A NEW THEORY ON THE STRUCTURE OF THE RUTHERFORD-SANTILLI NEUTRON

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Abstract

A theory of second-order phase transition from a normal to a superdense state in compressed H-atom is presented, based on two analogies. Firstly, from an analogy between classical mechanics and thermodynamics, the Gibbs function of temperature and pressure (generated from the internal energy via Legendre's transformation in classical thermodynamics) is related to the Birkhoffian function (characterizing a Hamiltonian with external velocity and acceleration terms) in Santilli's Hadronic Mechanics (HM). Secondly, from an analogy between the Landau-Ginsburg equation for the order parameter ψ (representing an electron-electron (Cooper) pair $(e^- \uparrow - e^- \downarrow)_{HM}$ wavefunction) in a superconductor and the isoschrodinger equation for the Rutherford-Santilli neutron as a compressed H-atom, $n = (e^- \downarrow - p^+)_{HM}$, in Hadronic Mechanics, we deduce that the realization of electron-electron (Cooper) pairing in the form $(e^- \uparrow - Cu^{z+} - e^- \downarrow)_{HM}$ around Cu^{z+} in a cuprate superconductor would be analogous to electron-spinion pairing $(e^- \downarrow - p^+ - e^0 \uparrow)_{HM}$ around p^+ in the compressed H-atom, where $(e^0 \uparrow)$ is an electron-like massive neutral spin-$\frac{1}{2}$ particle (called *spinion* in superconductivity theory) overlapping the 1s electron $(e^- \downarrow)$ wavefunction and interacting with the p^+ via an effective Hulthen potential. As a result we arrive at a realization of the Rutherford-Santilli model of the neutron in the form $(e^- \downarrow - p^+ - e^0 \uparrow)_{HM}$ which agrees with Barut's representation of the physical neutron as a $p^+ e^- \overline{\nu}_e$ bound state, $\overline{\nu}_e$ being the electron-like antineutrino. The implications of the existence of the massive electron-like spinion (e^0) as an elementary particle under appropriate conditions are discussed.

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1. INTRODUCTION

In 1920, Rutherford[1] hypothesized that a neutral particle, subsequently discovered twelve years later by Chadwick[2] as the *neutron*, could result by compressing hydrogen atom until the negatively charged orbital electron touches and neutralizes the positively charged proton. However, Rutherford's hypothesis was later dropped because of the following inconsistencies among others:

1. The neutron rest mass (m_n = 939.565 MeV) is greater than the sum of the rest masses of the proton (m_p = 938.273MeV) and the electron, (m_e = 0.511 MeV) thus requiring a *positive* binding energy which would be contrary to the laws of basic quantum mechanics.
2. The neutron's meanlife of 15 minutes is too large compared with other unstable elementary particles due to the inability of the very light electron to be bound inside the proton for such a long period of time.
3. The neutron spin $\frac{1}{2}$ would be inexplicable, because the quantum mechanical bound state of two spin-$\frac{1}{2}$ particles can only produce an integer total spin.

These objections prompted Santilli[3,4] to question the exact applicability of quantum mechanics for the physical conditions of compression envisaged by Rutherford and to propose a generalization of basic Quantum Mechanics to a new discipline called Hadronic Mechanics (HM). The argument runs briefly as follows: If one starts to compress normal hydrogen (gas) in an enclosure, one would expect the gas to go first into a liquid phase (which would behave like a liquid monovalent metal) and finally into a solid (amorphous or crystalline) phase. If the compression is continued, a superdense phase (believed to exist in the interior of stars) would result. Indeed, according to the schematic representation [5]of 1.4M_S neutron star, M_S being the mass of the Sun (see, Fig.1), the interior of a typical neutron star is usually divided into five regions, namely the *surface* where the properties of matter can be strongly affected by the temperature and the magnetic field; the *outer crust* consisting of solid lattice of nuclei embedded in a sea of degenerate relativistic electrons; the *inner crust* consisting of that part interior to the neutron drip point (a superfluid phase in which neutrons exist outside the nuclei), so that one has a solid state lattice of nuclei immersed in a sea of electrons and neutrons; the *liquid core* consisting mainly of neutrons but with some superconducting electrons, protons, and a few muons; and possibly a distinct *plasma core* which could consist of condensed pions, kaons, quark matter, etc.

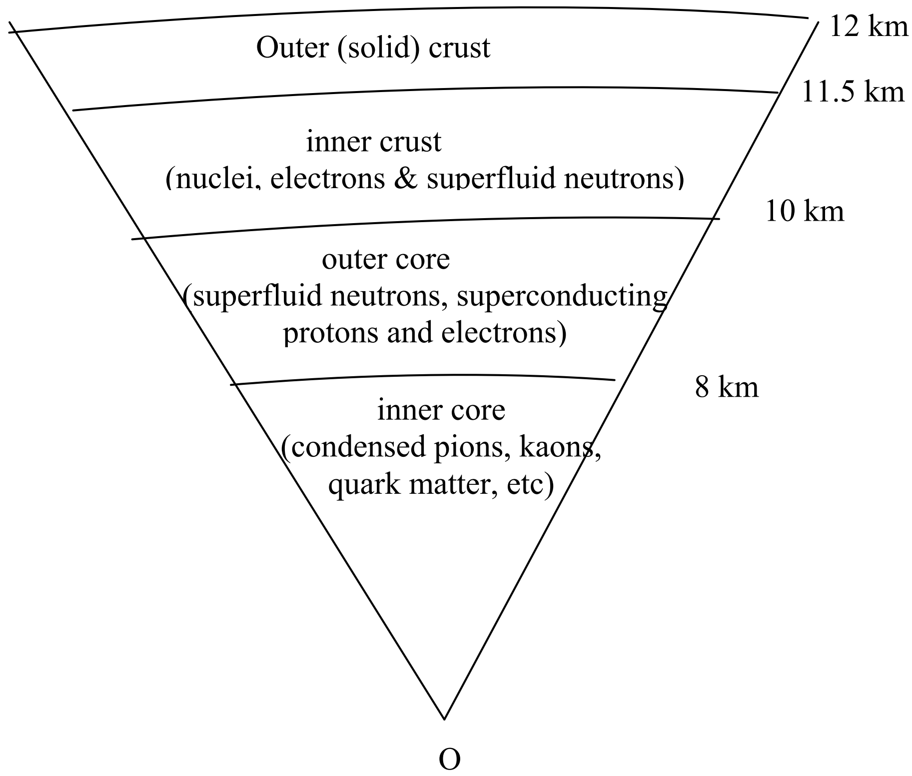


Fig. 1: Schematic representation (Urama[5]) of the cross-section of a $1.4M_S$ neutron star based on an equation of state.

Doubting the validity of the basic laws of quantum mechanics under such superdense conditions, Santilli proceeded in his 1978 memoir [3] to account for all the physical characteristics of the neutral pion (π^0) as a nonlocal-nonhamiltonian structure $\pi^0 = (e^+ \uparrow, e^- \downarrow)_{HM}$ of a compressed positronium atom described by Hadronic Mechanics (see, Fig. 2(a)). Subsequently, Santilli[4] presented and solved exactly a similar representation of the neutron (n) as a nonlocal-nonhamiltonian structure of a compressed hydrogen atom, $n = (p\uparrow, e^-\downarrow)_{HM}$ (see, Fig. 2(b)), hereunder referred to as the *Rutherford-Santilli neutron*.

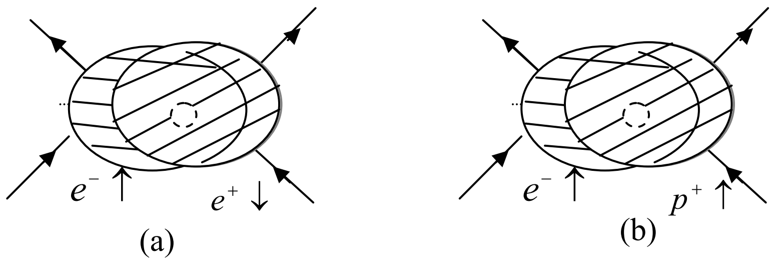
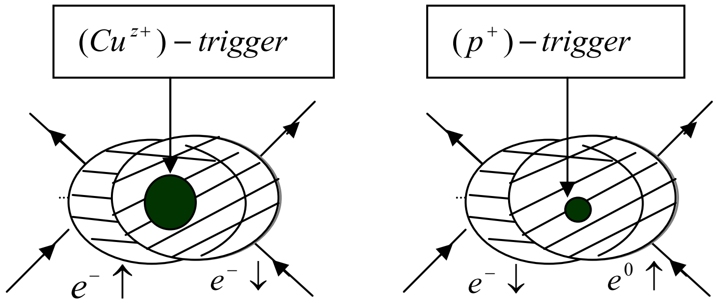


Fig.2: Models of (a) the neutral pion as compressed positronium atom and (b) the neutron as compressed H-atom showing mutual overlapping of the wavepackets of the constituent particles, as proposed by Santilli in refs.[3] and [4].

Prompted by the similarity between Santilli's concept of isotopic lifting transformation in Hadronic Mechanics and pseudopotential transformation in metal physics, Animalu[6,7] proposed in 1991 and 1994 a similar nonlocal-nonhamiltonian structure $(e^-\uparrow, e^-\downarrow)_{HM}$ for the Cooper pair in superconductors. However, in setting up the basic equations for the Cooper pair in the Nambu representation, the presence of a Cu^{z+} "trigger" was made evident in the structure model $(e^-\uparrow - Cu^{z+} - e^-\downarrow)_{HM}$ shown in Fig. 3(a), which led to a prediction of the transition temperatures (T_c) of the family of cuprate materials in remarkably good agreement with experimental data. Animalu and Santilli[8] subsequently verified the consistency of the $(e^-\uparrow - Cu^{z+} - e^-\downarrow)_{HM}$ model with the axioms of Hadronic Mechanics.



(a) (b)

Fig.3: Models of (a) $(e^{-}\uparrow, e^{-}\downarrow)_{HM}$ pairing in a cuprate superconductor and (b) $(e^{-}\downarrow, e^{0}\uparrow)_{HM}$ pairing in compressed H-atom, due to wave overlapping around the respective Cu^{z+} and p^{+} triggers" as indicated in the diagrams.

Another feature that arose from the analysis of charge fluctuations within the Cooper pair in ref.[7] was the possibility of accommodating an electron-like massive neutral spin-$\frac{1}{2}$ quasiparticle $(e^{0}\uparrow)$, called *spinion*[9], in anologous structure models for pairing. For this reason, one can hardly avoid characterizing $(e^{-}\downarrow, e^{0}\uparrow)_{HM}$ pairing in the form $(e^{-}\downarrow - p^{+} - e^{0}\uparrow)_{HM}$ with the positively charge proton p^{+} as a "trigger" in a structure model of the neutron as a 2nd order (superfluid) phase of compressed H-atom shown in Fig. 3(b).This representation of the neutron will be explored in this paper.

We shall start by reviewing in Sec.2, the recent paper by A.O.E. Animalu and C.N. Animalu[10] on an extension of the analogy between thermodynamics and classical mechanics, leading to an identification of Gibbs free energy function (of temperature and pressure) with the *Birkhoffian function* representing a Hamiltonian with external (velocity and acceleration) terms in Hadronic Mechanics. As a consequence, we shall identify the Landau-Ginsburg equation for order parameter (representing Cooper pair wavefunction) in superconductivity theory which is a 2nd order phase transition from a normal to a superfluid phase with the isoschrodinger equation for the system $(e^{-}\downarrow - p^{+} - e^{0}\uparrow)_{HM}$ representing a 2nd order (superfluid) phase of the compressed H-atom. In Sec.3 we shall discuss the solutions of the isoschrodinger equation for the Rutherford-Santilli neutron as a compressed H-atom, and in sec. 4, we shall present experimental verification of the physical characteristics of the neutron. Finally, in Sec. 5, we shall draw the attendant conclusions.

2. ANALOGY BETWEEN THERMODYNAMICS AND MECHANICS

2.1 Equivalence of the Gibbs Function and the Birkhoffian

In an extension of the analogy between thermodynamics and classical mechanics described in ref.[10], the thermodynamic variables (volume *V*, entropy *S*, temperature *T*, and pressure *P*) correspond to the dynamical variables of the classical mechanics of a one-dimensional system (coordinate (*q*), momentum (*p*), velocity $(\dot{q})$, and acceleration $(\dot{p})$ respectively), while the internal energy *(U)*, the Helmholtz free energy ($\mathcal{H}$) and Gibbs free energy *(G),* which are related to one another by Legendre's transformation, correspond to the Hamiltonian *(H)*, the Lagrangian *(L)*, and the Birkhoffian *(B)* respectively. We wish in this section to review the analogy between the Gibbs function and the Birkhoffian at the classical level, as a prelude for describing 2nd order phase transitions in a compressed hydrogen atom.

At the classical level, the Gibbs free energy (and its associated thermodynamic differential relations),

$$G(P,T) = U + PV - TS,\ \ (dG = VdP - SdT;\ V = \partial G/\partial P, S = -\partial G/\partial T) \tag{2.1}$$

correspond to the Birkhoffian (and its associated dynamical equations of motion)

$$B(\dot{p},\dot{q}) = H - \dot{q}p + q\dot{p},\ \ (dB = qd\dot{p} - pd\dot{q};\ q = \partial B/\partial \dot{p}, p = -\partial B/\partial \dot{q})\ \ , \tag{2.2a}$$

It is thus evident that when the Birkhoffian system is described in terms of Hamilton's equations of motion, one would obtain equations having the form

$$\dot{q} = \partial H / \partial p + f_1, \, \dot{p} = -\partial H / \partial q + f_2 \tag{2.2b}$$

where $f_1 = -\dot{q}, f_2 = -\dot{p}$ represent the contributions from the external [velocity and acceleration] terms, $-\dot{q}p + q\dot{p}$. Thus, instead of the conventional Hamilton's equations and the Liouville equation for the Hamiltonian system,

$$\dot{a}^{\mu} = \omega^{\mu\nu} \frac{\partial H}{\partial a^{\nu}}, \; \dot{A} = \frac{\partial A}{\partial a^{\mu}} \omega^{\mu\nu} \frac{\partial H}{\partial a^{\nu}}, \text{with } (\omega^{\mu\nu}) \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{2.3}$$

where $(a^1, a^2) = (q, p)$, one obtains the generalization for the Birkhoffian system

$$\dot{a}^{\mu} = S^{\mu\nu} \frac{\partial B}{\partial a^{\nu}}, \; \dot{A} = \frac{\partial A}{\partial a^{\mu}} S^{\mu\nu} \frac{\partial B}{\partial a^{\nu}} \tag{2.4a}$$

in which the determinant of the inverse of the tensor $S^{\mu\nu}$ generalizing $\omega^{\mu\nu}$,

$$\det \left\| (S^{-1})^{\mu\nu} \right\| = \frac{\partial \dot{q}}{\partial q} \frac{\partial \dot{p}}{\partial p} - \frac{\partial \dot{p}}{\partial q} \frac{\partial \dot{q}}{\partial p} \equiv [\dot{q}, \dot{p}]_c, \tag{2.4b}$$

is the non-zero classical Poisson bracket of $(\dot{q}, \dot{p})$. Accordingly, if $S^{\mu\nu}$ is rewritten in the form

$$S^{\mu\nu} \equiv \tfrac{1}{2}(S^{\mu\nu} - S^{\nu\mu}) + \tfrac{1}{2}(S^{\mu\nu} + S^{\nu\mu}) \equiv \Omega^{\mu\nu} + T^{\mu\nu} \tag{2.5}$$

where $\Omega^{\mu\nu}$ is antisymmetric and $T^{\mu\nu}$ symmetric, then the generalized Liouville equation in (2.4) takes the form

$$\dot{A} = \frac{\partial A}{\partial a^{\mu}} \Omega^{\mu\nu} \frac{\partial B}{\partial a^{\nu}} + \frac{\partial A}{\partial a^{\mu}} T^{\mu\nu} \frac{\partial B}{\partial a^{\nu}} \equiv [A, B]^* + \{A, B\}^*, \tag{2.6}$$

which involves the generalized Poisson bracket $[A, B]^*$ defining the generalized Lie-algebraic products used by Santilli[3] in his generalization of classical Birkhoffian mechanics to the new form of quantum mechanics known as *hadronic mechanics.* It is thus apparent [by comparing the Liouville equation in (2.3) and its generalization in (2.6)] that it is the *symmetric tensor* ($T^{\mu\nu}$) part of $\dot{A}$ that explicitly corresponds to the external temperature and pressure terms in the Gibbs free energy.

2.2 Equivalence of the Landau-Ginsburg equation and the Isoschrodinger equation for the Compressed H-Atom

Suppose that the phase transition in the compressed H-atom is of 2nd order so that it may be characterized classically by a free energy $\int g(\psi,T)d^3r$, where $\psi=\psi(r)$ is a complex order parameter. Suppose further that the free energy density $g(\psi,T)$ has the Ginsburg-Landau form [see, Eq.(6.5.13) ref.[11] p. 462]:

$$g = g_0(T) + \varepsilon|\psi|^2 + \tfrac{1}{2}\kappa|\psi|^4 + \gamma|\vec{\nabla}\psi|^2 \qquad (2.7)$$

ε,κ being functions of the temperature *T*, and γ is defined as

$$\gamma \equiv \frac{\hbar^2 b^{-2}}{2\overline{m}}, \text{ with } \overline{m} \equiv \frac{m_e m_p}{m_e + m_p}, \text{ i.e., } \frac{1}{\overline{m}} = \frac{1}{m_e} + \frac{1}{m_p}, \qquad (2.8)$$

where b is a parameter characterizing compression so that $\overline{m}^* \equiv \overline{m}b^2$ may be interpreted as an effective reduced mass. Then, if this free energy is minimized with respect to variations in the order parameter by requiring that

$$\frac{\partial}{\partial\psi^*}\int g(\psi,T)d^3r = 0 \qquad (2.9)$$

and the divergence theorem is applied, one would get the Schrodinger-like equation:

$$-\frac{\hbar^2}{2\overline{m}^*}\nabla^2\psi + \kappa|\psi|^2\psi = -\varepsilon\psi, \qquad (2.10)$$

which is the Ginsburg-Landau equation. In superconductivity theory, this has the significance that it provides the temperature dependence of the coherence length (ξ) given by (Eq.(6.5.17) of ref.[11] p. 463)

$$\xi = \left(\frac{\gamma}{\varepsilon}\right)^{1/2} = \left(-\frac{b^{-2}\hbar^2}{2\overline{m}^*\varepsilon}\right)^{1/2} \propto (T_c - T)^{1/2}. \qquad (2.11)$$

as well as the temperature dependence of the density, n_s, of Cooper pairs which is just the square of the order parameter (Eq.(6.5.16) of ref.[11] p. 462):

$$|\psi|^2 \propto n_s = A(T_c - T) \qquad (2.12a)$$

Since Cooper pairs are bosons, such a temperature dependence of n_s would arise from an expansion of a Bose-Einstein distribution function in the following way:

$$n_s(T) = \frac{1}{e^{E/k_B(T_c - T)} - 1} \approx \frac{k_B(T_c - T)}{E} \propto (T_c - T) \tag{2.12b}$$

Moreover, suppose that the dependence of $|\psi|^2 \propto n_s$ on inter-particle separation can be determined, as in the Ornstein-Zernike theory of direct and total correlation functions of fluid particles in liquids, by an integral equation of the convolution type (cf, Eq.(4.5.73) of ref.[11] p. 300):

$$n_s(r) = n_0(r) + \rho \int_0^r n_s(r') n_s(|\vec{r} - \vec{r}'|) d^3\vec{r}', \tag{2.13}$$

which can be solved by Laplace transformation. To this end, we multiply both sides of this integral equation by $e^{\vec{q}.\vec{r}}$ and integrate over d^3r, using the definition of the Laplace transform of $n_s(r)$ as follows,

$$\widetilde{n}_s(q) \equiv \int \exp(\vec{q}.\vec{r}) n_s(r) d^3r,$$

to find

$$\widetilde{n}_s(q) = \widetilde{n}_0(q) + \rho \iint \widetilde{n}_s(r') \widetilde{n}_0(|\vec{r} - \vec{r}'|) \exp(\vec{q}.\vec{r}) d^3\vec{r} d^3\vec{r}' .$$

Thus

$$\begin{aligned}\widetilde{n}_s(q) &= \widetilde{n}_0(q) + \rho \iint \widetilde{n}_s(r') \widetilde{n}_0(\xi) \exp[\vec{q}.(\vec{\xi} + \vec{r}')d^3r' d^3\xi \\ &= \widetilde{n}_0(q) + \rho \int \widetilde{n}_s(r') \exp[\vec{q}.\vec{r}']d^3r' \int \widetilde{n}_0(\xi) \exp[\vec{q}.\vec{\xi}]d^3\xi \\ &= \widetilde{n}_0(q) + \rho \widetilde{n}_0(q) \widetilde{n}_s(q)\end{aligned}$$

i.e., $\widetilde{n}_s(q) = \widetilde{n}_0(q) + \rho \widetilde{n}_0(q) \widetilde{n}_s(q),$

and hence,

$$\widetilde{n}_s(q) = \widetilde{n}_0(q) / [1 - \rho \widetilde{n}_0(q)] \tag{2.14}$$

Accordingly, if we use as input, the point-particle number density, $n_0(r) = \delta(|\vec{r} - \vec{r}'|)$, so that

$$\widetilde{n}_0(q) \equiv \int_0^r \exp(\vec{q}.\vec{r}') \delta(|\vec{r} - \vec{r}'|) d^3r' = e^{-qr} \tag{2.15}$$

then, on substituting for $\widetilde{n}_0(q)$ in Eq.(2.14) we find the result

$$|\psi|^2 \propto \tilde{n}_s(q) = e^{-qr} /[1-\rho e^{-qr}]\,. \tag{2.16}$$

This enables us (by setting $\rho = 1$) to replace the potential energy term $\kappa|\psi|^2$ in Eq.(2.10) by a Hulthen potential:

$$V_H(r) = \kappa|\psi|^2 \propto \frac{e^{-qr}}{1-e^{-qr}} \tag{2.17}$$

leading to the isoschrodinger equation proposed by Santilli[4] for the compressed H-atom in Hadronic Mechanics[4]:

$$-\frac{\hbar^2}{2\overline{m}\,*}\nabla^2\psi + V_H\psi = -\varepsilon\psi\,. \tag{2.18}$$

Having arrived at this equation from our analogy between thermodynamic and classical mechanics, the stage is now set to search, in the next section, for a microscopic interpretation of the order parameter $\psi = \psi(r)$ as a pair wavefunction in the compressed H-atom analogous to the order parameter for Cooper $(e^-\uparrow, e^-\downarrow)_{HM}$ pairing in superconductors. We shall show in the next that just as the Cooper pairing occurs in the form, $(e^-\uparrow - Cu^{z+} - e^-\downarrow)_{HM}$, around Cu^{z+} "trigger" in a cuprate superconductor so also will the analogous pairing occur in the form , $(e^-\downarrow - p^+ - e^0\uparrow)_{HM}$, around p^+ "trigger" in the compressed H-atom, where $(e^0\uparrow)$ is an electron-like neutral elementary excitation in the outer core of a neutron star, (see, Fig.1) containing superfluid neutrons and superconducting electrons and protons.

3. SOLUTION OF THE COMPRESSED H-ATOM PROBLEM

In order to obtain a microscopic interpretation of the order parameter ψ as a pair wavefunction for the compressed H-atom, we introduce the isotopic lifting operator of Hadronic Mechanics[6,7]:

$$\hat{T} = 1 - \left|\psi_\uparrow^*\right\rangle\left\langle\psi_\uparrow\right|, \tag{3.1}$$

where $\left\langle\psi_\uparrow^*\middle|\psi_\uparrow\right\rangle = 1$ but $\left\langle\psi_\uparrow^*\middle|\psi_\downarrow\right\rangle \equiv Z \neq 0$, so that $\left\langle\psi_\downarrow^*\middle|\hat{T}\middle|\psi_\downarrow\right\rangle = 1 - Z$, while $\left\langle\psi_\uparrow^*\middle|\hat{T}\middle|\psi_\uparrow\right\rangle = 0$. These properties of $\hat{T}$ imply that the charge on the particle $\psi_\downarrow$ is depleted by an amount Z whereas the charge on the particle

$\psi_\uparrow$ appears to vanish altogether. Accordingly, we may identify $\psi_\downarrow$ with a down-spin fractionally-charged electron $(e^- \downarrow)$, called *anyon*, and $\psi_\uparrow$ with an up-spin neutral particle $(e^0 \uparrow)$, called *spinion* (in superconductivity theory[9]), and unify them into a two-component (Nambu) spinor field,

$$\Psi = \begin{bmatrix} \psi_\uparrow \\ \psi_\downarrow^* \end{bmatrix}, \Psi^+ = [\psi_\uparrow^*, \psi_\downarrow]. \tag{3.2}$$

The Ψ field obeys an iso-Lurie-Cremer[12] wave equation replacing Eq.(2.18):

$$i\hbar \frac{\partial}{\partial t}\Psi(\vec{r},t) = \hat{H}\Psi(\vec{r},t), \ \ \hat{H} \equiv HS\hat{T} = \left(\frac{p^2}{2\overline{m}\,*} - \frac{e^2}{r}\right)\hat{\tau}_3, \tag{3.3}$$

where, $\hat{\tau}_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -S\hat{T} \end{pmatrix}$, and $S \Rightarrow$ scale transformation of space-time coordinates: $(c_0 t, \vec{r}) \to (b_4 c_0 t, b\vec{r})$, such that isorelativistic transformation law holds in the form:

$$c_0^2 b_4^2 dt'^2 - b^2(dx'^2 + dy'^2 + dz'^2) = c_0^2 b_4^2 dt^2 - b^2(dx^2 + dy^2 + dz^2) \tag{3.4}$$

(b_4, b) being parameters representing the effects of external pressure and temperature, and c_0 the speed of light in vacuum. Eq.(3.3) reduces to the pair of explicit equations:

$$\left(-\frac{1}{2\overline{m}}\frac{\hbar^2}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - \frac{e^2}{r}\right)\psi_\downarrow(r) = E_\downarrow \psi_\downarrow(r) \tag{3.5a}$$

$$\left(-\frac{b^{-2}}{2\overline{m}}\frac{\hbar^2}{r^2}\frac{d}{dr}r^2\frac{d}{dr} + V_H(r)\right)\psi_\uparrow^*(r) = E_\uparrow \psi_\uparrow^*(r) \tag{3.5b}$$

where the Hulthen potential $V_H(r)$ is defined via the replacement:

$$-\frac{e^2}{br} - |E_\downarrow| \left|\langle \psi_\uparrow^* | \psi_\downarrow \rangle^*\right| \frac{\psi_\downarrow(r)}{\psi_\uparrow^*(r)} \to -Mc_0^2 \frac{\exp(-r/R)}{1-\exp(-r/R)} \equiv V_H(r). \tag{3.6}$$

The pair of Eqs.(3.5) characterizing motions of the electron $(e^- \uparrow)$ and the spinion $(e^0 \downarrow)$ relative to the proton (p^+) constitute our structure model of the neutron as $(e^- \uparrow - p^+ - e^0 \downarrow)_{HM}$ bound state. This is an exactly soluble non-relativistic model of the neutron which we are after.

The *exact* solutions of Eq.(3.5a) for $\psi_\downarrow$ and Eq.(3.5b) for $\psi_\uparrow$ with appropriate boundary conditions for a bound state have the forms (p. 175 of ref.[13]):

$$\psi_{e^-\downarrow}(r) = A\exp(-r/R_e) \tag{3.7a}$$

$$\psi_{e^0\uparrow}(r) = {}_2F_1(2\beta + 1 + n, 1 - n, 2\beta + 1, e^{-r/R})e^{-r/R}(1 - e^{-r/R})/r, \tag{3.7b}$$

while the spectra of energy eigenvalues are given for (3.5a) by the Rydberg formula,

$$E_{n\downarrow} = -\frac{e^2}{2R_{e^-}n^2}, \quad n = 1,2,... \quad , \tag{3.8a}$$

where $R_{e^-} = \dfrac{\hbar^2}{\overline{m}e^2}$ is the Bohr radius; and for (3.5b) by the formula

$$E_{n\uparrow} = -\frac{\hbar^2(bR)^{-2}}{4\overline{m}}\left(\frac{\overline{m}Mc_0^2(bR)^2}{n\hbar^2} - n\right)^2, \quad n = 1,2,... \tag{3.8b}$$

where $\beta = \dfrac{\overline{m}|E_{n\uparrow}|}{\hbar^2(bR)^{-2}}$.

Consequently, assuming a helium-like $1s^2$ configuration of $e^- \downarrow$ and $e^0 \uparrow$ around p^+, the total energy E_T of the bound system, $(e^- \downarrow - p^+ - e^0 \uparrow)_{HM}$, is:

$$\begin{aligned} E_T &= [m_{e^-}c_o^2 + E_{kin\downarrow} - E_\downarrow^0] + [m_p c_0^2 + m_{e^-}b^2 b_4^2 c_0^2 + E_{kin\uparrow} - E_\uparrow^0] \\ &\equiv [m_{e^-}c_o^2 + E_{B\downarrow}] + [m_p c_0^2 + m_{e^-}b^2 b_4^2 c_0^2 + E_{B\uparrow}] \end{aligned} \tag{3.10}$$

where $m_{e^-} = m_{e^-\downarrow} = m_{e^0\uparrow}$, ($m_{e^-\downarrow}, m_{e^0\uparrow}$ being the rest masses of $e^- \downarrow$ and $e^0 \uparrow$ in the uncompressed system), and their ground state energies are given by:

$$E_\downarrow^0 = -\frac{e^2}{2R_{e^-}}, \quad E_\uparrow^0 = -\frac{\hbar^2(bR)^{-2}}{4\overline{m}}\left(\frac{\overline{m}Mc_0^2(bR)^2}{\hbar^2} - 1\right)^2. \tag{3.11a}$$

while their kinetic energies are given formally by:

$$E_{kin\downarrow} = \frac{\hbar^2 R_{e^-}^{-2}}{2\overline{m}}\ ,\ E_{kin\uparrow} = \frac{\hbar^2 (bR)^{-2}}{2\overline{m}} \tag{3.11b}$$

In terms of the dimensionless quantities, k_1 and k_2, defined as follows

$$k_1 \hbar c_0 / R = \hbar^2 (bR)^{-2} / 2\overline{m}\ ,\ \ k_2 = \overline{m} M c_0^2 (bR)^2 / \hbar^2\ , \tag{3.12}$$

the binding energies for the electron $e^- \downarrow$ and the spinion $e^0 \uparrow$ are given by

$$(R/2\hbar c_0) E_{B\downarrow} \equiv (R/2\hbar c_0)[E_{kin\downarrow} - E_{\downarrow}^0] = k_1 (R/R_e) + \overline{m} b^2 \alpha_0 (R/2R_e);$$
$$(R/2\hbar c_0) E_{B\uparrow} \equiv (R/2\hbar c_0)[E_{kin\uparrow} - E_{\uparrow}^0] = k_1 + (k_1/8)(k_2 - 1)^2. \tag{3.13}$$

Altogether, $E_{B\downarrow}, E_{B\uparrow}$ and E_T involves five parameters, namely the $e^- - p^+$ separation R_{e^-}, the $e^0 - p^+$ separation, R, and the parameters, b_4, b and M which we now proceed to determine from experimental data as a verification of the model.

4. EXPERIMENTAL VERIFICATION

The simplest way to carry out an experimental verification of our model of the neutron as an $(e^- \downarrow - p^+ - e^0 \uparrow)_{HM}$ bound state is to compare its total energy in Eq.(3.10) with that of Rutherford-Santilli model of the neutron as an $(e^- \downarrow, p^+)$ bound state given by:

$$E_T^S = (m_p + m_{e^-} b_4^2 b^2) c_0^2 + [E_{kin\uparrow} - E_{\uparrow}^0] \equiv (m_p + m_{e^-} b_4^2 b^2) c_0^2 + E_{B\uparrow}\ . \tag{4.1}$$

It follows by subtracting Eq.(4.1) from Eq.(3.10) that essentially,

$$E_T - E_T^S \cong m_{e^-} c_o^2 + E_{B\downarrow} \equiv m_{e^-} c_o^2 + E_{kin\downarrow} - e^2 / 2R_e\ . \tag{4.2}$$

Here, we observe that there is a fortuitous cancellation of the electron rest mass energy with the (H-atom) binding energy of $e^- \downarrow$, i.e.,

$$m_{e^-} c_0^2 - e^2 / 2R_e = 0. \tag{4.3}$$

at $e^- - p^+$ separation R_{e^-} given by a *coherence length* ξ of the order of the Compton wavelength of the pion:

$$R_{e^-} \equiv \xi \approx \hbar / m_\pi c_0 = 10^{-13}\, cm = 1 fm\ . \tag{4.4}$$

This follows from the fact that

$$\left|E_{\downarrow}^{0}\right| = \frac{e^2}{2R_{e^-}} = \frac{e^2}{2}\left(\frac{m_\pi c_0}{\hbar}\right) = \left(\frac{m_\pi c_0^2 \alpha_0}{2}\right) \approx m_{e^-} c_0^2 . \qquad (4.5)$$

We also note (see, Fig. 4) that

$$R_{e^-} = \frac{e^2}{2\overline{m}c_0^2} \equiv \left(\frac{e^2}{2m_e c_0^2} + \frac{e^2}{2m_p c_0^2}\right) \approx 1 fm = \frac{\hbar}{m_\pi c_0}, \ \left(m_e = \tfrac{1}{2} m_\pi \alpha_0\right) \quad (4.6)$$

implies occurrence of contact between the classical electric charge spheres of the electron and the proton, and ultimately mutual interpenetration of the electron and the proton as envisaged in the Rutherford-Santilli model[4].

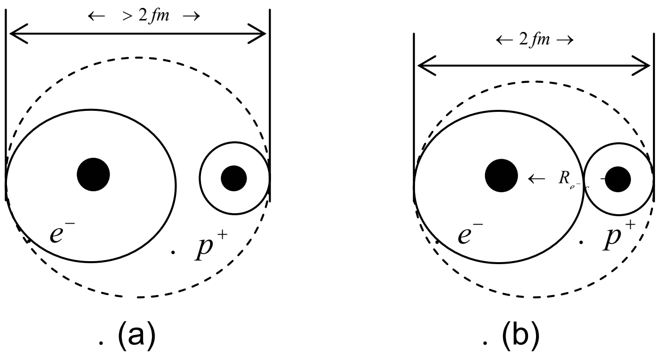


Fig. 4: Configuration of $(e^- - p^+)_{HM}$ when the sum of the classical electric charge radii of the electron and the proton is (a) greater than $1\ fm$ (in absence of compression) and (b) equal to $1\ fm$ (under compression).

Accordingly, if the kinetic energy of $e^- \downarrow$ *is presumed to vanish* upon the contact of its classical electric charge sphere with that of the proton at rest, then we would have $E_{kin\downarrow} \approx 0$ in Eq.(4.2), leading to the result,

$E_T - E_T^S \approx 0$.

However, a *naïve* substitution of $R_{e^-} \equiv \xi \approx \hbar / m_\pi c_0$ in the expression for $E_{kin\downarrow}$ in Eq.(3.11b) would have resulted in a rather large value:

$$E_{kin\downarrow} = (m_\pi^2 / m_{e^-})c_0^2 = [(140)^2 / 0.511]GeV = 38.4GeV . \qquad (4.7)$$

This, it so happens, is the value of the empirical mass M characterizing the Hulthen potential which is determined by the requirement that a minimum size of the potential hole is required before a bound state can be formed, i.e. from Eq.(3.11b),

$$\left(\frac{\overline{m} M c_0^2 (bR)^2}{\hbar^2} - 1 \right) = 0 , \qquad (4.8)$$

giving, for $Rb = \xi = \hbar / m_\pi c_0$, the value, $M = m_\pi^2 / \overline{m}_e$. This result is consistent with the fact that the Hulthen potential defined by Eq.(3.6) is proportional to $E_\downarrow$, and therefore, appears to have absorbed $E_{kin\downarrow}$ in the characteristic mass parameter, M. Moreover,

$$M = m_\pi^2 / \overline{m}_e = \tfrac{1}{2}(e^2 / \sqrt{2} G_F) = 38GeV / c_0^2 \qquad (4.9)$$

where G_F is the Fermi coupling constant for weak interactions; and we may interpret an optimum value of the Hulthen potential (by analogy with our $(e^- \uparrow - Cu^{z+} - e^- \downarrow)_{HM}$ model of Cooper pairing in superconductors) as a transition temperature (related to the mass of Cu^{z+} in the cuprate superconductors) or mass of the proton in $(e^- \downarrow - p^+ - e^0 \uparrow)_{HM}$ and given quite accurately by

$$m_p = M e^{-\bar{r} / R_e} \cong \tfrac{1}{2}(e^2 / \sqrt{2} G_F)^{\frac{1}{2}} e^{-1/3\alpha_0} . \qquad (4.10)$$

In effect, therefore, we have a repetition of the parallelism between Santilli's $\pi^0 = (e^+ \uparrow, e^- \downarrow)_{HM}$ and Animalu's Cooper pairing, $(e^- \uparrow - Cu^{z+} - e^- \downarrow)_{HM}$ on one hand, and between Rutherford-Santilli neutron, $n = (e^- \downarrow, p^+)_{HM}$ and our new pairing, $(e^- \downarrow - p^+ - e^0 \uparrow)_{HM}$ on the other hand. The advantage of the new pairing scheme is that, apart from the spin and charge fluctuations associated with the isotopic lifting transformation defined in Eq.(3.1), the spin-statistics theorem is applicable to the system $(e^- \downarrow - p^+ - e^0 \uparrow)_{HM}$, in agreement with Barut[14] representation of the neutron as $p^+ e^- \overline{\nu}_e$ bound state, $\overline{\nu}_e$ being the electron-like antineutrino. Following Barut[14], one can hardly resist proposing a similar model for another electrically neutral member of the SU(3) baryon octet, $\Xi^0 = (\mu^- \downarrow - p^+ - \mu^0 \uparrow)_{HM}$, where μ^- is the muon and μ^0 a muon-like spinion analogous to the electron-like spinion, e^0.

5. CONCLUSION

In this paper, we have reformulated the theory of the non-local-nonhamiltonian structure of the Rutherford-Santilli neutron as a compressed H-atom from both classical thermodynamics (Ginsburg-Landau equation) and hadronic mechanics (isoschrodinger equation) points of view. The new model is exactly soluble and, therefore, has not only provided a framework for physical interpretation of the theoretical results but also elucidated the origins of the Hulthen potential from an Ornstein-Zernike-like integral equation for the superfluid density in the former and from an isotopic lifting (transformation) operator in the latter. The existence of a neutral spin-$\frac{1}{2}$ quasiparticle (called spinion, e^0) associated with the charge structure of the isotopic lifting transformation which first arose in ref.[7] and in superconductivity theory[9] appears to have resolved the question of the proper role for a neutral spin-$\frac{1}{2}$ particle usually identified with the neutrino as a constituent of matter. We have also shown that the neutron mass can be accounted for, in a self-consistent manner, as the total energy of the $(e^- \downarrow - p^+ - e^0 \uparrow)_{HM}$ system as previously demonstrated by Santilli[4] while the proton mass, like the critical temperature T_c for the superconducting phase transition, is predicted in Eq.(4.10) as the critical mass for the second-order transition from normal to the superfluid (neutron) phase of the compressed H-atom which occurs in the interior of a neutron star.

It is, therefore, our hope that the model presented in this paper should be investigated further by theoreticians and experimentalists as a prelude to establishing it as a viable structure of all other baryons, as hadronic bound states of the types $(l^- \downarrow - p^+ - l^0 \uparrow)_{HM}$ where $l = e, \mu, \tau, \ldots$. Experimental evidence for the existence of spinions (l^0) is already provided by the observation of the fractional quantum Hall effect in supercodncutors[9] which we have discussed in ref.[7].

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