BRST SYMMETRY IN COHOMOLOGICAL HAMILTONIAN MECHANICS

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April 28, 1994
Hadronic J. 17 (1994) 429-439.

Abstract

We present BRST gauge fixing approach to cohomological Hamiltonian mechanics. Considered as one-dimensional field theory, the Hamiltonian mechanics appeared to be an example of topological field theory, with the trivial underlying Lagrangian. Twisted (anti-)BRST symmetry is related to an exterior algebra. Correlation functions of the BRST observables are studied.

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1 The BRST approach

We consider one-dimensional field theory, in which the dynamical variable is the map, $a^i(t) : M^1 \to M^{2n}$, from one-dimensional space M^1, $t \in M^1$, to $2n$-dimensional symplectic manifold M^{2n}.

The commuting fields $a^i = (p_1, \dots, p_n, q^1, \dots, q^n)$ are local coordinates on the target space, phase space of Hamiltonian mechanics equipped with the closed non-degenerate symplectic two-form, $\omega = \frac{1}{2}\omega_{ij} da^i \wedge da^j$; $\omega_{ij} = -\omega_{ji} = const$, $\omega_{ij}\omega^{jk} = \delta^i_k$.

We start with the partition function

$$Z = \int Da \ \exp(iI_0), \tag{1}$$

where $I_0 = \int dt\mathcal{L}_0$, and take the Lagrangian to be trivial, $\mathcal{L}_0 = 0$. This Lagrangian has symmetries more than the usual diffeomorphism invariance.

Trivial Lagrangians are known to be of much significance in the cohomological quantum field theories[1]-[4], which can be derived by an appropriate BRST gauge fixing of a theory in which the underlying Lagrangian is zero.

We use the BRST gauge fixing scheme[5, 6] to fix the symmetry in (1) by introducing appropriate ghost and anti-ghost fields. The diffeomorphisms of M^{2n} we are interested in are the symplectic diffeomorphisms, which leave the symplectic tensor ω_{ij} form invariant[7],

$$\delta a^i = \ell_h a^i, \tag{2}$$

where $\ell_h = h^i\partial_i$ is a Lie-derivative along the vector field h^i. To garantee the invariance of ω_{ij}, we take h^i to be Hamiltonian vector field[8], $h^i = \omega^{ij}\partial_j H(a)$, where we assume H to be Hamiltonian of classical mechanics.

By introducing the ghost field $c^i(t)$ and the anti-ghost field $\bar{c}_i(t)$, we write the BRST version of the diffeomorphism (2),

$$sa^i = ic^i, \quad sc^i = 0, \quad s\bar{c}_i = q_i, \quad sq_i = 0, \tag{3}$$

where the BRST operator is nilpotent, $s^2 = 0$, and q_i is a Lagrange multiplier. By an obvious mirror symmetry to the BRST transformations (3), we demand the following anti-BRST transformations hold:

$$\bar{s}a^i = i\bar{c}_i, \quad \bar{s}\bar{c}_i = 0, \quad \bar{s}c^i = q_i, \quad \bar{s}q_i = 0, \tag{4}$$

Evidently, $\bar{s}^2 = 0$, and it can be easily checked that $s\bar{s} + \bar{s}s = 0$.

The partition function (1) then becomes

$$Z = \int DX \, \exp(iI), \tag{5}$$

where DX represents the path integral over the fields a, q, c, and $\bar{c}$. The action I is trivial action I_0 plus s-exact part,

$$I = I_0 + \int dt \, sB. \tag{6}$$

Since s is nilpotent, I is BRST invariant for any choice of B, with sB having ghost number zero. We face with the restriction implied by antisymmetric property of ω_{ij}, and choose judiciously B to be linear in the fields,

$$B = \bar{c}_i(\partial_t a^i - h^i + \alpha a^i + \gamma \omega^{ij} q_j). \tag{7}$$

where α and γ are real parameters. The first two term in (7) give rise to the terms of the form (Lagrange multiplier)$\times$(gauge fixing condition) and the ghost dependent part,

$$sB = q_i(\partial_t a^i - h^i) + i\bar{c}_i(\partial_t c^i - sh^i) + \alpha(q_i a^i + i\bar{c}_i c^i). \tag{8}$$

As a feature of the theory under consideration, the γ-dependent term vanishes because ω_{ij} is antisymmetric. To evaluate sh^i, we note that $\delta h^i = (\partial_k h^i)\delta a^k$ and, therefore, $sh^i = c^k \partial_k h^i$. Thus, the resulting Lagrangian becomes $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{gf}$,

$$\mathcal{L} = \mathcal{L}_0 + q_i(\partial_t a^i - h^i) + i\bar{c}_i(\partial_t \delta^i_k - \partial_k h^i)c^k + \alpha(q_i a^i + i\bar{c}_i c^i). \tag{9}$$

$\mathcal{L}$ is BRST invariant by construction, and it can be readily checked that it is also anti-BRST invariant, $\bar{s}\mathcal{L} = 0$.

In the delta function gauge, *i.e.* at $\alpha = 0$, it reproduces exactly, up to $\mathcal{L}_0$, the Lagrangian, which has been derived in the path integral approach to Hamiltonian mechanics by Gozzi and Reuter[9]-[11], via the Faddeev-Popov method. Indeed, by integrating out the fields q, c and $\bar{c}$ we obtain from (5) the partition function $Z = \int Da \, \delta(a - a_{cl}) \exp(iI_0)$, where a^i_{cl} denotes solutions of the Hamilton's equation $\partial_t a^i = h^i$, which had been used as a starting point of the approach, with $I_0 = 0$. The delta function constraint plays, evidently, the role of the Faddeev-Popov gauge fixing condition, which is in effect the Hamilton's equation.

Thus, the theory (5) represents an example of one-dimensional cohomological field theory, in the sense that the Lagrangian (9) is BRST exact, with trivial underlying Lagrangian $\mathcal{L}_0$. The theory (5) can be thought as a topological phase of the Hamiltonian mechanics.

The (anti-)BRST symmetry is an inhomogeneous part of larger symmetry of the theory, inhomogeneous symplectic $ISp(2)$ group symmetry, generated by the charges, $Q = ic^i q_i$, $\bar{Q} = i\bar{c}_i\omega^{ij}q_j$, $C = c^i\bar{c}_i$, $K = \frac{1}{2}\omega_{ij}c^ic^j$, and $\bar{K} = \frac{1}{2}\omega^{ij}\bar{c}_i\bar{c}_j$[9]. Here, Q and $\bar{Q}$ are the BRST and anti-BRST operators, respectively. $ISp(2)$ algebra reflects the Cartan calculus on symplectic manifold M^{2n}, with the correspondences, $c^i \leftrightarrow da^i \in TM^{2n}$ and $q_i \leftrightarrow -i\partial_i \in T^*M^{2n}$[11].

2 Physical states

If we consider deformations δa^i along the solution of the Hamilton's equation, then in order for $a^i + \delta a^i$ to still be a solution it has to satisfy the deformation equation $\partial_t \delta a^i = \delta h^i$. This equation is the equation for Jacobi field, $\delta a^i \in TM^{2n}$, which can be thought of as the "bosonic zero mode". This mode is just compensated by anti-commuting zero-mode through the ghost dependent term, in the Lagrangian (9), $\alpha = 0$.

The Hamilton function $\mathcal{H}$ associated with (9),

$$\mathcal{H} = q_i h^i + i\bar{c}_i c^k \partial_k h^i - \alpha a^i q_i - i\alpha C, \tag{10}$$

covers, at $\alpha = 0$, in the ghost-free part the usual Liouvillian $L = -h^i\partial_i$ of ordinary classical mechanics derived by von Neumann[12], in the operator approach. $\mathcal{H}$ is the generalization of the Liouvillian to describe an evolution of the p-form (p-ghost) probability distributions, $\rho = \rho(a, c, t)$, instead of the usual distribution function (zero-form), governed by the Liouville equation, $\partial_t\rho(a, t) = -L\rho(a, t)$.

In the following, we use the delta function gauge omitting the α-dependent terms in (10), one of which is the ghost number operator C.

To study the physical states, that is the states which are BRST and anti-BRST invariant, $Q\rho = \bar{Q}\rho = 0$, we exploit the identification of "twisted" (anti-)BRST operator algebra with an exterior algebra[13]. Conventionally, twist is used to obtain BRST theory from a supersymmetric one[1]. Below, we use a kind of twist to obtain, conversely, supersymmetry from the BRST invariance. Defining the twisted operators[14]

$$Q_\beta = e^{\beta H} Q e^{-\beta H}, \quad \bar{Q}_\beta = e^{-\beta H} \bar{Q} e^{\beta H}, \tag{11}$$

where $\beta \geq 0$ is a real parameter, one can easily find that they are conserved nilpotent supercharges, and $\{Q_\beta, \bar{Q}_\beta\} = 2i\beta\mathcal{H}$. Consequently, these supercharges, together with $\mathcal{H}$, build up $N = 2$ supersymmetry, which has been found[9] to be a fundamental property of the Hamiltonian mechanics. Particularly, this supersymmetry is related[16] to the regular-unregular motion transitions in Hamiltonian systems. Also, it has been proven[14] that the cohomologies of Q_β and $\bar{Q}_\beta$ are both isomorphic to the de Rham cohomology so one can associate, in a standard way, some elliptic complex to them.

Thus, the following identifications can be made: $d_\beta \leftrightarrow Q_\beta$, $d^*_\beta \leftrightarrow \bar{Q}_\beta$, $\Delta_\beta = d_\beta d^*_\beta + d^*_\beta d_\beta \leftrightarrow \{Q_\beta, \bar{Q}_\beta\} = 2i\beta\mathcal{H}$, and $(-1)^p \leftrightarrow (-1)^C$, where exterior derivative d_β acts on p-forms, $\rho \in \Lambda^p$, and C is the ghost number. The cohomology groups $H^p(M^{2n}) = \{\ker d_\beta / \mathrm{im} d_\beta \cap \Lambda^p\}$ are finite when M^{2n} is compact. According to the Hodge-de Rham theorem, canonical representatives of the cohomology classes $H^p(M^{2n})$ are harmonic p-forms, $\Delta_\beta \rho = 0$. They are closed p-forms minimizing the functional $\mathcal{E}(\rho) = \int_{M^{2n}} |\rho|^2$, *i.e.* $d_\beta \rho = 0$ and $d^*_\beta \rho = 0$. One can then define B_p as the number of independent harmonic forms, *i.e.* $B_p(\beta) = \dim\{\ker \Delta_\beta \cap \Lambda^p\}$. Formally, B_p continuosly varies with β but, being a discrete function, it is independent on β so that one can find B_p by studying the vacua of the Hamilton function, $\mathcal{H}\rho = 0$.

In the Hamiltonian mechanics, such an equation plays the role of the *ergodicity* condi- tion[15, 17]. Thus, the extended ergodicity condition can be thought of as the condition for ρ to be harmonic form, or, equivalently, supersymmetric (Ramond) ground state.

To ensure the solution in the p-ghost sector to be non-degenerate one therefore should have $B_p = 1$. For the standard de Rham complex, B_p's are simply Betti numbers, with $\sum(-1)^p B_p$ being Euler characteristic of M^{2n}. Recent studies of the physical states[17, 18] showed that the only physically relevant solution comes from the $2n$-ghost sector, and has specifically the Gibbs state form, $\rho = \kappa K^n \exp(-\beta H)$. The other ghost sectors yield solutions of the form $\rho = \kappa K^p \exp(+\beta H)$, for the even-ghost sectors. In two-dimensional case, analysis shows that ρ does not depend on β, for the odd-ghost sectors (see Appendix).

Note that in the limit $\beta \to 0$, we recover the classical Poincare integral invariants K^p, $p = 1, \dots n$, as the solutions of $\mathcal{H}\rho = 0$. They are fundamental BRST invariant observables of the theory, $\{Q, K^p\} = 0$, since in the untwisting limit, $\beta \to 0$, the supersymmetry generators Q_β and $\bar{Q}_\beta$ become the BRST and anti-BRST generators, respectively. Geometrically, this follows from $dK^p = 0$ since $K^p = \omega^p$ and $d\omega = 0$. Similarly, $\bar{K}^p$'s are anti-BRST invariant observables.

Also, it should be noted that there seems to be no relation of the supersymmetric Hamilton function $\mathcal{H}$ to the Morse theory[13] due to the generic absence of terms quadratic in h^i. The reason is that the symplectic two-form ω is closed so that this does not allow one to construct, or obtain by the BRST gauge fixing procedure, non-vanishing terms quadratic in fields, except for the ghost-antighost. On the other hand, such terms, which are natural in (cohomological) quantum field theories, would produce stochastic contribution (Gaussian noise) to the equations of motion[19] that would, clearly, spoil the deterministic character of the Hamiltonian mechanics.

3 Correlation functions

Let us now turn to consideration of the correlation functions of the BRST observables. The BRST invariant observables of interest are of the form $\mathcal{O}_A = A_{i_1 \cdots i_p}(a) c^{i_1} \cdots c^{i_p}$, which are p-forms on M^{2n}, $A \in \Lambda^p$. One can easily find that $\{Q, \mathcal{O}_A\} = 0$ if and only if A is closed since $\{Q, \mathcal{O}_A\} = \mathcal{O}_{dA}$. Therefore, the BRST observables correspond to the de Rham cohomology.

The basic field $a^i(t)$ is characterized by homotopy classes of the map $M^1 \to M^{2n}$. Clearly, they are classes of conjugated elements of the fundamental homotopy group $\pi_1(M^{2n})$, in our one-dimensional field theory, since closed paths make $M^1 = S^1$ ($M^1 = R^1$ is homotopically trivial). Hence, we should study (quasi-)periodical trajectories in M^{2n} characterized by a period τ, $t \in S^1$. This case is of much importance in a general framework since it allows one to relate the correlation functions to the Lyapunov exponents[20], positive values of which are well known to be a strong indication of chaos in Hamiltonian systems, and to the Kolmogorov-Sinai entropy.

If N is a closed submanifold of (compact) M^{2n} representing some homology class of codimension m ($2n$-m cycle), then, by Poincare duality, we have m-dimensional cohomology class A (m-cocycle), which can be taken to have delta function support on N[21]. Thus, any closed form A is cohomologious to a linear combination of the Poincare duals of appropriate N's. The general correlation function is then of the form,

$$\langle \mathcal{O}_{A_1}(t_1) \cdots \mathcal{O}_{A_m}(t_m) \rangle, \tag{12}$$

where A_k are the Poincare duals of the N's. Our aim is to find the contribution to this correlation function on S^1 coming from a given homotopy class of the maps $S^1 \to M^{2n}$. The conventional techniques with the moduli space $\mathcal{M}$[1] consisting of the fields $a^i(t)$ of the above topological type can be used here owing to the BRST symmetry. The non-vanishing contribution to (12) can only come from the intersection of the submanifolds $L_k \in \mathcal{M}$ consisting of a's such that $a^i(t) \in N_k$, and we obtain familiar formula[22], $\langle \mathcal{O}_{A_1}(t_1) \cdots \mathcal{O}_{A_m}(t_m) \rangle_{S^1} = \#\Big(\sum^m \cap L_k\Big)$, relating the correlation function to the number of intersections. As it was expected, the correlation function does not depend on time but only on the indeces of the BRST observables. The Poincare invariants, K^p's, as the BRST observables, correspond to homotopically trivial sector since ω_{ij} are constant coefficients, for which case a's are homotopically constant maps, $a^i(t) = a^i(t_0)$, and, therefore, the correlation function (12) can be presented as $\int_{M^{2n}} A_1 \wedge \cdots \wedge A_m$.

Particular kind of observables, we turn to consider, is of the form[20]

$$\mathcal{O}_A = \bar{c}_{i_1} \cdots \bar{c}_{i_p} \delta(a(t_0) - a_0) c^{i_1} \cdots c^{i_p}$$

. After normal ordering, this observable can be thought of as the operator creating p ghosts (p-volume form in TM^{2n}) at some time t_0 and point $a_0 \in M^{2n}$, and then annihilating them at some later time t. Certainly, we should introduce also time-ordering to define this operator correctly, but this will not be a matter in the correlation function since we dealing with $t \in S^1$. Indeed, the correlation function $\langle \mathcal{O}_A(t) \rangle_{S^1} \equiv \Gamma_p(\tau, a_0)$ does not depend on specific time, and we indicate this fact analytically by labeling Γ_p with the period τ. An important result[20] is that the higher order largest Lyapunov exponents are found to be related to this correlation function by

$$l_p(a_0) = \sum_{m=1}^{p} \lim_{\tau \to \infty} \sup \frac{1}{\tau} ln\, \Gamma_m(\tau, a_0). \tag{13}$$

The partition function (5), for the p-form sector, $Z_p(\tau) = Tr_{S^1} \exp(-i\mathcal{H}_p t)$, takes the normalized form[20]

$$Z_p(\tau) = Tr_{S^1} \Gamma_p(t, a) / Tr_{S^1} 1, \tag{14}$$

where Tr_{S^1} denotes integral over all the solutions a^i characterized by period τ.

The problem in computing $Z_p(\tau)$ arises due to the fact that the integral over all the a's is obviously abandoned. The finite value can be obtained by realizing that the integral can be replaced with a sum over all the homotopy classes of a's mentioned above. Explicit computation can be performed, for example, by realizing M^{2n} as a covering space, $f : M^{2n} \to Y^{2n}$, of a suitable lineary connected manifold Y^{2n} having the same fundamental group as S^1, $\pi_1(Y^{2n}, t_0) \simeq \pi_1(S^1, t_0)$. Since Y^{2n} has $\pi_0(Y^{2n}, t_0) = 0$, the set of preimages is *discrete*, $f^{-1}(t_0) = \{a_0, a_1, \ldots\}$. The number of elements of $f^{-1}(t_0)$ and of the monodromy group $G = \pi_1(Y^{2n}, t_0)/f_*\pi_1(M^{2n}, a_0)$ coincides due to canonical one-to-one correspondence between $f^{-1}(t_0)$ and G, and does not depend on t_0 due to $\pi_0(Y^{2n}, t_0) = 0$. Hence,

$$Z_p(\tau) = \sum_{\alpha \in G} \Gamma_p(\tau, a_\alpha). \tag{15}$$

which is finite if G is a finite group. Clearly, Z_p's are topological entities, which can be used to define *topological* entropy[20] of the Hamiltonian system, with $\sum^{2n}(-1)^p Z_p$ being Euler characteristic of the symplectic manifold M^{2n}[11].

4 Concluding remarks

After having analyzed the main ingredients of the construction, we make few comments.

Note that the α-dependent terms in the Lagrangian can be absorbed by slightly generalizing of the time derivative, $\partial_t \to D_t = \partial_t + \alpha$, which looks like a covariant derivative, in one-dimensional case.

As we have already mentioned, the symmetries of the resulting Hamiltonian (10) appeared to be even more than the (anti-)BRST symmetry we have demanded upon. So, the reason of the appearence of these additional symmetries, K, $\bar{K}$, and C, should be clarified, in the context of the BRST gauge fixing approach. Surely, these symmetries are natural, and establish the Poincare integral invariants, its conjugates, and the ghost-number conservation.

We note that there is a tempting possibility to start with a non-trivial *topologically* invariant action I_0, if exist, instead of the trivial one. The problem is to construct an appropriate topological Lagrangian $\mathcal{L}_0$, for which I_0 will not be dependent on the metric on M^1, a positive definite function of time, $g = g(t)$, that is, $\delta g = arbitrary$, $\delta I_0 = 0$. In this way, by the use of the BRST gauge fixing scheme, which is presently known to be the only method to deal with topologically invariant theories, one would construct a kind of topological classical mechanics.

Also, it is interesting to make BRST gauge fixing for the theory, with an explicit accounting for the energy conservation. Due to the fact that the $(2n-1)$-dimensional submanifold, $M^{2n-1} \subset M^{2n}$, of constant energy, $H(a) = E$, is invariant under the Hamiltonian flow, and the p-forms evolve to p-forms on M^{2n-1}[20], one deals in effect with a "reducible" action of the symplectic diffeomorphisms, for which case more refined Batalin-Vilkovisky gauge fixing method[23] should be applied, instead of the usual BRST one used in this paper.

Appendix

Two-dimensional phase space, $n = 1$, is characterized by the coefficients of the symplectic tensor $\omega^{12} = -\omega^{21} = 1$; $a^1 = p$, $a^2 = q$. The general expansion of the distribution reads

$$\rho(a, c) = \rho_0 + \rho_1 c^1 + \rho_2 c^2 + \rho_{12} c^1 c^2.$$

The general system of the ground state equations, $Q_\beta \rho = \bar{Q}_\beta \rho = 0$, reduces, in this case, to (see Ref.[18] for details)

$$c^1(\partial_1 - \beta h_1)\rho_0 = 0, \quad c^2(\partial_2 - \beta h_2)\rho_0 = 0,$$

$$c^1(\partial_1 + \beta h_1)\rho_{12} = 0, \quad c^2(\partial_2 + \beta h_2)\rho_{12} = 0,$$

$$c^1 c^2[(\partial_1 - \beta h_1)\rho_2 - (\partial_2 - \beta h_2)\rho_1)] = 0,$$

$$(\partial_1 + \beta h_1)\rho_2 - (\partial_2 + \beta h_2)\rho_1 = 0.$$

Here, $\partial_i \equiv \partial/\partial a^i$ and $h_i \equiv \partial H(a^1, a^2)/\partial a^i$ $(i = 1, 2)$. This system of equations immediately implies, for the ghost-free sector ρ_0 and the two-ghost sector ρ_{12},

$$\rho_0 = \kappa_0 e^{+\beta H}, \quad \rho_{12} = \kappa e^{-\beta H}$$

These ghost sectors define scalar distributions, with commuting coefficients ρ_0 and ρ_{12}.

The equations for the "vector" distribution, $\vec{\rho} \equiv (\rho_1, \rho_2)$, can be rewritten in the following form:

$$\partial_1\rho_2 - \partial_2\rho_1 = 0, \quad 2\beta(h_1\rho_2 - h_2\rho_1) = 0,$$

or, taking $\beta > 0$,

$$\frac{h_2}{h_1}\partial_1\rho_1 - \partial_2\rho_1 = -\rho_1\partial_1\frac{h_2}{h_1}, \quad \rho_2 = \frac{h_2}{h_1}\rho_1.$$

The characteristic equations for the non-homogeneous first-order partial differential equation above are

$$\frac{da^1}{ds} = \frac{h_2}{h_1}, \quad \frac{da^2}{ds} = -1, \quad \frac{d\rho_1}{ds} = -\rho_1\partial_1\frac{h_2}{h_1},$$

from which one can easily find the first and the second integrals

$$U_1 = \int(h_1 da^1 + h_2 da^2), \quad U_2 = \frac{h_2}{h_1}\rho_1.$$

The general solution then is of the form $\Phi(U_1, U_2) = 0$, that is, we can write

$$\rho_1 = \frac{h_1}{h_2}f(U_1),$$

and, accordingly, $\rho_2 = f(U_1)$, where f is an arbitrary function. We see that the solutions for the one-ghost (odd-ghost) sector, ρ_1 and ρ_2, characterizing the vector distribution $\vec{\rho}$, do not depend on the parameter $\beta > 0$. This remarkable property might go beyond the two-dimensional case.

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