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INCONSISTENCIES OF GENERAL RELATIVITY AND THEIR APPARENT RESOLUTION VIA THE POINCARÉ INVARIANT ISOGRAVITATION

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Abstract

In preceding works we pointed out serious axiomatic inconsistencies of grand unified theories when gravitation is included in its conventional Riemannian formulation. In this note we present a number of additional inconsistencies of general relativity and show that they ultimately originate from the the Riemannian curvature. In fact, the latter implies a noncanonical structure at the classical level and a nonunitary structure at the operator level, with consequential structural problems at both classical and operator levels, such as the lack of invariance in time of basic units. In turn the latter features imply the lack of well defined *invariance* (rather than the customary covariance), with consequential lack of invariance in time of numerical predictions. These problematic aspects suggest the construction of a new theory of gravitation based on the conditions of admitting a universal symmetry without curvature. The compatibility of gravitation with special relativity then uniquely identifies the needed invariance as a symmetry isomorphic to the Poincaré symmetry. It is shown that the latter conditions do not admit a solution within the context of the conventional Lie theory and its underlying mathematics. It is shown that the use instead of the novel isomathematics for matter and its isodual for antimatter, the related Lie-Santilli isotheory and its isodual and the resulting isotopies and isodualities of the Poincaré symmetry allow a geometric unification of general and special relativity via the axioms of the special, in which case gravitation does indeed emerge as possessing a universal symmetry without curvature. We indicate the apparent resolution of the inconsistencies of general relativity permitted by its isotopic reformulation, we point out some intriguing cosmological implications, and we show that the new invariant isogravitation is a concrete and explicit realization of the theory of "hidden variables", with a natural, axiomatically consistent operator form.

1. Introduction

As it is well known, electroweak theories have an outstanding scientific consistency (see, e.g., Refs. [1]), while the achievement of a grand unification with the inclusion of gravity as represented by general relativity [2] has remained elusive despite attempts dating back to Einstein.

In preceding works [3], we have pointed out a number of axiomatic inconsistencies of grand unifications in the representation of matter as well as of antimatter whenever gravity is represented via curvature in a Riemannian space, such as:

1) The admission by electroweak interactions of the fundamental Poincaré symmetry compared to the absence of a symmetry for any Riemannian treatment of gravitation in favor of the well known covariance;

2) The essentially flat, thus canonical structure of electroweak interactions compared to the curved, thus noncanonical structure of Riemannian gravitation, with consequential nonunitary character of quantum gravity and related well known problems of consistency;

3) The admission by electroweak interactions of negative-energy solutions for antimatter as compared to the strict absence of negative energies for any Riemannian treatment of gravitation.

An axiomatically consistent grand unification was then attempted in Refs. [3] via the isominkowskian representation of gravity [4] because: i) isominkowskian gravity admits a symmetry for matter that is isomorphic to the Poincaré symmetry, thus resolving inconsistency 1); ii) isominkowskian gravity replaces the Riemannian curvature with a covering notion compatible with the flatness of electroweak theories, thus resolving inconsistency 2); and iii) inconsistency 3) is resolved via the isodual theories of antimatter [5], including the isodual isominkowskian geometry [5g] that permits negative-energy solutions for the gravitational field of antimatter.

In this note, we study a number of additional inconsistencies of general relativity published in refereed journals, yet generally ignored in the vast literature in the field.

We then show that the only resolution of these additional inconsistencies known to the author is that proposed in Refs. [3], thus confirming the fundamental character of the Poincaré symmetry in its isotopic formulation for matter and its isodual for antimatter.

2. Consistency and Limitations of Special Relativity.

As it is well known, thanks to historical contributions by Lorentz, Poincaré, Einstein, Minkowski, Weyl and others, *special relativity* (see, e.g., the historical accounts [2f,2g]) achieved a majestic axiomatical and physical consistency.

After one century of studies, we can safely identify the origins of this consistency in the following crucial properties:

1) Special relativity is formulated in the Minkowski spacetime $M(x, \eta, R)$ with local spacetime coordinates, metric, line element and basic unit given respectively by

$$x = \{x^\mu\} = (r^k, t), \quad k = 1, 2, 3, \quad \mu = 1, 2, 3, 0, \quad c_o = 1, \tag{2.1a}$$

$$\eta = Diag.(1, 1, 1, -1), \tag{2.1b}$$

$$(x - y)^2 = (x^\mu - y^\mu) \times \eta_{\mu\nu} \times (x^\nu - y^\nu), \tag{2.1c}$$

$$I = Diag.(1, 1, 1, 1, 1), \tag{2.1d}$$

over the field of real numbers R, where we identify the conventional associative multiplication with the symbol $\times$ in order to distinguish it from the numerous additional multiplications used in the studies herein considered [3-10];

2) All laws of special relativity, beginning with the above line element, are *invariant* (rather than covariant) under the fundamental *Poincaré symmetry*

$$\mathcal{P}(3.1) = \mathcal{L}(3.1) \times \mathcal{T}(3.1), \tag{2.2}$$

where $\mathcal{L}(3.1)$ is the *Lorentz group* and $\mathcal{T}(3.1)$ is the *Abelian group of translations* in spacetime; and

3) The Poincaré transformations are *canonical* at the classical level and *unitary* at the operator level with implications crucial for physical consistency, such as the invariance of the assumed basic units (as per the very definition of a canonical or unitary transformation),

$$\mathcal{P} \times [Diag.(1cm, 1cm, 1cm, 1sec)] \times \mathcal{P}^t \equiv \\ \equiv Diag.(1cm, 1cm, 1cm, 1sec), \tag{2.3}$$

with the consequential fundamental property that *special relativity admits basic units and numerical predictions that are invariant in time.* In fact, the quantities characterizing the dynamical equations are the *Casimir invariants* of the Poincaré symmetry.

As a result of the above features, special relativity has been and can be confidently applied to experimental measurements because the units selected by the experimenter do not change in time, and the numerical predictions of the theory can be tested at any desired time under the same conditions without fear of internal axiomatic inconsistencies.

Despite these historical results, it should be stressed that, as is the fate for all theories, *special relativity has its own well defined limits of applicability.* What is well established at this moment is that special relativity is indeed valid for the arena of its original conception, the classical and operator treatment of "point-like particles" moving in vacuum.

Nevertheless, special relativity is *inapplicable* for the *classical* treatment of antiparticles as shown in detail in Ref. [5g]. This is essentially due due to the existence of only one quantization channel. Therefore, the quantization of a *classical antiparticle* characterized by special relativity (essentially via the sole change of the sign of the charge) clearly leads to a *quantum particle* with the wrong sign of the change, and definitely not to the appropriate charge conjugated state, resulting in endless inconsistencies.

In fact, the achievement of the correct antiparticle at the quantum level has requested the construction of the new *isodual mathematics* as an anti-isomorphic image of conventional mathematics, including its own *isodual quantization* and, inevitably, the construction of the new *isodual special relativity* (for brevity, see Ref. [7d] and quoted literature). In this case the isodual characterization of a classical antiparticle does indeed lead, under the isodual (rather than conventional) quantization, to the correct antiparticle as a charge conjugated state.

Special relativity has also been shown to be *inapplicable* (rather than violated) for the treatment of both, particles and antiparticles, such as hadrons, represented as they are in the physical reality, extended, generally nonspherical and deformable (such as protons or antiprotons), particularly when interacting at very short distances. In fact, these conditions imply the mutual penetration of the wavepackets and/or the hyperdense media constituting the particles, resulting in nonlocal integro-differential interactions that cannot be entirely reduced to potential interactions among point-like constituents (for mathematical studies of these aspects see Refs. [6], for comprehensive treatments see Refs. [7] and for independent works see Refs. [8-10]).

Note that the use of the words "violation of special relativity" would be here inappropriate because special relativity was specifically conceived for *point-like particles (and not antiparticles) moving in vacuum under sole action-at-a-distance interactions* [2f]. As a matter of fact, antiparticles were still unknown at the time of the conception and construction of special relativity. Similarly, states of deep mutual penetrations of extended hadrons, as occurring in the core of neutron stars or black holes, where simply unthinkable at the inception of special relativity.

3. Inconsistencies of General Relativity due to the Lack of Sources.

By comparison with special relativity, despite widespread popular support, there is no doubt that *general relativity* has been the most controversial theory of the 20-th century. In this and in the next section we shall review some of the major mathematical, theoretical and experimental inconsistencies of general relativity published in the refereed technical literature, yet generally ignored by scientists in the field.

There exist subtle distinctions between "general relativity", "Einstein's Gravitation", and "Riemannian" formulation of gravity. For our needs, we here define *Einstein's gravitation* as the reduction of exterior gravitation in vacuum to pure geometry, namely, gravitation is solely represented via curvature in a Riemannian space $\mathcal{R}(x, g, R)$ with spacetime coordinates (2.1a) and nowhere singular real-valued and symmetric metric $g(x)$ over the reals R, with field equations [2b,2c]

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = 0, \tag{3.1}$$

in which, as a central condition to have Einstein's gravitation, *there are no sources for the exterior gravitational field in vacuum of a body with null total electromagnetic field (null total charge and magnetic moment).*

For our needs, we define as *general relativity* any description of gravity on a Riemannian space over the reals with Einstein-Hilbert field equations with a source due to the presence of electric and magnetic fields,

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times t_{\mu\nu}, \tag{3.2}$$

where k is a constant depending on the selected unit whose value is here irrelevant. For the scope of this paper it is sufficient to assume that the *Riemannian description of gravity* coincides with general relativity according to the above definition.

In the following, we shall first study the inconsistencies of Einstein gravitation, that is, the inconsistencies in the entire reduction of gravity to curvature without source, and then study the inconsistency of general relativity, that is, the inconsistencies caused by curvature itself even in the presence of sources.

It should be stressed that a technical appraisal of the content of this paper can only be reached following the study of the axiomatic inconsistencies of grand unified theories of electroweak and gravitational interactions whenever gravity is represented with curvature on a Riemannian space irrespective of whether with or without sources [3].

THEOREM 3.1 [11a]: Einstein's gravitation and general relativity at large are incompatible with the electromagnetic origin of mass established by quantum electrodynamics, thus being inconsistent with experimental evidence.

Proof. Quantum electrodynamics has established that the mass of all elementary particles, whether charged or neutral, has a primary electromagnetic origin, that is, all masses have a first-order origin given by the volume integral of the 00-component of the energy-momentum tensor $t_{\mu\nu}$ of electromagnetic origin,

$$m = \int d^4x \times t_{oo}^{elm}. \tag{3.3a}$$

$$t_{\alpha\beta} = \frac{1}{4\pi}(F_{\alpha}^{\mu} F_{\mu\beta} + \frac{1}{4} g_{\alpha\beta} F_{\mu\nu} F^{\mu\nu}), \tag{3.3b}$$

where $t_{\alpha\beta}$ is the *electromagnetic tensor,* and $F_{\alpha\beta}$ is the *electromagnetic field* (see Ref. [11a] for explicit forms of the latter with retarded and advanced potentials).

Therefore, quantum electrodynamics requires the presence of a *first-order source tensor* in the *exterior field equations* in vacuum as in Eqs. (3.2). Such a source tensor is absent in Einstein's gravitation (3.1) by conception. Consequently, Einstein's gravitation is incompatible with quantum electrodynamics.

The incompatibility of general relativity with quantum electrodynamics is established by the fact that the source tensor in Eqs. (3.2) is of *higher order in magnitude*, thus being ignorable in first approximation with respect to the gravitational field, while according to quantum electrodynamics said source tensor is of first order, thus not being ignorable in first approximation.

The inconsistency of both Einstein's gravitation and general relativity is finally established by the fact that, for the case when the total charge and magnetic moment of the body considered are null, Einstein's gravitation and general relativity allows no source at all. By contrast, as illustrated in ref. [11a], quantum electrodynamics requires a first-order source tensor even when the total charge and magnetic moments are null due to the charge structure of matter. **q.e.d.**

The first consequence of the above property can be expressed via the following:

COROLLARY 3.1A [11a]: Einstein's reduction of gravitation in vacuum to pure curvature without source is incompatible with physical reality.

A few comments are now in order. As is well known, the mass of the electron is entirely of electromagnetic origin, as described by Eq. (3.3), therefore requiring a first-order source tensor in vacuum as in Eqs. (3.2). Therefore, Einstein's gravitation for the case of the electron is inconsistent with nature. Also, the electron has a point charge. Consequently, *the electron has no interior problem at all, in which case the gravitational and inertial masses coincide,*

$$m_{Electron}^{Grav.} \equiv m_{Electron}^{Iner}. \tag{3.4}$$

Next, Ref. [11a] proved Theorem 3.1 for the case of a neutral particle by showing that the π^o meson also needs a first-order source tensor in the exterior gravitational problem in vacuum since its structure is composed of one charged particle and one charged antiparticle in high dynamical conditions.

In particular, the said source tensor has such a large value to account for the entire *gravitational mass* of the particle [11a]

$$m_{\pi^o}^{Grav.} = \int d^4x \times t_{00}^{Elm}. \tag{3.5}$$

For the case of the interior problem of the π^o , we have the additional presence of short range weak and strong interactions representable with a new tensor $\tau_{\mu\nu}$. We, therefore, have the following

COROLLARY 3.1B [11a]: In order to achieve compatibility with electromagnetic, weak and strong interactions, any gravitational theory must admit two source tensors, a traceless tensor for the representation of the electromagnetic origin of mass in the exterior gravitational problem, and a second tensor to represent the contribution to interior gravitation of the short range interactions according to the field equations

$$G^{Int.}_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times (t^{Elm}_{\mu\nu} + \tau^{ShortRange}_{\mu\nu}). \tag{3.6}$$

A main difference of the two source tensors is that the electromagnetic tensor $t^{Elm}_{\mu\nu}$ is notoriously traceless, while the second tensor $\tau^{ShortRange}_{\mu\nu}$ is not. A more rigorous definition of these two tensors will be given shortly.

It should be indicated that, for a possible solution of Eqs. (3.6), various explicit forms of the electromagnetic fields as well as of the short range fields originating the electromagnetic and short range energy momentum tensors are given in Ref. [11a].

Since both sources tensors are positive-definite, Ref. [11a] concluded that the interior gravitational problem characterizes the *inertial mass* according to the expression

$$m^{Iner} = \int d^4x \times (t^{Elm}_{00} + \tau^{ShortRange}_{00}), \tag{3.7}$$

with consequential general law

$$m^{Inert.} \geq m^{Grav.}, \tag{3.8}$$

where the equality solely applies for the electron.

Finally, Ref. [11a] proved Theorem 3.1 for the exterior gravitational problem of a neutral massive body, such as a star, by showing that the situation is essentially the same as that for the π^o. The sole difference is that the electromagnetic field requires the sum of the contributions from *all* elementary constituents of the star,

$$m^{Grav.}_{Star} = \Sigma_{p=1,2,...} \int d^4x \times t^{Elem.}_{p00}. \tag{3.9}$$

In this case, Ref. [11a] provided methods for the approximate evaluation of the sum that resulted to be of first-order also for stars with null total charge.

When studying a charged body, there is no need to alter equations (3.6) since that particular contribution is automatically contained in the indicated field equations.

Once the incompatibility of general relativity at large with quantum electrodynamics has been established, the interested reader can easily prove the incompatibility of general relativity with quantum field theory and quantum chromodynamics, as implicitly contained in Corollary 3.1.B.

An important property apparently first reached in Ref. [11a] in 1974 is the following:

COROLLARY 3.1C [11a]: The exterior gravitational field of a mass originates entirely from the total energy-momentum tensor (3.3b) of the electromagnetic field of all elementary constituents of said mass.

In different terms, a reason for the failure to achieve a "unification" of gravitational and electromagnetic interactions initiated by Einstein himself is that the said interactions can be "identified" with each other and, as such, they cannot be unified. In fact, in all unifications attempted until now, the gravitational and electromagnetic fields preserve their identity, and the unification is attempted via geometric and other means resulting in redundancies that eventually cause inconsistencies.

Note that conventional electromagnetism is represented with the tensor $F_{\mu\nu}$ and related Maxwell's equations. When electromagnetism is identified with exterior gravitation, it is represented with the energy-momentum tensor $t_{\mu\nu}$ and related equations (3.6).

In this way, *gravitation results as a mere additional manifestation of electromagnetism.* The important point is that, besides the transition from the field tensor $F_{\mu\nu}$ to the energy-momentum tensor $T_{\mu\nu}$, there is no need to introduce a new interaction to represent gravity.

Note finally the irreconcilable alternatives emerging from the studies herein considered:

ALTERNATIVE I. Einstein's gravitation is assumed as being correct, in which case quantum electrodynamics must be revised in such a way to avoid the electromagnetic origin of mass; or

ALTERNATIVE II: Quantum electrodynamics is assumed as being correct, in which case Einstein's gravitation must be irreconcilably abandoned in favor of a more adequate theory.

By remembering that quantum electrodynamics is one of the most solid and experimentally verified theories in scientific history, it is evident that the rather widespread assumption of Einstein's gravitation as having final and universal character is non-scientific.

THEOREM 3.2 [11b,7d]: Einstein's gravitation (3.1) is incompatible with the Freud identity of the Riemannian geometry, thus being inconsistent on geometric grounds.

Proof. The Freud identity [11b] can be written

$$R^\alpha_\beta - \frac{1}{2} \times \delta^\alpha_\beta \times R - \frac{1}{2} \times \delta^\alpha_\beta \times \Theta \ = \ U^\alpha_\beta + \partial V^{\alpha\rho}_\beta / \partial x^\rho \ = \ k \times (t^\alpha_\beta + \tau^\alpha_\beta), \tag{3.10}$$

where

$$\Theta \ = \ g^{\alpha\beta} g^{\gamma\delta} (\Gamma_{\rho\alpha\beta} \Gamma^\rho_{\gamma\beta} - \Gamma_{\rho\alpha\beta} \Gamma^\rho_{\gamma\delta}), \tag{3.11a}$$

$$U^\alpha_\beta \ = \ -\frac{1}{2} \frac{\partial \Theta}{\partial g^{\rho\alpha}_{|\rho}} g^{\gamma\beta} \uparrow_\gamma, \tag{3.11b}$$

$$V^{\alpha\rho}_\beta \ = \ \frac{1}{2} [g^{\gamma\delta} (\delta^\alpha_\beta \Gamma^\rho_{\alpha\gamma\delta} - \delta^\rho_\beta \Gamma^\rho_{\alpha\delta}) +$$
$$+ (\delta^\rho_\beta g^{\alpha\gamma} - \delta^\alpha_\beta g^{\rho\gamma}) \Gamma^\delta_{\gamma\delta} + g^{\rho\gamma} \Gamma^\alpha_{\beta\gamma} - g^{\alpha\gamma} \Gamma^\rho_{\beta\gamma}]. \tag{3.11c}$$

Therefore, the Freud identity requires two first order source tensors for the exterior gravitational problems in vacuum as in Eqs. (3.6) of Ref. [11a]. These terms are absent in Einstein's gravitation (3.1) that, consequently, violates the Freud identity of the Riemannian geometry. **q.e.d.**

By noting that trace terms can be transferred from one tensor to the other in the r.h.s. of Eqs. (3.10), it is easy to prove the following:

COROLLARY 3.2A [7d]: Except for possible factorization of common terms, the t- and τ-tensors of Theorem 3.2 coincide with the electromagnetic and short range tensors, respectively, of Corollary 3.1B.

A few historical comments regarding the Freud identity are in order. It has been popularly believed throughout the 20-th century that the Riemannian geometry possesses only *four identities* (see, e.g., Ref. [2h]). In reality, Freud [11b] identified in 1939 a *fifth identity* that, unfortunately, was not aligned with Einstein's doctrines and, as such, the identity was ignored in virtually the entire literature on gravitation of the 20-th century.

However, as repeatedly illustrated by scientific history, structural problems simply do not disappear with their suppression, and actually grow in time. In fact, the Freud identity did not escape Pauli who quoted it in a footnote of his celebrated book of 1958 [2g]. Santilli became aware of the Freud identity via an accurate reading of Pauli's book (including its important footnotes) and assumed the Freud identity as the geometric foundation of the gravitational studies presented in Ref. [7d].

Subsequently, in his capacity as Editor in Chief of *Algebras, Groups and Geometries,* Santilli requested the mathematician Hanno Rund, a known authority in Riemannian geometry [2i], to inspect the Freud identity for the scope of ascertaining whether the said identity was indeed a new identity. Rund kindly accepted Santilli's invitation and released paper [11c] of 1991 (the last paper prior to his departure) in which Rund confirmed indeed the character of Eqs. (3.10) as a genuine, independent, fifth identity of the Riemannian geometry.

The Freud identity was also rediscovered by Yilmaz (see Ref. [11d] and papers quoted therein) who used the identity for his own broadening of Einstein's gravitation via an external *stress-energy tensor* that is essentially equivalent to the source tensor with non-null trace of Ref. [11a], Eqs. 3.6).

Despite these efforts, the presentation of the Freud identity to various meetings and several personal mailings to colleagues in gravitation, the Freud identity continues to remain vastly ignored to this day, with very rare exceptions (the indication by colleagues of additional studies on the Freud identify not quoted herein would be gratefully appreciated.)

Theorems 3.1 and 3.2 complete our presentation on the catastrophic inconsistencies of Einstein's gravitation due to the lack of a first-order source in the exterior gravitational problem in vacuum. These theorems, by no means, exhaust all inconsistencies of Einstein's gravitation, and numerous additional inconsistencies do indeed exist.

For instance, Yilmaz [11d] has proved that Einstein's gravitation explains the 43" of the precession of Mercury, but cannot explain the basic Newtonian contribution. This result can also be seen from Ref. [11a] because the lack of source implies the impossibility of importing into the theory the basic Newtonian potential. Under these conditions the representation of the Newtonian contribution is reduced to a religious belief, rather than a serious scientific statement.

For these and numerous additional inconsistencies of general relativity we refer the reader to Yilmaz [11d], Wilhelm [11e-11g], Santilli [11h], Alfvén [11i-11j], Fock [11k], Nordensen [11l], and large literature quoted therein.

4. Inconsistencies of General Relativity due to Curvature

We now pass to the study of the structural inconsistencies of general relativity caused by the very use of the Riemannian *curvature*, irrespective of the selected field equations, including those fully compatible with the Freud identity.

THEOREM 4.1 [11m]: Gravitational theories on a Riemannian space over a field of real numbers do not possess time invariant basic units and numerical predictions, thus having serious mathematical and physical inconsistencies.

Proof. The map from Minkowski to Riemannian spaces is known to be *noncanonical,*

$$\eta = Diag.(1, 1, 1, -1) \ \rightarrow \ g(x) = U(x) \times \eta \times U(x)^{\dagger}, \tag{4.1a}$$

$$U(x) \times U(x)^{\dagger} \neq I. \tag{4.1b}$$

Thus, the time evolution of Riemannian theories is necessarily noncanonical, with consequential lack of invariance in time of the basic units of the theory, such as

$$I_{t=0} = Diag.(1cm, 1cm, 1cm, 1sec) \rightarrow I'_{t>0} = U_t \times I \times U_t^{\dagger} \ \neq I_{t=0}. \tag{4.2}$$

The lack of invariance in time of numerical predictions then follows from the known "covariance", that is, lack of time invariance of the line element. **q.e.d.**

As an illustration, suppose that an experimentalist assumes at the initial time $t = 0$ the units 1 cm and 1 sec. Then, all Riemannian formulations of gravitation, including Einstein's gravitation, predict that at the later time $t > 0$ said units have a different numerical value.

Similarly, suppose that a Riemannian theory predicts a numerical value at the initial time $t = 0$, such as the 43" for the precession of the perihelion of Mercury. One can prove that the same prediction at a later time $t > 0$ is numerically different precisely in view of the "covariance", rather than invariance as intended in special relativity, thus preventing a serious application of the theory to physical reality. We therefore have the following:

COROLLARY 4.1A [11m]: Riemannian theories of gravitation in general, and Einstein's gravitation in particular, can at best describe physical reality at a fixed value of time, without a consistent dynamical evolution.

Interested readers can independently prove the latter occurrence from the *lack of existence of a Hamiltonian in Einstein's gravitation.* It is known in analytic mechanics (see, e.g., Refs. [2l,7b]) that Lagrangian theories not admitting an equivalent Hamiltonian counterpart, as is the case for Einstein's gravitation, are inconsistent under time evolution, unless there are suitable subsidiary constraints that are absent in general relativity.

It should be indicated that the inconsistencies are much deeper than that indicated above. For consistency, the Riemannian geometry must be defined on the field of numbers $R(n, +, \times)$ that, in turn, is fundamentally dependent on the basic unit I. But the Riemannian geometry does not leave time invariant the basic unit I due to its noncanonical character. The loss in time of the basic unit I then implies the consequential loss in time of the base field R, with consequential catastrophic collapse of the entire geometry [11m].

In conclusion, not only is Einstein's reduction of gravity to pure curvature inconsistent with nature because of the lack of sources, but also the ultimate origin of the inconsistencies rests in the curvature itself when assumed for the representation of gravity, due to its inherent noncanonical character at the classical level with consequential nonunitary structure at the operator level.

serious mathematical and physical inconsistencies are then unavoidable under these premises, thus establishing the impossibility of any credible use of general relativity, for instance, as an argument against the test on antigravity predicted for antimatter in the field of matter [5], as well as establishing the need for a profound revision of our current views on gravitation.

THEOREM 4.2: Einstein's gravitation is incompatible with experimental evidence because it predicts a bending of the speed of light that is double the experimental value.

Proof. Light carries energy, thus being subjected to a bending due to the conventional Newtonian gravitational attraction, while Einstein's gravitation predicts that the bending of light is due to curvature, thus resulting in a bending twice the experimentally measured value, the first being incompatible with the latter. **q.e.d.**

COROLLARY 4.2.A: the lack of curvature in gravitation is established by the free fall of masses that necessarily occurs along straight radial lines.

In fact, a consistent representation of the free fall of a mass along a straight radial line requires that the Newtonian attraction be represented the field equations necessarily without curvature, thus disproving the customary belief needed to avoid Corollary 4.2.A that said Newtonian attraction emerges at the level of the PPN approximation of Eqs. (3.1).

THEOREM 4.3. Gravitational experimental measurements do not verify Einstein's gravitation uniquely.

Proof. All claimed "experimental verifications" of Einstein's gravitation are based on the PPN "expansion" (or linearization) of the field equations that, as such, is not unique. In fact, Eqs. (3.1) admit a variety of inequivalent expansions depending on the selected parameter, the selected expansion and the selected truncation. It is then easy to show that the selection of an expansion of the same equations (3.1) but different from the PPN approximation leads to dramatic departures from experimental values. **q.e.d.**

A comparison between special and general relativities is here in order. Special relativity can be safely claimed to be "verified by experiments" because the said experiments verify numerical values uniquely and unambiguously predicted by special relativity. By contrast, no such statement can be made for general relativity since the latter does not uniquely and unambiguously predict given numerical values due, again, to the variety of possible expansions and linearization.

The origin of such a drastic difference is due to the fact that *the numerical predictions of special relativity are rigorously controlled by the basic Poincaré "invariance". By* *contrast, one of the several drawbacks of the "covariance" of general relativity is precisely the impossibility of predicting numerical values in a unique and unambiguous way, thus preventing serious claims of true "experimental verifications" of general relativity.*

By no means, the inconsistencies expressed by Theorems 3.1, 3.2, 4.1, 4.2 and 4.3 constitute all inconsistencies of general relativity. In the author's opinion, additional deep inconsistencies are caused by the fact that *general relativity does not possess a well defined Minkowskian limit,* while the admission of the Minkowski space as a tangent space is basically insufficient on dynamical grounds (trivially, because on said tangent space gravitation is absent).

As an illustration, we should recall the controversy on conservation laws that raged during the 20-th century [11]. Special relativity has rigidly defined total conservation laws because they are the Casimir invariants of the fundamental Poincaré symmetry. By contrast, there exist several definitions of total conservation laws in a Riemannian representation of gravity due to various ambiguities evidently caused by the absence of a symmetry in favor of covariance.

Moreover, none of the gravitational conservation laws yields the conservation laws of special relativity in a clear and unambiguous way, precisely because of the lack of any limit of a Riemannian into the Minkowskian space. Under these conditions, the compatibility of general relativity with the special reduces to personal beliefs outside a rigorous scientific process.

Another controversy that remained unresolved in the 20-th century (primarily because of lack of sufficient consideration by scholars in the field) is that, during its early stages, gravitation was divided into the *exterior and interior problems.* For instance, Schwartzchild wrote *two* articles on gravitation, one on the exterior and one on the interior problem [2d].

However, it soon became apparent that general relativity was structurally unable to represent interior problems for numerous reasons, such as the impossibility of incorporating shape, density, local variations of the speed of light within physical media via the familiar law we study in high school $c = c_o/n$ (which variation cannot be ignored classically), inability to represent interior contact interactions with a first-order Lagrangian, structural inability to represent interior nonconservation laws (such as the vortices in Jupiter's atmosphere with variable angular momenta), structural inability to represent entropy, its increase and other thermodynamical laws, etc. (see Ref. [7d] for brevity).

Consequently, Schwartzchild's solution for the *exterior* problem became part of history (evidently because aligned with general relativity), while his *interior* solution has remained vastly ignored to this day (evidently because not aligned with general relativity). In particular, the constituents of all astrophysical bodies have been abstracted as being point-like, an abstraction that is beyond the boundaries of science for classical treatments; all distinctions between exterior and interior problems have been ignored by the vast majority of the vast literature in the field; and gravitation has been tacitly reduced to one single problem.

Nevertheless, as indicated earlier, major structural problems grow in time when ignored, rather than disappearing. The lack of addressing the interior gravitational problem is causing major distortions in astrophysics, cosmology and other branches of science (see also next section). We have, therefore, the following important result:

THEOREM 4.4 [7d]: General relativity is incompatible with the experimental evidence on interior gravitational problems.

By no means the above analysis exhaust all inconsistencies of general relativity, and numerous additional ones do indeed exist, such as that expressed by the following:

THEOREM 4.5 [11m]: Operator images of Riemannian formulations of gravitation are inconsistent on mathematical and physical grounds.

Proof. As established by Theorem 4.1, classical formulations of Riemannian gravitation are noncanonical. Consequently, all their operator counterparts must be nonunitary for evident reasons of compatibility. But nonunitary theories are known to be inconsistent on both mathematical and physical grounds [11m]. In fact, on mathematical grounds, nonunitary theories of quantum gravity (see, e.g., Refs. [2j,2k]) do not preserve in time the basic units, fields and spaces, while, on physical grounds, the said theories do not possess time invariant numerical predictions, do not possess time invariant Hermiticity (thus having no acceptable observables), and violate causality. **q.e.d**

The reader should keep in mind the additional well known inconsistencies of quantum gravity, such as the historical incompatibility with quantum mechanics, the lack of a credible PCT theorem, etc.

To avoid raising issues of scientific ethics, all these inconsistencies establish beyond a scientific, or otherwise credible, doubt the need for a profound revision of the gravitational views of the 20-th century.

5. Apparent Resolution of the Inconsistencies via the Poincaré Invariant Isogravitation.

Following decades of studies, in order to achieve a resolution of the above inconsistencies, this author recommends the construction of a new theory of gravitation under the central conditions of admitting a *basic invariance of the line element without curvature.* In fact, such properties would resolve most of the inconsistencies studied in the preceding sections. The condition of compatibility of any gravitational theory with special relativity then restricts the said symmetry to a form isomorphic to the Poincaré symmetry (2.2).

The biggest technical difficulty in the realization of the above proposal rests on the fact that the achievement of the needed symmetry without curvature admits no solution within the context of the conventional Lie theory, and this illustrates the reasons gravitation departed from special relativity for one century.

In fact, any meaningful representation of gravitation requires a generalization of the Minkowskian metric $\eta = Diag.(+1, +1, +1, -1)$ into a nonsingular 4×4-matrix g that preserves the Minkowskian signature $(+1, +1, +1, -1)$, but possesses an otherwise unrestricted functional dependence on the local coordinates and, possibly, other variables. Symmetrization of the g matrix (permitted by its non singularity) then leads to a Riemannian metric. In this way, we the familiar Riemannian line element

$$(x - y)^2 = (x^\mu - y^\mu) \times g_{\mu\nu}(x, ...) \times (x^\nu - y^\nu) \in R, \tag{5.1}$$

for which no universal symmetry is known to exist within the context of the conventional

Lie theory, and for which curvature is unavoidable.

In order to achieve a universal symmetry without curvature, the author was forced to construct a *new mathematics* first proposed in Refs. [12a,12b] of 1978, then studied in various works [3-7]] and today known as *Santilli isomathematics* for the treatment of matter (with the *isodual isomathematics* for the treatment of antimatter).

The main main assumption for the case of matter is the generalization (called *lifting*) of the N-dimensional unit I of Lie's theory into a nowhere singular, N-dimensional and positive-definite matrix $\hat{I}$, called *isounit* [12a], with an arbitrary functional dependence on the local coordinate x, velocities v, accelerations a, densities d, temperatures τ and any other needed variables. Jointly, the conventional associative product $A \times B$ of generic quantities A, B (e.g., numbers, matrices, vector fields, operators, etc.) must be lifted into a form admitting $\hat{I}$, rather than I, as the correct left and right unit,

$$I > 0 \rightarrow \hat{I}(x, v, a, d.\tau, ...) = 1/\hat{T}(x, v, a, d, \tau, ...) > 0, \tag{5.2a}$$

$$A \times B \rightarrow A \hat{\times} B = A \times \hat{T} \times B, \tag{5.2b}$$

$$I \times A = A \times I = A \rightarrow \hat{I} \hat{\times} A = A \hat{\times} \hat{I} = A, \tag{5.2c}$$

for all A of the set considered, where $\hat{T}(x, ...)$ is called the *isotopic element,* and the prefix "iso" stands for "isotopic" and denotes the preservation of the original axioms although under a broader realization [12a].

The isodual isomathematics for antimatter can be simply constructed by subjecting all quantities of isomathematics and their operations to the *isodual map* given by the anti-Hermitean transform that, for a generic quantity $Q(x, \psi, ...)$ is given by $Q^d = -Q^\dagger(-x^\dagger, -\psi^\dagger, ...)$. The following presentation is restricted to the isomathematical treatment of matter. Its isodual to antimatter can be easily worked out by interested readers.

The lifting of the basic unit requires a compatible lifting of the totality of the mathematics used in Lie's theory, resulting in new numbers, new spaces, new algebras, etc, known as *isonumbers, isospaces, isoalgebras, isogroups, isosymmetries, isotopologies, etc.*

Following these lines, Santilli proposed since the original memoirs [12a] the isotopic lifting of all main branches of Lie's theory, including the isotopies of the universal enveloping algebras, Lie's algebras, Lie's group and the representation theory.

The emerging new theory was then studied in various papers and monograph and is today known as the *Lie-Santilli isotheory for matter and its isodual for antimatter* (see memoirs [6] for mathematical works, papers [3-5] for various applications, monographs [7a,7b] for a review up to 1982, monograph [7c,7d] for a review up to 1995 and independent studies [8-10]).

A geometric unification of gravitation and special relativity was first proposed in Ref. [12b] of 1996 (see also memoir [12c]) under the name of *isogravitation.* The basic assumption is the decomposition of any given Riemannian metric $g(x)$ (for instance, Schwartzchild's *exterior* metric [2d]) into the product of the Minkowski metric η and a 4×4-dimensional matrix $\hat{T}(x)$ that is necessarily positive-definite (from the nowhere degeneracy of $g(x)$). Isogravitation then occurs when $\hat{T}(x)$ is assumed as the inverse of the isounit,

$$g(x) = \hat{T}(x) \times \eta, \quad \hat{I}(x) = 1/\hat{T}(x), \quad Detg(x) \neq 0, \quad \hat{T}(x) > 0, \tag{5.3}$$

in which case $\hat{I}(x)$ and $\hat{T}(x)$ are called the *gravitational isounit and isotopic element,* respectively.

The basic numbers of isogravitation are then given by the isofield $\hat{R}(\hat{n}, \hat{+}, \hat{\times})$ of isonumbers $\hat{n} = n \times \hat{I}$ with the above isounit [6a].

The basic spaces of isogravitation are given by the isotopies $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ of the Minkowski space $M(x, \eta, R)$ first introduced by Santilli in Ref. [4a] of 1983 with *isocoordinates* $\hat{x} = x \times \hat{I}$, *isometric* $\hat{\eta}(x) = \hat{T}(x) \times \eta = g(x)$ now defined over $\hat{R}$, rather than R. The basic isotopic line element of isogravitation is then given by

$$\hat{x} = x \times \hat{I}, \quad \hat{y} = y \times \hat{I}, \quad \hat{N}_{\mu\nu} = \hat{\eta} \times \hat{I}, \tag{5.4a}$$

$$(\hat{x} - \hat{y})^{\hat{2}} = (\hat{x}^\mu - \hat{y}^\nu)\hat{\times}\hat{N}_{\mu\nu}\hat{\times}(\hat{x}^\nu - \hat{y}^\nu) =$$
$$= \{(x^\mu - y^\mu) \times [\hat{T}(x) \times \eta]_{\mu\nu} \times (x^\nu - y^\nu)\} \times \hat{I} \in \hat{R}, \tag{5.4b}$$

where the lifting of the isometric $\hat{\eta}$ into the form $\hat{N}_{\mu\nu} = \hat{\eta} \times \hat{I}$ is necessary for mathematical consistency on $\hat{M}$ due to the condition that the elements of the isometric must be isonumbers.

Therefore, the first expression of Eqs. (5.4b) depicts the isoline element properly written on $\hat{M}$ over $\hat{R}$, while the second expression of Eqs. (5.4b) is its *projection* on M. The reader should acquire a familiarity with this dual interpretation because typical of all isotheories (although absent for conventional theories), thus applying also for the field equations of isogravitation (see below).

As one can see, the main mechanism of the isotopies is to turn any given Riemannian line element in $\mathcal{R}$ over R into an *identical* form merely written on isospace $\hat{M}$ over $\hat{R}$. Despite its simplicity, this mechanism does indeed achieves the desired objectives.

To begin, despite the assumption of an arbitrary Riemannian metric as the isometric, *isominkowskian spaces are isomorphic to the Minkowski space and, therefore, are isoflat* (see memoir [6c] for geometric studies).

This feature can be empirically seen from the fact that the conventional Minkowski metric η is deformed by the product of the Riemannian isotopic element $\hat{T}(x)$ but, jointly, the basic unit I of η is deformed by the *inverse* amount $\hat{T}(x)^{-1}$, thus verifying the abstract axiom of flatness.

Next, the above mechanism does indeed permit the construction of the *universal isosymmetry of all infinitely possible Riemannian line elements and that symmetry is isomorphic to the Poincaré symmetry, thus being without curvature.* This isosymmetry was first proposed by Santilli in Ref. [4a] of 1983, then studied in various works [4b-4g,5-7] and is today called the *Poincaré-Santilli isosymmetry* for matter (see studies [8] and monographs [11]),

$$\hat{\mathcal{P}}(3.1) = [\hat{\mathcal{L}}(3.1)\hat{\times}\hat{\mathcal{T}}(3.1)] \times \hat{\mathcal{S}}, \tag{5.5}$$

where $\hat{\mathcal{L}}(3.1)$ is the *Lorentz-Santilli isogroup*, $\hat{\mathcal{T}}(3.1)$ is the group of *isotranslations*, and $\hat{\mathcal{S}}$ is the following novel one-dimensional isosymmetry

$$(\hat{x}^\mu\hat{\times}\hat{\eta}_{\mu\nu}\hat{\times}\hat{x}^\nu) \times \hat{I} \equiv [\hat{x}^\mu\hat{\times}(\hat{n}^{-2}\hat{\times}\hat{\eta})_{\mu\nu}\hat{\times}\hat{x}^\nu] \times (\hat{n}^2\hat{\times}\hat{I}), \tag{5.6}$$

that is evidently in the center of the isogroup. Note that the latter essentially acts as the isotopic image of the conventional "scalar extension" of Lie's symmetries, as familiar for the Galileo's (but not for the Poincaré) symmetry.

Contrary to the popular belief throughout the 20-th century that the Poincaré symmetry is ten-dimensional, isosymmetry (5.6) also applies to the conventional Minkowskian line element, and we have the following

LEMMA 5.1 [4e]: The Poincaré symmetry and the Poincaré-Santilli isosymmetry are eleven dimensional.

Despite its simplicity, the discovery of the 11-th dimensionality of conventional spacetime symmetries has far reaching implications. In fact, the iso-grand-unification of electroweak and gravitational interactions of Refs. [3] is precisely permitted by the above 11-th dimensionality.

The reader should be aware that the identification of the new symmetry (5.6) required the prior discovery of *new numbers,* those with arbitrary units [6a], and this illustrates the reason isosymmetry (5.6) escaped detection for about one century (see below for the "hidden" character of the isosymmetries as well as connection with the E-P-R argument).

The explicit construction of the universal invariance of the isogravitational line element (5.4) is elementary. The *isogenerators* and *isoparameters* are given by the conventional quantities of $\mathcal{P}(3.1)$ merely written on isospace $\hat{M}$ over isofield $\hat{R}$ (see below for their explicit form), and can be written

$$\hat{X} = \{\hat{X}_k\} = \{\hat{M}_{\mu\nu} = \hat{x}_\mu \hat{\times} \hat{p}_\nu - \hat{x}_\nu \hat{\times} \hat{p}_\mu, \hat{p}_\alpha, \hat{\mathcal{S}}\}, \quad \hat{w} = w \times \hat{I} \in \hat{R}, \tag{5.7a}$$

$$\hat{p}_\mu \hat{\times} |\hat{\psi} >= -\hat{i} \hat{\times} \hat{\partial}_\mu |\hat{\psi} >= i \times \hat{I}^\nu_\mu \times \partial_\nu |\hat{\psi} >, \tag{5.7b}$$

where: $\mu, \nu = 1, 2, 3, ; k = 1, 2, ..., 11$; and expression (5.7b) characterizes the realization of the isomomentum permitted by the *isodifferential calculus* on a *iso-Hilbert space* with isostates $|\hat{\psi} >$, *isoexpectation values* of a Hermitean operator $< \hat{\psi} | \hat{\times} \hat{O} \hat{\times} | \hat{\psi} > \hat{/} < \hat{\psi} | \hat{\times} | \hat{\psi} >$ and *isonormalization* $< \hat{\psi} | \hat{\times} | \hat{\psi} >= \hat{I}$ (see Refs. [6b, 7c,12c] for brevity).

Some of the important features of the above operator isotopies (that should be confronted with the inconsistencies of Theorem 4.5) are given by: the identity of conventional and isotopic hermiticity, thus assuring that all observables of conventional quantum mechanics remain observables under isotopies; the preservation of Hermiticity under the time evolution, thus assuring the existence of acceptable observables and the strict verification of causality guaranteed by the isounitary structure of the liftings (see below).

The connected component of the isosymmetry can be written

$$\hat{\mathcal{P}}_o(3.1) : \hat{A}(\hat{w}) = \Pi_{k=1,...,10} \hat{e}^{\hat{i} \hat{\times} \hat{X} \hat{\times} \hat{w}} = (\Pi_k e^{i \times X \times \hat{T} \times w}) \times \hat{I} =$$

$$= \tilde{A}(x, v, d, \tau, \psi, ...) \times \hat{I}. \tag{5.8}$$

where the *isoexponentiation* is given by

$$\hat{e}^{\hat{A}} = \hat{I} + \hat{A} \hat{/} \hat{1}! + \hat{A} \hat{\times} \hat{A} \hat{/} \hat{2}! + ... = (e^{A \times \hat{T}}) \times \hat{I}. \tag{5.9}$$

Note the appearance of the gravitational isotopic element $\hat{T}(x)$ in the *exponent* of the group structure. This illustrates the nontriviality of the Lie-Santilli isotheory and, in particular, its *nonlinear, nonlocal and nonunitary* characters when projected on conventional spaces over conventional fields. However, the Lorentz-Poincaré-Santilli isosymmetry recovers linearity, locality and unitarity on $\hat{M}$ over $\hat{R}$, as the reader is encouraged to verify [*loc. cit.*].

Conventional linear transforms on M *violate* isolinearity on $\hat{M}$. Consequently, they must be replaced with the *isotransforms*

$$\hat{x}' = \hat{A}(\hat{w})\hat{\times}\hat{x} = \hat{A}(\hat{w}) \times \hat{T}(x) \times \hat{x} = \tilde{A}(w, ...) \times \hat{x}, \tag{5.10}$$

that verify the laws of the *Lie-Santilli isogroups*

$$\hat{A}(\hat{w})\hat{\times}\hat{A}(\hat{w}') = \hat{A}(\hat{w}')\hat{\times}\hat{A}(\hat{w}) = \hat{A}(\hat{w} + \hat{w}'), \hat{A}(\hat{w})\hat{\times}\hat{A}(-\hat{w}) = \hat{A}(0) = \hat{I}. \tag{5.11}$$

The use of the isodifferential calculus on $\hat{M}$ [6b] then yields the *Poincaré-Santilli isoalgebra* $\hat{\mathbf{P}}(3.1)$ [4]

$$[\hat{M}_{\mu\nu} \hat{,} \hat{M}_{\alpha\beta}] = i \times (\hat{\eta}_{\nu\alpha} \times \hat{M}_{\mu\beta} - \hat{\eta}_{\mu\alpha} \times \hat{M}_{\nu\beta} - \hat{\eta}_{\nu\beta} \times \hat{M}_{\mu\alpha} + \hat{\eta}_{\mu\beta} \times \hat{M}_{\alpha\nu}), \tag{5.12a}$$

$$[\hat{M}_{\mu\nu}, \ \hat{p}_\alpha] = i \times (\hat{\eta}_{\mu\alpha} \times \hat{p}_\nu - \hat{\eta}_{\nu\alpha} \times \hat{p}_\mu), \tag{5.12b}$$

$$[\hat{p}_\alpha \hat{,} \hat{p}_\beta] = [\hat{M}_{\mu\nu} \hat{,} \hat{\mathcal{S}}] = [\hat{p}_\mu \hat{,} \hat{\mathcal{S}}] = 0, \tag{5.12c}$$

$$[\hat{A} \hat{,} \hat{B}] = \hat{A}\hat{\times}\hat{B} - \hat{B}\hat{\times}\hat{A} = \hat{A} \times \hat{T} \times \hat{B} - \hat{B} \times \hat{T} \times \hat{A}, \tag{5.12d}$$

where $[A \hat{,} B]$ is the *Lie-Santilli isoproduct* first proposed in [12a] (that satisfies the Lie axioms in isospace, as one can verify), and we have written the isocommutation rules in their projection on conventional spaces for simplicity.

More technically, the isoalgebra $\hat{\mathbf{P}}(3.1)$ is characterized by the *universal enveloping isoassociative algebra* $\hat{\mathcal{A}}(\hat{\mathbf{P}})$ with isoproduct (5.2b) such that the attached antisymmetric algebra $[\hat{\mathcal{A}}(\hat{\mathbf{P}})]^-$ is locally isomorphic to $\hat{\mathbf{P}}$ with underlying *Poincaré-Birkhoff-Witt-Santilli isotheorem* first introduced in Ref. [12a].

Note the appearance of the Riemannian metric as the *structure isofunctions* of the theory.

The local isomorphism

$$\hat{\mathbf{P}}(3.1) \approx \mathbf{P}(3.1), \tag{5.13}$$

is ensured by the positive–definiteness of $\hat{T}$.

The *isocasimir invariants* of $\hat{\mathbf{P}}(3.1)$ are the simple isotopic images of the conventional invariants

$$C^o = \hat{I} = [\hat{T}(x, v, d, \tau, \psi, ...)]^{-1}, \tag{5.14a}$$

$$C^{(2)} = \hat{p}^{\hat{2}} = \hat{p}_\mu \hat{\times} \hat{p}^\mu = \hat{\eta}^{\mu\nu} \times \hat{p}_\mu \hat{\times} \hat{p}_\nu, \tag{5.14b}$$

$$C^{(4)} = \hat{W}_\mu \hat{\times} \hat{W}^\mu, \hat{W}_\mu = \in_{\mu\alpha\beta\pi} \hat{M}^{\alpha\beta} \hat{\times} \hat{p}^\pi, \tag{5.14c}$$

and they can be used for the construction of isorelativistic equations with the inclusion of gravitation (see below for an example).

It should be noted that the above setting characterizes the isotopies of relativistic quantum mechanics first proposed in ref, [12a] of 1978 under the name of *relativistic hadronic mechanics,* and then developed in the references of this work by various scholars (see, e.g., memoir [12c] or monographs [7c,7d] for details).

The reader should be aware that we are presenting here *operator isogravitation* as a particular realization of the relativistic hadronic mechanics characterized by the *restriction* of the isounit to the gravitational expression (5.3).

The explicit form of the Poincaré-Santilli isotransformations can be easily constructed from Eqs. (5.8) and are given by:

1) Isorotations [4c], that can be written for the isorotation in the $(1,2)$-plane

$$x' = x \times \cos(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3) - y \times \hat{T}_{11}^{-\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \sin(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3), \tag{5.15a}$$

$$y' = x \times \hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{-\frac{1}{2}} \times \sin(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3) + y \times \cos(\hat{T}_{11}^{\frac{1}{2}} \times \hat{T}_{22}^{\frac{1}{2}} \times \theta_3), \tag{5.15b}$$

(see Ref. [7d] for general isorotations in all there Euler angles).

2) Lorentz-Santilli isotransformations [4a], characterized by the isorotations and the *isoboosts* in the $(3,4)$–plane

$$x^{3\prime} = x^3 \times \sinh(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times v) - x^4 \times \hat{T}_{33}^{-\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times \cosh(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44} \times v) =$$

$$\tilde{\gamma} \times (x^3 - \hat{T}_{33}^{-\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times \hat{\beta} \times x^4), \tag{5.16a}$$

$$x^{4\prime} = -x^3 \times \hat{T}_{33} \times c_0^{-1} \times \hat{T}_{44}^{-\frac{1}{2}} \times \sinh(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44} \times v) + x^4 \times \cosh(\hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44}^{\frac{1}{2}} \times v) =$$

$$\tilde{\gamma} \times (x^4 - \hat{T}_{33}^{\frac{1}{2}} \times \hat{T}_{44}^{-\frac{1}{2}} \times \tilde{\beta} \times x^3), \tag{5.16b}$$

$$\tilde{\beta} = v_k \times \hat{T}_{44}^{\frac{1}{2}} / c_0 \times \hat{T}_{44}^{\frac{1}{2}}, \tilde{\gamma} = (1 - \tilde{\beta}^2)^{-\frac{1}{2}}. \tag{5.16c}$$

3) Isotranslations [4w], that can be written

$$\hat{x}'_\mu = (\hat{e}^{\hat{i} \hat{\times} \hat{p} \hat{\times} \hat{a}}) \hat{\times} \hat{x}_\mu = [x_\mu + a_\mu \times A_\mu(x, v, d, ...)] \times \hat{I} =, \hat{p}' = (\hat{e}^{\hat{i} \hat{\times} \hat{p} \hat{\times} \hat{a}}) \hat{\times} \hat{p} = \hat{p}, \tag{5.17a}$$

$$A_\mu = \hat{T}_{\mu\mu}^{1/2} + a^\alpha \times [\hat{T}_{\mu\mu}^{1/2}, \hat{p}_\alpha]/1! + ... \tag{5.17b}$$

and they are also nonlinear, as expected.

4) Isoinversions [4e], given by

$$\hat{\pi} \hat{\times} \hat{x} = (\pi \times x) \times \hat{I} = (-r, x^4) \times \hat{I}, \hat{\tau} \hat{\times} \hat{x} = (\tau \times x) \times \hat{I} = (r, -x^4) \times \hat{I}, \tag{5.18}$$

where $\hat{\pi} = \pi \times \hat{I}, \hat{\tau} = \tau \times \hat{I}$, and π, τ are the conventional inversion operators.

5) Isoscalar transforms [4e], characterized by invariances (5.6), that can be written

$$\hat{I} \rightarrow \hat{I}' = \hat{n}^{\hat{2}} \hat{\times} \hat{I} = n^2 \times \hat{I}, \hat{\eta} \rightarrow \hat{\eta}' = \hat{n}^{-\hat{2}} \hat{\times} \hat{\eta} = n^{-2} \times \hat{\eta}, \tag{5.19}$$

where $\hat{n}^{\hat{2}} = \hat{w}_{11}$ is the parameter characterizing the novel 11-th dimension.

A few comments are now in order. Note first the universal character of the Poincaré-Santilli isosymmetry and related isotransforms for all possible Riemannian metrics. In particular, *there is nothing to compute for the invariance of any given Riemannian metric,* except the identification of the gravitational element $\hat{T}(x)$ and its plotting in the above isotransforms.

Note also that the isorotations leave invariant all ellipsoidal deformations of the sphere, as the reader is encouraged to verify. The local isomorphism between $\hat{O}(3)$ and O(3) then confirms the perfect spheridicity of ellipsoids when formulated on the Euclidean isospace, called *isosphere.*

The mechanism for the reconstruction of the perfect spheridicity (that is, for the reconstruction of the exact rotational symmetry) is essentially the same as that for the elimination of curvature. In fact, we have the deformation of the sphere with semiaxes $(1, 1, 1)$ into the ellipsoid with semiaxes $(n_1^{-2}, n_2^{-2}, n_3^{-2})$ while, jointly, the units are deformed from the trivial value of the sphere $(1, 1, 1)$ to the *inverse* of the deformations (n_1^2, n_2^2, n_3^2), thus preserving the perfect spheridicity because the structure of the isoinvariant is given by $[length]^2 \times [unit]^2$, as shown by invariance (5.6).

In particular, the space components of all gravitational theories, including Schwartzchild's solution, characterize an isosphere when reformulated on isoeuclidean spaces over isofields.

Despite their simplicity, the physical implications of the isoinversions are not trivial because of *the possibility of reconstructing as exact discrete symmetries when believed to be broken.* This reconstruction can be achieved by merely embedding all symmetry breaking terms in the isounit.

For instance, it has been shown in Ref. [7d] that *parity is indeed an exact symmetry for weak interactions.* The widespread belief parity violation is merely due to the use of a mathematics insufficient for the problem at hand.

The reconstruction of exact symmetries generally applies for all conventional spacetime symmetries when believed to be broken. In fact, the isorotational symmetry reconstructs the exact rotational symmetry under conditions for which the latter is manifestly broken, such as for deformable ellipsoids.

Similarly, the Lorentz-Santilli isosymmetry reconstructs the exact Lorentz symmetry when the latter is believed to be broken by signature-preserving deformations of the Minkowski spacetime. As a matter of fact, the Lie-Santilli isotheory has permitted the Lorentz and Poincaré symmetries to become "universal" because exact for all infinitely possible space-times.

Next, it should be noted that, thanks to the fundamental isodifferential calculus [6b], the isominkowskian geometry admits an isotopic image of the entire formalism of the Riemannian geometry, such as Christoffel's symbols, covariant derivative, etc. [6c].

Consequently, *the isominkowskian gravitation preserves the Einstein-Hilbert field equations,* although in their covering isotopic form compatible with the iso-Freud identity. By keeping in mind the analysis of Sections 3 and 4, we therefore have the following *basic field equations of isogravitation* (see Ref. [6c] for details)

$$\hat{G}^{Int.}_{\mu\nu} = \hat{R}_{\mu\nu} - \hat{N}_{\mu\nu} \hat{\times} \hat{R} \hat{/} \hat{2} = \hat{k} \hat{\times} (\hat{t}^{Elm}_{\mu\nu} + \hat{\tau}^{ShortRange}_{\mu\nu}). \qquad (5.20)$$

From the above treatment, one can construct any needed *isorelativistic equation*, such as the following *Dirac-Santilli-Schwartzchild isoequation* including electromagnetic and gravitational interactions [4e,7d]

$$(\hat{\gamma}^\mu \hat{\times} \hat{p}_\mu + \hat{i} \hat{\times} \hat{m}) \hat{\times} | >= [\hat{\eta}_{\mu\nu}(x, v, ...) \times \hat{\gamma}^\mu \times \hat{T} \times \hat{p}^\nu - i \times m \times \hat{I}] \times \hat{T} \times | >= 0, \quad (5.21a)$$

$$\{\hat{\gamma}^\mu, \hat{\gamma}^\nu\} = \hat{\gamma}^\mu \times \hat{T} \times \hat{\gamma}^\nu + \hat{\gamma}^\nu \times \hat{T} \times \hat{\gamma}^\mu = 2 \times \hat{\eta}^{\mu\nu}, \hat{\gamma}^\mu = \hat{T}^{1/2}_{\mu\mu} \times \gamma^\mu \times \hat{I}, \qquad (5.21b)$$

where γ^μ represents the conventional gammas, $\hat{\gamma}^\mu$ represents the *isogamma matrices*, and the gravitational isotopic element is that of the Schwartzchild's metric,

$$\hat{\gamma}_k = \frac{\gamma_k}{(1-2M/r)^{1/2}} \times \hat{I}, \tag{5.22a}$$

$$\hat{\gamma}_4 = \gamma_4 \times (1-2M/r)^{1/2} \times \hat{I}. \tag{5.22b}$$

Note that Eqs. (5.21) belong to hadronic (and not quantum) mechanics. Note also that, again for the particular case $\eta(x, v, d, ...) = g(x)$, *the anti-isocommutators of the isogamma matrices yield twice the Riemannian metric*, thus confirming the representation of any desired Riemannian metric in the *structure* of Dirac's equation. Consequently, one can similarly construct the isogravitational version of all other equations of relativistic quantum mechanics.

Equations (5.21) are not a mere mathematical curiosity because they establish the compatibility of operator isogravity with experimental data in particle physics (with the understanding that compatibility with gravitational data requires a separate inspection). In fact, the much smaller value of gravitational over electromagnetic, weak and strong interactions establishes the compatibility with currently available experimental data in particle physics of the isogravitational equations of type (5.21).

It should be indicated also that, as one can verify via the isotopic decomposition of Schwartzchild's metric or Eqs. (5.22), *gravitational singularities are characterized by the zeros of the fourth component of Santilli's isounit, or, equivalently, by the zeros of the space component of the gravitational isotopic element,*

$$\hat{I}(x)_{44} = 0, \quad \hat{T}(x)_{kk} = 0. \tag{5.23}$$

The explicit construction of the entire theory of isogravitation, including its isosymmetry $\hat{\mathcal{P}}(3.1)$, can be simply done by identifying a *nonunitary transform* with the gravitational isounit,

$$U \times U^\dagger = \hat{I}(x) \neq U, \tag{5.24}$$

and then applying such a transform to the *totality* of the quantity and their operations of special relativity (with no known exception to avoid major structural inconsistencies comparable to those emerging if quantum mechanics is treated with isomathematics), resulting in the isotopies introduced above, e.g.,

$$I \to U \times I \times U^\dagger = \hat{I}, \tag{5.25a}$$

$$A \times B \to U \times (A \times B) \times U^\dagger = \hat{A} \hat{\times} \hat{B}, \hat{A} = U \times A \times U^\dagger, \ etc. \tag{5.25b}$$

The invariance of isogravitation can be proved by rewriting nonunitary transform (5.25) in the correct *isounitary form* (that is, by reconstructing unitarity on isospaces)

$$\hat{U} = U \times \hat{T}^{1/2}, \quad \hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}, \tag{5.26}$$

under which we have the isoinvariances

$$\hat{I} \to \hat{U} \hat{\times} \hat{I} \hat{\times} \hat{U}^\dagger \equiv \hat{I}, \quad \hat{A} \hat{\times} \hat{B} \to \hat{U} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{U}^\dagger \equiv \hat{A}' \hat{\times} \hat{B}', \ etc. \tag{5.27}$$

The above results can be summarized with the following:

THEOREM 5.1 [4]: The 11-dimensional Poincaré-Santilli isosymmetry on isominkowski spaces over real isofields with common, 4×4-dimensional, positive-definite isounits is directly universal for nonsingular, signature preserving generalizations of the Minkowskian spacetime, where "universal" represents the the validity of the isosymmetry for all infinitely possible spacetimes of the class admitted, and "directly universal" represents its applicability in the fixed coordinates of the observer, without any use of coordinate or other transforms.

The isosymmetries for the lifting of the de Sitter, Finslerian, non-Desarguesian and other nondiagonal and diagonal metrics with signatures different from $(+,+,+,-)$ is straightforward and it is ignored here. The case of nondiagonal metrics requires a structural generalization of the Lie-Santilli isotheory into the broader*Lie-admissible theory*, and it is not considered here for simplicity (for these broader cases the interested reader may consult Ref. [6d-6f,7d]).

It is easy to see that isogravitation does resolve most of the inconsistencies studied in this paper, such as:

I. Lack of compatibility of Riemannian gravity with special relativity. This is the fundamental insufficiency of general relativity for whose solution the Lie-Santilli isotheory and isogravitation were built. In fact, the isominkowskian space and related Lorentz-Poincaré-Santilli isosymmetry admit the simple, unique and unambiguous limit into conventional structures

$$\hat{I}(x) \rightarrow I, \quad \hat{\eta}(x) \rightarrow \eta, \quad \hat{M} \rightarrow M, \;\; \hat{\mathcal{P}}(3.1) \rightarrow \mathcal{P}(3.1), \;\; etc. \tag{5.28}$$

In particular, the proposed Poincaré invariant gravitation constitutes a geometric unification of general and special relativities since the said relativities are merely differentiated by the basic positive-definite unit while all abstract axioms are the same for both relativities.

The century old controversies on the incompatibility of the Riemannian conservation laws with those of special relativity are uniquely and unambiguously resolved by isogravitation because *the total conservation laws of isogravitation are characterized by the same generators of the Poincaré symmetry, only written on isospace over isofield.* Consequently, the total conservation laws of isogravitation can be uniquely and unambiguously reduced to those of special relativity under the limit

$$Lim_{\hat{I}\rightarrow I}(\hat{M}_{\mu\nu}, \hat{P}_\mu) = M_{\mu\nu}, P_\mu. \tag{5.29}$$

II. Inconsistencies due to curvature (Theorems 4.1, 4.2). Isogravitation is invariant under the Poincaré-Santilli isosymmetry in the same way as occurring for a Poincaré invariant theory on Minkowski space. In particular, isogravitation preserves the numerical values of the isounit and of the isoproduct as explicitly proved by Eqs. (5.27), with evident invariance of the numerical predictions. Unlike general relativity, isogravitation can indeed be safely applied to experiments without fear that the numerical predictions have been lost under the time evolution.

Moreover, isogravitation is isoflat, that is, *there exists no curvature on the isominkowskian space,* as proved, for instance, by the fact that *the isomomenta isocommute,* Eqs. (5.12c), while it is well known that momenta do not commute on a curved space.

The removal of the curvature as the basic notion representing gravity also resolves other controversies that have raged throughout the 20-the century. For instance, the Newtonian attraction remains fundamental in isogravity and it is formulated at the isorelativistic level, thus including relativistic corrections. Consequently, isogravitation correctly predicts and represents one single bending of light and provides the first and only consistent representation known to this author of the free fall of a massive body along a straight radial line, since no curvature can be credibly used in this setting.

III. Inconsistencies due to lack of sources and the Freud identity, (Theorems 3.1 and 3.2). these inconsistency are readily resolved by the "identification" of the gravitational and electromagnetic fields [11a], resulting in isofield equations (5.20) in which the electromagnetic and short range isotensors are of first-order in magnitude even for a body with null total electromagnetism. As such, these isotensors cannot be eliminated even in first approximation. Other technical or epistemological controversies that raged during the 20-th century are also resolved by isogravitation.

IV) Inconsistencies for interior gravitation (Theorem 4.4). We have recalled earlier the complete inability of general relativity to represent even minimal features of interior gravitational problems, such as the shape and density of the considered body, the locally varying character of the speed of light in interior conditions $c = c_o/n$, the local nonconservation of the angular momentum, the entropy and its increase, and other interior features [7d].

These inconsistencies too are resolved by isogravitation due to the unrestricted functional dependence of the isometric. As an illustration, any given gravitational isotopic element representing a conventional exterior gravitation

$$\hat{T}(x)^{Ext.} = Diag.(\hat{T}_{11}, \hat{T}_{22}, \hat{T}_{33}, \hat{T}_{44}) > 0, \tag{5.30}$$

can be easily lifted to the interior form

$$\hat{T}(x, v, a, d, \tau, ...)^{Int.} = Diag.(\hat{T}_{11}/n_1^2, \hat{T}_{22}/n_2^2, \hat{T}_{33}/n_3^2, \hat{T}_{44}/n_4^2), \tag{5.31}$$

where n_1^2, n_2^2, n_3^2 can represent the *shape* of the body considered (generally a spheroidal ellipsoid), and n_4^2 can represent its density or, equivalently, the local variation of the speed of light (since all n's are normalized to 1 to represent the vacuum). The central point is that, for the case of general relativity, the transition from the exterior to the interior problem causes serious structural inconsistencies, while for isogravitation the same transition causes no problem of any type, since all basic axioms, symmetries, etc. remain completely unaffected.

V) Inconsistencies due to quantum gravity (Theorem 4.5). As recalled in Section 4, another controversy that raged during the 20-th century is the quantum version of general relativity, due to the resulting incompatibility of gravity with quantum mechanics.

One of the most significant advances permitted by isogravitation is the resolution of this century old controversy. In fact, isogravitation was first and most naturally formulated at the *operator interior gravitational problems* as a simple particular case of relativistic hadronic mechanics [12c], and then its classical counterpart was identified [6c].

The resolution of the controversies is then assured by the fact that the basic abstract axioms remain those of quantum mechanics. A clear understanding is that one should not expect conventional quantization of energy levels (thus justifying a new name), since the latter have no physical meaning in the core of a star.

Despite the above results, a number of additional aspects remain to be studied. For instance, Euclidean-PPN expansions are notoriously insufficient within a relativistic setting. Consequently, a basically new isorelativistic expansion of Eqs. (5.20) has to be worked out and compared with experiments. It is hoped that the rigid implementation of invariance under the Poincaré-Santilli isosymmetry will restrict such an expansion to a unique form, thus avoiding the century old controversy on the lack of uniqueness of the Euclidean-PPN expansion, while replacing it with a *bona fide* relativistic expansion. Studies on these and related aspects are under way and they will be reported in some future paper.

The cosmological implications of the (apparently only known) axiomatically consistent, classical and operator treatment of interior gravitation are significant. They are studied as part of the new *isoselfdual cosmology* [5f], namely, a cosmology in which the universe is assumed, as a limit case, to be composed half of matter and half of antimatter under the universal isosymmetry

$$\hat{S} = \hat{\mathcal{P}}(3.1) \times \hat{\mathcal{P}}(3.1)^d = \hat{S}^d. \tag{3.32}$$

that is isoselfdual (as the Dirac equation and its isotopies), namely the symmetry is invariant under the anti-Hermitean isodual transform [5].

The primary implications of the above new cosmology relevant for this paper are:

1) The isoselfdual cosmology provides the only explanation known to this author, not only of the expansion of the universe, but also of the recently reported increase of the expansion itself, due to the necessary gravitational repulsion between matter and antimatter galaxies implied by the isodual theory of antimatter [5,7f];

2) The isoselfdual cosmology eliminates any need for very large values of the missing mass because the maximal causal speed for all interior astrophysical problems is predicted to be bigger than that in vacuum, as confirmed by recent astrophysical and other evidence. Consequently, the total energy of a galaxy is characterized by values $E = m \times c^2 = m \times c_o^2/n_4^2$ that are much bigger than those currently believed under the rather simplistic assumption that the speed of light *in vacuum* c_o remains valid in the interior of hyperdense stars and quasars [7d]; and

3) The isoselfdual cosmology eliminates the immense singularity at the creation of the universe that is implied by the "big bang" because, under the isodual representation of antimatter, the universe has identically null total characteristics, that is, identically null total time, identically null total mass, identically null total energy, etc. [*loc. cit*].

As a final comment, it should be indicated that *the isotopies are an explicit and concrete realization of "hidden variables"* λ (see Ref. [12g] for the "hidden variables", and Ref. [12c] for their isotopic realization), as evident from the fact that the isotopies are hidden in conventional relativistic axioms. In fact, the conventional and isotopic eigenvalue equations

$$H \times |\psi >= E \times |\psi > \rightarrow \hat{H} \hat{\times} |\hat{\psi} >= \hat{H} \times \hat{T} \times |\hat{\psi} >= \hat{E}' \hat{\times} |\hat{\psi} >= E' \times |\hat{\psi} >, \quad (5.33)$$

coincide at the abstract, realization-free level, because both actions are modular, associative and to the right. Consequently, the isotopic element constitutes an explicit and concrete realization of the "hidden variable", $\lambda = \hat{T}(x, ...)$.

The geometric unification of general and special relativity at the basis of these studies is also an explicit and concrete realization of the degrees of freedom, this time, hidden in the axioms of special relativity. Intriguingly, classical images of quantum mechanics are restricted by Bell's inequality [12h], while the same inequality does not hold under isotopies (due to the nonunitarity - isounitarity of the lifting), with intriguing epistemological implications, such as a necessary revision of local realism, studied in Ref. [12f].

In the author's view, one of the most important and thought provoking intuitions of Albert Einstein has been his vision of the *lack of completion of quantum mechanics,* a vision today known as the *E-P-R argument* [12l] (the author elected to become a physicist mostly stimulated by this vision). In the final analysis, the isotopies in general, including isogravitation, have been conceived and constructed to achieve an axiom-preserving "completion" of special relativity and relativistic quantum mechanics precisely along the lines of Einstein's vision.

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References

[1] C. N. Yang and R. Mills, Phys. Rev. 96, 191 (1954) [1a]. S. L. Glashow, Nuc. Phys. **22**, 579 (1961) [1b]. S. Weinberg Phys. Rev. Lett. **19**,1264 (1967) [1c]. A. Salam, in *Elementary Particle Physics* (Nobel Symp. No. 8), N. Svartholm, Almquist and Wiksell, ed., Stockholm (1968) [1d]. J. C. Pati and A. Salam, Phys. D Rev. **10**, 275 (1974) [1e]. M. Günaydin and F. Gürsey, J. Math. Phys. **14**, 1651 (1973) [1f]. L. P. Horwitz and L. C. Biedenharn, J. Math. Phys. **20**, 269 (1979) [1g].

[2] B. Riemann, Gött. Nachr. **13,** 133 (1868) and *Collected Works*, H. Weber, ed. (Dover, New York, 1953) [2a]. D. Hilbert, Nachr. Kgl. Ges. Wissench. Gottingen, 1915, p. 395 [2b]. A. Einstein, Sitz. Ber. Preuss. Akad. Wissssench Berlin, 1915, p. 844 [2c]. K. Schwartzschild, Sitzber. Deut. Akad. Wiss. Berlin, K1. Math.–Phys.

Tech., 189 and 424 (1916) [2d]. H. Weyl, Raum–Zeit–Materie (Springer, Berlin, 1916) [2e]. A. Einstein, H. Minkowski and H. Weyl, *The Principle of Relativity: A collection of original memoirs* (Methuen, London, 1923) [2f]. W. Pauli, *Theory of Relativity*, Pergamon Press, London (1958) [2g]. C. W. Misner, K. S. Thorne and A. Wheeler, *Gravitation*, Freeman, San Francisco (1970) [2h]. D. Lovelock and H. Rund, *Tensors, Differential Forms and Variational Principles*, Wiley, New York (1975) [2i]. M. J. G. Veltman, in *Methods in Field Theory*, R. Ballan and J. Zinn–Justin, eds. (North–Holland, Amsterdam, 1976) [2j]. C. J.Isham, R. Penrose and D. W. Sciama, Editors, *Quantum Gravity 2* (Oxford University Press, Oxford, 1981) [2k]. E. C. G. Sudarshan and N. Mukunda, *Classical Mechanics: A Modern Perspective*, Wiley & Sons, New York (1974) [2l].

[3] R. M. Santilli, Found. Phys. Letters **10**, 305 (1997) [3a]; contributed paper in the *Proceedings of the Eight Marcel Grossmann Meeting in Gravitation,* T. Piran, and R. Ruffini, Editors, World Scientific, pages 473-475 (1999) [3c]; Annales Fondation L. de Broglie, **29**, 1 (2004) [3d].

[4] R. M. Santilli, Nuovo Cimento Lett. **37,** 545 (1983) [4a]; Lett. Nuovo Cimento **38,** 509, (1983) [4b]; Hadronic J. **8**, 25 and 36 (1985) [4c]; JINR Rapid Comm. **6**, 24 (1993) [4d]; J. Moscow Phys. Soc. **3,** 255 (1993) [4e]; JINR Comm. No. E4-93-352 [1993] and Chinese J. Syst. Eng. and Electr. & Electr. **6,** 177 (1996) [4f]; Intern. J. Modern Phys. A **14**, 2205 (1999) [4g].

[5] R. M. Santilli, Hadronic J. **8**, 25 and 36 (1985) [5a]; Comm. Theor. Phys. **3**, 153 (1993) [5b]; Hadronic J. **17**, 257 (1994) [5c]; contributed paper to *New Frontiers in Hadronic Mechanics,* T. L. Gill, Editor, Hadronic Press (1996), pages 343-416 [5d]. Hyperfine Interactions, **109**, 63 (1997) [5e]; contributed paper to proceedings of the *International Workshop on Modern Modified Theories of Gravitation and Cosmology,* E. I. Guendelman, Editor, Hadronic Press, pages 113-169 (1998) [5f]; Intern. J. Modern Phys. A **14**, 2205 (1999) [5g];

[6] R. M. Santilli, Algebras, Groups and Geometries **10**, 273 (1993) [6a]; Rendiconti Circolo Matematico di Palermo, Supplemento **42**, 7 (1996); [6b]; Intern. J. Modern Phys. D **7**, 351 (1998) [6c]; Found. Phys. **27**, 1159 (1997) [6d]; Advances in Algebras, **21**, 121 (2003) [6e]; Journal of Dynamical Systems and Geometric Theories, **1**, 121 (2003) [6f].

[7] R. M. Santilli, *Foundations of Theoretical Mechanics,* Vol. I (1978) [7a], Vol. II (1982) [7b], Springer Verlag, Heidelberg-New York; *Elements of Hadronic Mechanics*, Vol. I (1995) [7c], Vol. II (1995) [7d], and Vol. III (in preparation) [7e], Ukrainian Academy of Sciences, Kiev; *Isodual Theory of Antimatter with Applications to Antigravity, Grand Unification and Cosmology,* Kluwer Academic Publisher (to appear) [7f].

[8] J. V. Kadeisvili, *Algebras, Groups and Geometries* **9**, 283 and 319 (1992) [8a]; Math. Methods in Applied Sciences **19**, 1349 [1996] [8b]; contributed paper in *Photon: old Problems in Light of New Ideas,* V. V. Dvoeglazov, Editor, Niva

Science, Hungtigton, N. Y. (2000) [8c]. Gr. T. Tsagas and D. S. Sourlas, Algebras, Groups and Geometries **12**, 1 and 67 (1995) [8d]. R. Aslaner and S. Keles, Algebras, Groups and Geometries **14,** 211 (1997) [8e]. S. Vacaru, Algebras, Groups and Geometries **14**, 225 (1997) [8f].

[9] S. L. Adler, Phys. Rev. **17**, 3212 (1978) [9a]; Cl. George, F. Henin, F. Mayne and I. Prigogine, Hadronic J. **1**, 520 (1978) [9b]; S. Okubo, Hadronic J. **3**, 1 (1979) [9c]; J. Fronteau, A. Tellez Arenas and R. M. Santilli, Hadronic J. **3**, 130 (1978) [9d] H. C. Myung and R. M. Santilli, Hadronic J. **5**, 1277 (1982) [9e]; C. N. Ktorides, H. C. Myung, and R. M. Santilli, Phys. Rev. D **22**, 892 (1982) [9f]. A. J. Kalnay, Hadronic J. **6**, 1 (1983) [9g]. R. Mignani, Nuovo Cimento Lett. **39**, 413 (1984) [9h]. J. D. Constantoupoulos and C. N. Ktorides, J. Phys. A **17**, L29 (1984) [9i]. E. B. Lin, Hadronic J. **11**, 81 (1988) [9l]. M. Nishioka, Nuovo Cimento A **82**, 351 (1984) [9m]. A. K. Aringazin, Hadronic J. **12**, 71 (1989) [9n]. D. Rapoport-Campodonico, Algebras, Groups and Geometries **8**, 1 (1991) [9o]. A. Jannussis, G. Brodimas, and R. Mignani, J. Phys. A **24**, L775 (1991) [9p]. A. Jannussis, M. Miatovic and B. Veljanowski, Physics Essays **4**, 202 (1991) [9q]. R. Mignani, Physics Essays **5**, 531 (1992) [9r]; F. Cardone, R. Mignani and R. M. Santilli, J. Phys. G **18**, L61 and L141 (1992) [9s]. T. Gill, J. Lindesay, and W. W. Zachary, Hadronic J. **17**, 449 (1994) [9t]; A. O. E. Animalu, Hadronic J. **17**, 349 (1995) [9u]. A. O. E. Aniamalu and R. M. Santilli, Int. J. Quantum Chemistry **29**, 175 (1995) [9v]. D. Schuch, Phys. Rev. A **55**, 955 (1997) [9x].

[10] A. K. Aringazin, A. Jannussis, D. F. Lopez, M. Nishioka and B. Veljanosky, *Santilli's Lie–Isotopic Generalization of Galilei's Relativities,* Kostarakis Publisher, Athens, Greece (1980) [10a]. J. V. Kadeisvili, *Santilli's Isotopies of Contemporary Algebras, Geometries and Relativities*, Second Edition, Ukraine Academy of Sciences, Kiev (1997) [10b]. D. S. Sourlas and G. T. Tsagas, *Mathematical Foundations of the Lie-Santilli Theory*, Ukraine Academy of Sciences, Kiev (1993) [10c]. J. Lôhmus, E. Paal and L. Sorgsepp, *Nonassociative Algebras in Physics*, Hadronic Press, Palm Harbor, FL, (1994) [10d]. R. M. Falcon Ganfornina and J. Nunez Valdes, *Fondamentos de la Isoteoria de Lie-Santilli,* (in Spanish) International Academic Press, America-Europe-Asia, (2001), also available in the pdf file http://www.i-b-r.org/docs/spanish.pdf [10e]. Chun-Xuan Jiang, *Foundations of Santilli's Isonumber Theory,* International Academic Press, America-Europe-Asia (2002), also available in the pdf file http://www.i-b-r.org/docs/jiang.pdf [10f].

[11] R. M. Santilli, Ann. Phys. **83**, 108 (1974) [11a]. P. Freud, Ann. Math. **40** (2), 417 (1939) [11b]. H. Rund, Algebras, Groups and Geometries **8**, 267 (1991) [11c]. H. Yilmaz, Hadronic J. **11**, 179 (1988) [11d] H. E. Wilhelm, Chinese J. Syst. Eng. & Electr. **6,** 59 (1965) [11e]. H. E. Wilhelm, Hadronic J. **19,** 1 (1996) [11f]. H. E. Wilhelm, Hadronic J. **27,** 349 (2004) [11g]. R. M. Santilli, Chinese J. Syst. Eng. & Electr. **6,** 155 (1965) [11h]. H. Alfvén, contributed paper in *Cosmology, Myth and Theology,* W. Yourgrau and A. D. Breck, Editors, Plenum Press, New York (1977) [11i]. H. Alfvén, American Scientist **76,** 249 (1988) [11j]. V. Fock, *Theory of Space, Time and Gravitation,* Pergamon Press, London (1969) [11k]. H.

Nordenson, *Relativity, Time and Reality: A Logical Analysis,* Allen and Unwin, London (1969) [11l]. R. M. Santilli, Intern. J. Modern Phys. A **20,** 3157 (1999) [11m].

[12] R. M. Santilli, *Hadronic J.* **1,** 224, 574 and 1267 [12a]; contributed paper in *Proceedings of the Seventh M. Grossmann Meeting on General Relativity*, R. T. Jantzen, G. Mac Kaiser and R. Ruffinni, Editors, World Scientific, Singapore (1996), p. 500 [12b]; *Found. Phys.* **27**, 691 (1997) [12c]; contributed paper in *Gravity, Particles and Space–Time*, P. Pronin and G. Sardanashvily, eds. (World Scientific, Singapore, 1995), p. 369 [12d]; Comm. Theor. Phys. **4,** 1 (1995) [12e]; Acta Applicanbdae Math. **50,** 177 (1998) [12f]. D. Bohm, *Quantum Theory* (Dover Publications, New York, 1979) [12g]. J. S. Bell, Physica **1,** 195 (1965) [12h]. A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47,** 777 (1935) [12l].