Thus, this work is organized as follows. In Sec. 2, we summarize the necessary equations and parameters. In Sec. 3, for a better intelligibility of this work, we give a detailed review for the universe and the time-reversal anti-universe as an eternal cycle of evolution, whereat we derive the transition from the final state of the massive universes in the direction to the big bang of the massless universe (Sec. 3.1), the lifetime of the sterile neutrinos (Sec. 3.2), the constant volume of the sterile neutrinos (Sec. 3.3), the heat accumulation in the final state of the universe as a result of the very high energy density (Sec. 3.4), the mean (maximum) energy of the "massless" and massive universe (Sec. (3.5), the zero-point oscillations as an existence form of the massless universes (Sec. 3.6), the particle horizon and the zero-point oscillations for the early and late massive universe (Sec. 3.7), the greatest possible gravitational energy and the hypothetical superforce of the particle interactions (Sec. 3.8), the very high temperature of the big bang (Sec. 3.9), the time reversal solution for anti-universe and universe (Sec. 3.10) as well as the eternal cyclic evolution for anti-universe and universe (Sec. 3.11). In Sec. 4, we explain the present dark matter and dark energy via the special properties of the sterile neutrinos. In Sec. 5, we give a reasonable argument for the new thermal equilibrium via the light neutrinos. In Sec. 6, we estimate the rest energy of photons and gravitons including conclusions. In Sec. 7, we calculate the different Hubble "constants" as a function of the cosmic evolution epochs. In Sec. 8, the time dependence of the cosmological "constant" is derived. In Sec. 9, we give a short summary. The values of physical constants, used in this work, are given in Refs. [7, 10].

2 The necessary equations and parameters

In Refs. [2-5], we have generally derived $\mathrm{k}=0$ by $\Omega_{\mathrm{tot}}=1$ and $\Omega_{\mathrm{tot}}(z)=1$, so that in the $\Lambda\mathrm{CDM}$ model the Friedmann-Lemaitre Equations are given by

$$H^2=\left(\frac{\dot{R}}{R}\right)^2=\frac{8\pi G_N}{3}\rho+\frac{c^2}{3}\Lambda \tag{2.1}$$

and

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{c^2}{3}\Lambda, \tag{2.2}$$

where $H = H(t)$ is the Hubble parameter, $R = R(t)$ is the scale factor, G_N is the gravitational constant, ρ is the mean mass density, Λ is the cosmological constant, P is the isotropic pressure and c is the speed of light in vacuum.

It is usual to introduce the mass density [1-3, 5-8, 10]:

$$\rho = \rho_\Lambda + \rho_m + \rho_r = \left[\Omega_\Lambda + \Omega_m (R_0/R)^3 + \Omega_r (R_0/R)^4\right]\rho_{0C} =$$

$$= \left[\Omega_\Lambda + \Omega_m (1+z)^3 + \Omega_r (1+z)^4\right]\frac{3H_0^2}{8\pi G_N}, \tag{2.3}$$

where according to Table I ρ_{0C} is the present, critical density

$$\rho_{0C} = \frac{3H_0^2}{8\pi G_N}. \tag{2.4}$$

The radiation density parameter Ω_r is defined by

$$\Omega_r = \frac{1}{2}N(T)\,\Omega_\gamma, \tag{2.5}$$

where the statistical particle weights $N(T)$ are given in Refs. [1-3, 10]. For example, in Eq. (2.8), at temperatures $T << 0.5\,\mathrm{MeV}$, $N(T)$ has the value

$$N(T) = 3.362644 \quad \text{(see also Sec. 8)}, \tag{2.6}$$

since the only relativistic species are photons at temperature T as well as massless or nearly massless neutrinos and antineutrinos of three different types at the temperature $T_\nu = (4/11)^{1/3}T$ because of the entropy conservation [1-3, 6, 8], i.e. $N(T)$ of Eq. (2.5) has the value 3.362644 [6, 8] at the present CMB temperature $T = T_0 = 2.7255\,\mathrm{K}$ (see Table I) and increases at higher temperatures (see Refs. [1-3, 10]). Thus, the present, massive universe contains a relic background of light neutrinos [6] with the temperature

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_0 = 1.9454\,\mathrm{K}. \tag{2.7}$$

The radiation density parameter Ω_γ [1-3, 5-8, 10] is defined by

$$\Omega_\gamma = \frac{\rho_0(\gamma)}{\rho_{0C}} = 5.46 \cdot 10^{-5} \tag{2.8}$$

with the radiation density [1-3, 5-8, 10, 11]:

$$\rho_0(\gamma) = \{\pi^2/15\,(\hbar^3 c^5)\}\,(kT_0)^4\,, \tag{2.9}$$

where $\hbar$ and k describe the reduced Planck constant and the Boltzmann constant, respectively.

In an expanding universe, the observed wavelength λ of the light emitted (e) from a distant source with λ_e, is shifted towards the red. Via the Friedmann-Robertson-Walker metric [1-3, 5-8, 10, 11], this so-called redshift z is defined for these photons by

$$1+z = \frac{\lambda}{\lambda_e} = \frac{R_0}{R} = \frac{T}{T_0}\,, \tag{2.10}$$

where R_0 and R are the scale factors of absorption and emission of light, respectively. Then, by Eqs. (2.3) and (2.4), the relativistic formula (2.1) yields

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = H_0^2\left[\Omega_\Lambda + \Omega_m\left(\frac{R_0}{R}\right)^3 + \Omega_r\left(\frac{R_0}{R}\right)^4\right] \tag{2.11}$$

with the general solution

$$t = t(z) = \frac{1}{H_0}\int_0^{1/(1+z)} \frac{dx}{x\,(\Omega_\Lambda + \Omega_m x^{-3} + \Omega_r x^{-4})^{1/2}}\,, \tag{2.12}$$

where $x \equiv (R/R_0) = (1/[1+z])$.

The most important parameter values, which were partly above introduced, are summarized in the Table I, which contains the critically analyzed observed (present) cosmological parameter values for the massive universe according to Ref. [7]. For better comparison of observed and estimated cosmological parameter values, we mention here also their measured Planck 2013 data [12], summarized in Table II. In Ref. [1, 2], we have shown that for the description of the massive anti-universe the (present), cosmological parameter values of Tables I and II are also valid, except the values H_0 and h, which are here negative. This behavior can be attributed to the known properties of particles and antiparticles according to Table III [1, 2]. Using these special properties for the Friedmann-Lemaitre Equations, we have also assumed the most important quantities in Table IV, which are necessary for the description of the "massless" and massive universe and anti-universe (see Sec. 1). In Refs. [1-4], we have estimated particle-defined (present) cosmological parameter values for universe and anti-universe, given in Table V. They agree well with the observed data in Table I.

It is usual to take apart Eq. (2.3) in the dominant density terms in dependence on the redshift evolution (2.10). Then, for example, if the universe expands adiabatically, the entropy per comoving volume is constant, so that we can describe the mass density of the radiation-dominated universe for $z \geq 10^5$ ($T \geq 3 \cdot 10^5$ K) by

$$\rho = \rho_{\rm r} = \tfrac{1}{2} N(T)\, \Omega_\gamma (1+z)^4 \rho_{0\rm C} = \tfrac{1}{2} N(T)\,(1+z)^4 \rho_0(\gamma)\,. \tag{2.13}$$

By the new inflation model [1-5], for the redshift evolution $1+z$, we have

$$\widetilde{R} = \frac{R}{1+z_{\rm M}} = \frac{\widetilde{R}_0}{1+z} \tag{2.14}$$

$$R = (1+z_{\rm M})\widetilde{R} = \frac{R_0}{1+z}\,, \tag{2.15}$$

where $\widetilde{R}$ and R characterize the scale factors for the emission of light of prior to and after the inflation, whereas $\widetilde{R}_0$ and R_0 define the constant scale factors for the absorption of light before and after the inflation, respectively.

In Refs. [1-4], we have proved that these cosmological parameters of Tables I and II are excellently described by particle-defined, (present), cosmological parameter values, calculated by the light neutrino density parameters as well as the ratios of the relativistic energy to the rest energy of the SUSY GUT particles. In detail, we have summarized these particle-defined, (present), cosmological parameters in Table V (see above). In Ref. [1, 2], by aid of the data of Tables III and IV, we have shown that these particle-defined, (present), cosmological parameters are also valid for the anti-universe. Because of the excellent agreement between these observed (Tables I and II) and calculated (Table V) cosmological parameters, we apply always the cosmological parameter values of Table I in this work at the calculations except for the second half of Sec. 7.

TABLE I. The most important, critically analyzed, (present), cosmological parameter values [7] of the massive universe. In this work, H_0 is interpreted as the present CMB Hubble "constant" (see also Sec. 1). a) Calculated by us for $h = 0.673 \pm 0.012$. b) Different fits [7]

Quantity	Symbol, equation	Value
present day CMB temperature	T_0	$2.7255(6)\ \mathrm{K}$
present day CMB Hubble expansion rate	H_0	$67.3 \pm 1.2\ \mathrm{km\,s^{-1}\,Mpc^{-1}} =$ $= h \times (9.777752\ \mathrm{Gyr})^{-1} =$ $= (2.181 \pm 0.039) \cdot 10^{-18}\ \mathrm{s}^{-1}$
scale factor for Hubble expansion rate	h	0.673 ± 0.012
critical present density of the universe	$\rho_{0C} = 3H_0^2/8\pi G_N$	$1.05375(13) \cdot 10^{-5}\, h^2\, (\mathrm{GeV}/c^2)\ \mathrm{cm}^{-3} =$ $= 4.77(17) \cdot 10^3\, (\mathrm{eV}/c^2)\ \mathrm{cm}^{-3}$
baryon (proton) density of the universe	$\Omega_\mathrm{b} = \rho_\mathrm{b}/\rho_{0C}$	$0.02207(27)\, h^{-2} = 0.0487(23)$ a), $0.0499(22)$ b)
(cold) dark matter density of the universe	$\Omega_\mathrm{dm} = \rho_\mathrm{dm}/\rho_{0C}$	$0.1198(26)\, h^{-2} = 0.265(15)$ a), $0.265(11)$ b)
dark energy density of the universe	$\Omega_\Lambda = \rho_\Lambda/\rho_{0C}$	$0.685^{+0.017}_{-0.016}$

TABLE I, continued.

Quantity	Symbol, equation	Value
pressureless matter density of the universe	$\Omega_{\mathrm{m}} = \Omega_{\mathrm{b}} + \Omega_{\mathrm{dm}}$	$0.315^{+0.016}_{-0.017}$
CMB radiation density of the universe	$\Omega_{\gamma} = \rho_{\gamma}/\rho_{0\mathrm{C}}$	$2.473 \cdot 10^{-5} (T/2.7255\ \mathrm{K})^4 h^{-2} =$ $= 5.46(19) \cdot 10^{-5}$
curvature	$\Omega_{\mathrm{tot}} = \Omega_{\mathrm{m}} + \Omega_{\Lambda} + \cdots$	$0.96^{+0.4}_{-0.5}$ (95% CL); 1.000(7) (95% CL; CMB + BAO)
sum of neutrino masses	$\sum m_{\nu} c^2$	< 0.23 eV
neutrino density of the universe	$\Omega_{\nu} = \rho_{\nu}/\rho_{0\mathrm{C}}$	$< 0.0025\, h^{-2}$; < 0.0055
redshift of matter-radiation equality	z_{eq}	3360 ± 70
redshift of photon decoupling	z_{dec}	1090.2 ± 0.7
redshift at half reionization	z_{reion}	11.1 ± 1.1
age of the universe	t_0	13.81 ± 0.05 Gyr = $= (4.358 \pm 0.016) \cdot 10^{17}$ s
Hubble length	$R_0 = c/H_0$	$0.9250629 \cdot 10^{26} h^{-1}$ m = $= (1.375 \pm 0.025) \cdot 10^{26}$ m [a)]

TABLE II. The measured, cosmological parameter values of Planck 2013 [12]. In column 1, the corresponding parameter symbols are given. The column 2 gives results for the Planck temperature data alone. Column 3, denoted as Planck+WP, combines these Planck data and the WMAP polarization data at low multipoles.

Parameter	Planck Best fit [68% limits]	Planck+WP Best fit [68% limits]
H_0 [km/s Mpc]	67.11 $[67.4 \pm 1.4]$	67.04 $[67.3 \pm 1.2]$
Ω_Λ	0.6825 $[0.686 \pm 0.020]$	0.6817 $[0.685^{+0.018}_{-0.016}]$
$100\, \Omega_b h^2$	2.2068 $[2.207 \pm 0.033]$	2.2032 $[2.205 \pm 0.028]$
$\Omega_{dm} h^2$	0.12029 $[0.1196 \pm 0.0031]$	0.12038 $[0.1199 \pm 0.0027]$
$\Omega_m h^2$	0.14300 $[0.1423 \pm 0.0029]$	0.14305 $[0.1426 \pm 0.0025]$
Age t_0 [Gyr]	13.819 $[13.813 \pm 0.058]$	13.8242 $[13.817 \pm 0.048]$
z_{eq}	3402 $[3386 \pm 69]$	3403 $[3391 \pm 60]$
z_{dec}	1090.43 $[1090.37 \pm 0.65]$	1090.48 $[1090.43 \pm 0.54]$
z_{reion}		11.37 $[11.1 \pm 1.1]$

TABLE III. The most important known properties of particles and antiparticles.

Property	**Particle**	**Anti-particle**
energy	E	E
mass	M	M
time	t	$-t$
momentum	p	$-p$
velocity	v	$-v$
elementary electric charge	q	$-q$

TABLE IV. The most important quantities of the "massless" and massive universe and anti-universe, derived via the known properties of the particles and the antiparticles (in connection with the time reversal). [a)]

Quantity	**Universe**	**Anti-universe**
energy	E	E
temperature	T	T
Boltzmann constant	k	k
mass density	ρ	ρ
gravitational constant	G_N	G_N
age	t_0	t_0
distance, scale factor	R	R
acceleration	$\ddot{R}$	$\ddot{R}$

TABLE IV, continued.

Quantity	Universe	Anti-universe
velocity including c	$\dot{R}$	$-\dot{R}$
reduced Planck constant	$\hbar$	$-\hbar$
time	t	$-t$
Hubble expansion rate	H	$-H$
present CMB Hubble constant	H_0	$-H_0$
scale factor of H_0	h	$-h$
ionized charges	q	$-q$

[a] In work [2], for the total (massive and "massless) anti-universe, all assumptions derived from the known properties of the antiparticle, are correct. Unfortunately, in contrast to Ref. [2], the work [1] gives a negative distance and scale factor $-R$ as one incorrect assumption instead of a correct positive scale factor R. We have here corrected this quantity.

Table V. The estimated particle-defined (present) cosmological parameter values for universe and anti-universe according to Refs. [1-4] (compare with Tables I and II). [a] Universe: "+"sign, anti-universe: "$-$"sign.

Symbol, equation	Value for universe and anti-universe
$\lvert\pm h\rvert$ [a]	$0.6736^{+0.0105}_{-0.0096}$
$\lvert\pm H_0\rvert$ [a]	$100\,h$ km s^{-1}Mpc^{-1} $= 67.36^{+1.05}_{-0.96}$ km s^{-1}Mpc^{-1} $=$
	$= h\times(9.777752\,\mathrm{Gyr})^{-1} = (2.183^{+0.034}_{-0.031})\cdot 10^{-18}\,\mathrm{s}^{-1}$

Table V, continued.

Symbol, equation	Value for universe and anti-universe
$R_0 = c/H_0$	$(1.373^{+0.020}_{-0.021})\cdot 10^{26}$ m
$\rho_{0C} = 3H_0^2/8\pi G_N$	$(4.78^{+0.15}_{-0.14})\cdot 10^3 (\mathrm{eV}/c^2)\ \mathrm{cm}^{-3}$
t_0	$13.82^{+0.06}_{-0.07}$ Gyr
Ω_{tot}	1
$\Omega_{\mathrm{b}} = \rho_{\mathrm{b}}/\rho_{0C}$	$0.02211^{+0.00089}_{-0.00091}\,h^{-2} = 0.0487 \pm 0.0020$
$\Omega_{\mathrm{dm}} = \rho_{\mathrm{dm}}/\rho_{0C}$	$0.1202^{+0.0059}_{-0.0051}\,h^{-2} = 0.265^{+0.013}_{-0.012}$
$\Omega_{\Lambda} = \rho_{\Lambda}/\rho_{0C}$	$0.311^{+0.014}_{-0.013}\,h^{-2} = 0.686^{+0.020}_{-0.021}$
$\Omega_{\mathrm{m}} = \rho_{\mathrm{m}}/\rho_{0C}$	$0.1423^{+0.0059}_{-0.0052}\,h^{-2} = 0.314^{+0.013}_{-0.012}$
$\Omega_{\gamma} = \rho_{\gamma}/\rho_{0C}$	$(2.4728 \pm 0.0025)\cdot 10^{-5}\,h^{-2} = (5.45^{+0.15}_{-0.17})\cdot 10^{-5}$
$\Omega_{\nu} = \rho_{\nu}/\rho_{0C}$	$(6.35^{+0.16}_{-0.14})\cdot 10^{-4}\,h^{-2} = (1.402^{+0.040}_{-0.044})\cdot 10^{-3}$
$\sum m_\nu c^2$	$(5.97^{+0.14}_{-0.13})\cdot 10^{-2}$ eV
$z_{\mathrm{eq}} = \dfrac{2\Omega_{\mathrm{m}}}{N(T)\Omega_{\gamma}} - 1$	3423^{+107}_{-94}
z_{dec}	1090.7
$z_{\mathrm{reion}}(\nu_\tau)$	$11.60^{+0.31}_{-0.26}$

In Ref. [1], for the anti-universe, the negative sign of the scale factor is incorrect. In this work, using the time reversal of the time component of the 4-vectors and the tensors, we have corrected this inadmissibility, which is based on a careless application of the CPT conservation.

According to Refs. [1-5], via the present critical mass density (2.4), for the conditions (2.14) and (2.15), the quantities $\widetilde{R}_0$ and R_0 were found to

$$\widetilde{R}_0 = (R_{\text{Pl}}\, c/H_0)^{1/2} = (4.713 \pm 0.042) \cdot 10^{-5}\ \text{m} \tag{2.16}$$

and

$$R_0 = c/H_0 = (1.375 \pm 0.025) \cdot 10^{26}\ \text{m}\,. \tag{2.17}$$

The quantity

$$1 + z_{\text{M}} = \frac{T_{\text{M}}}{T_0} = \frac{\widetilde{R}_0}{R_{\text{Pl}}} = \frac{R_0}{\widetilde{R}_0} = (2.916 \pm 0.026) \cdot 10^{30} \tag{2.18}$$

enlarges the early (massive) universe ($R_{\text{Pl}} \leq \widetilde{R} \leq \widetilde{R}_0$) by the enormous factor $1 + z_{\text{M}} = 2.916 \cdot 10^{30}$ to the late (massive) universe ($\widetilde{R}_0 \leq R$), whereat T_{M} is the temperature, which is defined by the magnetic monopoles (M) via their rest energy (see Eq. (2.58) or Refs. [1-5]).

In Refs. [1-5], by aid of the blueshift $1 + z(\nu_{\text{e}}) = 0.406^{+0.020}_{-0.025}$ of the light electron neutrino ν_{e}, for the final (f) state of the universe, we have found its scale factor to

$$R = R_{\text{f}} = R(\nu_{\text{e}}) = \frac{R_0}{1 + z(\nu_{\text{e}})} = (3.39^{+0.27}_{-0.23}) \cdot 10^{26}\ \text{m}\,. \tag{2.19}$$

In Eq. (2.4), according to Refs. [1-3, 5], the influence of the dark mass density $\Omega_\Lambda \rho_{0\text{C}}$ begins at

$$1 + z_\Lambda = 18.90^{+0.48}_{-0.49}\,, \tag{2.20}$$

so that by Eq. (2.15) we find its minimum scale factor

$$R = R_\Lambda = \frac{R_0}{1 + z_\Lambda} = (7.28 \pm 0.32) \cdot 10^{24}\ \text{m}\,. \tag{2.21}$$

Because of the expressions (2.19) and (2.21), which define the range of influence of the dark energy (Ω_Λ), for the mean, negative acceleration (MNA), we must assume the average of their redshift conditions [1-3, 5]:

$$1+z_{\rm MNA}=\frac{R_0}{\sqrt{R_\Lambda R(\nu_{\rm e})}}=\left([1+z_\Lambda][1+z(\nu_{\rm e})]\right)^{1/2}=2.77^{+0.10}_{-0.12}, \qquad (2.22)$$

so that we obtain the corresponding scale factor [1-3, 5]:

$$R=R_{\rm MNA}=\frac{R_0}{1+z_{\rm MNA}}=(4.96^{+0.31}_{-0.27})\cdot 10^{25}\ {\rm m}. \qquad (2.23)$$

Consequently, according to Refs [1-3, 5], by Eq. (2.2), for $\Lambda=0$, at the pressure $P\cong 0$, in the matter-dominated universe, we obtain this mean negative acceleration to

$$\ddot{R}_{\rm MNA}=a=-\frac{1}{2}H_0^2\left[\Omega_{\rm m}(R_0/R_{\rm MNA})^3+\Omega_\Lambda\right]R_{\rm MNA}=$$
$$=(-8.71^{+0.76}_{-0.87})\cdot 10^{-8}\ {\rm cm\ s^{-2}}. \qquad (2.24)$$

The value (2.24) agrees excellently with the Pioneer anomaly [14, 15]. Then, by Eq. (2.2), for $\Lambda=0$ (wCDM model), according to $\rho=\rho_{\rm vac,\Lambda}=\Omega_\Lambda\rho_{\rm 0C}$ (see Eq. (2.3)) and $P=-\rho_{\rm vac,\Lambda}c^2=-\Omega_\Lambda\rho_{\rm 0C}c^2$, the acceleration of the "present" accelerated expansion (see below) is positive [1, 2] because of

$$\ddot{R}=\Omega_\Lambda H_0^2 R. \qquad (2.25)$$

Thus, for the end of this "present" accelerated expansion, the effective (eff) equilibrium condition $\ddot{R}+\ddot{R}_{\rm MNA}=0$ yields the effective scale factor [1, 2]:

$$R_{\rm eff}=\frac{-\ddot{R}_{\rm MNA}}{\Omega_\Lambda H_0^2}=2.67\cdot 10^{26}\ {\rm m}. \qquad (2.26)$$

Because the "present" accelerated expansion [1, 2] is defined by

$$R=R_0\,{\rm e}^{\Omega_\Lambda^{1/2}H_0\,(t-t_0)}=R_0\,{\rm e}^{H(t-t_0)}\quad {\rm at}\quad H=\Omega_\Lambda^{1/2}H_0, \qquad (2.27)$$

the effective equilibrium takes place at the time

$$t_{\rm eff}=t_0+\frac{1}{\Omega_\Lambda^{1/2}H_0}\ln\frac{R_{\rm eff}}{R_0}=8.034\cdot 10^{17}\ {\rm s}=25.46\ {\rm Gyr}, \qquad (2.28)$$

where t_0 describes the present age of the universe (see Table I).

Consequently, after this effective equilibrium, according to Refs. [1, 2], we have only still a slow linear expansion. By this slow linear expansion, in Refs. [1, 2], we find the provisional age of the final state of the universe to

$$t_{\mathrm{f}} = 2.136 \cdot 10^{20}\,\mathrm{s} = 6768\,\mathrm{Gyr}\,. \qquad (2.29)$$

This age $6768\,\mathrm{Gyr}$ is much greater than the age $t = t_{\exp}(\nu_{\mathrm{e}}) = 29.63\,\mathrm{Gyr}$ in Refs. [3-5], which is alone defined by the "present" exponential expansion of the universe, i.e. for $R = R_{\mathrm{f}} = R(\nu_{\mathrm{e}}) = 3.39 \cdot 10^{26}\,\mathrm{m}$ in Eq. (2.27).

However, to the result (2.29), where we must still take into account the influence of the general vacuum energy density or the corresponding cosmological "constant" of the massive universe, we will return in Sec. 3.2.

According to Refs. [1, 2], the general vacuum energy density or the cosmological "constant" of the massive universe were found to

$$\rho_{\mathrm{vac}}(T)\,c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} \qquad (2.30)$$

or

$$\Lambda = \widetilde{\Lambda} = \frac{8\pi G_N}{c^2}\,\rho_{\mathrm{vac}}(T) = \frac{8}{45}\pi^3\frac{(kT)^4}{E_{\mathrm{Pl}}^2(\hbar c)^2}\frac{1}{N(T)}\,, \qquad (2.31)$$

respectively. Using Eqs. (2.30) and (2.31), we can introduce the two conditions

$$\rho_{\mathrm{vac}}(T)\,c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} \geq \rho_{\mathrm{vac},\Lambda}\,c^2 = \Omega_\Lambda \rho_{0\mathrm{C}} c^2 \qquad (2.32)$$

and

$$\widetilde{\Lambda} = \frac{8}{45}\pi^3\frac{(kT)^4}{E_{\mathrm{Pl}}^2(\hbar c)^2}\frac{1}{N(T)} \geq \Lambda = \Lambda_\Lambda = \frac{3\Omega_\Lambda}{R_0^2}\,, \qquad (2.33)$$

where on the right-hand side the terms describe the corresponding values of the "present" accelerated expansion of the massive universe [1, 2].

Using $N(T) = 3.362644$ (see Eq. (2.6)), by the two conditions (2.32) and (2.33), we get the lower limiting temperature

$$T = 51.41\,\mathrm{K}\,. \qquad (2.34)$$

This temperature agrees excellently with the value $T = (1+z_\Lambda)T_0 \cong 51.45\,\mathrm{K}$, where for $1+z_\Lambda \cong 18.90$ the influence of the vacuum mass density $\Omega_\Lambda \rho_{0\mathrm{C}}$

begins according to Eq. (2.20). Thus, we assume that the vacuum energy density (2.30) influences the "present" exponential expansion, which therefore is considered as follows.

According to Refs. [1, 2], at the redshift condition $1+z_{\mathrm{acc}}=1.632$, the "present" accelerated expansion began after the big bang at the time

$$t(z_{\mathrm{acc}})=2.43\cdot 10^{17}\,\mathrm{s}=7.70\,\mathrm{Gyr}\,. \tag{2.35}$$

Then, by Eq. (2.15)), we find the corresponding scale factor

$$R=R(z_{\mathrm{acc}})=R_0/(1+z_{\mathrm{acc}})=8.43\cdot 10^{25}\,\mathrm{m}\,. \tag{2.36}$$

Using Eqs. (2.27) and (2.36), the fictitious beginning of the "present" accelerated expansion takes place before the time

$$t_{\mathrm{acc}}=t_0-t=2.72\cdot 10^{17}\,\mathrm{s}=8.62\,\mathrm{Gyr}\,, \tag{2.37}$$

seen backward from the present age $t_0=4.358\cdot 10^{17}\,\mathrm{s}$ of the universe (see Table I). We can solve the discrepancy between Eqs. (2.35) and (2.37) if we use the cosmological "constant" (2.31) in the following expression

$$(t_0-\tilde{t})=(t_0-t)\,\frac{\Omega_\Lambda^{1/2}H_0}{(c^2\tilde{\Lambda}/3)^{1/2}}=$$

$$=(t_0-t)\,\frac{\Omega_\Lambda^{1/2}H_0}{(8/135\,\pi^3)^{1/2}(k^4T^4/N(T)\,E_{\mathrm{Pl}}^2\,\hbar^2)^{1/2}}\,, \tag{2.38}$$

derived in Refs. [1, 2].

Thus, we can solve the discrepancy between Eqs. (2.35) and (2.37) if in Eq. (2.38) we select $\tilde{t}=t(z_{\mathrm{acc}})=2.43\cdot 10^{17}\,\mathrm{s}$, so that for $t_0-t=2.72\cdot 10^{17}\,\mathrm{s}$ the result (2.38) provides $(c^2\tilde{\Lambda}/3)^{1/2}=2.55\cdot 10^{-18}\,\mathrm{s}^{-1}$ with $\tilde{\Lambda}=2.16\cdot 10^{-52}\,\mathrm{m}^{-2}$ or $\rho_{\mathrm{vac}}(T)\,c^2=6.50\cdot 10^3\,\mathrm{eV\,cm}^{-3}$ for $N(T)=3.362644$ at $T=61\,\mathrm{K}$.

Therefore, in the matter-dominated universe, by this constant temperature $T=61\,\mathrm{K}$, the influence of the dark energy (vacuum energy) begins a little earlier for the "present" accelerated expansion

$$R=R(z_{\mathrm{acc}})=R_0\,\mathrm{e}^{(c^2\tilde{\Lambda}/3)^{1/2}(\tilde{t}-t_0)}\,. \tag{2.39}$$

Finally, we summarize still the results of the quantum gravity [1, 2] for the massless universe ($R\le R_{\mathrm{Pl}}$). According to Refs. [1, 2], we have obtained here the following expressions for the connection between distance R as well as thermal energy kT or particle (P) rest energy $E_0(\mathrm{P})$:

$$R = \frac{G_N}{c^4} kT = \frac{\hbar c}{E_{\mathrm{Pl}}^2} kT = \frac{G_N}{c^4} E_0(\mathrm{P}), \tag{2.40}$$

for the vacuum energy density:

$$\rho_{\mathrm{vac}}(R)\, c^2 = \hat{\rho}_{\mathrm{vac}}\, c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{E_{\mathrm{Pl}}^2}{\hbar c\, R^2}, \tag{2.41}$$

for the cosmological "constant":

$$\Lambda = \hat{\Lambda} = \frac{16}{45}\pi^3 \Omega_\gamma \frac{1}{R^2}, \tag{2.42}$$

for the distance:

$$R = ct, \tag{2.43}$$

for the thermal particle number density:

$$n_{\mathrm{P}}(R) = \frac{\rho_{\mathrm{vac}}(R)\, c^2}{kT} = \frac{\rho_{\mathrm{vac}}(R)\, c^2}{E_0(\mathrm{P})} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{1}{R^3}, \tag{2.44}$$

for the kinetic photon energy:

$$E_{\mathrm{K}}(\gamma) = 2.701178\, kT = 2.701178 \frac{E_{\mathrm{Pl}}^2}{\hbar c} R \tag{2.45}$$

and for the thermal photon number density:

$$n_\gamma(R) = 2.701178 \frac{\rho_{\mathrm{vac}}(R)\, c^2}{E_{\mathrm{K}}(\gamma)} = \frac{\rho_{\mathrm{vac}}(R)\, c^2}{kT} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{1}{R^3}. \tag{2.46}$$

Consequently, we have $n_{\mathrm{P}}(R) = n_\gamma(R)$.

The usefulness of the expressions (2.40) to (2.46) is demonstrated at the big bang, for example, in Sec. 3.1.

In Refs. [1-3, 5], for the three light (left handed) neutrinos (electron neutrino ν_{e}, muon neutrino ν_μ and tauon neutrino ν_τ), we have estimated the following rest energies for these neutrinos:

$$E_0(\nu_{\mathrm{e}}) \cong (1.589^{+0.078}_{-0.098}) \cdot 10^{-3}\mathrm{eV}, \tag{2.47}$$

$$E_0(\nu_\mu) \cong (8.85^{+0.14}_{-0.16}) \cdot 10^{-3}\mathrm{eV}, \tag{2.48}$$

$$E_0(\nu_\tau) \cong (4.93^{+0.12}_{-0.10}) \cdot 10^{-2}\mathrm{eV}. \tag{2.49}$$

Then, the sum of the neutrino rest energies (see Eqs. (2.47) to (2.49)) gives the approximate solution

$$\sum_i E_0(\nu_i) = E_0(\nu_e) + E_0(\nu_\mu) + E_0(\nu_\tau) \cong (5.97^{+0.14}_{-0.13}) \cdot 10^{-2}\,\text{eV}, \qquad (2.50)$$

where the subscript $i = \mathrm{e}, \mu, \tau$ characterizes the e, μ and τ neutrino.

According to Refs. [1-3, 5], we have derived the following general expression for the light neutrino density parameters:

$$\Omega_\nu(\nu_i) = (429.889 \pm 0.095)\, E_0(\nu_i)\, \Omega_\gamma =$$
$$= (0.010630 \pm 0.000011)\, E_0(\nu_i)\, h^{-2}, \qquad (2.51)$$

where

$$\Omega_\gamma = (2.4728 \pm 0.0025) \cdot 10^{-5} h^{-2}. \qquad (2.52)$$

Then, using the rest energies (2.47) to (2.49) together with the radiation density $\Omega_\gamma = (5.46 \pm 0.19) \cdot 10^{-5}$ (see Table I), by Eqs. (2.51) and (2.52), we get the following neutrino density parameters:

$$\Omega_\nu(\nu_e) \cong (0.683^{+0.034}_{-0.042})\, \Omega_\gamma =$$
$$= (1.69^{+0.09}_{-0.11}) \cdot 10^{-5} h^{-2} = (3.73^{+0.32}_{-0.36}) \cdot 10^{-5}, \qquad (2.53)$$
$$\Omega_\nu(\nu_\mu) \cong (3.805^{+0.061}_{-0.070})\, \Omega_\gamma =$$
$$= (9.41^{+0.16}_{-0.18}) \cdot 10^{-5} h^{-2} = (2.08 \pm 0.11) \cdot 10^{-4}, \qquad (2.54)$$
$$\Omega_\nu(\nu_\tau) \cong (21.19^{+0.52}_{-0.44})\, \Omega_\gamma =$$
$$= (5.24^{+0.13}_{-0.11}) \cdot 10^{-4} h^{-2} = (1.157^{+0.069}_{-0.064}) \cdot 10^{-3}. \qquad (2.55)$$

The sum of Eqs. (2.53) to (2.55) determines the total neutrino density parameter as follows

$$\Omega_\nu = \sum_i \Omega_\nu(\nu_i) \cong (25.68^{+0.62}_{-0.55})\, \Omega_\gamma =$$
$$= (6.35^{+0.16}_{-0.14}) \cdot 10^{-4} h^{-2} = (1.402^{+0.083}_{-0.079}) \cdot 10^{-3}. \qquad (2.56)$$

In Refs. [1-3, 5], we have estimated the rest energy of the X and Y gauge bosons as well as the magnetic monopoles to

$$E_0(\mathrm{X}) = E_0(\mathrm{Y}) = (2.675^{+0.058}_{-0.063}) \cdot 10^{16}\,\text{GeV} \qquad (2.57)$$

and

$$E_0(\mathrm{M}) = (6.849 \pm 0.063) \cdot 10^{17}\ \mathrm{GeV}, \tag{2.58}$$

respectively. According to Refs. [1-3, 5], the correctness of the estimations of the rest energies of these particles of the supersymmetric grand unification theories (SUSY GUTs) (see Eqs. (2.57) and (2.58)) can be proved by their coupling constant α_{GUT}, which is defined by

$$\alpha_{\mathrm{GUT}} = \frac{E_0(\mathrm{X})}{E_0(\mathrm{M})} = 0.0391^{+0.0012}_{-0.0013} \tag{2.59}$$

in excellent agreement with the value $\alpha_{\mathrm{GUT}} \approx 0.04$ obtained by a completely different way via the extrapolation of the gauge couplings constants of the standard model to very high energies [7].

By the Gunn-Peterson effect [1-6], the intergalactic neutral hydrogen (HI) gas, which has the density $n_{\mathrm{HI}}(z) \approx 2.42 \cdot 10^{-11} \tau(z) h E(z)\ \mathrm{cm}^{-3}$ (see Ref. [6]) {using $E(z) = \left[\Omega_\Lambda + \Omega_{\mathrm{m}}(1+z)^3 + \Omega_{\mathrm{r}}(1+z)^4\right]^{1/2}$ (see Refs. [1-3, 5])}, can be estimated according to Ref. [6] by the measurement of its optical depth $\tau(z)$ from flux decrement in quasar spectra at the wavelength $\lambda_{\mathrm{Ly}\alpha} = 121.6\,\mathrm{nm}$ in $\mathrm{Ly}\alpha$ absorption (of the neutral hydrogen), which has a very large cross-section for

$$T_{\mathrm{IGM}} << T_{\mathrm{Ly}\alpha} = \frac{2\pi \hbar c}{k\, \lambda_{\mathrm{Ly}\alpha}} = 1.183 \cdot 10^5\ \mathrm{K}. \tag{2.60}$$

The observations show that at a redshift of order $z \approx 5\text{-}6$ the neutral hydrogen, left over from the time of recombination, becomes reionized by this ultraviolet light ($\lambda_{\mathrm{Ly}\alpha} = 121.6\,\mathrm{nm}$) from the massive stars (quasars) [6, 8].

Consequently, according to Refs. [1-3, 5], via Eqs. (2.54) to (2.56), at $N(T) = 3.362644$ (see Eq. (2.6)), for this reionization, we assume that the redshifts z_{reion} of the neutrinos

$$z_{\mathrm{reion}}\left(\textstyle\sum_i \nu_i\right) = \frac{2 \sum_i \Omega_\nu(\nu_i)}{N(T)\, \Omega_\gamma} - 1 = 14.27^{+0.37}_{-0.33}, \tag{2.61}$$

$$z_{\mathrm{reion}}(\nu_\tau) = \frac{2\, \Omega_\nu(\nu_\tau)}{N(T)\, \Omega_\gamma} - 1 = 11.60^{+0.31}_{-0.26} \tag{2.62}$$

and

$$z_{\text{reion}}(\nu_\mu) = \frac{2\,\Omega_\nu(\nu_\mu)}{N(T)\,\Omega_\gamma} - 1 = 1.263^{+0.036}_{-0.042} \tag{2.63}$$

define the beginning, the half and the end of the reionization, respectively. This assumption is supported by the neutrino temperatures

$$T(\textstyle\sum_i \nu_i) = \left\{1 + z_{\text{reion}}(\textstyle\sum_i \nu_i)\right\} T_0 = 41.62\ \text{K}\,, \tag{2.64}$$

$$T(\nu_\tau) = \left\{1 + z_{\text{reion}}(\nu_\tau)\right\} T_0 = 34.34\ \text{K} \tag{2.65}$$

and

$$T(\nu_\mu) = \left\{1 + z_{\text{reion}}(\nu_\mu)\right\} T_0 = 6.168\ \text{K}\,, \tag{2.66}$$

since they fulfil the condition (2.60).

We see that the value (2.62) agrees excellently with the redshifts of the half reionization in the Tables I and II, which were found on completely other way. This fact is a strong argument for the correctness of the assumptions (2.61) and (2.63) as the beginning and the end of the reionization, respectively.

At the electron neutrino, its blueshift [1-3, 5] is defined by

$$1 + z(\nu_e) = \frac{2\,\Omega_\nu(\nu_e)}{N(T)\,\Omega_\gamma} = 0.406^{+0.020}_{-0.025}\,, \tag{2.67}$$

so that the scale factor of the final state of the massive universe is determined via the expression (2.19).

In this work, the results of the most recent analysis of the observed cosmological parameters and the 3 neutrino oscillation parameters of Ref [16] are not applied, since within the error limits these most recent data agree well with the corresponding cosmological parameters of Ref. [7] and the 3 neutrino oscillation parameters of Ref [10]. Therefore, a renewed calculation of all astrophysical parameters, determined in Refs [1-5], where they were derived by aid of the cosmological parameters of Ref. [7] and the 3 neutrino oscillation parameters of Ref. [10], is first necessary, if new better measurements are available, since within the error limits the application of the cosmological parameters and 3 neutrino oscillation parameters from Ref. [16] gives similar results. Thus, in this work, we use always as basis the cosmological parameters from Table I, given by Ref. [7]. We will return to this problem in Sec. 4, where we have exceptionally derived correspondingly new cosmological parameters

(Table VI) as a result of a new interpretation of the dark matter and dark energy, which are needed in the second half of Sec. 7.

Using the data of Tables III to V, these considerations, derived in this Sec. 2 for the universe, are also valid for the anti-universe.

3 Final state and big bang as well as the time reversal solution and the eternal cyclic evolution of universe and anti-universe

In this Sec. 3, via results of Refs. [1, 2], we give a detailed review, whereat we show the relationship between the final state of the universe and the big bang.

3.1 The final state of the universe and the big bang

According to the results of Refs. [1, 2], in this chapter, via the relationship between the final state of the universe and the big bang (see also Sec. 3.11), we calculate their parameters.

At the time $t_{\mathrm{eff}} = 8.034 \cdot 10^{17}\,\mathrm{s} = 25.46\,\mathrm{Gyr}$ (see Eq. (2.28)) or the scale factor $R_{\mathrm{eff}} = 2.67 \cdot 10^{26}\,\mathrm{m}$ (see Eq. (2.26)), we have the end of the "present" accelerated expansion because of the effective equilibrium (see Eq. (2.26)), so that here the final value of the dark (d) energy E_{d} is found via the vacuum energy density $\rho_{\mathrm{vac},\Lambda}\, c^2 = \Omega_\Lambda \rho_{0\mathrm{C}}\, c^2 = 3.27 \cdot 10^3\,\mathrm{eV\,cm^{-3}}$ (see Eq. (2.32)) to

$$E_{\mathrm{d}} = \Omega_\Lambda \rho_{0\mathrm{C}} c^2\, \tfrac{4}{3} \pi\, d_{\mathrm{eff}}^3 , \tag{3.1}$$

where the proper distance d_{eff} is defined by

$$d_{\mathrm{eff}} = R_{\mathrm{eff}}\, r_{\mathrm{eff}} = R_{\mathrm{eff}}\, r \tag{3.2}$$

with $r = r_{\mathrm{eff}}$ as the dimensionless, time-independent, comoving coordinate distance (see Refs. [1, 2, 6, 8, 11]).

Using the hypothesis of the joint origin of the dark matter and dark energy by the three sterile, neutrino types $\hat{\nu}_\Lambda$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$ (see, e.g., Ref. [1]), we can assume that this dark energy E_{d} must be distributed among the decay products