cong 5.03\cdot10^{212}\,\text{eV}\,, \tag{3.133}$$

so that for the universe alone we find the greatest possible gravitational energy $\overline{E}_{\text{max}}$ to

$$\overline{E}_{\text{max}} = \frac{G_N}{c^4}\frac{\{\frac{1}{4}E_{\text{d}}\}\times\{\frac{1}{4}E_{\text{d}}\}}{2\times2R_{\text{BB}}} \cong 2.51\cdot10^{212}\,\text{eV}\,. \tag{3.134}$$

Then, this enormous energy (3.134), which can be interpreted in the extreme case as the potential energy for a gravitational collapse at the transition of the total ("massless" and massive) universe from its final state to the big bang, is so identical with the mean (maximum) energy (3.78) {see also the end of Sec. 3.5}. In this extreme case, the potential energy $\overline{E}_{\text{max}} \cong 2.51\cdot10^{212}\,\text{eV}$ leads to a very fast transition via the reduction of the dimension of the final state of the universe (see Eq. (3.37)) to the distance of the big bang (see Eq. (3.23)) in the time $t_{\text{BB}} = 6.901\cdot10^{-117}\,\text{s}$ (see Eq. (3.24)), i.e. the start of an eternal cycle of evolution is found. However, by a deeper analysis of the result (3.131), in connection with the zero-point oscillations (see also Eqs. (3.128) to (3.130)), we can exclude this extreme case of a gravitational collapse, so that the gigantic potential energy $\overline{E}_{\text{max}} \cong 2.51\cdot10^{212}\,\text{eV}$ must be responsible for a very slow transition from the final state of the universe to the big bang in the time $t_{\text{f2}} = 6883\,\text{Gyr}$ (see Eqs. (3.36) and (3.45)). i.e. we obtain the start of an eternal cyclic evolution of the universe by the corresponding reversal of its expansion. Consequently, we have proved finally the relationship between the final state of the universe and the big bang.

Because we have used the assumption $\hat{\rho}_{\text{vac}} = \rho_{\text{f1}} = \rho_{\text{f2}}\times(T_1/T_2)^3$ for the result (3.131), we must still explain the application of the dark mass density ρ_{f1} as $\rho_{\text{f2}}\times(T_1/T_2)^3$ (see Eq. (3.75 a)), i.e. we have here the application of $d_{\text{f2}} = 6.512\cdot10^{30}\,\text{cm}$ (see Eq. (3.38)) instead of $d_{\text{f1}} = 6.404\cdot10^{30}\,\text{cm}$ (see Eq. (3.37)). Then, according to uncertainty relation, via the quantum fluctuation

$d_{f21} = d_{f2} - d_{f1} \cong 1.08 \cdot 10^{29}$ cm of the distance d_{f1}, we obtain the energy uncertainty

$$E_{12} = 6\,kT_{BB} = \frac{\hbar c}{2d_{f21}} \cong 9.14 \cdot 10^{-35}\ \text{eV} \tag{3.135}$$

for the decay products of the 3 sterile neutrino types of the dark energy (see Sec. 3.1). Therefore, in the final state d_{f2} of the (massive) universe, for $E_0^{\hat{\nu}}(\gamma) = E_0^{\hat{\nu}}(\frac{1}{2}\hat{\nu})$, the energy uncertainty $6\,kT_{BB} = 3 \times E_0^{\hat{\nu}}(\gamma) + 3 \times E_0^{\hat{\nu}}(\frac{1}{2}\hat{\nu})$ of these decay products is correspondingly identical with the triple sum of the rest energies of the photons $E_0(\gamma) = kT_{BB} \cong 1.563 \cdot 10^{-35}$ eV (see Eq. (6.1 b)) and of the gravitons $E_0(G) = kT_{BB} \cong 1.563 \cdot 10^{-35}$ eV (see Eq. (6.1 a))}. These two new assumptions are based on the existence of the sterile neutrino-antineutrino pairs in the final state of the universe because of the Dirac theory, since one sterile neutrino-antineutrino pair determines these decay properties of the sterile neutrinos. Thus, the final state d_{f2} of the universe is a result of these photons and gravitons in an unstable equilibrium, which is destroyed by the gigantic potential energy (3.134), so that it gives the transition from the final state of the universe in the direction to the big bang. Then, the proper distance d_{f2} is determined by the quantum fluctuation $d_{f21} = d_{f2} - d_{f1} \cong 1.08 \cdot 10^{29}$ cm, i.e. the assumption $\hat{\rho}_{vac} = \rho_{f1} = \rho_{f2} \times (T_1/T_2)^3$ is a function of $d_{f2} = c\,t_{f2}$ (see Eqs. (3.75 a) or (3.118)).

Using the data of Tables III to V, all these events are also valid for the transition from the final state of the anti-universe in direction to the big bang.

With that, we have precisely and uniquely proved the eternal cyclic evolution of the anti-universe and the universe.

4 The explanation of the present dark matter and dark energy

Because of several incorrect interpretations in Ref. [1], we perform here once more the explanation of the present dark matter and dark energy, whereat these