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New Problematic Aspects of Current String Theories and Their Invariant Resolution

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We identify new, rather serious, physical and mathematical inconsistencies of the current formulation of noncanonical or nonunitary string theories due to the lack of invariant units necessary for consistent measurements, lack of preservation in time of Hermiticity-observability, and other shortcomings. We propose three novel reformulations of string theories for matter *of progressively increasing complexity via the novel iso-, geno-, and hyper-mathematics of hadronic mechanics, which resolve the current inconsistencies, while offering new intriguing possibilities, such as: an axiomatically consistent and invariant inclusion of gravity, the reduction of macroscopic irreversibility to the most primitive level of vibrations of the universal substratum (ether), or the treatment of multi-valued, irreducible, biological structures. We then identify three corresponding* classical *formulations of string theories for* antimatter *via the novel anti-isomorphic isodual mathematics. We finally outline the intriguing features of the emerging new cosmologies (including biological structures, as it should be for all cosmologies), such as: universal invariance (rather than covariance) under a symmetry isomorphic to the Poincaré group and its isodual; equal distributions of matter and antimatter in the universe (as a limit case); continuous creation; no need for the missing mass; significantly reduced dimensions; possibility of experimental identification of antimatter in the universe; identically null total characteristics of time, energy, linear and angular momentum, charge, etc.; and other intriguing features.*

KEY WORDS: ▮▮▮

1. CATASTROPHIC INCONSISTENCIES OF NONCANONICAL-NONUNITARY THEORIES

As it is well known, the origin of the majestic axiomatic consistency of special relativity (SR) and relativistic quantum mechanics (RQM) is its Lie

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structure, which we express for subsequent needs with the following finite and infinitesimal versions, and related conjugation,

$$A(w) = U(w) \times A(0) \times U^{\dagger}(w)$$
$$= e^{iX \times w} \times A(0) \times e^{-iw \times X} = e_{>}^{iX > w} > A(0) < e_{<}^{-iw < X}, \quad \text{(1a)}$$

$$idA/dw = A \times X - X \times A = A < X - X > A, \quad \text{(1b)}$$

$$e_{>}^{iX > w} = [e_{<}^{-iw < X}]^{\dagger}, \quad \text{(1c)}$$

where $>$ ($<$) denotes the conventional modular associative product to the right (left), and the symmetry is realized via *unitary transformations* defined on a Hilbert space $\mathscr{H}$ over the field $C(c, +, \times)$ of complex numbers c with conventional sum $+$, associative product $\times$, additive unit 0, and multiplicative unit I.

The unitary structure implies the following well known basic invariances:

$$U \times U^{\dagger} = U^{\dagger} \times U = I, \quad \text{(2a)}$$

$$I \to U \times I \times U^{\dagger} = I' = I, \quad \text{(2b)}$$

$$A \times B \to U \times (A \times B) \times U^{\dagger}$$
$$= (U \times A \times U^{\dagger}) \times (U \times B \times U^{\dagger}) = A' \times B', \quad \text{(2c)}$$

$$H \times |\psi\rangle = E \times |\psi\rangle \to U \times H \times |\psi\rangle$$
$$= (U \times H \times U^{\dagger}) \times (U \times |\psi\rangle) = H' \times |\psi'\rangle$$
$$= U \times E \times |\psi\rangle = E' \times |\psi'\rangle, \qquad E' = E. \quad \text{(2d)}$$

It then follows that *all theories with a unitary structure defined on a Hilbert space over the field of complex numbers possess numerically invariant units, products and eigenvalues, thus being suitable to represent physical reality*.

By comparison, theories with a *nonunitary structure* have serious flaws studied in detail in Refs. 1, because invariances (2) are turned into the following noninvariances,

$$U \times U^{\dagger} \neq I, \quad \text{(3a)}$$

$$I \to U \times I \times U^{\dagger} = I' \neq I, \quad \text{(3b)}$$

$$A \times B \to U \times (A \times B) \times U^{\dagger}$$
$$= (U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger})$$
$$= A' \times T \times B' \neq A' \times B', T = (U \times U^{\dagger})^{-1}, \quad \text{(3c)}$$

$$H \times |\psi\rangle = E \times |\psi\rangle \rightarrow U \times H \times |\psi\rangle$$
$$= (U \times H \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times |\psi\rangle)$$
$$= H' \times T \times |\psi'\rangle = \; = U \times E \times |\psi\rangle = E' \times |\psi'\rangle, E' \neq E, \quad (3d)$$

which imply rather serious *physical inconsistencies*, such as: the lack of invariance of the basic units of time, space, energy, etc, which invariance is necessary for consistent measurements; the lack of preservation in time of Hermiticity, which Hermiticity is necessary to have physically acceptable observables; lack of uniqueness and invariance in time of the physical predictions of the theory; and other flaws (for a comprehensive treatment, see Ref. 1g).

Noninvariances (3) also have seemingly catastrophic mathematical inconsistencies at both classical and operator levels. Recall that all axiomatic structures of physical theories (such as vector and metric spaces, functional analysis, algebras and groups, etc.) are formulated over a given field of numbers. which formulation, in turn, is crucially dependent on the unit. The alteration in time of the basic unit then implies the loss of the original field at subsequent times. This is due to the fact that the noncanonical-nonunitary transform must be applied, for consistency, to the *totality* of the original structure (see below), including basic unit, numbers, spaces, etc., and cannot be applied only to part of the original structure to please personal preferences.

In this way, noncanonical-nonunitary theories lose the basic unit I of the field during their evolution in time, yet they continue to be generally expressed over the original field, with a consequential manifest inconsistency. In fact, the lack of invariance of the basic unit implies the inapplicability of the basic field, with consequential loss of the entire axiomatic structure, without any exception known to this author (for details, see Refs. 1g and 3g).

As an elementary illustration, consider a nonunitary theory predicting at time $t = 0$ a stable bound state with 3 MeV. Suppose that the nonunitary time evolution is such to yield the diagonal and dimensionless value 5 at time $t = 10$ sec. Then, from Eqs. (3), the value of the same energy under the same conditions become 15 MeV,

$$[U(t) \times U^{\dagger}(t)]_{t=0\,\mathrm{sec}} = 1, \qquad [U(t) \times U^{\dagger}(t)]_{t=10\,\mathrm{sec}} = 5, \quad (4a)$$

$$H \times |\psi\rangle = (3\ \mathrm{MeV}) \times |\psi\rangle \rightarrow H' \times |\psi'\rangle = (15\ \mathrm{MeV}) \times |\psi'\rangle, \quad (4b)$$

thus proving the lack of known physical meaning of theories with nonunitary time evolutions. The reader can identify even more serious inconsistencies for nonunitary theory in which $U \times U^{\dagger}$ is not diagonal or constant.

An insidious aspect which appears to be known only to few experts on noninvariance problems, is that *the inconsistencies of nonunitary theories also occur for Hermitean Hamiltonians*, because the nonunitary character is represented by the *structure of the time evolution*, and not necessarily by the Hamiltonian. As we shall see shortly, the new Hamiltonian $H' = U \times H \times U^{\dagger}$ can be fully Hermitean under nonunitary transforms; yet the theory is nonunitary, thus generally noninvariant (because, as we shall see, H and $T = [U \times U^{\dagger}]^{-1}$ do not generally commute).

The above physical and axiomatic inconsistencies reach their climax for theories formulated on a *curved manifold*.[1g, 3g] In fact, the map from the Minkowski metric $\eta = \text{diag}(1, -1, -1, -1) = \text{constant}$ to a Riemannian metric $g(x) = \text{function}$ is transparently a *noncanonical transform*,

$$\eta_{\text{Minkowski}} \to g(x)_{\text{Riemann}} = U(x) \times \eta \times U^{t}(x),\ U(x) \times U^{t}(x) \neq I. \qquad (5)$$

Operator theories on curved manifolds must then be necessarily nonunitary, for consistency under naive or symplectic quantization. As a result, the above physical and axiomatic inconsistencies hold at both classical and operator levels (see also Refs. 1g and 3g for brevity).

We therefore have the following

Theorem 1 (Ref. 1). All operator theories with a nonunitary structure formulated on a conventional Hilbert space over the field of complex numbers, including (but not limiting to) all operator theories of gravity on a manifold with non-null curvature, possess the following physical inconsistencies:

(I) lack of invariant units of space, time, energy, etc., with consequential lack of consistent applications to measurement;

(II) lack of preservation of the original Hermiticity in time, with consequential absence of physically acceptable observables;

(III) general violation of probability and causality laws;

(IV) lack of invariance of conventional and special functions and transforms used in data elaborations;

(V) lack of uniqueness and invariance of numerical predictions;

(VI) General violation of the superposition principle with consequential inapplicability to composite systems;

(VII) General violation of Mackey imprimitivity theorem with consequential violation of Galilei's and Einstein's special relativities.

All classical noncanonical theories formulated on conventional spaces over conventional fields, including (but not limited to) all classical theories of gravity formulated on a manifold with non-null curvature, are afflicted by corresponding physical inconsistencies which prevent their consistent representation of physical reality.

Finally, classical-noncanonical, and operator-nonunitary theories formulated on conventional spaces over conventional fields are afflicted by additional catastrophic mathematical inconsistencies due to the lack of preservation in time of the basic unit, with consequential loss at subsequent times of the entire axiomatic structure (fields, spaces, geometries, topology, etc.)

The above physical inconsistencies have been identified to occur for numerous theories, such as (see Ref. 1g for details and literature): (1) Dissipative nuclear models with imaginary potentials; (2) Statistical models with external collisions terms; (3) q-, k- and *-deformations; (4) Certain quantum groups (evidently those with a nonunitary structure); (5) Weinberg's nonlinear theory; (6) Theories of classical and quantum gravity on curved manifolds; (7) Supersymmetric theories; (8) Kac–Moody theories; and other theories.

To avoid possible misinterpretations, it should be noted that Theorem 1 treats nonunitary theories *without* possible unitary reformulations, e.g., following a given renormalization or other procedure. Also, the ' violation of the superposition principle is primarily referred to nonlinear theories (for which reason it has been presented as a "general," rather than absolute violation). To avoid a prohibitive length, as well as un-necessary duplications, the interested reader is suggested to consult Ref. 1g.

2. CATASTROPHIC INCONSISTENCIES OF STRING THEORIES WITH NONCANONICAL-NONUNITARY STRUCTURE

Despite their undeniable conceptual beauty, string theories (see, e.g., Refs. 2 and vast literature quoted therein) are afflicted by a number of problematic aspects or sheer inconsistencies already identified in the literature, and, thus not reviewed here for brevity (see, e.g., the readable account[2c]). The first objective of this note is to point out, apparently for the first time, *new physical and mathematical inconsistencies of classical-noncanonical and operator-nonunitary string theories* resulting from Theorem 1.

The features of contemporary string theories which trigger the applicability of Theorem 1 are numerous, such as:

(A) The known nonunitary character of string theories formulated via the Beta function according to Veneziano and Suzuki;

(B) The more recent supersymmetric formulations of string theories, because they imply the exiting from Lie's axioms (1), with consequential noninvariances (3);

(C) Recent formulations of string theories on curved manifolds, because they imply the additional, independent, rather serious inconsistencies mentioned earlier.

As one of the simplest possible examples, consider the string theory by Gasperini *et al.*[(2b)] Its central assumption is metric (1.3), p. 50, [*loc. cit.*], i.e.,

$$g_{\mu\nu} = \text{Diag}(1, -a^2\delta_{ij}, \qquad i, j = 1, 2, 3, \tag{6}$$

To understand the implications of the above basic assumption, one should recall the original words by Minkowski, namely, that his metric represents in a dimensionless form the basic units of time and space, $\eta =$ Diag(1 sec, -1 cm, -1 cm, -1 cm). Therefore, by its very conception, *the assumption of metric (6) implies the alteration of the basic unit of space*. In fact. according to Minkowski's original conception, metric (6) is the dimensionless version of the form $g = \text{Diag}(1 \text{ sec}, -a^2 \text{ cm}, -a^2 \text{ cm}, -a^2 \text{ cm})$.

The alteration of the basic unit of space is not necessarily negative *per se*, because, even though not addressed in Ref. 2b, it could be justified as a sort of rescaling of the basic unit of space, or via some other unconventional interpretation. The catastrophic inconsistencies indicated earlier emerge because of the *lack of invariance of the assumed new units*.

The map from the Minkowski metric to metric (6), $\eta \to g = U \times \eta \times U^\dagger$, is outside the classes of equivalence of the Poincaré symmetry. Therefore, the string theory of Ref. 2b is structurally noncanonical at the classical level and nonunitary at the operator level. It is then easy to see that the application of the same transform to the new metric does not leave the latter invariant, i.e., $g \to g' = U \times g \times U^\dagger \neq g$, and the same occurs under the time evolution of the theory.

In different words, a reason for the majestic physical and mathematical consistency of the Minkowski metric is that it is invariant under the time evolution of theories build on it. By comparison, a reason for the physical and mathematical inconsistencies of metric (6) and related theories is their lack of invariance.

Equivalently, we can say that a reason for the physical and mathematical consistency of theories with a canonical or unitary time evolution is that *canonical and unitary tranecorms leave invariant, by conception, the basic units of the theory, of course, in their dimensionless form,*

$$1(\mathrm{sec/cm/eV/etc.}) \to 1'(\mathrm{sec/cm/eV/etc.}) = U \times 1 \times U^{\dagger} = 1(\mathrm{sec/cm/eV/etc.}). \quad (7)$$

By comparison, noncanonical or nonunitary transforms do not preserve such basic units, thus preventing the application of the theories to physical reality.

Example (6) is one of the simplest possible cases. The inconsistencies for more complex examples of string theories, e.g., with a nontrivial functional dependence of generalized metrics, are then evident, and their study is left to the interested reader for brevity.

After considerable study of the problem, the only way found by this author to resolve the inconsistencies of contemporary string theories, and regain all invariances (2), is the formulation of string theories via the new formalism of *relativistic hadronic mechanics* (see Refs. 3, 4, and 5 for technical studies, and Ref. 3g for an outline).

More specifically, the indicated inconsistencies of string theories can be eliminated via the use of the novel *iso-, geno-, and hyper-mathematics* of relativistic hadronic mechanics (see, the first presentation of 1978[3c] and the more recent study[3f]). The main viewpoint here submitted is that the insufficiencies of string theories *are not* due to their physical conception, but rather to inadequate mathematics used for their treatment.

To avoid excessive complexities for non-experts, we recall that the axiomatic structure of isomathematics is fully reversible in time. Therefore, isomathematics will be solely used for the reformulation of string theories describing reversible systems, such as the structure of hadrons, nuclei, and atoms. The broader genomathematics is structurally irreversible, and it will be used for the invariant formulation of string theories intended for irreversible systems. Finally, the type of hypermathematics considered in this paper is irreversible as well as multi-valued. As such, it will be used for string theories intended for biological structures or multidimensional universes.

It should also be recalled that iso-, geno-, and hyper-mathematics coincide with conventional mathematics at the abstract, realization-free level. Therefore, on rigorous grounds, they do not constitute "new mathematics," but rather "new realizations" of conventional, abstract mathematical axioms. We should finally recall that the iso-, geno-, and hyper-branches of hadronic mechanics recover conventional quantum mechanics at large distances, while adding at short distance a "completion" due to nonlinear, nonlocal, and nonunitary effects beyond any dream of representation via a Hamiltonian.

3. ISOTOPIC UNIFICATION OF SPECIAL AND GENERAL RELATIVITIES

The main advance underlying the proposed invariant formulation of string theories is a geometric unification of special and general relativities first presented by the author at the VII M. Grossmann Meeting on General Relativity, Ref. 3i. Consider any signature-preserving generalization $g(x)$ of the Minkowski metric η, thus including all possible Riemannian metrics. Suppose, for simplicity, that $g(x)$ is diagonal. Then, the formulation of $g(x)$ and related theories in a way as invariant as η can be achieved via the following procedure [*loc. cit.*]:

(I) Factorize from the metric $g(x)$ the Minkowski metric η, resulting in a 4×4 matrix $\hat{T}(x)$ which is necessarily positive-definite (from the assumed local Minkowskian character which is verified for all possible Riemannian metrics),(3i)

$$g(x) = \hat{T}(x) \times \eta, \qquad \hat{T}(x) = \hat{T}^t(x) > 0; \tag{8}$$

(II) Assume the inverse of $\hat{T}(x)$, as the new basic unit of the theory,(3c)

$$\hat{I}(x) = 1/\hat{T}(x) = \hat{I}^t(x) > 0; \tag{9}$$

(III) Reconstruct the the mathematics in such a way that $\hat{I}$ is the correct left and right unit, beginning with the generalization (called *lifting*) of the conventional associative product $A \times B$ among generic quantities A, B, into a new product, called *isoassociative product* or *isoproduct* for short:(3c)

$$A \hat{\times} B = A \times \hat{T} \times B, \qquad \hat{I} \hat{\times} A = A \hat{\times} \hat{I} = A. \tag{10}$$

Invariance is achieved when the *totality* of the original mathematical treatment of gravity is lifted into a form admitting $\hat{I}(x)$, rather than I, as the left and right new unit, with no exception known to this author. This procedure yields the novel *isonumbers, isospaces, isomanifolds, isotopologies, isogeometries, isoalgebras, isosymmetries, etc.*,(3f) which are collectively known as *Santilli's isomathematics.*(5)

A first guiding principle here is that, *whether generalized or conventional, the unit is the basic invariant of all theories. Therefore, the representation with generalization units of deviations from canonical or unitary theories, assures, by conception, their invariance.*

Most notable for this study is the first achievement of the *universal invariance* of all possible Riemannian line elements. It is given by the

Poincaré–Santilli isosymmetry $\hat{P}(3.1)$,[3d, 3h] which is essentially a reconstruction of the conventional symmetry with respect to the new unit $\hat{I}n$. Not unexpectedly, $\hat{P}(3.1)$ results to be locally isomorphic to the conventional symmetry $P(3.1)$ (because $\hat{I}n$ is positive-definite).

In turn, the isomorphism of $\hat{P}(3.1)$ and $P(3.1)$ permits the *geometric unification of the special and general relativities*[3j] submitted by the author under the name of *isospecial relativity*,[3j, 3s] where special and general relativities are merely differentiated by the assumed unit. Such unification is an evident necessary pre-requisite for any general relativity to enjoy the majestic invariance properties of the special relativity.

Recall that isospecial relativity coincides at the abstract level with special relativity. Therefore, the former consists of a broader realization of the conventional axioms, and provides a rather remarkable broadening of the conditions of applicability of special relativity due to the unrestricted functional dependence of the isometric. For instance, in addition to including gravitation, isospecial relativity can directly represent the local variation of speed of light within physical media, study the gravitational effect of *density* (in addition to mass), because the latter can be directly included in the metric itself, and study other problems simply beyond any possible treatment by both Minkowskian and Riemannian geometries.

A second guiding principle (complementary to the preceding one) is that *the referral of a canonical-unitary deviation $\hat{T}(x)$ to a new unit which is its inverse, $\hat{I}(x) = 1/(x)$, leaves the original numerical values unchanged, thus assuring invariance*. The best illustration is given by the *isosphere*.[3] Consider a sphere of unit radii $R_k = 1$ cm, $k = 1, 2, 3$. Suppose that such a sphere is deformed into the ellipsoid with $R'_k = 1/n_k^2$. If the latter is referred to the new units $\hat{I}_k = n_k^2$, then it remains the perfect sphere on isospaces over isofields (isosphere). In particular, the resulting isosphere preserves the original *unit* value of the radii. A fully similar situation occurs in the modifications of the Minkowski light cone which becomes the *light isocone*.[3q] In particular, the maximal causal speed in isospacetime with an arbitrary Riemannian metric $g(x)$ remains *the speed of light c in vacuum,* thus assuring the causality of isotheories.

A third guiding principle (complementary to the preceding two) is the following *new invariance law of the conventional Minkowskian line element* characterized by the lifting of the unit,[3g]

$$I \to n^2 \times I = \hat{I}, \qquad \eta \to n^{-2} \times \eta = \hat{\eta}, \tag{11a}$$

$$\begin{aligned} x^2 &= [x^\mu \times \eta_{\mu\nu} \times x^\nu] \times I = \{x^\mu \times [n^{-2} \times \eta_{\mu\nu}] \times x^\nu\} \times (n^2 \times I) \\ &= [x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu] \times \hat{I}, \end{aligned} \tag{11b}$$

with corresponding *new invariance law of the conventional Hilbert product*,[3g]

$$\langle\psi| \times |\psi\rangle \times I = \langle\psi| \times n^{-2} \times |\psi\rangle \times (n^2 \times I) = \langle\psi| \hat{\times} |\psi\rangle \times \hat{I}. \qquad (12)$$

The above invariances establish that, contrary to popular belief, in its both classical and operator formulation, *the Poincaré symmetry is eleven, and not ten dimensional*. The reason why the eleventh-dimension was not discovered for about one century is that it required the prior discovery of *new numbers*, those with arbitrary (positive-definite) units $\hat{I}$.[3e]

Fully *invariant* (and not covariant) gravitational theories emerge by merely extending the parameter n to a functional dependence on spacetime similar to the transition from Abelian to non-Abelian gauge theories. Note that the invariance of Riemannian metrics rests on the existence of the hitherto unknown eleventh dimension of the Poincaré symmetry.

The most salient property of the isominkowskian formulation of gravity[3j] is the *elimination of the conventional notion of curvature,* due to its irreconcilable noncanonical-nonunitary character, as well as its irreconcilable structural incompatibility with all non-gravitational interactions.

In fact, the isominkowskian geometry is *isoflat*, that is, it admits no curvature when a Riemannian metric $g(x) = \hat{T}(x) \times \eta$ is referred to the isounit $\hat{I}(x) = 1/\hat{T}(x)$, although curvature occurs in its *projection* on conventional spaces over conventional fields, that is, when a Riemannian metric $g(x)$ is referred to the conventional unit $I = \text{Diag}(1, 1, 1, 1)$.

The isominkowskian geometry is a symbiotic unification of the Minkowskian and Riemannian geometries, because, on one side, it is isomorphic to the Minkowskian geometry, while, on the other side, it preserves all the machinery of Riemann (such as covariant derivatives, Christoffell's symbols, etc.), although formulated via the isodifferential calculus.

The above feature permits the preservation of Einstein–Hilbert (as well as any other) field equation, although gravitational and relativistic line elements formally coincide, and are merely differentiated by different interpretations of the differentials.[3j]

It should be also indicated for the noninitiated reader that the isominkowskian formulation of gravity has permitted the resolution of a number of controversies that have raged on gravitation for about one century, such as the compatibility of relativistic and gravitational conservation laws (which is guaranteed by the Poincaré–Santilli isosymmetry since its generators *coincide* with those of the conventional symmetry), absence of a relativistic limit, and others (see Ref. 3j for brevity).

The reconstruction of gravity on an isoflat space has permitted the achievement of an axiomatically consistent grand unification of electroweak and gravitational interactions, which was first proposed by the author at the VIII M. Grossmann Meeting on General Relativity Ref. 3k (see Ref. 3l for more details) under the name of *iso grand-unification* (IGU).

The new grand unification is essentially permitted by the resolution of three main structural incompatibilities of gravitation and electroweak interactions:

(1) The irreconcilable axiomatic incompatibility between curvature and the Minkowskian structure of electroweak interactions. This first incompatibility is resolved by embedding gravity in the unit of conventional gauge theories, thus achieving an isoflat formulation of gravity axiomatically equivalent to that of electroweak interactions;

(2) The irreconcilable axiomatic incompatibility between electroweak theories, which are rigidly controlled by the fundamental Poincaré symmetry, and gravitation, which does not possess any symmetry. This second incompatibility is resolved by the achievement of the universal Poincaré–Santilli isosymmetry for all possible gravitations, which is locally isomorphic to the conventional symmetry of electroweak interactions.

(3) The irreconcilable axiomatic incompatibility between the treatment of antimatter by gravitation, which requires positive-definite energy-momentum tensors, and the treatment of antiparticles by electroweak interactions, which require negative-definite energies. The resolution of this third inconsistency required the construction of a new theory of antimatter beginning at the *classical level*, which is outlined in Section 8.

As an incidental note, it should be mentioned that the achievement of the invariance studied in this paper escaped all efforts by this author for decades. After lifting all conceivable structures (fields, spaces, algebras, symmetries, etc.), invariance was still missing on rigorous grounds. The origin of the noninvariance was eventually identified were least expected, in the *ordinary differential calculus*, which also had to be isotopically lifted for consistency, as first presented in memoir[3f] of 1996. Therefore, *all papers on isotopies prior to the appearance of the isodifferential calculus in Ref. 3f (beginning with the articles by this author) are physically inconsistent because noninvariant, unless extended' with the isodifferential calculus.*

4. ISOTOPIC STRING THEORIES FOR REVERSIBLE SYSTEMS

We are now in a position to submit, apparently for the first time, *isostring theories* (IST) herein defined as locally Minkowskian theories with left and right generalized unit $\hat{I}(x) = 1/\hat{T}(x)$, where $\hat{T}(x)$ represents the departure of the metric $g(x) = \hat{T}(x) \times \eta$ from the Minkowskian form η, under the rigid constraints of the Poincaré–Santilli isosymmetry (identical to the rigid constraints of the Poincaré symmetry for conventional Minkowskian theories).

Isomathematics has reached an operational level permitting the following simple construction of IST. It consists in *assuming the nonunitary transform itself as the new basic unit, and then applying the same transform to the totality of the original mathematics*, thus reaching in this way isonumbers, isocoordinates, isodifferential and isoderivatives, iso-Hilbert products, isoeigenvalue equations, etc.,[3g]

$$I = \mathrm{diag}(1, 1, \ldots, 1) \rightarrow \hat{I}(t, r, p, \psi, \ldots) = U \times U^{\dagger} = 1/\hat{T} \neq I, \tag{13a}$$

$$U \times (A \times B) \times U^{\dagger} = (U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger}) = \hat{A} \hat{\times} \hat{B}, \tag{13b}$$

$$\hat{c} = U \times c \times U^{\dagger} = c \times \hat{I}, \qquad \hat{c}_1 \hat{+} \hat{c}_2 = (c_1 + c_2) \times \hat{I}, \qquad \hat{c}_1 \hat{\times} \hat{c}_2 = (c_1 \times c_2) \times \hat{I}, \tag{13c}$$

$$\hat{r}^k = r^k \times \hat{I}, \qquad \hat{d}\hat{r}^k = \hat{I}^k_i \times d\hat{r}^i, \qquad \hat{\partial}/\hat{\partial}\hat{r}^k = \hat{T}^i_k \times \partial/\partial r^i, \qquad \hat{\partial}\hat{r}^i/\hat{\partial}\hat{r}^j = \hat{\delta}^i_k = \delta^i_k \times \hat{I}, \tag{13d}$$

$$\begin{gathered} U \times \langle\phi| \times |\psi\rangle \times U^{\dagger} = \langle\hat{\phi}| \hat{\times} |\hat{\psi}\rangle \times \hat{I}, \qquad |\hat{\phi}\rangle = U \times |\phi\rangle, \\ U \times H \times |\phi\rangle = \hat{H} \hat{\times} |\hat{\phi}\rangle = U \times E \times |\phi\rangle = \hat{E} \hat{\times} |\hat{\phi}\rangle = E \times |\hat{\phi}\rangle, \\ \hat{H} = U \times H \times U^{\dagger}. \end{gathered} \tag{13e}$$

The transformation theory of the new IST is then strictly *nonunitary*. Therefore, the theory remains *noninvariant* when conventionally treated. However, for consistency, all possible nonunitary transforms must be rewritten as *isounitary transforms* on $\hat{\mathscr{H}}$ over $\hat{C}$, with consequential regaining of all original invariances (2),[3g] e.g.,

$$V \times V^{\dagger} = \hat{I} \neq I, \qquad V = \hat{V} \times \hat{T}^{1/2}, \qquad V \times V^{\dagger} = \hat{V} \hat{\times} \hat{V}^{\hat{\dagger}} = \hat{V}^{\hat{\dagger}} \hat{\times} \hat{V} = \hat{I}, \tag{14a}$$

$$\hat{I} \rightarrow \hat{V} \times \hat{I} \times \hat{V}^{\hat{\dagger}} = \hat{I}' = \hat{I}, \tag{14b}$$

$$\hat{A} \hat{\times} \hat{B} \rightarrow \hat{V} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{V}^{\hat{\dagger}} = (\hat{V} \hat{\times} \hat{A} \hat{\times} \hat{V}^{\hat{\dagger}}) \hat{\times} (\hat{V} \hat{\times} \hat{B} \hat{\times} \hat{V}^{\hat{\dagger}}) = \hat{A}' \hat{\times} \hat{B}', \tag{14c}$$

$$\begin{aligned} \hat{H} \hat{\times} |\hat{\psi}\rangle = \hat{E} \hat{\times} |\hat{\psi}\rangle \rightarrow \hat{V} \hat{\times} \hat{H} \hat{\times} |\hat{\psi}\rangle &= \hat{V} \hat{\times} \hat{H} \hat{\times} \hat{V}^{\hat{\dagger}} \hat{\times} \hat{V} \hat{\times} |\hat{\psi}\rangle = \hat{H}' \hat{\times} |\hat{\psi}'\rangle \\ &= \hat{V} \hat{\times} \hat{E} \hat{\times} |\hat{\psi}\rangle = \hat{E}' \hat{\times} |\hat{\psi}'\rangle, \qquad \hat{E}' = \hat{E}, \end{aligned} \tag{14d}$$

As one can see, the use of the isotopic formalism of hadronic mechanics implies the full regaining of the *numerical invariance* of the isounit, isoproduct, and isoeigenvalues, thus regaining necessary conditions for insistent physical applications. It is easy to prove that *isohermiticity coincides with the conventional Hermiticity*. As a result, all conventional observables of unitary theories remain observables under their isotopic lifting. The preservation of Hermiticity-observability in time is then ensured by the above isoinvariances. Detailed studies conducted in Ref. 3g then establish the resolution of all inconsistencies of Theorem 1.

The explicit isotopic reformulation of pre-existing string models is elementary. Consider, for instance, the theory of Ref. 2b with metric (6). The isotopic reformulation then merely requires the use of the isounit $\hat{I} = U \times U^{\dagger} = \mathrm{Diag}(1, a^{-2} \times \delta^i_j)$ and the reconstruction of the totality of the formalism with respect to $\hat{I}$, including numbers, fields, spaces, algebras, functional analysis, etc. Invariance and the resolution of the inconsistencies of Theorem 1 then follow. Note that the construction implies a mere *reformulation* of conventional string theories without alterations of their conception. However, the numerical results and their interpretations are altered, as we shall see in Section 9.

A few comments are now in order. The primary reason for the the resolution of the inconsistencies is the full regaining of the Lie axioms. In fact, under nonunitary transforms submitted to isotopic reformulation, we have the rules of the *Lie–Santilli isotheory*,[3p, 5]

$$U \times e^X \times U^{\dagger} = \hat{e}^{\hat{X}} = (e^{\hat{X} \times \hat{T}}) \times \hat{I} = \hat{I}(e^{\hat{T} \times \hat{X}}), \tag{15a}$$

$$\hat{A}(\hat{w}) = \hat{U} \hat{\times} \hat{A}(\hat{0}) \hat{\times} \hat{U}^{\hat{\dagger}} = \hat{e}^{i\hat{X} \hat{\times} w} \hat{\times} \hat{A}(\hat{0}) \hat{\times} \hat{e}^{-i\hat{w} \hat{\times} \hat{X}} = e^{(\hat{X} \times w) \times \hat{T}} \times \hat{A}(\hat{0}) \times e^{-i\hat{T} \times (\hat{w} \times \hat{X})}, \tag{15b}$$

$$i\hat{d}\hat{A} \hat{/} \hat{d}\hat{w} = \hat{A} \hat{\times} \hat{X} - \hat{X} \hat{\times} \hat{A} = \hat{A} \times \hat{T} \times \hat{X} - \hat{X} \times \hat{T} \times \hat{A} = [\hat{A} \hat{,} \hat{X}], \tag{15c}$$

$$\hat{e}^{i\hat{X} \hat{\times} \hat{w}} = [\hat{e}^{-i\hat{w} \hat{\times} \hat{X}}]^{\hat{\dagger}}. \tag{15d}$$

As one can see, the regaining of Lie's theory is so strong that the conventional and isotopic theories coincide at the abstract, realization-free level. In fact, the Lie–Santilli isotopic theory can be formulated by essentially "putting a hat" to the *totality* of symbols and operations of the conventional formulation of Lie's theory or, equivalently, by keeping the conventional formulation and subjecting *all* conventional symbols to the more general isotopic interpretation.

All Lie–Santilli and conventional Lie symmetries are then locally isomorphic due to the positive-definiteness of the isounit. This implies the preservation, not only of the fundamental Poincaré symmetry mentioned earlier, but also of gauge, SU(2), SU(3), and the other symmetries of contemporary use [3]. However, the latter are realized with hitherto unknown, novel degrees of freedom which constitute a concrete and specific realization of "hidden variables."[3s] As it is the case for IST and the Poincaré symmetry, the novel degrees of freedom permit novel applications which are impossible for conventional realizations of Lie symmetries.

In summary, the main point submitted at the VII M. Grossmann Meeting[3i] is that, apparently, an axiomatically consistent formulation of quantum gravity escaped identification for about three-quarter of a century, because occurring where nobody looked for: in the *unit* of relativistic quantum mechanics. The main point submitted at the VIII M. Grossmann meeting[3k] is that the achievement of a consistent grand unification has failed since Einstein's attempts because of the irreconcilable axiomatic incompatibility of curvature with all non-gravitational interactions. The abandonment of curvature in favor of the covering isoflatness permits the first known, axiomatically consistent grand-unification, thanks to the prior unifications of the special and general relativities (unification of the Minkowskian and Riemannian geometries) under the covering, eleven-dimensional Poincaré–Santilli isosymmetry. The proposed isotopic formulation of string theories incorporates all these results, thus permitting *the string formulation of grand unified theories.* In short, relativistic hadronic mechanics[3g] emerge as the only consistent operator formulation of string, gravitational, and electroweak theories known to this author.

5. INVARIANT FORMULATION OF IRREVERSIBILITY

It is nowadays established that the irreversibility of our macroscopic reality originates at the most elementary possible level of nature, in view of the following property of easy proof:

Theorem 2 (Refs. 3g and 3s). A classical irreversible system cannot be consistently reduced to a finite set of elementary reversible systems, and, vice-versa, a finite collection of reversible elementary systems cannot possibly yield a macroscopic irreversible ensemble.

The above new realities in irreversibility requires irreversible formulations for any consistent representation of irreversible structures and events, such as particle and chemical reactions, composite systems with internal entropy, etc.

On the other hand, all potentials (and, therefore, all Lagrangians and Hamiltonians) are fully reversible, with no meaningful exception known to this author.

This establishes that *irreversibility should be represented with anything except a Lagrangian or a Hamiltonian*. The approach suggested by the author[3b, 3c, 3g] is to represent irreversibility via the *structure* of the theory, in such a way that irreversibility occurs for all possible reversible Lagrangians or Hamiltonians.

The assumed main physical origin is the historical teaching by Lagrange and Hamilton (generally forgotten during the past century), according to which physical reality cannot be solely represented with functions we today call Lagrangians or Hamiltonians, for which reason Lagrange and Hamilton added *external terms* in their celebrated analytic equations.

The above occurrence essentially establishes that irreversibility is due to contact interactions of the type classically established in resistive forces, which are generally nonlinear, nonlocal, and noncanonical. These interactions are of "zero-range" by conception, thus requiring analytic representations dramatically beyond the familiar notion of action-at-a-distance, potential forces. Particle counterparts occurs for deep overlapping of the *extended wavepackets* of particles, beginning with those of *point-like charges*, as for the electrons, which interactions are at the foundations of hadronic mechanics.[3]

Lagrange's and Hamilton's historical external terms have been "truncated" in the virtual entire physics literature of the past century, thus resulting in now vexing, unresolved controversies on the origin of irreversibility, in which controversies, rather ironically, the most plausible origin is entirely missing.

Unfortunately, the addition of external terms to the basic analytic equations implies the loss, not only of invariance (because of their evident noncanonical-nonunitary structure), but actually implies the loss of *any* consistent algebra in the brackets of the time evolution (due to the violation of the right distributive and scalar laws).

The achievement of an algebraically consistent and invariant formulation of irreversibility along the historical teaching by Lagrange and Hamilton required decades of efforts. First, as part of his Ph.D. thesis, the author proposed in 1967[3b] the first known (p, q)*-parameter deformations of Lie's theory* with basic equations,

$$(A, B) = p \times A \times B - q \times B \times A$$

$$= m \times (A \times B - B \times A) + n \times (A \times B + B \times A), \tag{16a}$$

$$A(w) = (e^{iX \times q \times w}) \times A(0) \times (e^{-iw \times p \times X}), \tag{16b}$$

plus classical counterpart here omitted for brevity (see Ref. 3s). Deformations (16) resulted to be, jointly, *Lie- and Jordan-admissible*, in the sense of the American mathematician A. A. Albert, i.e., the attached antisymmetric and symmetric brackets

$$[A\hat{,}B] = (A, B) - (B, A), \{A\hat{,}B\} = (A, B) + (B, A), \tag{17}$$

are Lie and Jordan, respectively.

Despite countless referrals to authors and editors alike, the quotation of origination[3b] back in 1967 of the (p, q) deformations of Lie algebras continue to be absent in the otherwise vast literature of the river of papers in the field. Ironically, at the time of initiation in 1987 of the ongoing vast studies on q-deformations, this author had already abandoned the field, because of their lack of invariance, thus lack of known physical meaning.

Inspection of structure (16) revealed that it is not invariant under its own action, evidently because the latter is nonunitary. Application of nonunitary transforms to structure (16) then yielded in 1978 the first known (P, Q)-*operator deformations* of Lie's theory,[3c]

$$(A\hat{,}B) = A \times P \times B - B \times Q \times A$$
$$= (A \times M \times B - B \times M \times A) + (A \times N \times B + B \times N \times A), \tag{18a}$$
$$A(w) = (e^{iX \times Q \times w}) \times A(0) \times (e^{-iw \times P \times X}. \tag{18b}$$

The latter theory also result to be jointly Lie- and Jordan-admissible, although in a form broader than that by Albert (today called Third Condition of Lie- and Jordan-admissibility), because the attached algebras are not conventional Lie or Jordan, but rather their isotopic images.

It is easy to see that the product $(A\hat{,}B)$ is the most general conceivable product of an algebras as currently understood in mathematics. Therefore, theory (18) is *directly universal*, in the sense of admitting as particular case *all* infinitely possible algebras, including: Lie algebras; Jordan algebra; isotopic Lie and Jordan algebras; supersymmetric algebras; Kac–Moody algebras, etc. Despite that, the quotation of theory (18) continues to be generally absent in the vast literature of Lie deformations and quantum groups, despite countless referrals to authors and editors alike.

It is also easy to see that *theory (18), and all its possible nonunitary particularizations are not invariant under their own action, and, therefore, they have no meaningful physical applications*. In fact, the application of finite transform (18b) to brackets $A\hat{,}B)$ implies *the alteration of the P and Q operators*, as the reader can verify.

The achievement of invariance left no other option for the author, a physicist, than that of constructing another new mathematics specifically conceived for irreversibility, today known as *Santilli's Lie-admissible theory*, or, more technically, *Santilli's genomathematics*. Its construction was initiated in 1978,[3c] with subsequent results presented in memoir[3f] of 1996. Invariance was achieved only in the recent paper[3n] of 1997.

The main feature is the assumption of a *non-Hermitean generalized unit* $\hat{I}^{>} = 1/Q$, called *genounit*, and a related *ordered product to the right* for the representation of *motion forward in time*, with conjugate quantities and multiplications for *motion backward in time*, according to the rules [*loc. cit.*],

$$\hat{I}^{>} = 1/Q, \qquad A > B = A \times Q \times B, \qquad A > \hat{I}^{>} = \hat{I}^{>} > A = A, \tag{19a}$$

$$^{<}\hat{I} = 1/P, \qquad A < B = A \times P \times B, \qquad ^{<}\hat{I} < A = A < {}^{<}\hat{I} = A, \tag{19b}$$

$$\hat{I}^{>} = 1/Q = ({}^{<}\hat{I})^{\dagger} = (1/Q)^{\dagger}. \tag{19c}$$

Genomathematics is given by the reconstruction of the totality of isomathematics with respect to the two ordered products to the right and to the left, resulting in the novel *genonumbers and genofields, genospaces and genogeometries, genoalgebras and genosymmetries, etc.*.[3f]

The procedure yields the following Lie-admissible/genotopic realization of Lie's axioms at a fixed value of the parameter w, which we write in the form explaining the unusual notation of Eqs. (1)[3c, 3d]

$$\begin{aligned}\hat{A}(\hat{w}) &= e_{>}^{i\hat{X} > \hat{w}} > \hat{A}(\hat{0}) < e_{<}^{-i\hat{w} < \hat{X}} \\ &= [e^{(i\hat{X} > \hat{w}) \times \hat{S}} \times \hat{I}^{>}] \times \hat{S} \times \hat{A}(\hat{0}) \times \hat{R} \times [{}^{<}\hat{I} \times e_{<}^{-i\hat{R} \times (\hat{w} < \hat{X}}],\end{aligned} \tag{20a}$$

$$i\hat{d}\hat{A}/\hat{d}\hat{w} = (\hat{A}, \hat{X}) = \hat{A} < \hat{X} - \hat{X} > \hat{A} = \hat{A} \times \hat{R} \times \hat{X} - \hat{X} \times \hat{S} \times \hat{A}, \tag{20b}$$

$$\hat{X} = \hat{X}^{\dagger}, \qquad \hat{P} = \hat{Q}^{\dagger}. \tag{20c}$$

It should be stressed that structures (20) merely provide a *broader realization* of the original Lie axioms, which is the basic theme of hadronic mechanics.[3] In fact, the Lie axioms have been written in form (1) to illustrate their *bimodular associative structure*, with a modular-associative action to the right, and a separate modular-associative action to the left. Hadronic mechanics was born[3c] from the observation that these actions do not need to be realized by the simplest conceivable associative product studied since high school, because the axioms admit other associative, yet less trivial realizations of the associative product. The lack of necessary identity of the two modular-isotopic actions then yields genotopic structures (20).

Note that theory (20) is structurally irreversible, that is, it is irreversible for all possible reversible generators. However, axiomatic consistency is achieved only under conjugation (20c), as a broader realization of the original conjugation (1c). We discover in this way *additional catastrophic inconsistencies under time reversal of all deformations for which* $p \neq q^\dagger$ *or* $P \neq Q^\dagger$.

The regaining of *Lie's* axioms for *irreversible* structures then permitted the achievement of *invariance*, as treated in detail in memoirs.[1g, 3n]

It was only following the the above studies that this author was able to achieve *an invariant formulation of the true Lagrange and Hamilton's equation*, in which the external terms are embedded in the *forward genounit* for motion forward in time, or in the backward genounits for the conjugate motion backward in time. We reach in this way the following analytic equations, today called *Hamilton–Santilli Lie-admissible (or geno-) equations*:[3f]

$$\frac{d^> a^{>\mu}}{d^> t^>} = \omega^{>\mu\nu} > \frac{\partial^> H}{\partial^> a^{>\nu}}, \tag{21a}$$

$$\begin{gathered} t^> = t \times I_t^>, \qquad d^> = I^> \times d, \qquad a^> = a \times I_{ps}^>, \\ a = (r^k, p_k), \qquad k = 1, 2, 3, \qquad \partial^> / \partial^> a^\nu = T_\nu^{>\ \rho} \partial / \partial a^\rho, \end{gathered} \tag{21b}$$

$$\omega^> = \omega \times I_{ps}^>, \omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad I_{ps}^> = 1/T_{ps} = \begin{pmatrix} 1 & F^{\text{ext}}[1/(\partial H/\partial p)] \\ 0 & 1 \end{pmatrix}, \tag{21c}$$

where: $t^> = t \times I_t^>$ is the *forward genotime*; $I^>$ is the (one dimensional) *forward time genounit*; $a^> = a \times I_s^>$ is the *forward phase genospace* (geno-cotangent bundle); $I_s^>$ is the sixdimensional *genounit of geno-phase-space*; ω is the conventional Lie tensor (cosymplectic structure); r and p are the conventional phase space coordinates; H is a conventional Hamiltonian; $F^{\text{ext}}(t, r, p)$ are the (three-dimensional) historical external terms; the brackets of the time evolution of Eqs. (21),

$$\begin{aligned} (A \hat{,} B) &= \frac{\partial^> A}{\partial^> a^{>\mu}} > \omega^{>\mu\nu} > \frac{\partial^> B}{\partial^> a^{>\mu}} \\ &= \frac{\partial A}{\partial a^\mu} \times \omega^{\mu\rho} \times \hat{T}_\rho^{>\ \nu} \times \frac{\partial B}{\partial a^\nu}, \end{aligned} \tag{22}$$

first characterize a consistent algebra, and then such algebra results to be Lie-admissible. The corresponding *Lagrange–Santilli genoequations* are omitted for brevity (see Ref. 3f).

A simple genotopy of the conventional, naive or symplectic quantization then yields the fundamental equations of hadronic mechanics[3c, 3g] of which the isotopic ones are particular cases:

$$i\partial_t^> \,|\psi\rangle = i\hat{T}_t^> \partial_t \,|\psi\rangle = H > |\psi\rangle = H \times T^> \times |\psi\rangle = E^> \times |\psi\rangle, \tag{23a}$$

$$p_k > |\psi\rangle = p_k \times T^> \,|\psi\rangle = -i\partial_k^> \,|\psi\rangle = -i\hat{T}_k^{>\,i} \times \partial_i \,|\psi\rangle, \tag{23b}$$

$$\begin{aligned} d^> A/d^> t^> &= (A, H) = A < H - H > A \\ &= A \times {}^<T_s \times H - H \times T_s^> \times A, \end{aligned} \tag{23c}$$

$$(r^i, p^j)^> = \delta_j^{>\,i} = \delta_j^i \times I_s^>, \tag{23d}$$

$$I_s^> = 1/T_s^> = ({}^<I)^\dagger = (1/{}^<T)^\dagger, \tag{23e}$$

As one can see, all Hamiltonians of Eqs. (21) and (23) are fully reversible, yet the equations are structurally irreversible, because of the lack of symmetry of the genounit induced by the external terms. This occurrence is in full compliance with Lagrange's and Hamilton's historical (but generally forgotten) legacy on the origin of irreversibility.

A mention of the *genogeometries*[3f, 3s] is appropriate, because they allow the first known, invariant geometric treatment of irreversibility via *nonsymmetric metrics*, provided that they are treated via genomathematics.

Consider a conventional, symmetric, Riemannian metric in iso-minkowskian decomposition, $g(x) = \hat{T}(x) \times \eta$. When treated with respect to the isounit $\hat{I}(x) = 1/(Tx)$, the axioms of the geometry result to be Minkowskian. It is then easy to see that any *nonsymmetric metric* $g^>(x) = \hat{T}^>(x)\,\eta$, $\hat{T}^> \neq (\hat{T}^>)^t$ with $\hat{T}^>(x)$ nonsingular, equally obeys the *Minkowskian* axioms when referred to the forward genounit $\hat{I}^> = 1/\hat{T}^>$.

As a result, the entire formalism of the isominkowskian geometry[3j] equally holds when the symmetric character of the isometric is relaxed, by characterizing in this way the *genominkowskian geometry* [*loc. cit.*]. Note the crucial need of genomathematics for the achievement of a consistent geometry. In fact, nonsymmetric extensions $g^>(x) \neq [g^>(x)]^t$ of the Riemannian metrics $g(x) = [g(x)]^t$ are afflicted by known inconsistencies when formulated over conventional fields and related units.

It should also be mentioned that the conventional Poincaré symmetry P(3.1) and its isotopic extension $\hat{P}(3.1)$ admit Lie-admissible/genotopic, forward and backward coverings, $\hat{P}^>(3.1)$ and ${}^<\hat{P}(3.1)$, respectively, with structure (20), today known as the *Poincaré–Santilli Lie-admissible (or geno-) symmetries.*[3, 4, 5] The classical (operator) representations of the isotopic symmetry $\hat{P}(3.1)$ can be easily constructed via noncanonical (nonunitary) transforms of conventional representations. Those of the broader genosymmetry $\hat{P}^{>}(3.1)$ are constructed in a similar way, although, this time, via a nonsymmetric combination of *two* noncanonical (nonunitary) transforms of conventional representations (see next section).

In conclusion, the entire body of formulation of conventional or isotopic theories (including units, fields, spaces, geometries, algebras, symmetries, classical and quantum theories, etc.) admits a consistent genotopic extension for the invariant characterization of irreversibility along the historical teaching by Lagrange and Hamilton.

This remarkable result is due to a truly basic property of numbers, apparently identified for the first time in Ref. 3e, according to which the restriction of all multiplications among numbers to an ordered product to the right, or, separately, to an ordered product to the left, verifies all axioms of a field. This occurrence permitted the construction of genofields to the right and to the left as particular realizations of ordinary fields. Once the axioms of a field are preserved by Lie-admissible/genotopic theories, a consistent formulation of all other structures built on them is a mere consequence.

6. INVARIANT GENO- AND HYPER-STRUCTURAL STRING THEORIES FOR SINGLE-VALUED IRREVERSIBLE SYSTEMS

We are now in a position to introduce, apparently for the first time, *Lie-admissible string or genostring theories* (GST) for the representation of single-valued irreversible systems, such as an individual irreversible reaction, herein defined as string theories characterized by a non-Hermitean genounit and related ordered product to the right for motion forward in time, with conjugate expressions for motion backward in time, under the rigid constraints of the Poincaré–Santilli genosymmetry.

Genomathematics has today reached an operational level which permits a simple explicit construction of GST, via the use of *two* nonunitary transforms, according to the identifications

$$V \times V^{\dagger} \neq I, \qquad W \times W^{\dagger} \neq I, \tag{24a}$$

$$V \times W^{\dagger} = \hat{I}^{>} = 1/\hat{Q}, \qquad W \times V^{\dagger} = {}^{<}\hat{I} = 1/\hat{P}, \qquad \hat{I}^{>} = ({}^{<}\hat{I})^{\dagger}. \tag{24b}$$

Classical GST are then characterized by the Hamilton–Santilli genoequations (21) or their geno-Lagrangian counterpart, in which the forward and backward genounits have the respective forms,

$$\hat{I}^{>}_{ps} = \begin{pmatrix} \hat{T}_s & F^{\text{ext}}[1/(\partial H/\partial p)] \\ 0 & \hat{I}_s \end{pmatrix}, \qquad {}^{<}I_{ps} = \begin{pmatrix} \hat{T}_s & 0 \\ F^{\text{ext}}[1/(\partial H/\partial p)] & \hat{I}_s \end{pmatrix}, \tag{25}$$

where: $\hat{I}_s = 1/\hat{T}_s$ is the symmetric isounit of space of IST. The operator version of GST is characterized by Eqs. (23) in which representative examples of off-diagonal terms are collision terms used in statistical mechanics.

The above elements must then be completed, for necessary reasons of consistency, with the forward and backward genofields, genospaces, geno-differential calculus, genogeometries, etc. Invariance is achieved by reformulating nonunitary transforms in the forward or backward genounitary form, depending on the case at hand. For brevity, we refer the reader on invariance of genotopic theories to paper.[3n]

The reader should keep in mind that *the very assumption of a Lie-admissible time evolution for GST assures their supersymmetric character*, although, this time, in an invariant way. In fact, as indicated earlier, supersymmetric algebras are a simple particular case of general Lie-admissible algebras, Eq. (18a). Other aspects and applications of GST will be presented in subsequent works.

7. INVARIANT HYPERSTRUCTURAL STRING THEORIES FOR MULTIVALUED IRREVERSIBLE STRUCTURES

Physics is a discipline that will never admit "final theories." No matter how sound or general any given theory may appear, its structural generalization is only a matter of time.

In view of this teaching from the history of science, by no means GST should be considered as the most general possible theories. For completeness, we mention the existence of the *hyperstructural branch of hadronic mechanics*[3f, 3g] whose main characteristic is that of being *multi-valued* with *forward and backward hyperunits* and related *forward ordered hyperproduct to the right and to the left*,

$$\hat{I}^{>} = \{\hat{I}_1^{>}, \hat{I}_2^{>}, \hat{I}_3^{>}, \ldots\} = 1/\hat{Q}, \tag{26a}$$

$$A > B = \{A \times \hat{Q}_1 \times B, A \times \hat{Q}_2 \times B, A \times \hat{Q}_3 \times B, \ldots\},$$
$$\hat{I}^{>} > A = A > \hat{I}^{>} = A \times I, \tag{26b}$$

$$^{<}\hat{I} = \{^{<}\hat{I}_1, {}^{<}\hat{I}_2, {}^{<}\hat{I}_3, \ldots\} = 1/\hat{P}, \tag{26c}$$

$$A < B = \{A \times \hat{P}_1 \times B, A \times \hat{P}_2 \times B, A \times \hat{P}_3 \times B, \ldots\}\, ^{<}\hat{I} < A = A < {}^{<}\hat{I} = A \times I, \tag{26d}$$

$$A = A^{\dagger}, \qquad B = B^{\dagger}, \qquad \hat{P} = \hat{Q}^{\dagger}. \tag{26e}$$

All aspects of the dual Lie-admissible formalism admit a unique, and significant extension to the above hyperstructures, including *hypernumbers, hyperfields, hyperspaces, hyperdifferential calculus, etc.*[3f]

The construction of invariant *hyper-string theories* (HST) is then elementary and will be left to the interested reader. Their most salient (and intriguing) feature is that of *permitting the existence of a multi-valued universe beginning at the most elementary level of nature, and in a way compatible with our three-dimensional sensory perception.* In fact, the hyper-Euclidean geometry coincides with the conventional Euclidean geometry at the abstract, realization-free level by construction.[3t]

A suggestive illustrative application of HST is given by *sea shells.*[3t] All their possible *shapes* can indeed be fully represented in the conventional Euclidean space corresponding to our three Eustachian tubes. However, their *evolution in time* cannot be described via the Euclidean axioms, trivially, because sea shells are open-nonconservative-irreversible, while the Euclidean axioms are strictly closed-conservative-reversible. Computer visualizations have shown that the imposition of the latter to the time evolution of the former implies that sea shells first grow in a deformed way and then they crack. Studies conducted by Illert (see Ref. 3t and references quoted therein) have shown that the quantitative representation of the growth in time of sea shells requires at least a *six dimensional space*; i.e., the doubling of each reference axis.

However, we can directly observe sea shells in our hands as being three-dimensional. The only reconciliation of these seemingly dissonant occurrences known to this author is that via hyperformulations. In fact, the latter are multi-dimensional precisely as needed for the representation of the growth of sea shells. Yet the axioms remain conventional, thus achieving compatibility with our sensory perception.

Specifically, for the case of sea shells we have two-valued hypereuclidean formulations which are fully compatible with our sensory perception because *they are not six-dimensional*, and remain instead fully three-dimensional. After all, there is a dramatic topological difference between a conventionally *six-dimensional* space and the *two-valued three-dimensional* hyperspace.

Note that any beliefs in treating open-nonconservative-irreversible biological systems via conventional quantum mechanics implies exiting the boundaries of science, evidently because of the strictly closed-conservative-reversible character of that theory. This irreconcilable incompatibility was a main motivation for the construction of hadronic mechanics.[3]

It should be finally mentioned that the hypermathematics considered in this section is a particular form of that studied in contemporary mathematics. The latter is expressed in terms of the so-called *weak* *identities and operations* which are excessively abstract for the limited scope of this note.

8. ISODUAL STRING THEORIES FOR ANTIMATTER

By no means HST constitute the ultimate and most general formulation of string theory. Despite their remarkable generality, hyperformulations (including conventional, isotopic and, genotopic particularizations) cannot consistently represent *antimatter* at the *classical* level.

This is due to the sole existence of *one* quantization channel per each theory, as a consequence of which we have the following:

Theorem 3 (Refs. 3a and 3p). Whether conventional, isotopic, genotopic, or hyperstructural, classical formulations of antimatter are inconsistent, because, after quantization, they yield ordinary particles with the wrong sign of the charge, rather than the correct charge conjugate antiparticles.

The above occurrence is only the symptom of what can be safely called the biggest unbalance of theoretical physics of the past century: the treatment of matter at all possible levels, from Newton to second quantization, while antimatter is solely treated at the level of *second* quantization.

After a laborious search, the consistent *classical* representation of *antimatter* required the construction of yet another novel mathematics, today called *Santilli's isodual mathematics.*[3o, 3p] In essence, charge conjugation is an *anti-automorphic map*, although it is valid only in operator settings. Classical treatment of antimatter via conventional or iso-, geno-, and hyper-mathematics are then inconsistent, because lacking a basic anti-automorphic (or, more general, anti-isomorphic) conjugation at the classical level.

The simplest possible map of conventional or broader mathematics which is anti-isomorphic, as well as applicable at *all* levels, beginning with *Newton's* equations, is that characterized by the *change of the sign of the basic unit*, or, more generally, by the following map, called *isoduality*, here expressed for a generic quantity A, as well as for the underlying spaces and fields,[3o, 3p]

$$A(x, v, \psi, \ldots) \to A^d = -A^\dagger(-x^\dagger, -v^\dagger, -\psi^\dagger, \ldots) \tag{27}$$

which characterizes the *isodual branches of hadronic mechanics.*[3g]

In order to achieve a consistent classical and operator representation of antimatter, isotopic, genotopic and hyper-string theories must be mapped into their respective isoduals. These isodualities have been already studied in the literature, and will not be repeated here for brevity. whose explicit construction is left to the interested reader for brevity.

Map (27) is mathematically nontrivial, e.g., because it implies the first known *numbers with negative units and norm.*[3e] Physically, the map is also nontrivial, because it implies an isodual image of our universe which coexists with our own, yet it is distinct from it. In other words, charge conjugation, maps spaces onto themselves, thus implying no new universe for antimatter. Isoduality, instead, maps our spaces into new independent spaces, which, as such, have to coexist with our own spaces. In this way, antimatter can be considered as the first possible evidence for a "multi-layered" (rather than multi-dimensional) structure of the universe more generally implied by hypertheories.

Isoduality has permitted the first known resolution of the scientific unbalance indicated above, by achieving a treatment of antimatter which is absolutely equivalent to that of matter at all levels. In particular, when applied at the operator level, isoduality is equivalent to charge conjugation.[3o] Also, *all physical quantities (and not only the charge) interconnected by isoduality have opposite signs, although referred to units also with opposite signs*. For instance, time for an isodual antiparticle is *negative,* although referred to a *negative unit of time* -1 sec, thus being as causal as our conventional positive time referred to the conventional positive unit $+1$ sec. The same happens for energies, which become negative for antiparticles, yet refereed to negative units of energy, thus resolving the old controversy on negative energies.

Most intriguingly, *the isodual theory of antimatter implies the existence of antigravity, intended as gravitational repulsion experienced by matter in the gravitational field of antimatter, or vice-versa* [*loc. cit.*]. This result is due to the *identification* (rather than unification) of electromagnetic and gravitational fields (in the sense that exterior gravitation can be entirely reduced to electromagnetic fields originating the gravitational masses of the individual particle constituents). The preservation by gravity of all features of electromagnetism, including attraction and repulsion, is then mandatory.

Note the crucial role of the novel isodual mathematics for the consistent formulation of antigravity, because, in its absence, antigravity is afflicted by a litany of inconsistencies well known in the specialized literature. All these objections against antigravity cannot be even formulated, let alone make physical sense, for the isodual treatment of antimatter, trivially, because they are all based on positive units.

The Kronecker product of a particle (or formulation) and its isodual is called *isoselfdual*, in the sense of coinciding with its isodual image. The simplest possible isodual quantity is the imaginary unit i, while isoselfdual particles are bound states of particles and antiparticles, such as the π^o (see Refs. 3o, and 3p for details). As discussed below, one of the biggest open problems in contemporary cosmology is whether the universe is isoselfdual or not.

9. EPISTEMOLOGICAL COMMENTS

The first paper written by this author in 1956,[(3a)] when a high school student, submitted the hypothesis that *the entire matter can be reduced to elementary oscillations-vibrations of the ether today called strings, as a necessary condition to eliminate the ethereal wind used at the time to dismiss the existence of a universal substratum.*

The main argument is that we would not be able to hear each-other voices without Earth's atmosphere. Similarly, we would be unable to see each-other faces without a universal medium (ether) propagating light. Since light is a *transversal* wave, the ether must necessarily be rigid, thus the title of paper of Ref. 3a "Perché lo spazio e' rigido" ("Why space is rigid"). Then, the only possibility for matter to freely move in space without the "ethereal wind," is that all electrons, protons and neutrons constituting matter are oscillations-vibrations of the ether.

According to this view, when matter is moved from one position to another, there is no actual motion of any "solid," but merely the transfer of the basic oscillations forming matter from one position in space to another. Stated in plain terms, a main view expressed in Ref. 3a is that *matter is totally empty and space is totally full.*

At the time of writing Ref. 3a the author was unaware that, over twenty years earlier, Schroedinger had proved that the variable "x" in Dirac's equations for the *free* electron describes precisely an oscillation which can only be that of the universal substratum. In this sense, Schroedinger can be considered one of the true founders of contemporary string theories.

The isotopic formulation of string theories permits quantitative studies along the view of Ref. 3a with increasing methodological complexity of methods for increasingly complex realities. In fact, *IST permit an axiomatically consistent and invariant reduction of reversible physical systems constituting matter to the most elementary entities in the universe, the vibrations of the universal substratum;* GST then permit the inclusion of irreversibility due to contact-nonpotential interactions; HST permit to extend the study to include multi-value structures and biological systems; isodual string theories permit a corresponding study of antimatter at all levels; while isoselfdual hypermodels permit the first formulation of cosmology as they should be according to the very meaning of "cosmos," inclusive of quantitative treatments of irreversible biological structure under the rigid restrictions of the universal isodual hypersymmetry ${}^{<}\hat{P}(3.1) \times \hat{P}^{>}(3.1)$.

The epistemological implications of these possibilities are far reaching. We here mention them as mere *consequences* of the new methods, without any claim that they are necessarily correct.

All equations based on conventional mathematics for the study of the micro- or macro-worlds (such as Schroedinger, Heisenberg, Einstein–Hilbert and other equations) use our own variable of time t and space r. The first implication of iso-, geno-, and hyper-theories and their isoduals which has not escaped the attentive reader is that *the intrinsic time and space of elementary particles or far away quasars may eventually result to be different than our own*.

The notions of isotime $\hat{t} = t \times \hat{I}_t$, genotime $\hat{I}^{>} = t \times \hat{I}^{>}$, and hypertime $\{I\}^{>} = t \times \{I\}^{>}$ have been introduced[3s] precisely for the purpose of admitting quantitative studies of the possible differentiation between our time "t" and that of elementary or far away systems. The same occurs for isocoordinates $\hat{r} = r \times \hat{I}_s$, genocoordinates $\hat{r}^{>} = r \times \hat{I}^{>}$ and hypercoordinates $\{\hat{r}\} = r \times \{\hat{I}^{>}\}$. Note that the differences occur in the *units* of time and space. Differences in their numerical values is a mere consequence.

A crucial property establishing the plausibility of the above view, is that *possible structural differences between our spacetime and those of other physical environments are fully compatible with our sensory perception, when treated with iso-, geno, and hyper-mathematics*. This feature is the very essence of the "axiom-preserving character" of the new theories. In fact, the central Minkowskian axioms of our spacetime are *identical* to those of all infinitely possible isominkowskian spacetimes.

The above important occurrence is established by the basic invariant of all line elements, which must include the unit of the field to identify whether a given line element is a scalar, or an isoscalar or a hyperscalar, and can be written,

$$[\text{Measurement}]^2 \times [\text{Unit}]^2 = \text{Invariant}. \qquad (28)$$

As a result, spacetime can indeed change in the transition from one physical reality to another without our perception of the same due to invariance (28).

The reader should be aware that the implications of the above properties of all formulations based on hadronic mechanics, including string theories, are far reaching. For example, the meanlife of a meson can is of the order of 10^{-16} sec in our own spacetime, while its intrinsic meanlife can be *dramatically bigger (smaller)*, depending on whether the local isounit of time is correspondingly smaller (bigger). Similarly, we can claim that Bohr's orbit is of the order of 10^{-8} cm only in our own spacetime, because its intrinsic value can be, again, different. Similar deviations of physical reality from our perception can occur for astrophysical times and lengths.

Greater departures from our sensory perception are implied by iso-, geno-, and hyper-theories in regard to *shapes*. Because of the equivalence of our Eustachian lobes, we perceive the identity of the units of length along all three space directions. However, elementary vibrations-strings of space are expected to have different units along different space directions (evidently due to their intrinsic anisotropy). It is then easy to see that an elementary particles which appears to us to be spherical, may in reality have dramatically different shapes.

The latter occurrence is illustrated in Ref. 3s via the *isobox*, namely, a transparent cube with two observers, an external observer in our spacetime, and an internal iso-observer with different units of space and time. Even though the external observer sees the internal one, it does not mean that the latter exists at the perceived time, because it could be dramatically earlier in past time, or ahead in future time. Moreover, a cube of one cm in side perceived by the external observer may have arbitrary dimensions and shapes for the interior observer.

The inclusion of *Eddington's time arrow*, and its representation via genotime, evidently multiplies the possible departures from our perception of reality, to such an extent that systems believe today to be reversible, such as atomic or planetary structures, may ultimately result to be themselves irreversible, again, in a way compatible with our perception of their reversibility.

When multi-valued biological structures are admitted, the potential ' differences between our perception and the actual reality becomes beyond our comprehension. As an illustration, it has been proved in Ref. 3t that all infinitely possible shapes of seashells can be unified into one single hypersphere in two dimensions with units varying from sea shell to sea shell.

The addition of isoduality adds further potential departures from our sensory perception. Note that isoduality is necessary to represent all possible "times arrows" according to the four possibilities of genotimes and their isoduals

$$\text{Motion Forward From Past Time } {}^{<}\hat{I};\ 0; \qquad \text{Motion Forward To Future Time } \hat{I}^{>}; \tag{29a}$$

$$\text{Motion Backward To Past Time } {}^{<}\hat{I}^{d};\ 0; \qquad \text{Motion Backward From Future Time } {}^{<}\hat{I}^{d}. \tag{29b}$$

An implication is the first known invariant formulation of *geometric isolocomotion*,[3s] that is, motion with respect to an outside observer without the use of Newtonian actions and reactions, merely realized via the anisotropic alteration of the geometry, and resulting alteration of distances in the selected direction. This new type of locomotion appears to be already realized in biological structure, e.g., in the transportation by tree of their sap at great elevations without any visible mechanical means. Note that there are several types of geometric models of locomotion in the existing literature, although at the time of altering the local geometry, they lose invariance and physical meaning. An important property of geometric isolocomotion is that of being invariant. But then, local variations of spacetime via variations of their units become mandatory.

Another implication is a *causal spacetime machine*,[3s] namely, a *mathematical* model which permits a complete closed loop in the forward light cone in a way compatible with the principle of causality. To our best understanding at this time, this "spacetime machine" is only possible for isoselfdual particles, since the latter acquire the isotime of their environment (as compared to particles or, separately, antiparticles, which have their intrinsic time). Apparently, this type of "spacetime machine" is already realized in biological systems, because the scientific (i.e., quantitative) representation of sea shell bifurcations requires the necessary use of all four possible directions of time (29).

Equally intriguing is the new cosmology offered by the HST and its isodual, here called *isoselfdual hyperstring cosmology* (IHSC), and patterned along the lines of Ref. 3m, with novel features such as:

(1) the first cosmology characterized by a symmetry (rather than covariance), the universal isodual Poincaré–Santilli hypersymmetry ${}^{<}\{\hat{P}\}(3.1) \times \{\hat{P}\}^{>}(3.1)$;

(2) the first cosmology capable of including physical and biological systems;

(3) same amount of matter and antimatter in the universe (as a limit conditions under Lie's axiom (1c));

(4) universe with all identically null total quantities, such as energy, linear momentum, etc.

(5) lack of meaning of the "age of the universe," because the total time is identically null, while that for matter, and, separately, antimatter, may vary dramatically from conditions to conditions;

(6) lack of curvature replaced by isoflatness, which permits the unified inclusion of gravitation and all other interactions [3l];

(7) lack of need of the "missing mass," because the average speed of light c within galaxies and quasars in law $E = m \times c^2$ is *bigger* the speed of light in vacuum c_o when including all interior dynamical effects;[3m]

(8) considerable reduction of the currently believed size of the universe (because of the decrease of the speed of light within the huge astrophysical chromospheres, by therefore emitting light in vacuum already redshifted);

(9) possibility of future experimental study whether a far away galaxy or quasar is made up of matter or of antimatter (due to the prediction that the photon emitted by antimatter, the *isodual photon*, is repelled by gravity and has new parity properties).[3o, 3p]

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