CHAPTER 6

THE MAXWELL-HEAVISIDE EQUATIONS

The gravitational field is set up[1],[2],[3] by a given distribution of - whether or not moving - masses and it is defined by a vector field with two components: the "*g-field*" characterized by the vector $\vec{E}_g$ and the "*g-induction*" characterized by the vector $\vec{B}_g$. These components each have a value defined at every point of space and time and are thus, relative to an inertial reference frame ***O***, regarded as functions of the space and time coordinates.

Let us focus on the contribution to a gravitational field of one of its sources: a certain mass *m*. We focus, more specifically, on the contribution of *m* to the flow of g-information at an arbitrary point *P* in the field. That flow is made up of informatons that pass near *P* in a specific direction with velocity $\vec{c}$ and it is characterized by *N*, the rate per unit area at which these informatons cross an elementary surface perpendicular to the direction in which they move. The cloud of these informatons in the vicinity of *P* is characterized by its density *n*: *n* is the number of informatons per unit volume. *N* and *n* are linked by the relationship:

$$n = \frac{N}{c} \qquad (1)$$

The definition in chapter 2 of an informaton implies that every informaton that passes near *P* is characterized by two attributes that refer to its emitter: its g-index $\vec{s}_g$ and its β-index $\vec{s}_\beta$. s_g, the magnitude of the g-index is the elementary quantity of g-information. It is a fundamental physical constant. $\vec{s}_\beta$ refers to the state of motion of the source of the informaton and is defined by the relationship

$$\vec{s}_\beta = \frac{\vec{c} \times \vec{s}_g}{c} \qquad (2)$$

The informatons emitted by *m* that pass near *P* with velocity $\vec{c}$ contribute there to the *density of the g-information flow* with an amount $(N.\vec{s}_g)$. That vectoral quantity is the rate per unit area at which g-information at *P* crosses an elementary surface perpendicular to the direction in which it moves. It is the contribution of *m* to the g-field at *P*. We put

$$\vec{E}_g = N.\vec{s}_g$$

And the same informatons contribute there to the *density of the g-information cloud* with an amount $(n.\vec{s}_\beta)$. That vectoral quantity determines at P the amount of β-information per volume unit. It is the contribution of m to the g-induction at P. We put:

$$\vec{B}_g = n.\vec{s}_\beta$$

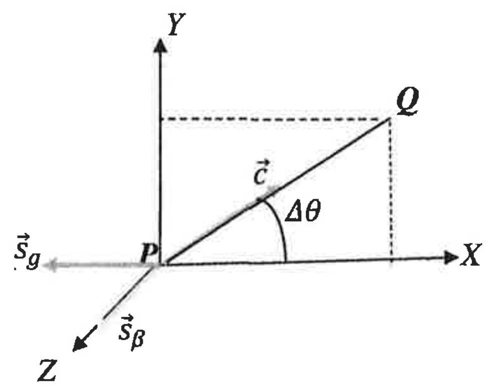

Fig 8

In fig 8, we consider the flow of informatons that - at the moment t - pass near P with velocity $\vec{c}$. These informatons are completely defined by their attributes $\vec{s}_g$ and $\vec{s}_\beta$, respectively their g-index and their β-index. $\Delta\theta$ is their characteristic angle: the angle between the lines carrying $\vec{s}_g$ and $\vec{c}$ that is characteristic for the movement of the emitter.

The infinitesimal change of the attributes of an informaton at P between the moments t and $(t + dt)$, is governed by the kinematics of that informaton. An informaton that at the moment t passes at P is at the moment $(t + dt)$ at Q, with $PQ = c.dt$. This implies that the spatial variation of the attributes of an informaton between P and Q at the moment t equals the change in time of those attributes at P between the moment $(t - dt)$ and the moment t.

On the macroscopic level, this implies that there must be a relationship between the change in time of the gravitational field $(\vec{E}_g, \vec{B}_g)$ *at a point P and the spatial variation of that field in the vicinity of P.*

The intensity of the spatial variation of the components of the gravitational field at P is characterized by $div\vec{E}_g$, $div\vec{B}_g$, $rot\vec{E}_g$ and by $rot\vec{B}_g$ and the rate at which these components change in time by $\dfrac{\partial\vec{E}_g}{\partial t}$ and by $\dfrac{\partial\vec{B}_g}{\partial t}$.

From the above it can be concluded that it makes sense to investigate the relationships between the quantities that characterize the spatial variations of $(\vec{E}_g, \vec{B}_g)$ and the rate's at which they change in time.

6.1 $div\vec{E}_g$ - THE FIRST EQUATION IN FREE SPACE

In chapter 3 it is shown that the physical fact that the rate at which g-information flows inward a closed empty space must be equal to the rate at which it flows outward, can be expressed as:

$$\oiint_S \vec{E}_g . \overrightarrow{dS} = 0$$

So (theorem of Ostrogradsky)[4]:

$$div\vec{E}_g = 0$$

In vacuum, the law of conservation of g-information can be expressed as follows:

(1) At a matter free point P of a gravitational field, the spatial variation of $\vec{E}_g$ obeys the law: $div\vec{E}_g = 0$

This is the first equation of Maxwell-Heaviside in vacuum.

Corollary: *At a matter free point P of a gravitational field* $\frac{\partial}{\partial t}[N.cos(\Delta\theta)] = 0$

Because[4]

$$div\vec{E}_g = div(N.\vec{s}_g) = grad(N).\vec{s}_g + N.div(\vec{s}_g) \qquad (3)$$

it follows from the first equation that:

$$grad(N).\vec{s}_g + N.div(\vec{s}_g) = 0$$

1. First we calculate: $grad(N).\vec{s}_g$.

Referring to fig 8:

$$grad(N) = \frac{N_Q - N_P}{PQ}.\vec{e}_c = \frac{N_Q - N_P}{c.dt}.\vec{e}_c$$

Because an informaton that at the moment t passes at P is at the moment $(t + dt)$ at Q, (with $PQ = c.dt$).

$$\frac{N_Q - N_P}{dt} = \frac{N(t-dt) - N(t)}{dt} = -\frac{\partial N}{\partial t}$$

So:

$$grad(N) = -\frac{1}{c}.\frac{\partial N}{\partial t}.\vec{e}_c = -\frac{1}{c}.\frac{\partial N}{\partial t}.\frac{\vec{c}}{c}$$

And:

$$grad(N).\vec{s}_g = -\frac{1}{c^2}.\frac{\partial N}{\partial t}.\ \vec{c}.\vec{s}_g = \frac{1}{c}.\frac{\partial N}{\partial t}.s_g.cos(\Delta\theta) \qquad (4)$$

2. Next, we calculate: $N.div(\vec{s}_g)$

$$div(\vec{s}_g) = \frac{\oiint \vec{s}_g.\overrightarrow{dS}}{dV}$$

For that purpose, we calculate the double integral over the closed surface S formed by the infinitesimal surfaces dS that are at P and Q perpendicular to the flow of informatons (perpendicular to $\vec{c}$) and by the tube that connects the edges of these surfaces (and that is parallel to $\vec{c}$). $dV = c.dt.dS$ is the infinitesimal volume enclosed by S:

$$div(\vec{s}_g) = \frac{\oiint \vec{s}_g.\overrightarrow{dS}}{dV} = \frac{s_g.dS.cos(\Delta\theta_P) - s_g.dS.cos(\Delta\theta_Q)}{dS.c.dt}$$

Because an informaton that at the moment t passes at P is at the moment $(t + dt)$ at Q, (with $PQ = c.dt$):

$$\frac{cos(\Delta\theta_P) - cos(\Delta\theta_Q)}{dt} = \frac{\cos[\Delta\theta(t)] - \cos[\Delta\theta(t-dt)]}{dt} = \frac{\partial[\cos(\Delta\theta)]}{\partial t}$$

$$div(\vec{s}_g) = \frac{1}{c}.s_g.\frac{\partial\{cos(\Delta\theta)\}}{\partial t}$$

And:

$$N.div(\vec{s}_g) = \frac{N}{c}.s_g.\frac{\partial\{cos(\Delta\theta)\}}{\partial t} \quad (5)$$

Substitution of (4) and (5) in (3) gives:

$$\frac{1}{c}.\frac{\partial N}{\partial t}.s_g.cos(\Delta\theta) + \frac{N}{c}.s_g.\frac{\partial\{cos(\Delta\theta)\}}{\partial t} = 0$$

Or:

$$\frac{\partial}{\partial t}[N.cos(\Delta\theta)] = 0 \quad \mathbf{(6)}$$

6.2 div$\vec{B}_g$ – THE SECOND EQUATION IN FREE SPACE

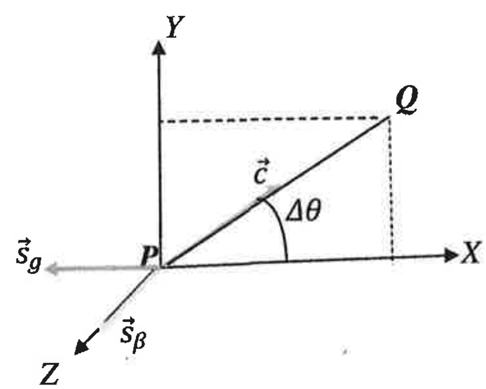

Fig 8

We refer again to fig 8 and notice that:

$\vec{s}_g = -s_g.\vec{e}_x$ and $\vec{s}_\beta = \frac{\vec{c}\times\vec{s}_g}{c} = s_g.sin(\Delta\theta).\vec{e}_z$

From mathematics[4] we know:

$$div\vec{B}_g = div(n.\vec{s}_\beta) = grad(n).\vec{s}_\beta + n.div(\vec{s}_\beta) \qquad (7)$$

1. First we calculate: $grad(n).\vec{s}_\beta$

$grad(n).\vec{s}_\beta = 0$ because *grad(n)* is perpendicular to $\vec{s}_\beta$. Indeed *n* changes only in the direction of the flow of informatons, so *grad(n)* has the same orientation as $\vec{c}$:

2. Next we calculate: $n.div(\vec{s}_\beta)$

$$div(\vec{s}_\beta) = \frac{\oiint \vec{s}_\beta.\overrightarrow{dS}}{dV}$$

We calculate the double integral over the closed surface *S* formed by the infinitesimal surfaces *dS* = *dz.dy* that are at *P* and at *Q* perpendicular to the *X*-axis and by the tube that connects the edges of these surfaces.

Because $\vec{s}_\beta$ is oriented along the Z-axis the flux of $\vec{s}_\beta$ through the planes *dz.dy* and *dx.dz* is zero, while the fluxes through the planes *dx.dy* are equal and opposite. So we can conclude that:

$$div(\vec{s}_\beta) = \frac{\oiint \vec{s}_\beta.\overrightarrow{dS}}{dV} = 0$$

Both terms of the expression (7) of $div\vec{B}_g$ are zero, so $div\vec{B}_g = 0$, what implies (theorem of Ostrogradsky) that for every closed surface *S* in a gravitational field:

$$\oiint_S \vec{B}_g.\overrightarrow{dS} = 0$$

We conclude:

(2) *At a matter free point P of a gravitational field, the spatial variation of* $\vec{B}_g$ *obeys the law:* $div\vec{B}_g = 0$

This is the second equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that the β-index of an informaton is always perpendicular to both its g-index $\vec{s}_g$ and to its velocity $\vec{c}$.

6.3 $rot\vec{E}_g$ - THE THIRD EQUATION IN FREE SPACE

The density of the flow of informatons that - at the moment *t* - passes near *P* with velocity $\vec{c}$ (fig 8) is defined as:

$$\vec{E}_g = N.\vec{s}_g = -N.s_g.\vec{e}_x$$

We know that[4]

$$rot\vec{E}_g = \{grad(N) \times \vec{s}_g\} + N.rot(\vec{s}_g) \qquad (8)$$

1. First we calculate: $\{grad(N) \times \vec{s}_g\}$

This expression describes the component of $rot\vec{E}_g$ caused by the spatial variation of *N* in the vicinity of *P* when $\Delta\theta$ remains constant.

N has the same value at all points of the infinitesimal surface that, at *P*, is perpendicular to the flow of informatons. So *grad(N)* is parallel to $\vec{c}$ and its magnitude is the increase of the magnitude of *N* per unit length. Thus, with $PQ = c.dt$, *grad(N)* is determined by:

$$grad(N) = \frac{N_Q - N_P}{PQ}.\frac{\vec{c}}{c} = \frac{N_Q - N_P}{c.dt}.\frac{\vec{c}}{c}$$

And:

$$grad(N) \times \vec{s}_g = \frac{N_Q - N_P}{c.dt}.\frac{\vec{c}}{c} \times \vec{s}_g = \frac{N_Q - N_P}{c.dt}.\vec{s}_\beta$$

The density of the flow of informatons at *Q* at the moment *t* is equal to the density of that flow at *P* at the moment (*t* - *dt*), so:

$$\frac{N_Q - N_P}{dt} = \frac{N(t-dt) - N(t)}{dt} = -\frac{\partial N}{\partial t}$$

And taking into account that :

$$\frac{N}{c} = n$$

we obtain:

$$grad(N) \times \vec{s}_g = -\frac{\partial n}{\partial t} \cdot \vec{s}_\beta \qquad (9)$$

2. Next we calculate: $\{N.rot(\vec{s}_g)\}$

This expression describes the component of $rot\vec{E}_g$ caused by the spatial variation of $\Delta\theta$ - the orientation of the g-index - in the vicinity of P - when N remains constant. $(\Delta\theta)_P$ is the characteristic angle of the informatons that pass near P and $(\Delta\theta)_Q$ is the characteristic angle of the informatons that at the same moment pass near Q. (fig 9)

For the calculation of

$$rot(\vec{s}_g) = \frac{\oint \vec{s}_g.\overrightarrow{dl}}{dS}$$

with dS the encircled area, we calculate $\oint \vec{s}_g.\overrightarrow{dl}$ along the closed path $PQqpP$ that encircles dS: $dS = PQ.Pp = c.dt.Pp$. (PQ and qp are parallel to the flow of the informatons, Qq and pP are perpendicular to it).

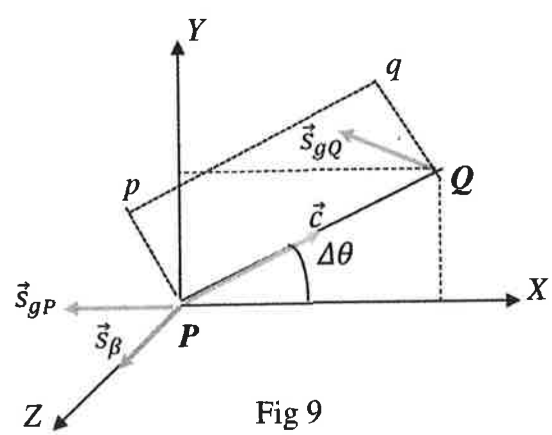

Fig 9

$$N.rot(\vec{s}_g) = N.\frac{s_g.sin[(\Delta\theta)_Q].Qq - s_g.sin[(\Delta\theta)_P)].pP}{c.dt.Pp}.\vec{e}_z$$

The characteristic angle of the informatons at Q at the moment t is equal to the characteristic angle of the informatons at P at the moment $(t - dt)$, so:

$$N.rot(\vec{s}_g) = N.\frac{s_g.\sin[\Delta\theta(t-dt)].Qq - s_g.\sin[\Delta\theta(t)].pP}{c.dt.Pp}.\vec{e}_z$$

The rate at which $\sin(\Delta\theta)$ in P changes at the moment t, is:

$$\frac{\partial\{sin(\Delta\theta)\}}{\partial t} = \frac{sin\{[\Delta\theta](t)\} - \sin\{[\Delta\theta](t-dt)\}}{dt}$$

And taking into account that

$$n = \frac{N}{c}$$

we obtain:

$$N.rot(\vec{s}_g) = -n.s_g.\frac{\partial\{sin(\Delta\theta)\}}{\partial t}.\vec{e}_z = -n.\frac{\partial}{\partial t}\{s_g.sin(\Delta\theta).\vec{e}_z\}$$

or

$$N.rot(\vec{s}_g) = -n.\frac{\partial\vec{s}_\beta}{\partial t} \qquad (10)$$

Combining the results (9) and (10), we obtain:

$$rot\vec{E}_g = grad(N) \times \vec{s}_g + N.rot(\vec{s}_g)$$

$$= -\left(\frac{\partial n}{\partial t}.\vec{s}_\beta + n.\frac{\partial\vec{s}_\beta}{\partial t}\right)$$

$$= -\frac{\partial(n.\vec{s}_\beta)}{\partial t} = -\frac{\partial\vec{B}_g}{\partial t} \qquad \mathbf{(11)}$$

We conclude:

(3) At a matter free point P of a gravitational field, the spatial variation of $\vec{E}_g$ and the rate at which $\vec{B}_g$ is changing are connected by the relation:

$$rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

This is the third equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that any change of the product $n.\vec{s}_\beta$ at a point of a gravitational field is related to a spatial variation of the product $N.\vec{s}_g$ in the vicinity of that point.

The relation

$$rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

implies (theorem of Stokes[4]):

$$\oint \vec{E}_g . \vec{dl} = -\iint_S \frac{\partial \vec{B}_g}{\partial t} . \vec{dS} = -\frac{\partial}{\partial t}\iint_S \vec{B}_g . \vec{dS} = -\frac{\partial \Phi_B}{\partial t}$$

The orientation of the surface vector $\vec{dS}$ is linked to the orientation of the path on L by the "rule of the corkscrew". $\Phi_B = \iint_S \vec{B}_g . \vec{dS}$ is called the "β-information-flux through *S*".

So, *in a gravitational field, the rate at which the surface integral of $\vec{B}_g$ over a surface S changes is equal and opposite to the line integral of $\vec{E}_g$ over the boundary L of that surface.*

6.4 $rot\vec{B}_g$ and $\frac{\partial \vec{E}_g}{\partial t}$ - THE FOURTH EQUATION IN FREE SPACE

We consider again $\vec{E}_g$ and $\vec{B}_g$, the contributions of the informatons that - at the moment t – pass with velocity $\vec{c}$ near P, to the g-field and to the g-induction at that point. (fig 10).

$$\vec{E}_g = N.\vec{s}_g = -N.s_g.\vec{e}_x$$

and

$$\vec{B}_g = n.\vec{s}_\beta = n.\frac{\vec{c} \times \vec{s}_g}{c} = n.s_g.sin(\Delta\theta).\vec{e}_z$$

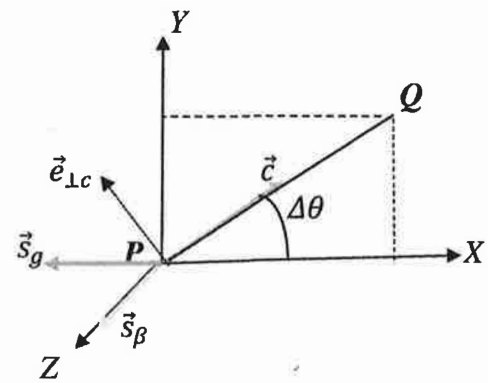

Fig 10

A. Let us calculate $rot\vec{B}_g$.

We know that[4]

$$rot\vec{B}_g = \{grad(n) \times \vec{s}_\beta\} + n.rot(\vec{s}_\beta) \qquad (12)$$

1. First we calculate: $\{grad(n) \times \vec{s}_\beta\}$

This expression describes the component of $rot\vec{B}_g$ caused by the spatial variation of *n* in the vicinity of *P* when $\Delta\theta$ remains constant.

n has the same value at all points of the infinitesimal surface that, at *P*, is perpendicular to the flow of informatons. So *grad(n)* is parallel to $\vec{c}$ and its magnitude is the increase of the magnitude of *n* per unit length.

With $PQ = c.dt$, *grad(n)* is determined by:

$$grad(n) = \frac{n_Q - n_P}{PQ}.\frac{\vec{c}}{c} = \frac{n_Q - n_P}{c.dt}.\frac{\vec{c}}{c}$$

The density of the cloud of informatons at *Q* at the moment *t* is equal to the density of that flow at *P* at the moment (*t* - *dt*), so:

$$\frac{n_Q - n_P}{dt} = \frac{n(t - dt) - n(t)}{dt} = -\frac{\partial n}{\partial t}$$

And

$$grad(n) = -\frac{1}{c}.\frac{\partial n}{\partial t}.\frac{\vec{c}}{c} = -\frac{1}{c}.\frac{\partial n}{\partial t}.\vec{e}_c$$

The vector $\{grad(n) \times \vec{s}_\beta\}$ is perpendicular to het plane determined by $\vec{c}$ and $\vec{s}_\beta$. So, it lies in the *XY*-plane and is there perpendicular to $\vec{c}$ forming an angle $\Delta\theta$ with the axis *OY*. Taking into account the definition of vectoral product we obtain:

$$grad(n) \times \vec{s}_\beta = -\frac{1}{c}.\frac{\partial n}{\partial t}.s_g.sin(\Delta\theta).(\vec{e}_c \times \vec{e}_z)$$

With

$$\vec{e}_c \times \vec{e}_z = -\vec{e}_{\perp c}$$

$$grad(n) \times \vec{s}_\beta = \frac{1}{c}.\frac{\partial n}{\partial t}.s_g.sin(\Delta\theta).\vec{e}_{\perp c}$$

And, taking into account that $n = \frac{N}{c}$, we obtain:

$$grad(n) \times \vec{s}_\beta = \frac{1}{c^2}.\frac{\partial N}{\partial t}.s_g.sin(\Delta\theta).\vec{e}_{\perp c} \qquad (13)$$

2. Next we calculate $\{n.rot(\vec{s}_\beta)\}$

This expression is the component of $rot\vec{B}_g$ caused by the spatial variation of $\vec{s}_\beta$ in the vicinity of *P* when *n* remains constant. For the calculation of

$$rot(\vec{s}_\beta) = \frac{\oint \vec{s}_\beta.\overrightarrow{dl}}{dS}.\vec{e}_{\perp c}$$

with dS the encircled area, we calculate $\oint \vec{s}_\beta . \overrightarrow{dl}$ along the closed path *PpqQP* that encircles *dS*: *dS= PQ.Pp = c.dt.Pp* (fig 11). (*PQ* and *qp* are parallel to the flow of the informatons, *Qq* and *pP* are perpendicular to it).

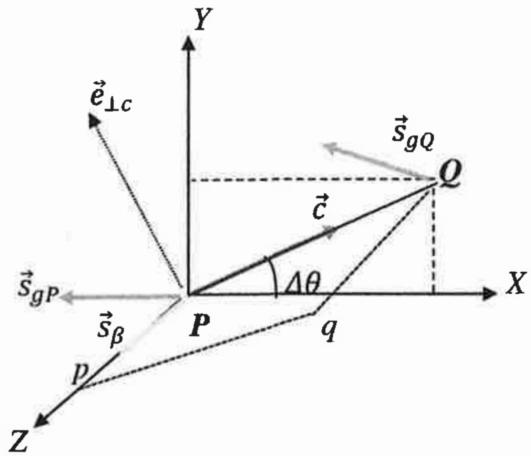

Fig 11

$$rot(\vec{s}_\beta) = \frac{\oint \vec{s}_\beta . \overrightarrow{dl}}{dS} . \vec{e}_{\perp c} = \frac{s_g . sin[(\Delta\theta)_P)]. Pp - s_g . \sin[(\Delta\theta)_Q]. qQ}{c. dt. Pp} \vec{e}_{\perp c}$$

The characteristic angle of the informatons at Q at the moment t is equal to the characteristic angle of the informatons at P at the moment $(t - dt)$, so:

$$rot(\vec{s}_\beta) = \frac{\oint \vec{s}_\beta . \overrightarrow{dl}}{dS} . \vec{e}_{\perp c} = \frac{s_g . \{\sin[\Delta\theta(t)]. Pp - s_g . \sin[\Delta\theta(t - dt)]\}. qQ}{c. dt. Pp} . \vec{e}_{\perp c}$$

The rate at which $\sin(\Delta\theta)$ at P changes at the moment t, is:

$$\frac{\partial\{sin(\Delta\theta)\}}{\partial t} = \frac{sin\{(\Delta\theta)[t]\} - \sin\{(\Delta\theta)[t - dt]\}}{dt}$$

So:

$$rot(\vec{s}_\beta) = s_g . \frac{1}{c} . \frac{\partial[\sin(\Delta\theta)]}{\partial t} . \vec{e}_{\perp c}$$

And with

$$n = \frac{N}{c}$$

we finally obtain:

$$n.rot(\vec{s}_{\beta}) = s_g.\frac{1}{c^2}.N.\frac{\partial[\sin(\Delta\theta)]}{\partial t}.\vec{e}_{\perp c} \qquad \textbf{(14)}$$

Substituting the results (13) and (14) in (12) gives:

$$rot\vec{B}_g = \frac{1}{c^2}.s_g.\{\frac{\partial N}{\partial t}.sin(\Delta\theta) + N.\frac{\partial[\sin(\Delta\theta)]}{\partial t}\}.\vec{e}_{\perp c}$$

$$= \frac{1}{c^2}.s_g.\frac{\partial}{\partial t}[N.\sin(\Delta\theta)].\vec{e}_{\perp c} \qquad \textbf{(15)}$$

B. Now we calculate $\frac{\partial\vec{E}_g}{\partial t}$

We know that[4]:

$$\frac{\partial\vec{E}_g}{\partial t} = \frac{\partial N}{\partial t}.\vec{s}_g + N.\frac{\partial\vec{s}_g}{\partial t}$$

And that:

$$\vec{s}_g = -s_g.\vec{e}_x \qquad \text{and} \qquad \frac{\partial\vec{s}_g}{\partial t} = s_g.\frac{\partial(\Delta\theta)}{\partial t}.\vec{e}_y$$

So:

$$\frac{\partial\vec{E}_g}{\partial t} = -\frac{\partial N}{\partial t}.s_g.\vec{e}_x + N.s_g.\frac{\partial(\Delta\theta)}{\partial t}.\vec{e}_y$$

Taking into account:

$$\vec{e}_x = cos(\Delta\theta).\vec{e}_c - sin(\Delta\theta).\vec{e}_{\perp c} \quad \text{and} \quad \vec{e}_y = sin(\Delta\theta).\vec{e}_c + cos(\Delta\theta).\vec{e}_{\perp c}$$

we obtain:

$$\frac{\partial \vec{E}_g}{\partial t} = \left[-\frac{\partial N}{\partial t}.s_g.cos(\Delta\theta) + N.s_g.\frac{\partial(\Delta\theta)}{\partial t}.sin(\Delta\theta)\right].\vec{e}_c$$

$$+\left[\frac{\partial N}{\partial t}.s_g.sin(\Delta\theta) + N.s_g.\frac{\partial(\Delta\theta)}{\partial t}.cos(\Delta\theta)\right].\vec{e}_{\perp c}$$

or:

$$\frac{\partial \vec{E}_g}{\partial t} = s_g.\left\{-\frac{\partial}{\partial t}[N.cos(\Delta\theta)\right].\vec{e}_c + \frac{\partial}{\partial t}[N.\sin(\Delta\theta)].\vec{e}_{\perp c}\}$$

Taking into account (6), we find:

$$\frac{\partial \vec{E}_g}{\partial t} = s_g.\frac{\partial}{\partial t}[N.\sin(\Delta\theta)].\vec{e}_{\perp c} \qquad \mathbf{(16)}$$

C. From (15) an (16), we conclude:

$$rot\vec{B}_g = \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t}$$

(4) *At a matter free point P of a gravitational field, the spatial variation of $\vec{B}_g$ and the rate at which $\vec{E}_g$ is changing are connected by the relation:*

$$rot\vec{B}_g = \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t}$$

This is the fourth equation of Maxwell-Heaviside in vacuum. It is the expression of the fact that any change of the product $N.\vec{s}_g$ at a point of a gravitational field is related to a spatial variation of the product $n.\vec{s}_g$ in the vicinity of that point.

This relation implies (theorem of Stokes): *In a gravitational field, the rate at which the surface integral of $\vec{E}_g$ over a surface S changes is proportional to the line integral of $\vec{B}_g$ over the boundary L of that surface:*

$$\oint \vec{B}_g.\vec{dl} = \frac{1}{c^2}\iint_S \frac{\partial \vec{E}_g}{\partial t}.\vec{dS} = \frac{1}{c^2}\frac{\partial}{\partial t}\iint_S \vec{E}_g.\vec{dS} = \frac{1}{c^2}\frac{\partial \Phi_G}{\partial t}$$

The orientation of the surface vector $\overrightarrow{dS}$ is linked to the orientation of the path on L by the "rule of the corkscrew". $\Phi_G = \iint_S \vec{E}_g \,.\, \overrightarrow{dS}$ is called the "g-information-flux through S".

6.5 THE MAXWELL-HEAVISIDE EQUATIONS

The volume-element at a point P inside a mass continuum is in any case a source of g-information and, if the mass is moving, also a source of β-information. According to §3.3, the instantaneous value of ρ_G - the mass density at P - contributes to the instantaneous value of $div\vec{E}_g$ at that point with an amount $-\frac{\rho_G}{\eta_0}$; and according to §4.7 the instantaneous value of $\vec{J}_G$ - the mass flow density - contributes to the instantaneous value of $rot\vec{B}_g$ at P with an amount $-\nu_0.\vec{J}_G$.

It is evident that at a point of a gravitational field - linked to an inertial reference frame $\boldsymbol{O}$ - one must take into account the contributions of the local values of $\rho_G(x, y, z; t)$ and of $\vec{J}_G(x, y, z; t)$. This results in the generalization and expansion of the laws at a mass free point. By superposition we obtain:

(1) *At a point P of a gravitational field, the spatial variation of $\vec{E}_g$ obeys the law:*

$$div\vec{E}_g = -\frac{\rho_G}{\eta_0}$$

In integral form:

$$\Phi_G = \oiint_S \vec{E}_g \,.\, \overrightarrow{dS} = -\frac{1}{\eta_0} . \iiint_G \rho_G . dV$$

(2) *At a point P of a gravitational field, the spatial variation of $\vec{B}_g$ obeys the law:*

$$div\vec{B}_g = 0$$

In integral form:

$$\Phi_B = \oiint_S \vec{B}_g \,.\, \overrightarrow{dS} = 0$$

(3) *At a point P of a gravitational field, the spatial variation of $\vec{E}_g$ and the rate at which $\vec{B}_g$ is changing are connected by the relation:*

$$rot\vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t}$$

In integral form:

$$\oint \vec{E}_g . \vec{dl} = -\iint_S \frac{\partial \vec{B}_g}{\partial t} . \vec{dS} = -\frac{\partial}{\partial t}\iint_S \vec{B}_g . \vec{dS} = -\frac{\partial \Phi_B}{\partial t}$$

(4) *At a point P of a gravitational field, the spatial variation of $\vec{B}_g$ and the rate at which $\vec{E}_g$ is changing are connected by the relation:*

$$rot\vec{B}_g = \frac{1}{c^2}\frac{\partial \vec{E}_g}{\partial t} - \nu_0 . \vec{J}_G$$

In integral form:

$$\oint \vec{B}_g . \vec{dl} = \frac{1}{c^2}\iint_S \frac{\partial \vec{E}_g}{\partial t} . \vec{dS} - \nu_0 . \iint_S \vec{J}_g . \vec{dS} = \frac{1}{c^2} . \frac{\partial}{\partial t}\iint_S \vec{E}_g . \vec{dS} - \nu_0 . \iint_S \vec{J}_G . \vec{dS}$$

These are the laws of Heaviside-Maxwell or the laws of GEM.

6.6 CONCLUSION

The mathematical deductions of the laws of GEM confirm that these equations indicate that there is no causal link between $\vec{E}_g$ and $\vec{B}_g$. Therefore, *we must conclude that a gravitational field is a dual entity always having a "field-" and an "induction-" component simultaneously created by their common sources: time-variable masses and mass flows•.*

The GEM equations are analogue to Maxwell's equations in EM and it is proved[5] that these are consistent with special relativity. Thus, *the Maxwell-Heaviside equations are invariant under a Lorentz transformation* and GEM is consistent with special relativity. In this context it should be noted that the fact that the rate at which a material body emits informatons is independent of its velocity and completely defined by its rest mass m_0, implies that in equation (1) the value of

• On the understanding that the induction-component equals zero if the source of the field is time independent.

$\rho_G = \frac{dm_0}{dV}$ depends on the state of motion – relative to the considered inertial reference system - of the mass element dm_0. Indeed in the case of a moving mass element, the Lorentz contraction must be taken into account in the determination of dV. Because a mass flow is made up of moving mass elements its density $\vec{J}_G$ also depends on the inertial reference frame in which it is considered. This implies that in equation (4) the expression of $\vec{J}_G$ also depends on the inertial reference frame.

In appendix B it is proven that the GEM equations are mathematically consistent.

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