CHAPTER 8

GRAVITATIONAL WAVES

We will show that the existence of gravitational waves is embedded in GEM. In the framework of the theory of informatons a gravitational wave is understood as the macroscopic manifestation of the fact that the "train" of informatons emitted by an oscillating source and travelling with the speed of light in a certain direction is a spatial sequence of informatons whose characteristic angle is harmonically fluctuating along the "train" what implies that the component of their g-index perpendicular to their velocity $\vec{c}$ and their β-index fluctuate harmonically in space. Gravitational waves transport gravitational energy because some of the informatons that constitute the "train" are carriers of energy. They are called gravitons.

8.1 THE WAVE EQUATION

In free space - where $\rho_G = \vec{J}_G = 0$ - the GEM equations are:

1. $div\vec{E}_g = 0$

2. $div\vec{B}_g = 0$

3. $rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$

4. $rot\vec{B}_g = \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t}$

To attempt a solution of a group of simultaneous equations, it is usually a good plan to separate the various functions of space to arrive at equations that give the distributions of each.

It follows from (3):

$$rot(rot\vec{E}_g) = -rot\left(\frac{\partial \vec{B}_g}{\partial t}\right) \quad (3')$$

Because[1] $rot(rot\vec{F}) = grad(div\vec{F}) - \nabla^2\vec{F}$, where $\nabla^2$is the Laplacian, (3')
leads to:

$$grad(div\vec{E}_g) - \nabla^2\vec{E}_g = -rot(\frac{\partial \vec{B}_g}{\partial t}) = -\frac{\partial}{\partial t}(rot\vec{B}_g)$$

And taking into account (1) and (4):

$$\nabla^2\vec{E}_g = \frac{1}{c^2}.\frac{\partial^2\vec{E}_g}{\partial t^2} \qquad (5)$$

This is the general form of the wave equation. This form applies as well to the g-induction, as is readily shown by taking first the rotor of (4) and then substituting (2) and (3):

$$\nabla^2\vec{B}_g = \frac{1}{c^2}.\frac{\partial^2\vec{B}_g}{\partial t^2} \qquad (5')$$

Solutions of this equation describe how disturbances of the gravitational field propagate as waves with speed c.

To illustrate this we consider the special case of space variation in one dimension only. If we take the *x*-component of (5) and have space variations only in the *z*-direction, the equation becomes simply:

$$\frac{\partial^2 E_{gx}}{\partial z^2} = \frac{1}{c^2}.\frac{\partial^2 E_{gx}}{\partial t^2}$$

This equation has a general solution of the form

$$E_{gx} = f_1\left(t - \frac{z}{c}\right) + f_2\left(t + \frac{z}{c}\right) \qquad (6)$$

The first term of (6) represents the wave or function f_1 traveling with velocity *c* and unchanged form in the positive *z*-direction, the second term represents the wave or function f_2 traveling with velocity *c* and unchanging form in the negative *z*-direction.

8.2 GRAVITATIONAL WAVE EMITTED BY A HARMONICALLY OSCILLATING PARTICLE

In fig 15 we consider a point mass *m* that harmonically oscillates around the origin of the inertial reference frame ***O*** with frequency $\nu = \frac{\omega}{2.\pi}$. At the moment *t* it passes at P_I. We suppose that the speed of the particle is always much smaller than the speed of light and that it is described by:

$$v(t) = V.\cos\omega\, t$$

The elongation *z(t)* and the acceleration *a(t)* are then expressed as:

$$z(t) = \frac{V}{\omega}.\cos(\omega t - \frac{\pi}{2}) \quad \text{and} \quad a(t) = \omega.V.\cos(\omega t + \frac{\pi}{2})$$

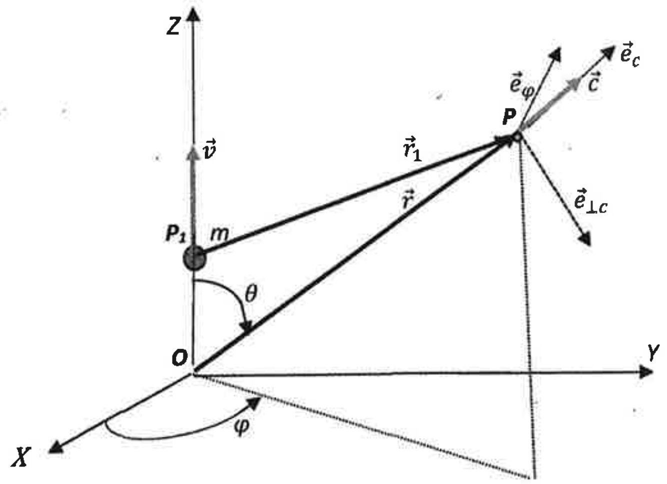

Fig 15

We restrict our considerations about the gravitational field of m to points P that are sufficiently far away from the origin O. Under that condition we can posit that the fluctuation of the length of the vector $\overrightarrow{P_1P} = \vec{r}_1$ is very small relative to the length of the time-independent position vector $\vec{r}$, that defines the position of P relative to the origin O. In other words: we assume that the amplitude of the oscillation is very small relative to the distances between the origin and the points P on which we focus.

8.2.1 The transversal gravitational field of a harmonically oscillating particle

Starting from the complex quantity $\bar{V} = V.e^{j.0}$ - that is representing $v(t)$ - $\bar{E}_{g\perp c}$, the complex representation of the time dependent part of the transversal component of $\vec{E}_g$, and $\bar{B}_{g\varphi}$, the complex representation of $B_{g\varphi}$, at P follow immediately from §5.2:

$$\bar{E}_{g\perp c} = -\frac{m.\bar{V}}{4\pi}.e^{-j.k.r}.(\frac{1}{\eta_0.c.r^2} + \frac{j.\omega.v_0}{r}).\sin\theta$$

$$\bar{B}_{g\varphi} = -\frac{v_0.m.\bar{V}}{4\pi}.e^{-j.k.r}.(\frac{1}{r^2} + \frac{j.k}{r}).\sin\theta$$

where $k = \frac{\omega}{c}$ the phase constant. Note that $\bar{B}_{g\varphi} = \frac{\bar{E}_{g\perp c}}{c}$.

Thus, relative to $\boldsymbol{O}$, $B_{g\varphi}$ and the time dependent part of $E_{g\perp c}$ are expressed as functions of the space and time coordinates as:

$$B_{g\varphi}(r,\theta;t) = \frac{E_{g\perp c}(r,\theta;t)}{c}$$
$$= \frac{v_0.m.V.\sin\theta.\sqrt{1+k^2r^2}}{4\pi r^2}.\cos(\omega t - kr + \Phi + \pi)$$

with $tg\Phi = kr$.

So, an harmonically oscillating particle emits a transversal "gravitomagnetic" wave that propagates out of the mass with the speed of light:

In points at a great distance from the oscillating mass, specifically there where $r >> \frac{1}{k} = \frac{c}{\omega}$, this expression asymptotically equals:

$$B_{g\varphi} = \frac{E_{g\perp c}}{c} = \frac{v_0.k.m.V.\sin\theta}{4\pi r}.\sin(\omega t - kr)$$
$$= \frac{v_0.m.\omega.V.\sin\theta}{4\pi c r}.\sin(\omega t - kr)$$

$$= -\frac{v_0.m.a\left(t - \frac{r}{c}\right).\sin\theta}{4\pi cr}$$

The intensity of the "far gravitational field" is inversely proportional to r, and is determined by the component of the acceleration of m, that is perpendicular to the direction of $\vec{e}_c$.

8.2.2 The longitudinal gravitational field of a harmonically oscillating particle

The oscillation of the point mass m along the Z-axis is responsible for the existence of a fluctuation of $r_0 = P_0P$, the distance travelled by the informatons at the moment t when they pass near P. Within the framework of our approximations:

$$r_0(t) \approx r_1(t) \approx r - z(t - \frac{r}{c}).\cos\theta = r.\left\{1 - \frac{z(t - \frac{r}{c})}{r}.\cos\theta\right\}$$

and

$$(\frac{1}{r_0})^2 \approx \frac{1}{r^2}.(1 + 2.\frac{z(t - \frac{r}{c})}{r}.\cos\theta)$$

From §5.2, it follows:

$$E_{gc} = -\frac{m}{4.\pi.\eta_0.r^2} - \frac{m}{4.\pi.\eta_{.0}.r^3}.2.z(t - \frac{r}{c}).\cos\theta$$

So $\bar{E}_{gc}$, the complex representation of the time dependant part of the longitudinal gravitationel field is:

$$\bar{E}_{gc} = -\frac{m.\bar{V}}{4\pi}.e^{-jkr}.\frac{2}{j.\omega.\eta_0.r^3}.\cos\theta$$

We conclude that an harmonically oscillating point mass emits a longitudinal gravitational wave that - relative to the position of the mass - expands with the speed of light:

$$E_{gc}(r,\theta;t) = \frac{m.V}{4.\pi.\eta_0.c.k}.\frac{2}{r^3}.sin(\omega t - kr)$$

Because its amplitude is proportional to $\frac{1}{r^3}$, at a great distance from the emitter the longitudinal field can be neglected relative to the transversal.

8.3 GRAVITATIONAL WAVE EMITTED BY AN OBJECT WITH VARIABLE REST MASS

Another phenomenon that is the source of a gravitational wave is the conversion of rest mass into energy (what per example happens in the case of radioactive processes). To illustrate this, let us - relative to an inertial reference frame - consider a particle with rest mass m_0 that - due to intern instability - during the period $(0, \Delta t)$ emits EM radiation.

This implies that that particle during that time interval is emitting electromagnetic energy U_{EM} carried by photons (and gravitational energy U_{GEM}* carried by gravitons) that propagate with the speed of light. Between the moment $t = 0$ and the moment $t = \Delta t$, the rest mass of the particle is, because of this event, decreasing *with* an amount $\frac{U_{EM}(+U_{GEM})}{c^2}$ from the value m_0 to the value m_0'. Because the gravitational field is determined by the rest mass, this implies that if $t < 0$ the source of the gravitational field of the particle is m_0 and for $t > \Delta t$ it is m_0'. It follows that at the moment t the gravitational field at a point P at a distance $r>c.t$ is proportional to m_0, and at a point at a distance $r < c.(t - \Delta t)$ to m_0'.

During the period $(t, t+\Delta t)$ the gravitational field at a point at a distance $r = c.t$ changes from the situation where it is determined by m_0 to the situation where it is determined by m_0'. So, the conversion of rest mass of an object into radiation is the cause of a kink in the gravitational field of that object, a kink that with the speed of light - together with the emitted radiation - propagates out of the object.

We can conclude that the conversion of (a part of) the rest mass of an object into radiation goes along with the emission by that object of a gravitational wave.

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* negligible in first approximation

The effect of the decrease - during the time interval $(0, \Delta t)$ - of the rest mass of a point mass on the magnitude of its g-field E_g at the point P at a distance r is shown in the plot of fig. 16.

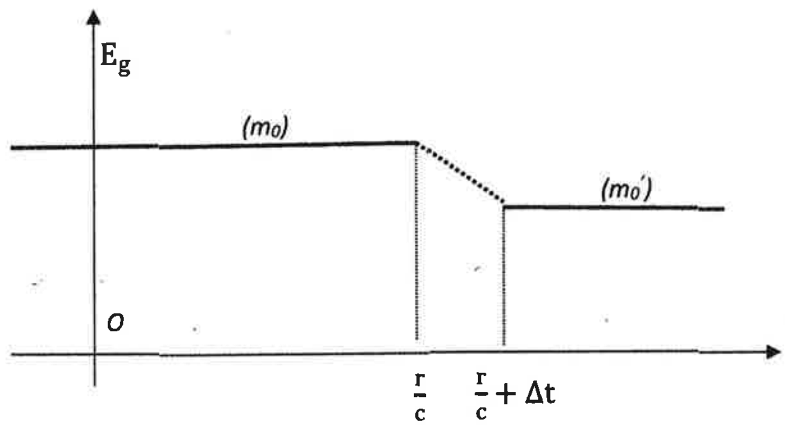

Fig 16

1. Until the moment $t = \frac{r}{c}$, the effect of the conversion of rest mass into radiation has not yet reached P. So, during the period $(0, \frac{r}{c})$ the quantity of mass-energy enclosed by an hypothetical sphere with radius r centered on the particle is still m_0 (the remaining part of the rest mass + all the radiation that during the mentioned period has arisen from the conversion of rest mass). From the first GEM equation it follows:

$$E_g = \frac{m_0}{4\pi\eta_0 . r^2}$$

2. From the moment $t = \frac{r}{c} + \Delta t$, the radiation generated by the conversion of rest mass has left the space enclosed by the hypothetical sphere with radius r, that from that moment only contains the remaining rest mass m_0'. From the first GEM equation it follows:

$$E_g = \frac{m_0{}'}{4\pi\eta_0 . r^2}$$

3. During the time interval $(\frac{r}{c}, \frac{r}{c} + \Delta t)$, the mass-energy enclosed by the hypothetical sphere with radius r is decreasing (not necessary linearly) because mass-energy flows out in the form of radiation. So, during that period E_g at P is decreasing.

8.4 ON THE DETECTION OF GRAVITATIONAL WAVES USING AN INTERFEROMETER

Let x and y be the directions of the arms L_1 and L_2 of an interferometer, and let z be the direction perpendicular to the plane defined by the arms. We consider the (optimized) situation where a uniform plane gravitational wave $(\vec{E}_g, \vec{B}_g)$ of sinusoidal form is travelling in the z-direction. We assume that the gravitational field $\vec{E}_g$ is in the x-direction and that the gravitational induction $\vec{B}_g$ is in the y-direction. If E_{MAX} is the amplitude of the gravitational field, than - according to GEM - $\vec{E}_g$ is given in magnitude by: $E_g = E_{MAX}.sin(\omega t - kz)$ with $k = \frac{\omega}{c}$, and the magnitude of $\vec{B}_g$ is given by: $B_g = \frac{E_g}{c}$.

When that gravitational wave is falling on the plane of the interferometer, the gravitational field $\vec{E}_g$ - being in the direction of L_1 - will induce a longitudinal mechanical wave in the tube of the arm L_1 what will result in a (very slight) oscillation of the mirror at the end. The mirror at the end of the arm L_2 will not react on $\vec{E}_g$ because that field is perpendicular to L_2. So, the effective length of the light beam that is travelling through L_1 will differ (in the manner of an oscillation) from the effective length of the light beam that is travelling through L_2, and the detector will record that the outgoing and reflected beams are out of phase. It is clear that this can be generalized and that *we can conclude that, according to GEM ,the interferometer will reacts on a gravitational wave*.

8.5 THE ENERGY RADIATED BY A HARMONICALLY OSCILLATING PARTICLE

8.5.1 Poynting's theorem

In free space a gravitational field is completely defined by the vectoral functions $\vec{E}_g(x,y,z;t)$ and $\vec{B}_g(x,y,z;t)$. It can be shown[2] that the spatial area G enclosed by the surface S - at the moment t - contains an amount of energy given by the expression:

$$U = \iiint_G (\frac{\eta_0.E_g^2}{2} + \frac{B_g^2}{2\nu_0}).dV$$

The rate at which the energy escapes from G is:

$$-\frac{\partial U}{\partial t} = -\iiint_V (\eta_0.\vec{E}_g.\frac{\partial \vec{E}_g}{\partial t} + \frac{1}{v_0}.\vec{B}_g.\frac{\partial \vec{B}_g}{\partial t}).dV$$

According to the third law of GEM:

$$rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

and according to the fourth law:

$$rot\frac{\vec{B}_g}{v_0} = \eta_0.\frac{\partial \vec{E}_g}{\partial t}$$

So:

$$-\frac{\partial U}{\partial t} = \iiint_G (\frac{\vec{B}_g}{v_0}.rot\vec{E}_g - \vec{E}_g.rot\frac{\vec{B}_g}{v_0}).dV = \iiint_G div(\frac{\vec{E}_g \times \vec{B}_g}{v_0}).dV$$

By application of the theorem of Ostrogradsky[1]: $\iiint_G div\vec{F}.dV = \oiint_S \vec{F}.\overrightarrow{dS}$, we can rewrite this as:

$$-\frac{\partial U}{\partial t} = \oiint_S \frac{\vec{E}_g \times \vec{B}_g}{v_0}.\overrightarrow{dS}$$

from which we can conclude that the expression

$$\frac{\vec{E}_g \times \vec{B}_g}{v_0}.\overrightarrow{dS}$$

defines the rate at which energy flows in the sense of the positive normal through the surface element dS at P .

So, the density of the energy flow at P is:

$$\frac{\vec{E}_g \times \vec{B}_g}{v_0}$$

This vectoral quantity is called the "Poynting's vector". It is represented by $\vec{P}$:

$$\vec{P} = \frac{\vec{E}_g \times \vec{B}_g}{\nu_0}$$

The amount of energy transported through the surface element *dS* in the sense of the positive normal during the time interval *dt* is:

$$dU = \frac{\vec{E}_g \times \vec{B}_g}{\nu_0} . \overrightarrow{dS} . dt$$

8.5.2 The energy radiated by a harmonically oscillating particle – gravitons

In §8.2 it is shown that an harmonically oscillating point mass *m* radiates a gravitomagnetic wave that at a far point *P* is defined by (see fig 15):

$$\vec{E}_g = E_{g\perp c} . \vec{e}_{\perp c} = \frac{\nu_0 . m . \omega . V . \sin\theta}{4\pi r} . \sin(\omega t - kr) . \vec{e}_{\perp c}$$

$$\vec{B}_g = B_{g\varphi} . \vec{e}_\varphi = \frac{\nu_0 . m . \omega . V . \sin\theta}{4\pi c r} . \sin(\omega t - kr) . \vec{e}_\varphi$$

The instantaneous value of Poynting's vector at *P* is:

$$\vec{P} = \frac{\nu_0 . m^2 . \omega^2 . V^2 . \sin^2\theta}{16 . \pi^2 . c . r^2} . \sin^2(\omega t - kr) . \vec{e}_c$$

The amount of energy that, during one period *T* , flows through the surface element *dS* that at *P* is perpendicular to the direction of the movement of the informatons, is:

$$dU = \int_0^T P . dt . dS = \frac{\nu_0 . m^2 . \omega^2 . V^2 . \sin^2\theta}{16 . \pi^2 . c . r^2} . \frac{T}{2} . dS$$

And, with $\omega = \frac{2.\pi}{T} = 2.\pi.\nu$:

$$dU = \frac{v_0.m^2.V^2.\sin^2\theta}{8c}.\nu.\frac{dS}{r^2}$$

$\frac{dS}{r^2} = d\Omega$ is the solid angle under which dS is "seen" from the origin. So, the oscillating mass radiates per unit of solid angle in the direction θ, per period, an amount of energy u_Ω:

$$u_\Omega = \frac{v_0.m^2.V^2.\sin^2\theta}{8c}.\nu \qquad (1)$$

This quantity is greatest in the direction perpendicular to the movement of the mass ($\theta = 90°$) and it is proportional to the frequency of the wave, thus proportional to the frequency at which the mass is oscillating.

We posit that the energy radiated by an oscillating point mass travels through space in the form of particle-like packets of energy, called "gravitons" and that the energy U_g transported by a graviton is proportional to the frequency of the oscillator, so:

$$Ug = h'.\nu \qquad (2)$$

h' plays the role of Planck's constant in electromagnetism.

A graviton can be understood as an information transporting a quantum of energy.

From (1) and (2), it follows that the number of gravitons emitted per period and per unit of solid angle in the direction θ by an oscillating point mass *m* is:

$$N_{g\Omega} = \frac{u_\Omega}{h'.\nu} = \frac{v_0.m^2.V^2.\sin^2\theta}{8.h'.c}$$

what is independent of the duration of a period.

If we assume that the number of gravitons and the number of photons emitted by an oscillating charged particle (e. g. an electron) are of the same order of magnitude, it turns out that the value of *h'* depends on the nature of the emitter and that the energy of a graviton is many orders smaller than that of a photon [2].

8.6 CONCLUSION

The existence of gravitational waves is embedded in the GEM description of gravity. According to the theory of informatons a gravitational wave is the macroscopic manifestation of the fact that the "train" of informatons emitted by an oscillating source and travelling with the speed of light in a certain direction is a spatial sequence of informatons whose characteristic angle is harmonically fluctuating along the "train" what implies that the component of their g-index perpendicular to their velocity $\vec{c}$ and their β-index fluctuate harmonically in space. Gravitational waves transport gravitational energy because some of the informatons that constitute the "train" are carriers of energy. They are called gravitons. However, the energy quantum carried by a graviton is small in such a way that it is very difficult to give experimental evidence of its existence.

References

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2.Acke, Antoine. *Gravitatie en elektromagnetisme*. Gent: Uitgeverij Nevelland, 2008