LIE-ISOTOPIC APPROACH TO A NEW HADRONIZATION MODEL

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Abstract

It is shown that the potential of a new hadronization model can be reproduced naturally in terms of Lie-isotopic generalization of the underlying operator algebra and Dirac equation. The equation provides quantum mechanical description of an eventual non- hamiltonian interaction of the quark fields in the hadronization regime, characterized as a nonunitary evolution. This allows us to mimic an effective quark confinement.

1 Introduction

In a recently proposed phenomenological hadronization model[1, 2] which has been successfully applied to the charm decay and other processes[3] (see also predictions for B mesons decay[4]), the quark-antiquark pairs obey, in the rest frame of the decaying meson, a generalized Dirac equation

$$(i\gamma^{\mu}\partial_{\mu} + i\vec{\gamma}\vec{x}/x_0^2 - m)\psi = 0, \tag{1}$$

According to this equation the quarks produced in a weak decay are described by a free wave damped by a gaussian, $\psi = \phi exp(-\vec{x}^2/2x_0^2)$, where the width x_0 ($x_0 = 0.2 - 0.3fm$[2]) is a separation distance beyond of which quarks hadronize and do not appear as asymptotic states. The non-hermitian part of the associated hamiltonian,

$$H = -\bar{\psi}(i\vec{\gamma}\cdot\vec{\partial} + i\vec{\gamma}\cdot\vec{x}/x_0^2 - m)\psi \tag{2}$$

leads to expected decreasing of the total probability $\langle\psi|\psi\rangle$ with time[2], $d\langle\psi|\psi\rangle/dt = (H - H^{\dagger})/i$, which is a desirable feature of the quark hadronization.

In the energy region $\sqrt{s} \sim 1{-}3GeV$, *i.e.* between the u, d, c, and s thresholds, perturbative QCD fails to describe carefully such a phenomenon[3] so that one can investigate this, as a first step, in terms of first quantization models. The BBPT model of Bediaga et al.[2] gives a simple minded mechanism which mimics an effective quark confinement and hadronization, and, on the other hand, provides a free particle limit for very short quark separation, $|x|/x_0 \ll 1$.

Various attempts[5]-[7] have been made recently to reproduce the non-hermitian potential of the Eq.(1), and justify the lack of Lorentz invariance of the BBPT model. Gasperini[5] considered a coupling quark field to the geometry of an anti-de Sitter vacuum[5] (cf.[8]). Cianiello et al.[6] supposed that some aspects of the hadronization process can be reproduced by coupling the quark field to the conformally flat metrics, $g_{\mu\nu} = \eta_{\mu\nu}exp(2\vec{x}^2/3x_0^2)$ (see also [7]).

Also, various aspects of a geometrical approach to the problem of strong interactions and confinement have been discussed by Hehl et al.[9], Sijacki[10], Santilli[11]; see also[12, 13].

In this paper, we use the Lie-isotopic generalization of the Dirac equation, and reproduce the BBPT potential of the Eq.(1) assuming that such a potential originates from non-hamiltonian interaction of the quark fields arising, as we suppose, at hadronization scale x_0 .

Without attempting here to construct a detailed quantum mechanical framework for hadronization, we shall simply show that the Lie-isotopic approach may give a natural interpretation for the model of hadronization under the same assumptions as those of the BBPT model[2]. We briefly discuss mathematical properties and immediate implications of the Lie-isotopic model restricting consideration on the new aspects arising from this approach. We try to display its relation with the other interpretation schemes of the original BBPT one.

Lie-isotopic generalization of quantum mechanics called hadronic mechanics is based on a natural generalization of representation of Hilbert space, and verify extended realization of algebra of observables via generalized (associative) product. The generalization provides, particularly, nonlinear equations and non-unitary time evolution so that tempting possibilities arise in this approach which may be needed for physics.

For more detailed and precise development of Lie-isotopic generalization we refer the reader to the original papers and recent review[14].

Another generalization of the conventional quantum mechanics has been attempted recently by Weinberg[20]. This generalization is based on a possible nonbilinear representation of observables, with the algebra being similar to Poisson one. The Poisson product is not associative so that the problem arises as to the integrability of (discrete) Hamiltonian systems. This leads to an important difference between ordinary quantum mechanics and the attempted nonlinear formalism[20]. Weinberg emphasized that generalizations are indeed necessary to test the quantum mechanics itself, for which modern experimental tests, ruled out conclusively local hidden variable theories, do not provide accuracy better than 2%.

The paper is organized as follows. In Sec.2, we briefly sketch some results of the Lie-isotopic technique. In Sec.3, we use the generalized Dirac equation in the BBPT model. In Sec.4, we discuss on the results.

2 Lie-isotopic generalization of the Dirac equation

According to the non-hermiticity of the associated hamiltonian (2), the BBPT model[2] describes, in fact, nonunitary time evolution of the quark-antiquark pair. We note that one can modify the conventional quantum mechanical description to include such a non-canonical time evolution by the Lie-isotopic generalization [11]-[16] of the underlying associative enveloping algebra A of operators $A,\ B,\cdots$ of Hilbert space equipped with the conventional product AB, and attached Lie algebra with the product $[A,B]=AB-BA$. Namely, the lifting $\mathcal{A}\rightarrow\hat{\mathcal{A}}$ is defined by the product $A*B=ATB$ and Lie-isotopic algebra with the product $[A,B]=A*B-B*A$, where T is a fixed element of algebra $\mathcal{A}$. In the limit $T\rightarrow 1$, the Lie-isotopic theory recovers the usual one.

In ref.[11], the following Lie-isotopic generalization of the Heisenberg equation has been proposed in order to describe closed non-hamiltonian systems: $idA/dt=[A,B]$, where B represents total energy of the system. The evolution can be presented then as $A(t)=exp(itX^{+})A(0)exp(-itX)$, where the nonhermitian operator X is defined by the decomposition $X=TB$, with the isotopic element T and the energy B being noncommuting hermitian operators, $T^{+}=T$, $B^{+}=B$, $[T,B]=X-X^{+}$. It is important to note here that these formulas as well as the new Lie bracket $[,]$ arise naturally when one deals with nonunitary evolution owing to the fact that any non-hermitian operator can be in general decomposed into two noncommuting hermitian ones[11].

The action of the T-isotopically lifted operators on physical states is defined by $A*\psi,\ A\in\mathcal{A}$[12]. This definition preserves the structure of the associated unital (left) modul since $I^{*}*\psi=\psi$, where $I^{*}=T^{-1}$ is the unit operator of algebra $\mathcal{A}$.

It should be noted that in spite of the nonunitarness of the time evolution the energy B is conserved due to the antisymmetry of the new bracket $[,]$, $dB/dt=0$. Energy nonconservation processes can be described by using of the Lie-admissible generalization (see [12] and references therein), which recovers the Lie-isotopic one, when one determines the bracket in the form $[A,B]^{\circ}=ARB-BSA$, with R and S being fixed operators, $R^{+}=S$, $R,S\in\mathcal{A}$.

Tending to provide a relativistic formulation of the Lie-isotopic technique one can construct the corresponding Lie-isotopic lifting of the Dirac equation written in Hamiltonian form as follows[13, 14]:

$$i\frac{\partial\psi}{\partial t} = H_D * \psi \tag{3}$$

where $H_D = \vec{\alpha}\vec{p} + m\beta$. This equation can be rewritten in covariant form,

$$(i\gamma^\mu\partial_\mu - m) * \psi = 0, \tag{4}$$

where γ-matrices verify the Lie-isotopic anticommutation relations

$$\{\gamma^\mu, \gamma^\nu\}^* = 2g^{\mu\nu}I^*, \tag{5}$$

and $\partial_0 I^* = 0$. Eq.(4) represents generalized Dirac equation for the spinor subjected to a non-hamiltonian interaction, which is defined by the (local) operator T entering the definition of the new Lie bracket $[,]^*$, and provides naturally a description of the nonunitary evolution.

3 Lie-isotopic approach to the BBPT model

In the conventional terms Eq.(4) takes the form

$$(i\gamma^\mu\partial_\mu + iT^{-1}\gamma^\mu\partial_\mu T - m)\psi = 0. \tag{6}$$

This equation implies that we can choose $T = exp(-\vec{x}^2/2x_0^2)$ to meet the BBPT model. We will discuss this specific representation below.

If this is not entirely due to the coincidence, it may indicate that the origin of the BBPT potential is a non-hamiltonian interaction of the quarks produced which perhaps begins manifest itself only at the low energy scale, $\sqrt{s} = 1 - 3$ GeV. Perhaps charmed mesons decays give an experimental evidence for such an interaction. In attributing such effects to the non-hamiltonian interaction, we are arguing in effect that the laws are quantum mechanical, but the evolution is not really unitary as the quark separation $|x|$ increases. The "non-hamiltonian" character of the interaction may be treated as the effect of overlapping the momenta distributions of the two quarks produced[2]. The latter is an essential point of the hadronization model.

Two comments are in order: (i) The T-isotopic lifting of the algebra $\mathcal{A}$ of the operators $A, B, ...$ acting on Hilbert space, spanned by bra and ket vectors $\langle\psi|$ and $|\psi\rangle$, assumes corresponding lifting of the conventional quantum mechanical Hilbert space[12]. In particular, the generalized Dirac equation (4) can be interpreted, in T-isotopic terms, simply as the Dirac equation for the isoket vector $*|\psi\rangle$ of the isoHilbert space[12, 14, 16]. An associated, hermitian conjugation is defined by $A^{\hat{+}} = TA^{+}T^{-1}$[12]. It is straightforward to verify that the (non-hermitian) operator X entering the evolution equation appears to be isohermitian, $X^{\hat{+}} = X$, under the definition $X = TB$.

(ii) In the context of the BBPT model, the isostate $T\psi$ describes exactly the free wave while the confining wave function comes with projection of the isoHilbert space to the conventional Hilbert one (see [2] for detailed physical analysis of the relation between the damped and free wave functions $\psi(x,t)$ and $\phi(x,t)$ in simple cases). In this respect, one can refer to intimate analogy of this with the fiber bundle description of the motion of particle subjected to a gauge interaction; see also gauge field interpretation[2] of the non-hermitian potential, and the connection between gauge theory and Lie-isotopic inner product (isonorm)[17]. In essence, the gauging away of the pure imaginary abelian gauge field $A = \nabla\lambda(\vec{x})$, $\lambda(\vec{x}) = i\vec{x}2x_0^2$, made in[2] corresponds to the Lie-isotopic lifting of the Hilbert space with $T = exp[i\lambda(\vec{x})$. The isostate $T\psi$ reduces trivially to the conventional one when $T \to 1$ or, equivalently, when $x \to \infty$ (no hadronization or lepton case). In general case T should be in the center of the algebra A to obtain the free particle limit.

4 Discussion

From phenomenological point of view, gaussian representation of the damping for the wave function of the quarks is certainly a convention[2]. We note that the specific form, in which the additional term in the generalized Dirac equation (6) appears, requires rather definitely a gaussian function for T to reproduce an effective linear confining BBPT potential. However, an essential point here is that a relation between the damped and free wave functions does arise naturally under the Lie-isotopic lifting of the conventional operator algebra describing nonunitary evolution and leading to a modification of the Dirac equation. On the other hand, both the damping of the free quark wave function and nonunitary evolution are specific points of the hadronization BBPT model. So, the formalism can provide an adequate interpretation, in the proper quantum mechanical terms, of the hadronization model.

Some comments on justification of the lack of Lorentz invariance of the model are in order. Despite of the formal covariance of the Eq.(4), we observe that the resulting Eq.(6) becomes manifestly non-covariant when T is not a scalar. This is clearly a consequence of the non-relativistic formulation of the T-isotopic technique used in this paper. However, trying to achieve an explicitly covariant description of the model, one can adopt the Eq.(4) as starting point of consideration assuming T be a covariant hermitian operator. The problem is then, schematically, to construct a relativistic formulation of the isostates, on which an appropriately lifted Dirac operator acts. On the other hand, in geometrical context $T^{-1} = \exp(-\vec{x}^2/2x_0^2)$ may play the role of factor of the conformally flat metrics[16, 18].

It is worthwhile to note that an operator nature of the isotopic element T can provide the energy dependence in the effective quark hadronization potential empirically introduced and discussed in[3]; see also[4, 19]. The particular form of the dependence (for the expected value), $T = T(\sqrt{s})$, can be taken to fit the data for the ratio $R = \sigma(e^+e^- \to hh)/(e^+e^- \to \mu^+\mu^-)$ in the energy region around and below 1 GeV, where, as it was emphasized in[3], care should be exerted to extrapolate the BBPT model without, at least, letting x_0 becomes function of energy. The effective distance x_0 has to decrease for lower energies.

We conclude with some remarks. The analysis made in[17] allows us to seek for another aspect in treating of the Lie-isotopic lifting. Namely, define the vacuum by $p|0\rangle = |0\rangle$, and consider the action, in configuration state, of the generalized (one-dimensional) momentum operator $p = -id/dq - d\lambda/dq$, where λ is a scalar function, $\lambda = \lambda(q)$, on the wave function of the vacuum state $T = \langle|0\rangle$,

$$(-i\frac{d}{dq} - \frac{d\lambda}{dq})T = 0, \tag{7}$$

The solution of the Eq.(7) is $T = N\exp(i\lambda)$ where N is a normalization constant. Under this definition of the vacuum, quantum states will be of the form $T|\psi\rangle$, just as the representation of the isoket vectors. Consequently, the underlying canonical commutation algebra, $[q, p] = i$, can be reformulated in T-isotopic terms[17]. Thus the element T can be understood as a vacuum state of the system described by the canonical commutation relations, with the generalized momentum operator p, which is hermitian if λ is pure imaginary. Clearly this is a sort of "minimal coupling" mentioned[2] in respect to the gauge field interpretation of the BBPT model.

Finally, we note that the replacement of the usual Dirac delta function conservation of three-momenta of the quarks produced by a gaussian distribution announced[2] as the main practical consequence of the BBPT model can be interpreted also as the result of general replacement of the usual norms by the isonorms, where T is the gaussian integration measure.

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